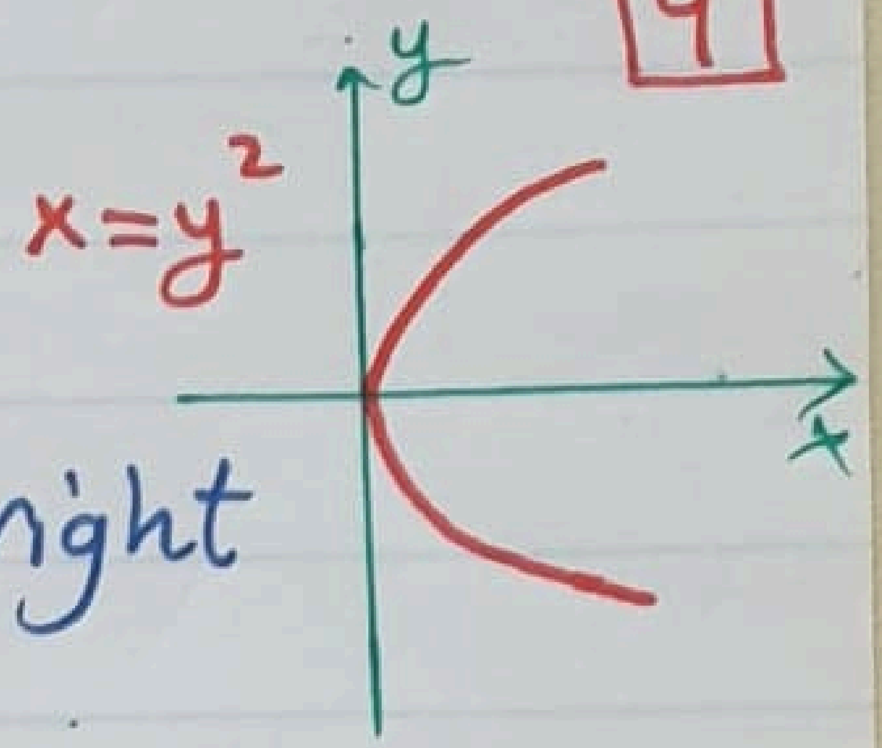
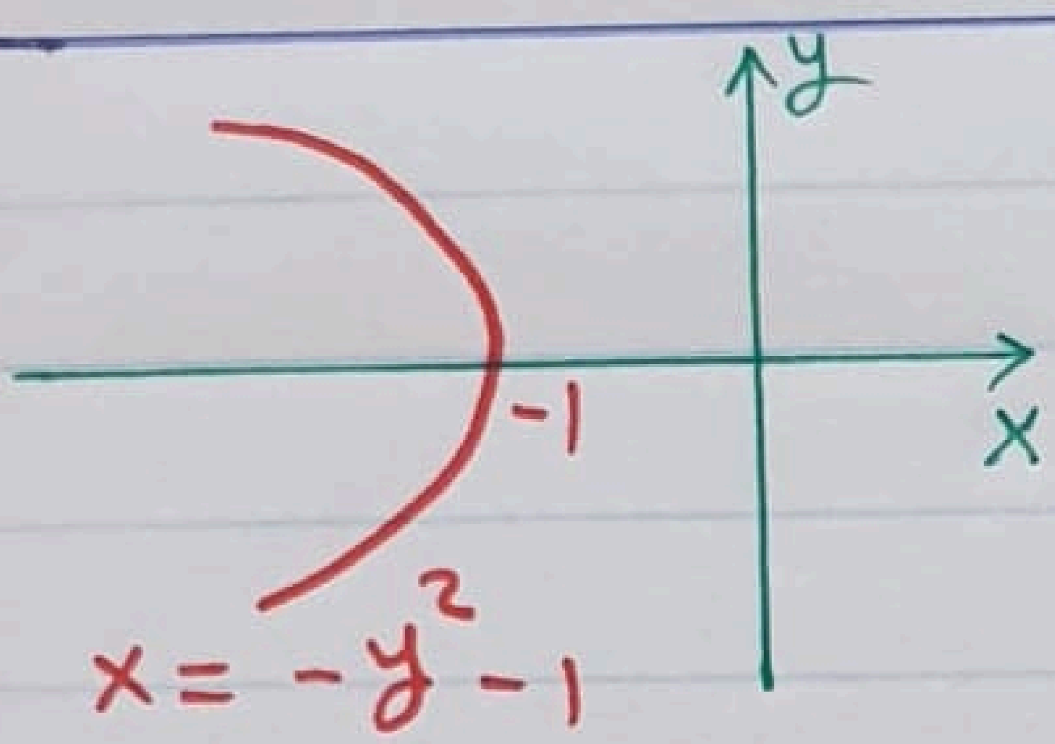
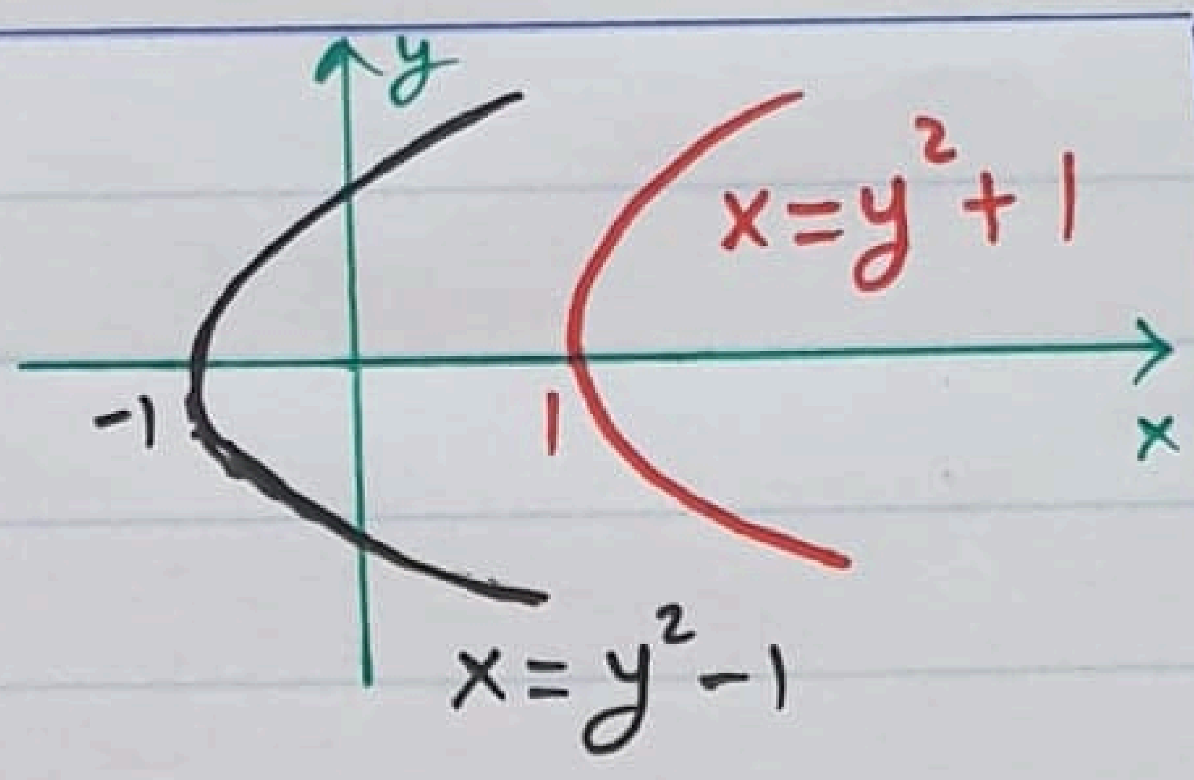
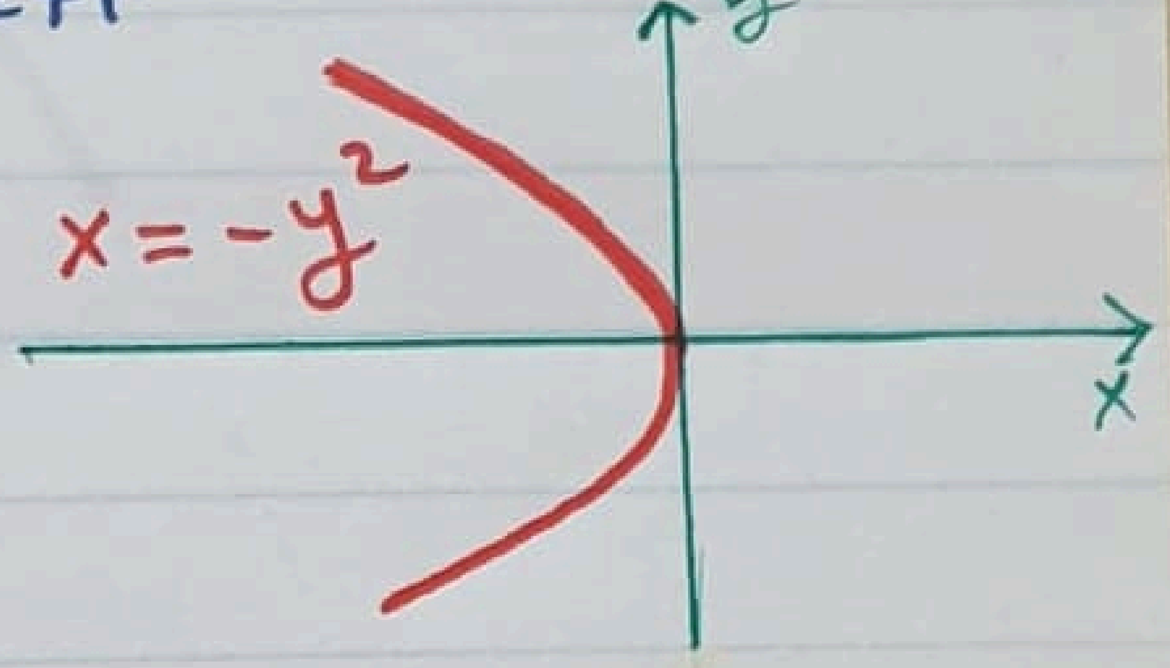


