7.4 Discussion

Wednesday, December 8, 2021 1:49 PM

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Modeling using exp e^x

25. First-order chemical reactions In some chemical reactions, the rate at which the amount of a substance changes with time is proportional to the amount present. For the change of δ-glucono lactone into gluconic acid, for example,

$$\frac{dy}{dt} = -0.6y \qquad y_0 = y(0)$$

when t is measured in hours. If there are 100 grams of δ-glucono lactone present when $t = 0$, how many grams will be left after the first hour?

$y(t)$: amount of lacton available at time t

Growth $k > 0$: Pop. ↑, Disease ↑

Decay $k < 0$: Radioactive ↓, Pop. ↓, DB ↓

$$y(t) = y_0 e^{kt}$$

$$= y_0 e^{-0.6t}$$

$$y(t) = 100\, e^{-0.6t}$$

$$y(1) = 100\, e^{-0.6(1)} = 100\, e^{-0.6} \approx 54.88 \text{ gm}$$

36. Polonium-210 The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

$\tau = 139$

$$\tau = \frac{\ln 2}{k}$$

Decay $\Rightarrow k < 0$

$$Q(t) = Q_0 e^{-kt}$$

$$k = \frac{\ln 2}{\tau} = \frac{\ln 2}{139} \approx 0.005$$

available: كمية المتبقية

$$= Q_0 e^{-0.005t}$$

Find time t^* such that $Q(t^*) = \frac{5}{100} Q_0$

use 5 % → Decay 95%

use 5 % → decay 95%

$$Q_0 e^{-0.005 t^*} = \frac{5}{100} Q_0$$

$$e^{-0.005 t^*} = \frac{5}{100}$$

$$\ln e^{-0.005 t^*} = \ln 0.05$$

$$-0.005 t^* (\ln e) = \ln 0.05 \quad (\ln e)^1$$

$$t^* = \frac{\ln 0.05}{-0.005}$$

$$\approx 600 \text{ days}$$