

Ch8 Integration By

Substitution

ch1-ch5
ch6
ch7

الاجزاء
Parts

8.1

Trigonometric
Substitution
8.3

Trigonometric
Power
8.2

Partial
fraction
8.4

8.7
Improper
Integral

8.1 Integration By Parts

$$\int u dv = uv - \int v du$$

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Exp

$$\int x \cos x dx = \int u dv$$

$$= uv - \int v du$$

$$\begin{array}{lcl} u = x & \rightarrow & dv = \cos x dx \\ du = dx & \rightarrow & v = \sin x \end{array}$$

$$= \underset{\uparrow}{u} \underset{\uparrow}{v} \ominus \int v du$$

$$= x \sin x - \int \sin x dx$$

$$= x \sin x - (-\cos x) + C$$

$$= x \sin x + \cos x + C$$

$$= x \cos x + \sin x - \sin x + 0$$

$$\int x \cos x dx$$

$$= x \sin x + \cos x + C$$

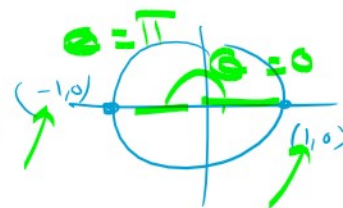
$\frac{d}{dx}$	$\int$
x	$\cos x$
1	$\sin x$
0	$-\cos x$

$$\int_0^{\pi} x \cos x dx = x \sin x + \cos x \Big|_0^{\pi}$$

$$= (\pi \sin \pi + \cos \pi) - (0 \sin 0 + \cos 0)$$

$$= (0 - 1) - (0 + 1)$$

$$= -1 - 1 = -2$$



$$\int \frac{1}{x} dx = \ln|x| + C$$

Exp $\int \frac{1}{x} dx = \ln|x| + c$

Exp $\int \ln x dx = \int u dv$

$u = \ln x$ $dv = dx$
 $du = \frac{1}{x} dx$ $\rightarrow v = x$

$= uv - \int v du$

$= (\ln x)x - \int x \cdot \frac{1}{x} dx$

$= \underline{x \ln x - x} + c$

Exp $\int x^3 e^x dx = \int u dv$

$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$

$\frac{d}{dx}$	$\int$
x^3	e^x
$3x^2$	$(+) e^x$
$6x$	$(-) e^x$
6	$(+) e^x$
0	$(-) e^x$

Exp $\int x^2 e^x dx$

$= \underline{x^2 e^x - 2x e^x + 2e^x + c}$

x^2	e^x
$2x$	$(+) e^x$
2	$(-) e^x$

or

or

$$\int x^2 e^x dx = \int u dv$$

$$= uv - \int v du$$

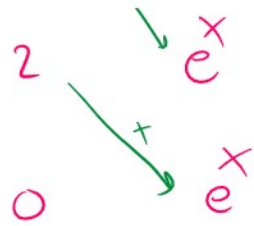
$$= \underline{x^2 e^x} - \int x e^x dx$$

$$= x^2 e^x - \int u dv$$

$$= x^2 e^x - [uv - \int v du]$$

$$= x^2 e^x - [2x e^x - \int 2e^x dx]$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$



$$u = x^2$$

$$dv = e^x dx$$

$$du = 2x dx$$

$$v = e^x$$

$$u = 2x$$

$$dv = e^x dx$$

$$du = 2 dx$$

$$v = e^x$$

Exp $\int e^x \sin x dx = \int u dv$

$$= uv - \int v du$$

$$= -e^x \cos x - \int -\cos x e^x dx$$

$$= -e^x \cos x + \int e^x \cos x dx$$

$$= -e^x \cos x + \int u dv$$

$$= -e^x \cos x + uv - \int v du$$

$$u = e^x$$

$$dv = \sin x dx$$

$$du = e^x dx$$

$$v = -\cos x$$

$$u = e^x$$

$$dv = \cos x dx$$

$$du = e^x dx$$

$$v = \sin x$$

$$\int e^x \sin x \, dx = -e^x \cos x + uv - \int v \, du$$

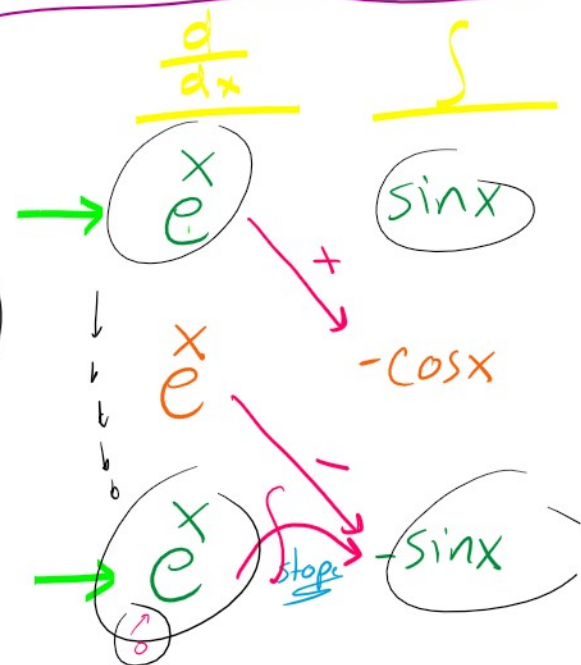
$$= -e^x \cos x + e^x \sin x - \int \sin x \, e^x \, dx$$

$$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\checkmark \int e^x \sin x \, dx = \frac{1}{2} [-\cos x + \sin x] e^x + C$$