- Special parabolas (curves)
- $x=y^2$ has vertex $(0,0)$ and open to the right
- $x=-y^2$ has vertex $(0,0)$ and open to the left



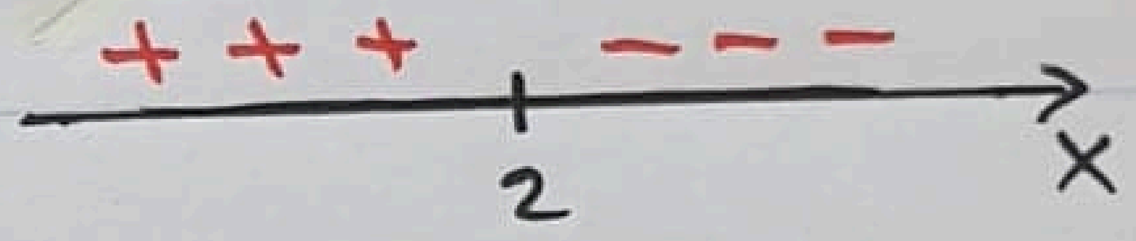




- Sign of functions $y=f(x)$ $\Rightarrow$ First find roots of $f(x)$

Exp $f(x)=6-3x$ "linear"

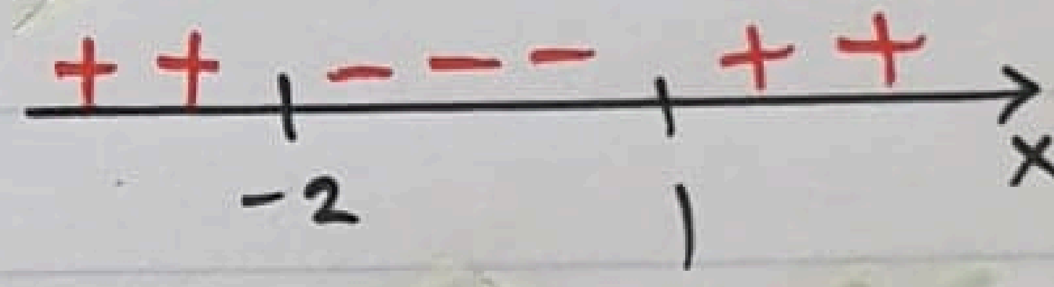
$f(x)=0 \Rightarrow 6-3x=0 \Rightarrow x=2$



