Numbering Systems

Lecture 2

Comp 230

Outline

- Converting Fractions.
- Adding Binary Fractions.
- Binary Subtraction.
- Data Representation.
- Characters and Integers Representation.
- Floating Point Representation.
- Summary

Converting Fractions

- When converting a fractional decimal value to binary, we need to use a slightly different approach. Instead of dividing by 2 ,we repeatedly multiply the decimal fraction by 2.

- Example:

  Convert 11.37510 to it's binary equivalents.

  First convert 11 to binary .

  We know from the last lecture $11_{10} = 1011_2$

  Now convert .37510 to binary

Converting Fractions

0.375	*	2	=	0.750
0.750	*	2	=	1.500
0.500	*	2	=	1.000

$.375_{10} = .011_2$

$11.375_{10} = 1011.011_2$

Converting Fractions

- Convert the following numbers to their binary equivalents.

- $(26.75)_{10} = 11010.11_2$
- $(37.375)_{10}$ = H.W

Converting Fractions

- Exercise:
- Convert the following decimal number to binary?

- $(0.2)_{10} = (0.\overline{0011})_2$
- $(0.3)_{10} = (0.0\overline{1001})_2$

Adding Binary Fractions

- Example:
- 1011.0 + 0.011 =

```
  1011.0
+    0.011
-------------
  1011.011
```

Adding Binary Fractions cont.

- Example:

110.01 + 1.011 =

```
  1
110.01
+  1.011
----------------
 111.101
```

Adding Binary Fractions cont.

- Example:

110.01 + 1.111 =

111 1

110.01

+ 1.111

1000.001

Binary Subtraction

- Solve the following 8-bit subtraction problem using 2's complement representation.

  $01111111_2 - 76_{10} = ???$

- Rewrite the above problem as $01111111_2 + (-76)_{10}$

76 ⇨ 01001100

1 1

1's complement ⇨ 10110011

2's complement ⇨ + 1

10110100 ⇨ (-76)

Binary Subtraction Cont.

- $01111111_2 + (-76)_{10}$

```
    1111
   01111111        127
 + 10110100        -76
   ---------------------- ----------
Overflow 1 00110011         51
```

Binary Subtraction Cont.

- Example: 00110010_2 + $(– 125)_{10}$

125	⇨	01111101
1's complement	⇨	10000010
2's complement	⇨	+ 1

		10000011 ⇨ (-125)

Binary Subtraction Cont.

- $00110010_2 + (-\ 125)_{10}$
-
```
              1
   00110010          50
+  10000011        -125
   ------------    ---------
    10110101         -75
```
- The 2's comp for the result (10110101) is 01001011 equivalent to $(75)_{10}$

Data Representation

- Computer understand two things: on and off .
- Data represented in binary form .
- Bit is the basic unit for storing data 0 ⇨ off ,1 ⇨ on .
- Byte is a group of 8 bits. That is, each byte has 256(2^8) possible values.
- Two bytes form a word

Parity bit

- Used for error detection
- Two types:
  1. Odd parity (number of 1’s are odd)
  2. Even parity (number of 1’s are even)

Characters Representation

- Using the even parity bit to represent the character Q (Q = 81 in ASCII) in memory (Hexadecimal) ?

  $(81)_{10}=(01010001)_2$

	Parity bit		
Q	1	1010001	$= D1_{16}$

Memory

D1

Note: ASCII for A=65 and a=97
American Standard Code for Information Interchange

A=65 B=66
a=97 b=98

. .
. .

Characters Representation

Using the **odd parity** bit to represent **your name** in memory ?

Ex. Ahmad

	Parity bit		
A	1	1000001	= $C1_{16}$

	Parity bit		
h	0	1101000	= 68_{16}

	Parity bit		
m	0	1101101	= $6D_{16}$

	Parity bit		
a	0	1100001	= 61_{16}

	Parity bit		
d	0	1100100	= 64_{16}

A 01000001
h 01101000
m 01101101
..

Memory

C1
68
6D
61
64

Integers Representation

- Represent the following integer in memory using 2 byte?

  92

  92 = 1011100

  Answer

  0000 0000 0101 1100

  0 0 5 C

Memory

5C
00

Integers Representation

- Represent the following integer in memory using 2 bytes?

-94

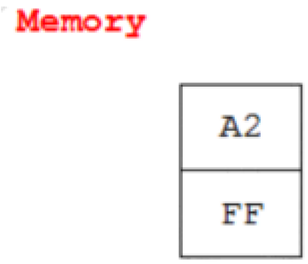
94 = 0000000001011110

1’s = 1111111110100001

2’s + 1

1111 1111 1010 0010

F F A 2



Floating Point Representation

32 bits divided into three sections

X	XXXXXXXX	XXXXXX.....X
1 bit For sign	8 bits For Exponent	23 bits For Mantissa

0 for Positive

1 for Negative

Floating Point Representation

32 bits divided into three sections

x	xxxxxxxx	xxxxxx.....x
1 bit For sign	8 bits For Exponent	23 bits For Mantissa

↓

2^8 =256

0-255

What about negative ??

Floating Point Representation

- 255/2=127.5 we take the integer part 127
- bias = 127 for 32 bit conversion. (2^{8-1} -1 = 128-1 = 127)

0---------------------- 255

0---------------------- 255 biased

-127 -127

-127 ----------------- 128 unbiased

Floating Point Representation

- Use the 32-bit floating representation to represent the following the binary number and show how it will represented in the memory?

$(26.75)_{10}$

Answer: Convert the number from decimal to binary

$(26.75)_{10} = (11010.11)_2$

Convert the result into Implicit Normalization

$(11010.11)_2 = (1.101011 * 2^4)_2$ Scientific notation

Exponent = 127+4=131

$(131)_{10} = (10000011)_2$

0	10000011	10101100000000000000000

4	1	D	6	0	0	0	0

Memory

00
00
D6
41

Floating Point Representation

- Convert the flowing 32-bit floating representation number into decimal
  0 10000011 10100000000000000000000

Sign = 0 = 1

Exponent =10000011 = $(131)_{10}$
131-127 = 4

The fractional part of mantissa is given by:

1*(1/2) + 0*(1/4) + 1*(1/8) + 0*(1/16) +……… = 0.625

Thus the mantissa will be 1 + 0.625 = 1.625

The decimal number hence given as:

Sign*Exponent*Mantissa = (1)*(16)*(1.625) = $(26)_{10}$

Floating Point Representation

- H.W

Lab 1 . P8,9

Q.5,6,7,9,11