Exp $\int e^x \sin x \, dx$

$$= -e^x \cos x + e^x \sin x + \int e^x (-\sin x) \, dx$$



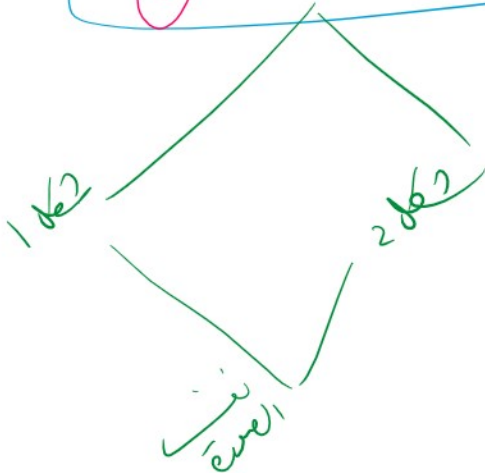
$$2 \int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

$$\checkmark \int e^x \sin x \, dx = \frac{e^x}{2} [-\cos x + \sin x] + C$$

$$\int x^{12} e^x dx$$

$$\int_0^{\pi} \sin x \cos x dx =$$

نفس
الجواب



$$y = \sqrt{5} \pi$$

$$\dot{y} = 0$$

مرد لایق

$$\int_1^2 x \ln x dx$$

$$= \int_1^2 u dv$$

$$= uv \Big|_1^2 - \int_1^2 v du$$

$$= \ln x \left(\frac{x^2}{2} \right) \Big|_1^2 - \int_1^2 \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$\begin{aligned} u &= x \\ du &= dx \end{aligned}$$

$$\begin{aligned} u &= \ln x \\ du &= \frac{1}{x} dx \end{aligned}$$

$$\begin{aligned} dv &= \ln x dx \\ v &= x \ln x - x \end{aligned}$$

$$\begin{aligned} dv &= x dx \\ v &= \frac{x^2}{2} \end{aligned}$$

$$\int \ln x dx = x \ln x - x + c$$

$$\begin{aligned}
 \textcircled{2} &= \ln x \left(\frac{x^2}{2} \right) \Big|_1^2 - \int_1^2 \frac{x^2}{2} \cdot \frac{1}{x} dx \\
 &= \left[(\ln 2)(2) - (\ln 1)\left(\frac{1}{2}\right) \right] - \frac{1}{2} \frac{x^2}{2} \Big|_1^2 \\
 &= 2 \ln 2 - \frac{1}{4} (2^2 - 1) \\
 &= 2 \ln 2 - \frac{1}{4} (3) = \ln 4 - \frac{3}{4}
 \end{aligned}$$

$$\int e^x \ln x \, dx$$

$\frac{d}{dx}$	$\int$
$\ln x$	e^x
$\frac{1}{x}$	e^x
$-\frac{1}{x^2}$	e^x

من دایره میزنیم

$$\int_1^2 x \ln x \, dx$$

$$\left(\int x \ln x \, dx = x(x \ln x - x) \right) \Big|_1^2 - \left(\int x \ln x \, dx - \frac{x^2}{2} \right)$$

$\frac{dx}{x}$	$\int$
x	$\ln x$
1	$x \ln x - x$
0	$\int x \ln x \, dx - \frac{x^2}{2}$

$$\left(\int_1^2 x \ln x \, dx = x(x \ln x - x) \right) - \left(\int_1^2 x \ln x \, dx - \frac{x^2}{2} \right) = 0$$

$$\boxed{\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4}}$$

$$\begin{aligned} \textcircled{2} \int_1^2 x \ln x \, dx &= \left. x^2 (\ln x - 1) + \frac{x^2}{2} \right|_1^2 \\ &= \left. x^2 \ln x - x^2 + \frac{x^2}{2} \right|_1^2 \\ &= \left. \frac{x^2}{2} \ln x - \frac{x^2}{4} \right|_1^2 \\ &= \left(\frac{4}{2} \ln 2 - \frac{1}{4} \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right) \\ &= 2 \ln 2 - \frac{1}{4} + \frac{1}{4} \end{aligned}$$