Exp $f(x)=x^2+x-2$

$f(x)=0 \Rightarrow x^2+x-2=0 \Rightarrow (x-1)(x+2)=0$

$x=1,\ x=-2$

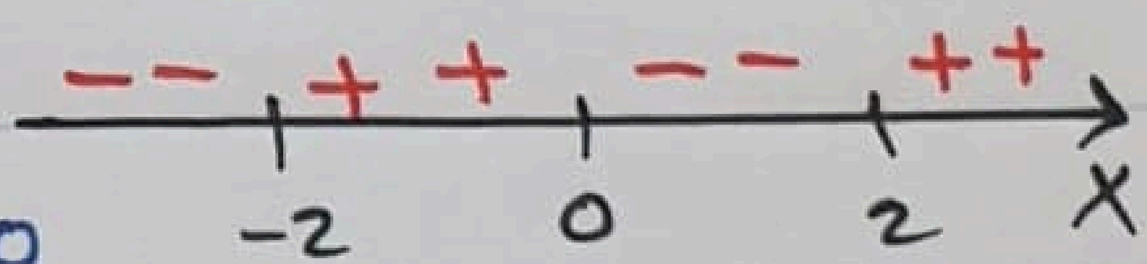


Exp $f(x)=x^3-4x$

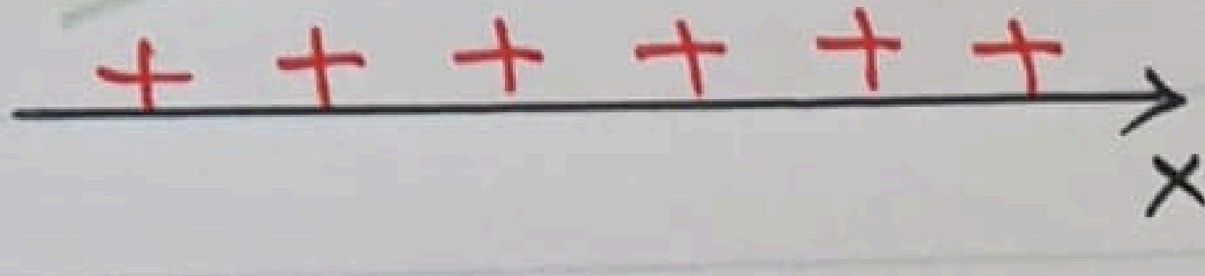
$f(x)=0 \Rightarrow x^3-4x=0 \Rightarrow x(x^2-4)=0$

$x(x-2)(x+2)=0$

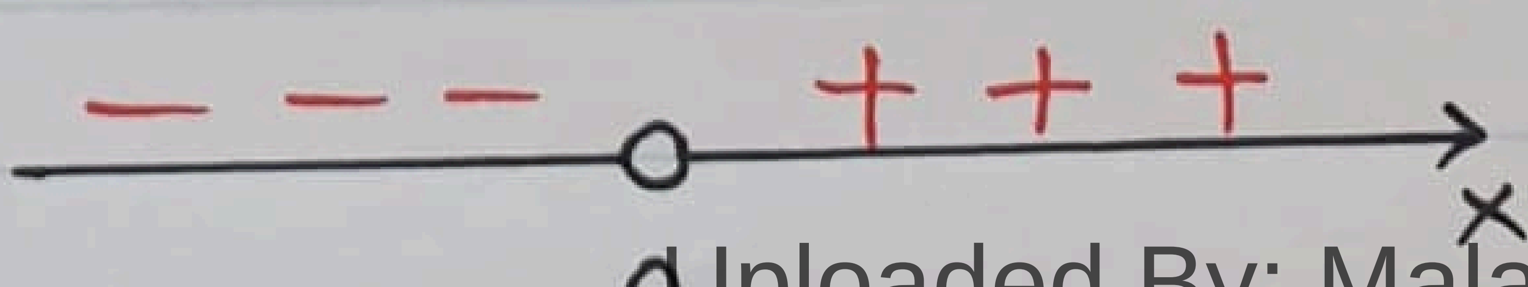
$x=0,\ x=2,\ x=-2$



Exp $f(x)=x^2+1$ has no roots



Exp $f(x)=\frac{1}{x}$



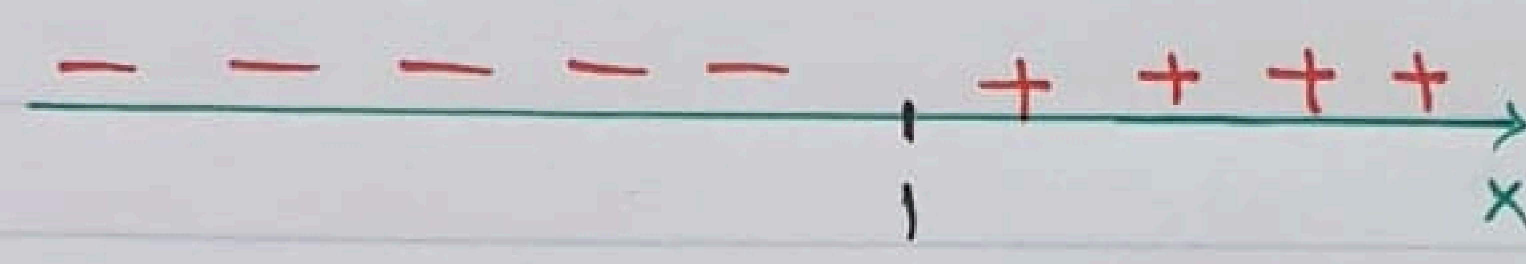
- Rational functions $= \dfrac{\text{Polynomial}}{\text{Polynomial}} = \dfrac{\text{Numerator}}{\text{Denominator}}$

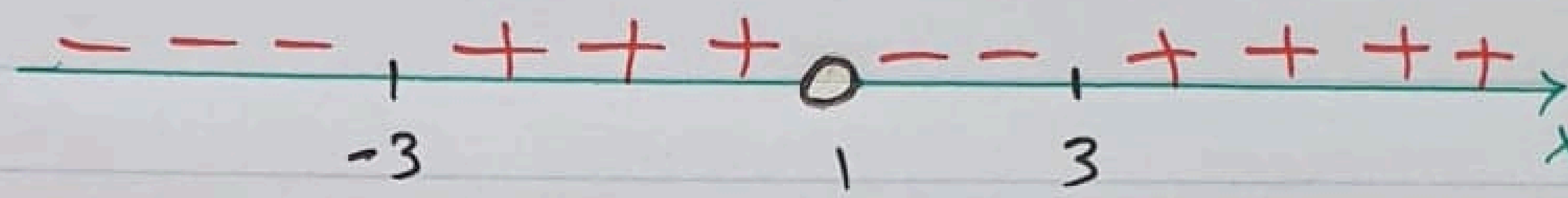
Exp $f(x) = \dfrac{x^2 - 9}{x - 1}$

Numerator $x^2 - 9 = 0 \Rightarrow (x-3)(x+3) = 0 \Rightarrow x = 3$ or $x = -3$

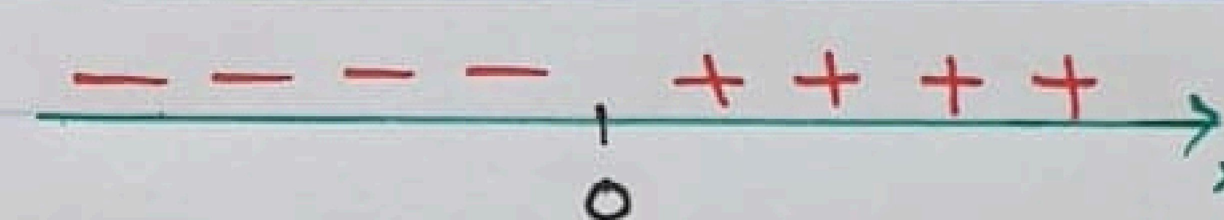
Denominator $x - 1 = 0 \Rightarrow x = 1$

