MODULE 2: DIFFUSION

LECTURE NO. 1

2.1 FICK'S LAW OF DIFFUSION

2.1.1 First law of diffusion (Steady state Law)

Adolf Fick (1955) first described the molecular diffusion in an isothermal, isobaric binary system of components A and B[1-3]. According to his idea of molecular diffusion, the molar flux of a species relative to an observer moving with molar average velocity is proportional to the concentration gradient in a certain direction.

$$J_A \propto \frac{dC_A}{dZ} \qquad (2.1)$$

Or

$$J_A = -D_{AB}\,\frac{dC_A}{dZ} \qquad (2.2)$$

Where, J_A is the molar flux of component A in the Z direction. C_A is the concentration of A and Z is the distance of diffusion. The proportionality constant, D_{AB} is the diffusion coefficient of the molecule A in B. This is valid only at steady state condition of diffusion. The Equation (2.2) is called Fick's first law of diffusion. If the concentration gradient is expressed as the gradient of mole fraction and in three dimensional cases, the molar flux can be expressed as

$$J_A = -CD_{AB}\left(\frac{\partial x_A}{\partial x} + \frac{\partial x_A}{\partial y} + \frac{\partial x_A}{\partial Z}\right) \qquad (2.3)$$

2.1.2 Prove that mutual diffusivities of species A and B are equal if gas mixture is ideal when total pressure is constant.

Substituting the Equation (2.2) for J_A into Equation (1.21) in module 1, the molar flux with negligible bulk movement of component *A* of the binary gas mixture can be represented as

$$N_A = -CD_{AB}\frac{dy_A}{dZ} + y_A N \tag{2.4}$$

Similarly for component *B*, it can be written as

$$N_B = -CD_{BA}\frac{dy_B}{dZ} + y_B N \tag{2.5}$$

Since $N_A + N_B = N$ and $y_A + y_B = 1$, addition of Equations (2.4) and (2.5) gives,

$$CD_{AB}\frac{dy_A}{dZ} = -CD_{BA}\frac{dy_B}{dZ} \tag{2.6}$$

Differentiation of the equality, $y_A + y_B = 1$ with respect to *Z*, gives

$$\frac{dy_A}{dZ} = -\frac{dy_B}{dZ} \tag{2.7}$$

Substituting the Equation (2.7) into Equation (2.6) one can get

$$D_{AB} = D_{BA} \tag{2.8}$$

From Equation (2.8) it is seen that for a binary gas mixture, the diffusivity of A in B equals the diffusivity of B in A.

2.1.3 Unsteady state Diffusion

If the change of concentration of a component *A* of the diffusive constituents in a mixture occurs over a time at a point, the Fick's law of diffusion at unsteady state condition can be expressed for *Z*-direction as

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial Z^2} \tag{2.9}$$

Both the diffusive and non-diffusive constituents affect the rate of unsteady state diffusion. The diffusivity at unsteady state condition can be expressed in terms of activation energy and the temperature as

$$D_{AB} = D_0 \exp(-\frac{E_D}{RT}) \qquad (2.10)$$

The activation energy (E_D) for the diffusion decreases the rate of diffusion whereas temperature increases the diffusion rate.