Numerator


Denominator


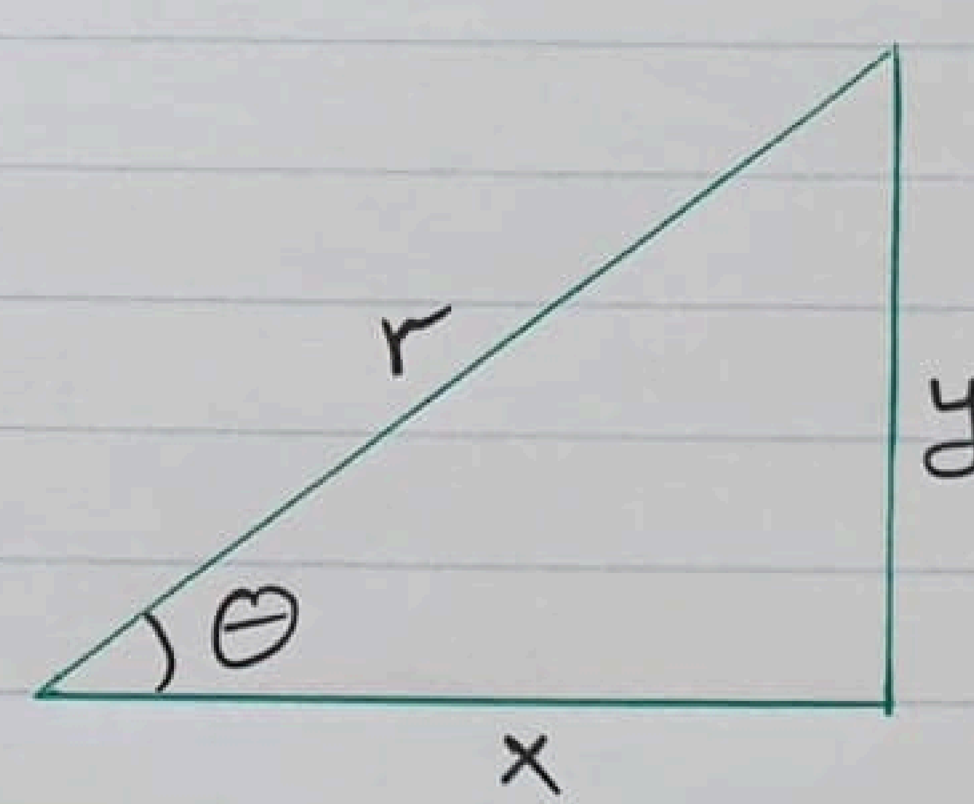
$f(x)$


Exp $f(x) = \dfrac{x}{x^2 + 1}$



- Trigonometric functions

x : Adjacent
y : Opposite
r : Hypotenuse



- $\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$
- $\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$
- $\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$
- $x^2 + y^2 = r^2 \Rightarrow \left(\dfrac{x}{r}\right)^2 + \left(\dfrac{y}{r}\right)^2 = 1$

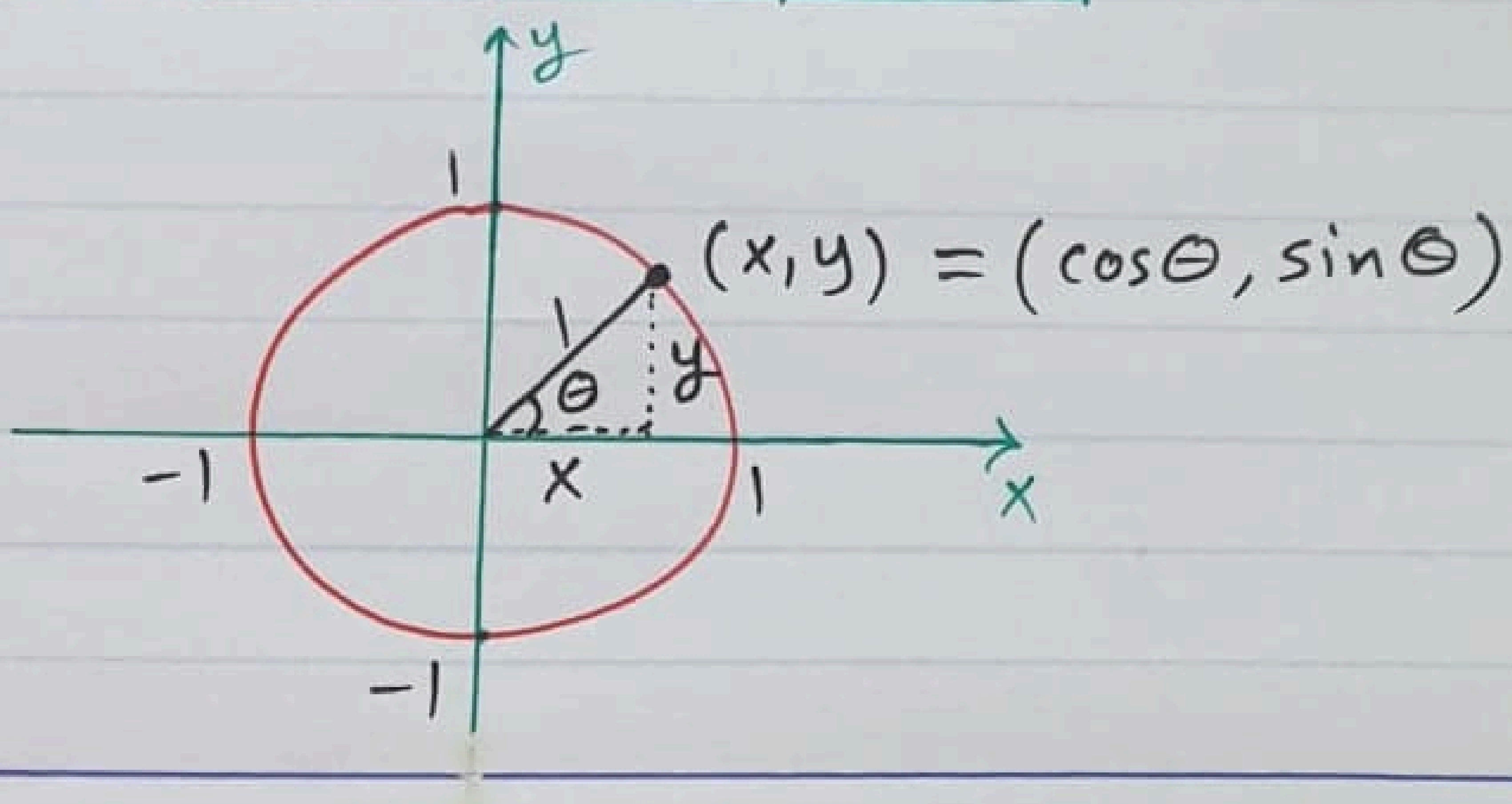
$\Rightarrow \cos^2\theta + \sin^2\theta = 1$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	...
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	...
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	...

• Unit Circle

$\cos\theta = \frac{x}{1} = x$

$\sin\theta = \frac{y}{1} = y$



• Trigonometric functions

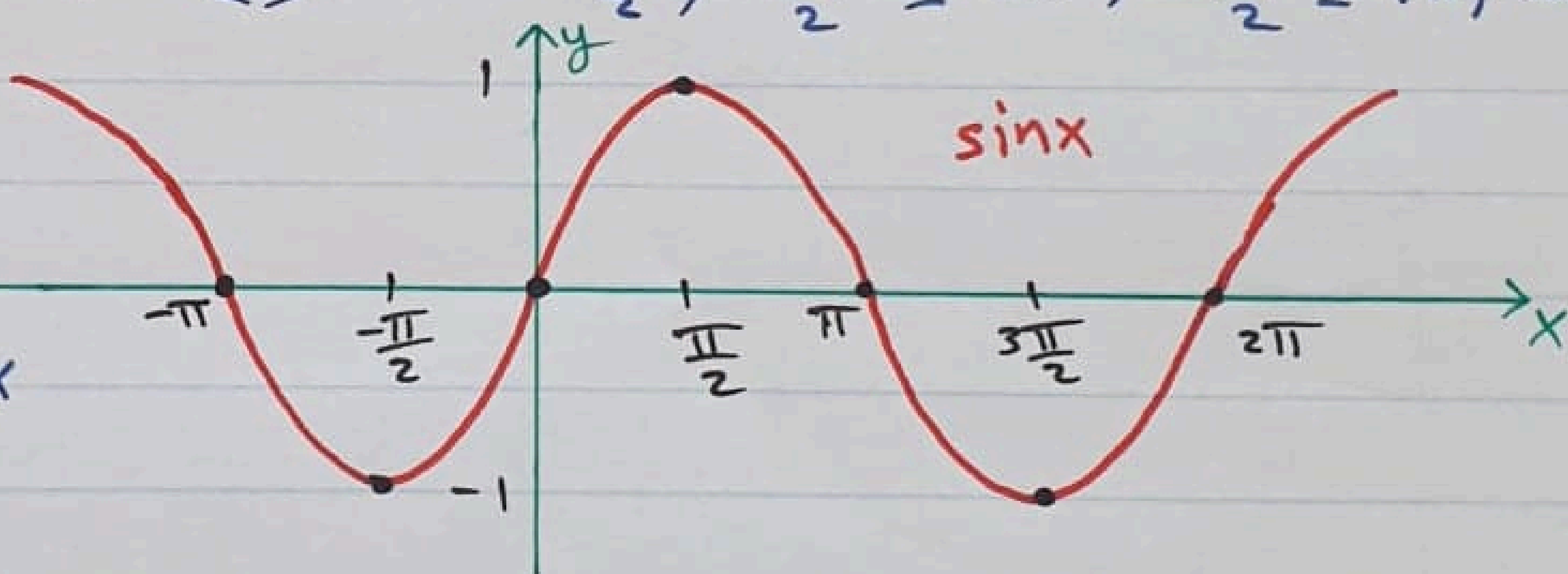
① $f(x) = \sin x \Rightarrow D = \mathbb{R} = (-\infty, \infty)$ and $R = [-1, 1]$

$\sin x = 0 \Leftrightarrow x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots$

$\sin x = 1 \Leftrightarrow x = \frac{\pi}{2}, \frac{\pi}{2} \pm 2\pi, \frac{\pi}{2} \pm 4\pi, \dots$

$\sin x = -1 \Leftrightarrow x = -\frac{\pi}{2}, -\frac{\pi}{2} \pm 2\pi, -\frac{\pi}{2} \pm 4\pi, \dots$

- period $= 2\pi$
- odd function
- $\sin(x + 2\pi) = \sin x$



② $f(x) = \csc x$ "Cosecant"

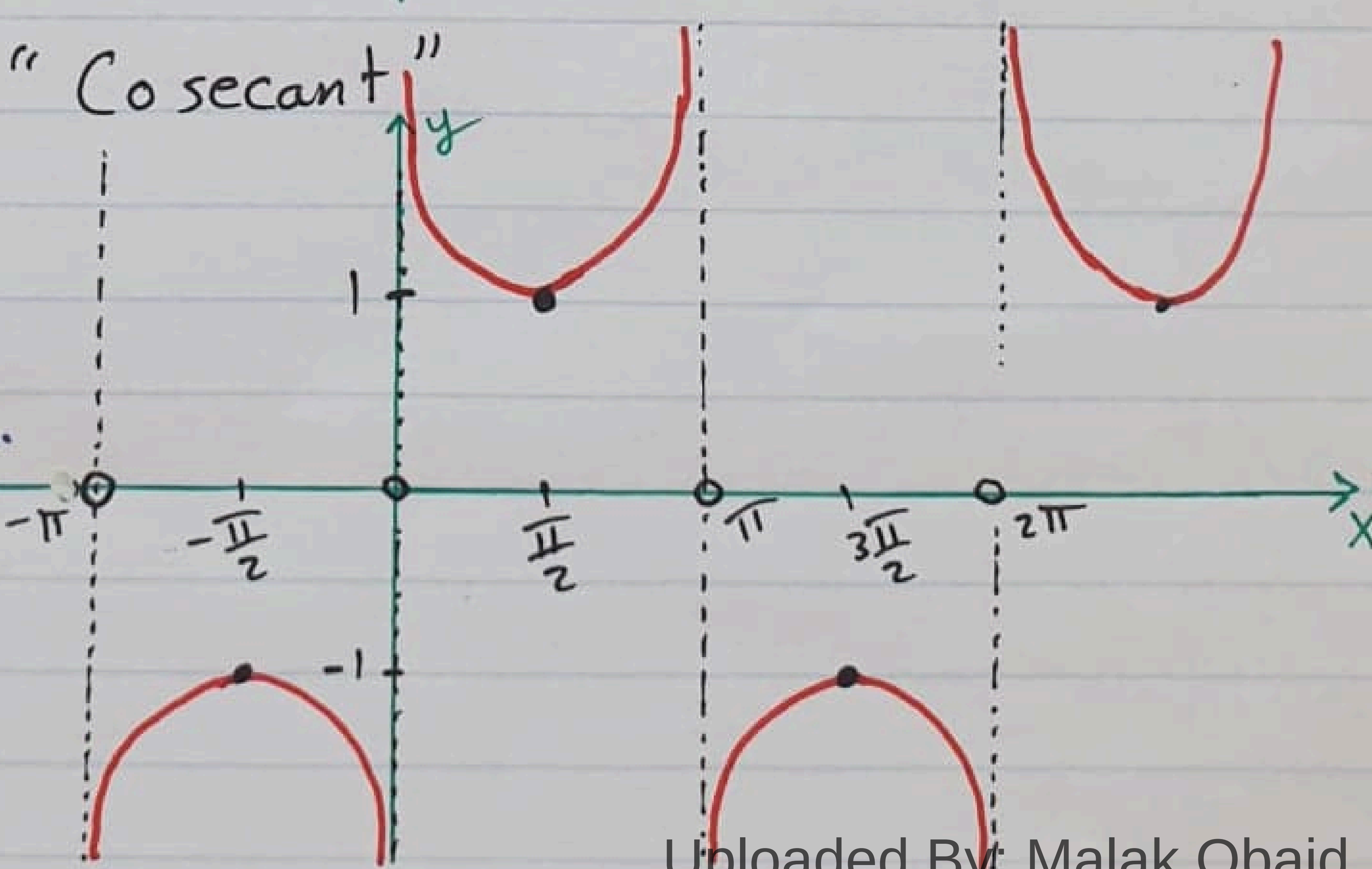
$= \frac{1}{\sin x}$

$D = \mathbb{R} \setminus \{0, \pm\pi, \pm 2\pi, \dots\}$

$= \mathbb{R} \setminus \{\pm n\pi\}, n = 0, 1, 2, \dots$

$R = (-\infty, -1] \cup [1, \infty)$

- period $= 2\pi$
- odd function
- $\csc(x + 2\pi) = \csc x$



$\boxed{3}$ $f(x) = \cos x \quad \Rightarrow D = \mathbb{R} = (-\infty, \infty) \quad , \quad R = [-1, 1]$

$\cos x = 0$ at $x = \pm \frac{\pi}{2}, \pm 3\frac{\pi}{2}, \pm -5\frac{\pi}{2}, \dots$

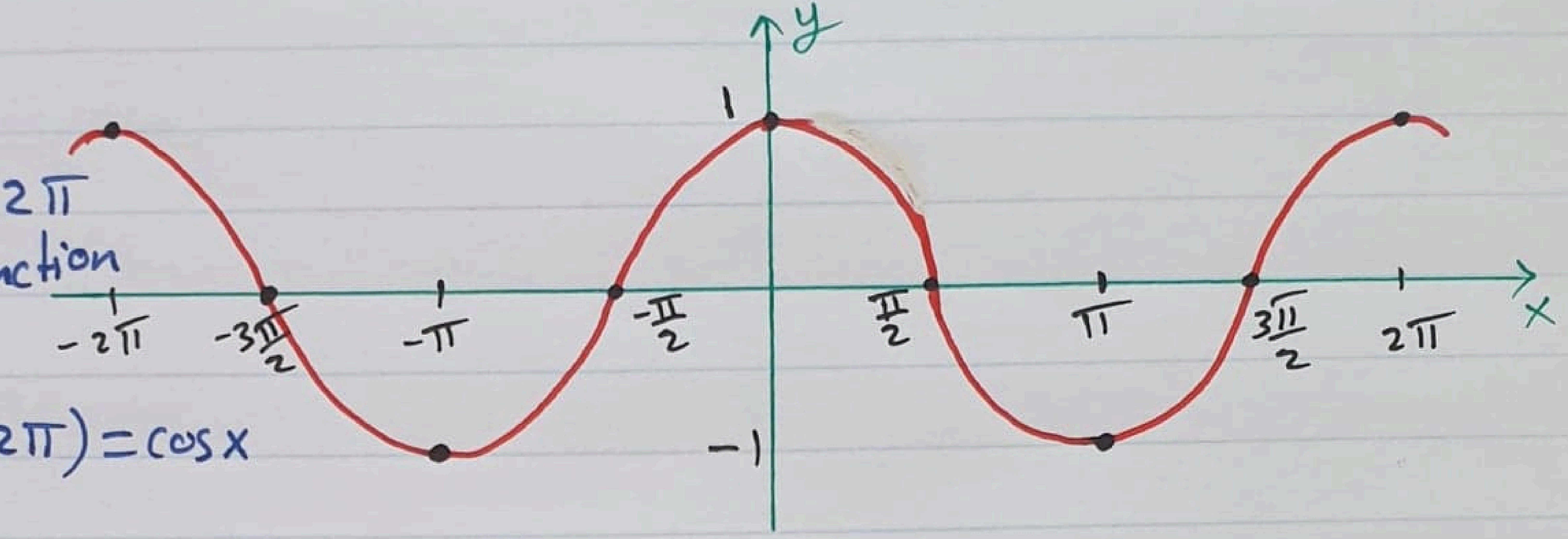
$\cos x = 1$ at $x = 0, \pm 2\pi, \pm 4\pi, \dots$

$\cos x = -1$ at $x = \pm \pi, \pm 3\pi, \pm 5\pi, \dots$

period $= 2\pi$

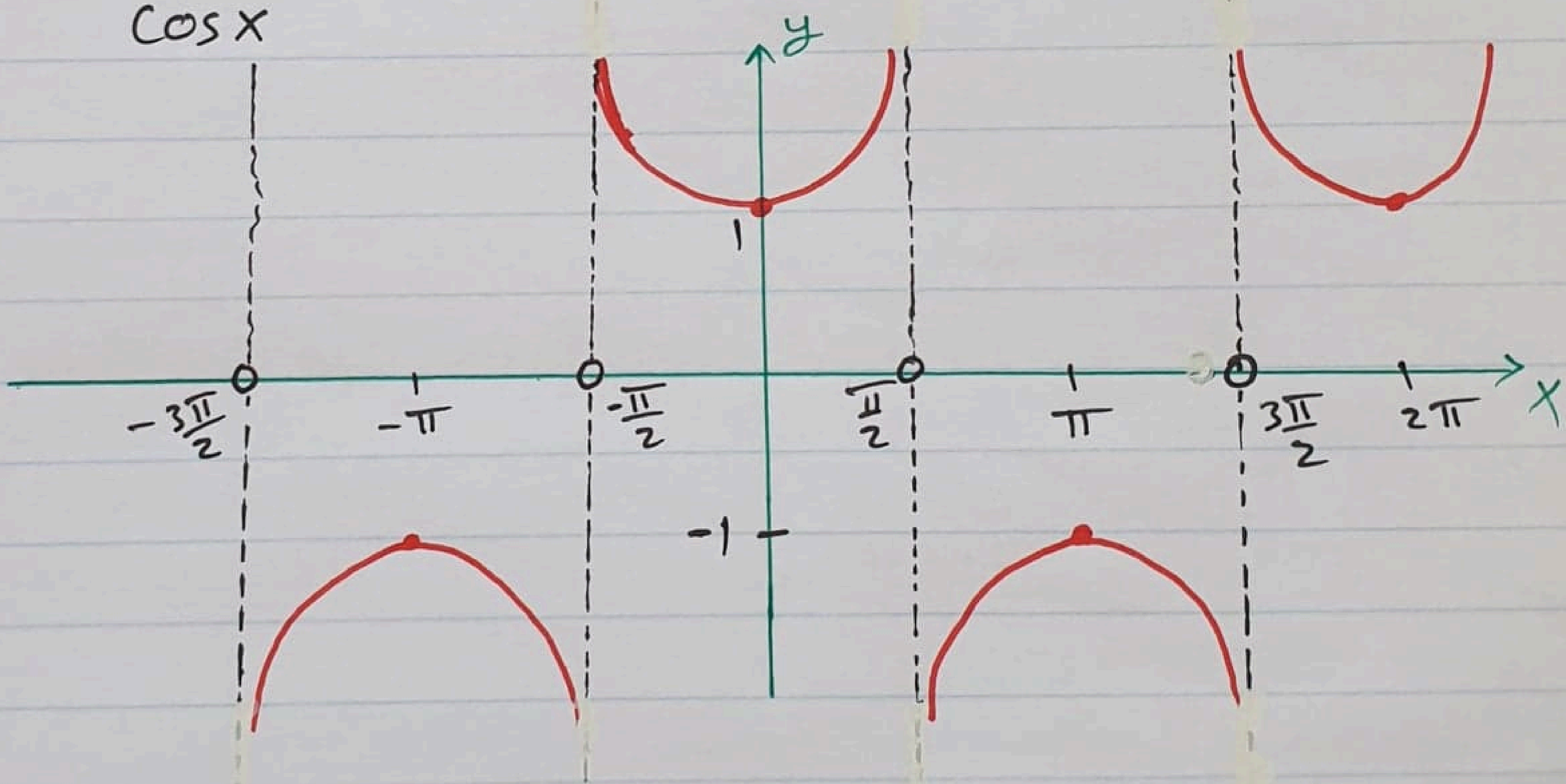
Even function

$\cos(x + 2\pi) = \cos x$



$\boxed{4}$ $f(x) = \sec x$ "secant"

$= \dfrac{1}{\cos x}$



$D = \mathbb{R} \setminus \{\pm\frac{\pi}{2}, \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, \dots\} = \mathbb{R} \setminus \{\frac{\pi}{2} \pm \pi n\}, \; n = 0, 1, 2, \dots$

$R = (-\infty, -1] \cup [1, \infty)$

period $= 2\pi \quad \Rightarrow \sec(x + 2\pi) = \sec x$

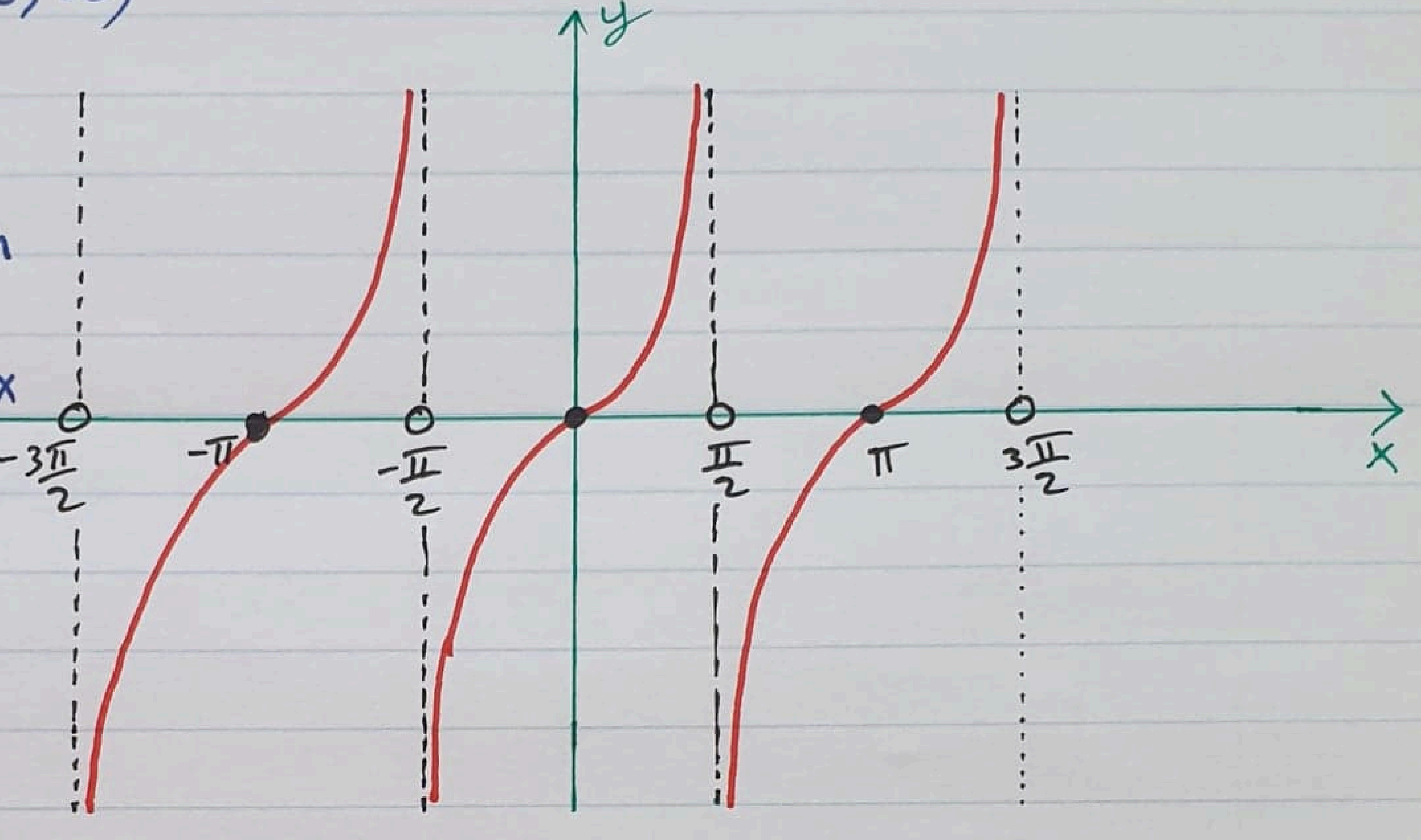
Even function

$\boxed{5}$ $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$D = \mathbb{R} \setminus \{\pm\frac{\pi}{2}, \pm 3\frac{\pi}{2}, \pm -\frac{5\pi}{2}, \dots\} = \mathbb{R} \setminus \{\frac{\pi}{2} \pm n\pi\}$
$n = 0, 1, 2, \dots$

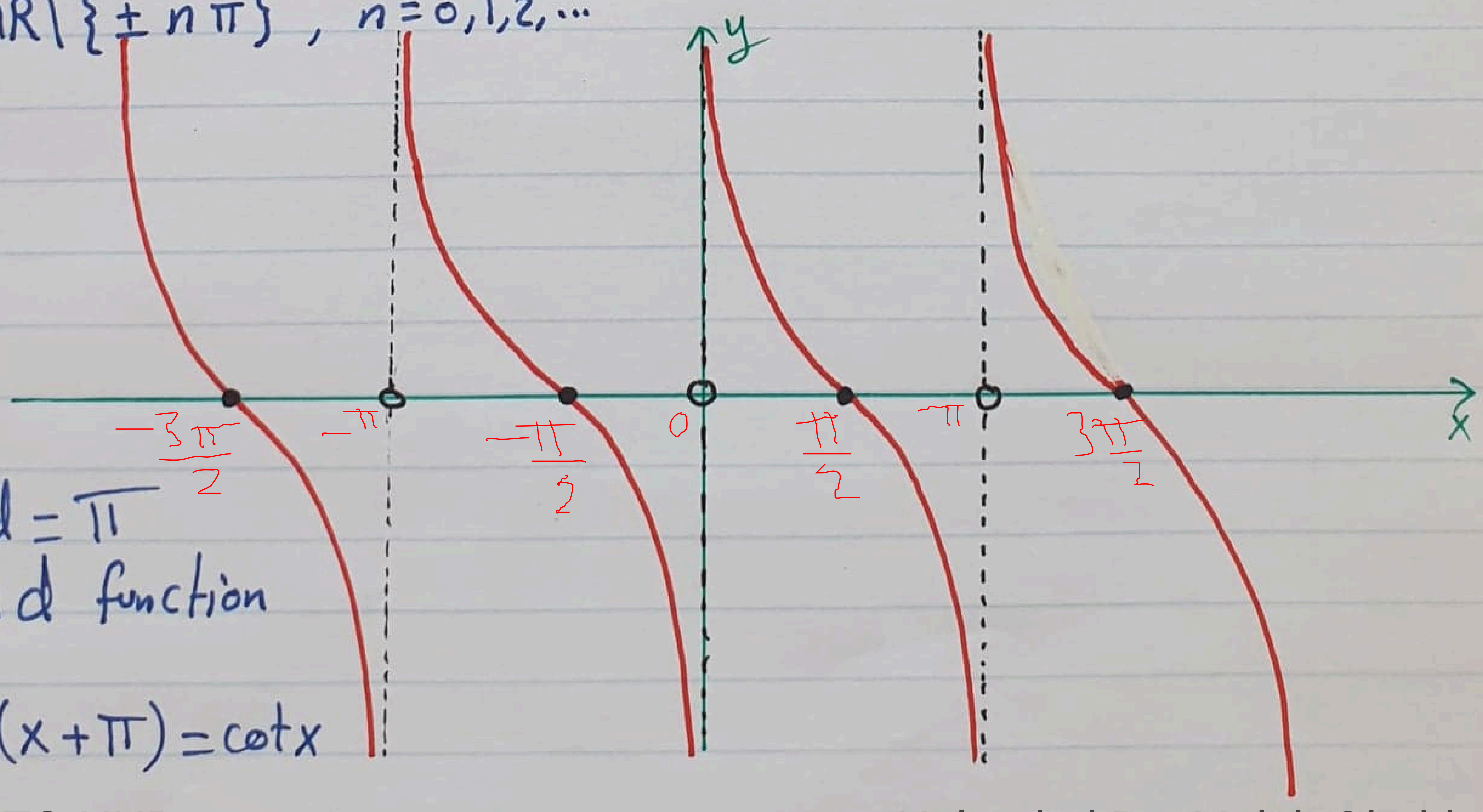
$R = \mathbb{R} = (-\infty, \infty)$

period $= \pi$
odd function
$\tan(x+\pi) = \tan x$



$\boxed{6}$ $f(x) = \cot x = \dfrac{\cos x}{\sin x}$ "cotan"

$D = \mathbb{R} \setminus \{0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots\}$ and $R = (-\infty, \infty) = \mathbb{R}$
$= \mathbb{R} \setminus \{\pm n\pi\}$, $n = 0, 1, 2, \dots$



period $= \pi$
odd function
$\cot(x+\pi) = \cot x$

Trigonometric Identities

- $\sin^2 x + \cos^2 x = 1$ ... *
- $\tan^2 x + 1 = \sec^2 x$ Divide * by $\cos^2 x$
- $1 + \cot^2 x = \csc^2 x$ Divide * by $\sin^2 x$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
  $= 2\cos^2 x - 1 \Rightarrow \cos^2 x = \frac{1+\cos 2x}{2}$ ✓
  $= 1 - 2\sin^2 x \Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$ ✓
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Exp $\sin(x+2\pi) = \sin x \cos(2\pi) + \cos x \sin(2\pi) = \sin x$ ✓
$\sin(x+\pi) = \sin x \cos\pi + \cos x \sin\pi = -\sin x$
$\cos(x+\pi) = \cos x \cos\pi - \sin x \sin\pi = -\cos x$
$\cos(x+\frac{\pi}{2}) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2} = -\sin x$

- **Even function** defined on interval I is symmetric about y-axis and satisfy $f(-x) = f(x)\ \forall x \in I$.

Exp $f(x) = x^2$, $y = x^4$, $g(x) = x^6$, $h(x) = |x|$, $r(x) = \cos x$, $m(x) = \sec x$ ... are even

- **Odd function** defined on interval I is symmetric about origin (0,0) and satisfy $f(-x) = -f(x)\ \forall x \in I$.

Exp $f(x) = x$, $y = x^3$, $g(x) = x^5$, $h(x) = \frac{1}{x}$, $r(x) = \sin x$, $m(x) = \csc x$, ... are odd

Exp ① show that $f(x) = \frac{x}{x^2-1}$ is odd function

$$f(-x) = \frac{(-x)}{(-x)^2-1} = \frac{-x}{x^2-1} = -\frac{x}{x^2-1} = -f(x)$$

② show that $g(x) = \frac{1}{x^2-1}$ is even function

$$g(-x) = \frac{1}{(-x)^2-1} = \frac{1}{x^2-1} = g(x)$$

• Composition $(f \circ g)(x) = f(g(x))$

Exp $f(x) = \sqrt{x}$, $g(x) = x^2$ ① Find $f \circ g$ and its domain
$D(f) = [0, \infty)$ $D(g) = \mathbb{R}$ ② Find $g \circ f$ and its domain

① $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2} = |x| \Rightarrow D = \mathbb{R}$ ✓

② $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x \Rightarrow D = [0, \infty)$ ✓

• $y = A \sin(B(x+C)) + D$

$|A|$: Amplitude

period $= \frac{2\pi}{B}$

C : Horizontal shift $\begin{cases} \to \text{to the left if } c > 0 \\ \to \text{to the right if } c < 0 \end{cases}$

D : Vertical shift $\begin{cases} \to \text{up if } D > 0 \\ \to \text{down if } D < 0 \end{cases}$