$$\textcircled{2} = 2 \ln 2 - \frac{3}{4}$$

$$\textcircled{11} \int \tan^{-1} y \, dy$$

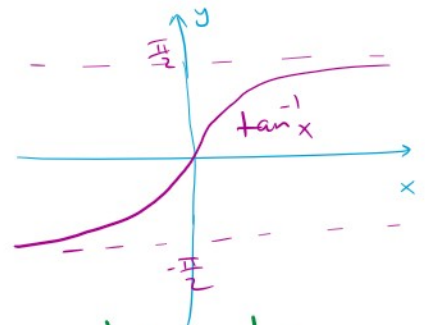
$$\int u \, dv$$

$$u = \tan^{-1} y$$

$$dv = dy$$

$$du = \frac{dy}{1+y^2}$$

$$v = y$$



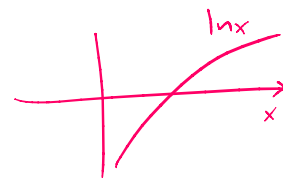
$$= uv - \int v du$$

$$= y \tan^{-1} y - \int y \frac{dy}{1+y^2}$$

$$= y \tan^{-1} y - \frac{1}{2} \int \frac{2y}{1+y^2} dy$$

$$= y \tan^{-1} y - \frac{1}{2} \ln |1+y^2| + C$$

$$= y \tan^{-1} y - \frac{1}{2} \ln(1+y^2) + C$$



$$\tan^{-1} y \neq (\tan y)^{-1} = \frac{1}{\tan y} = \cot y$$

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$$\int e^{\sqrt{3x+9}} ds = \int e^{\sqrt{3x+9}} dx$$

$$u = e^{\sqrt{3x+9}}$$

$$dv = dx$$

$$du = \frac{3}{2\sqrt{3x+9}} e^{\sqrt{3x+9}}$$

$$v = x$$

$$du = \frac{3}{2\sqrt{3x+9}} e$$

$$\int e^{\sqrt{3x+9}} dx = \int u du = uv - \int v du$$

$$= x e^{\sqrt{3x+9}} - \int x \frac{3}{2\sqrt{3x+9}} e^{\sqrt{3x+9}}$$

cor

$$\int e^{\sqrt{3x+9}} dx$$

$$y = \sqrt{3x+9}$$

$$dy = \frac{3}{2\sqrt{3x+9}} dx$$

$$\int e^y \frac{2y dy}{3}$$

$$= \frac{3}{2y} dx$$

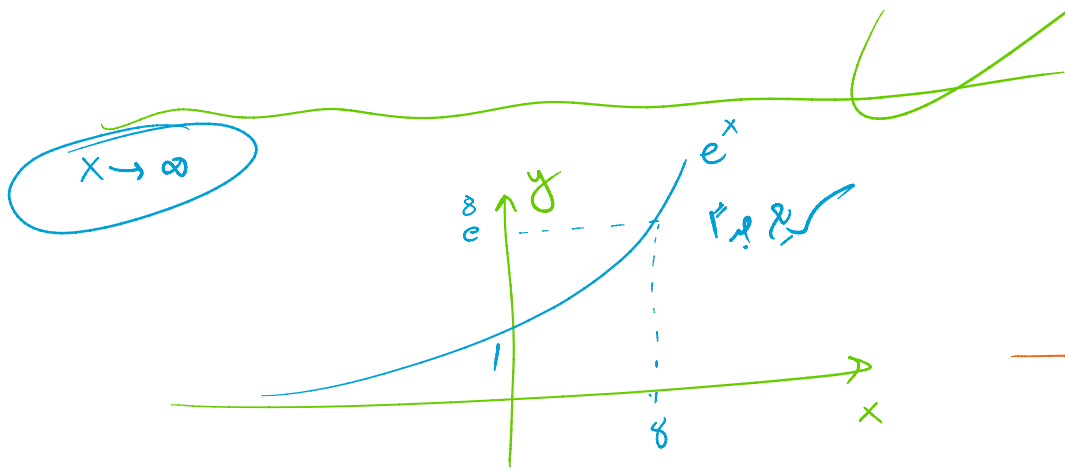
$$\int \frac{2}{3} y e^y dy$$

$$= \frac{2}{3} y e^y - \frac{2}{3} e^y + c$$

$$= \frac{2}{3} e^y [y - 1] + c$$

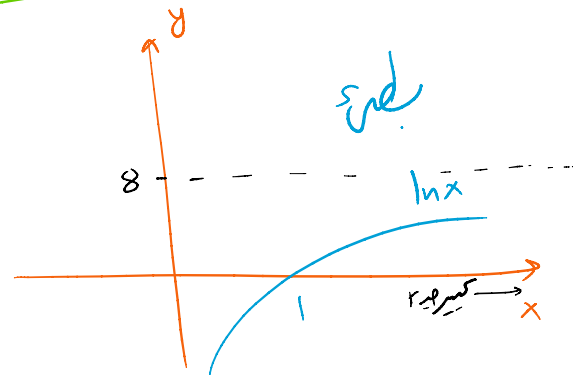
$$= \frac{2}{3} e^{\sqrt{3x+9}} [\sqrt{3x+9} - 1] + c$$

$$\begin{array}{r} \frac{2}{3} y \quad e^y \\ \swarrow + \\ \frac{2}{3} \quad e^y \\ \swarrow - \\ 0 \quad e^y \end{array}$$



$$\lim_{x \rightarrow \infty} e^x = \infty$$

7.8



$$\lim_{x \rightarrow \infty} \ln x = \infty$$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \begin{cases} 0 & \Rightarrow g \text{ goes faster than } f \text{ as } x \rightarrow \infty \\ \infty & \Rightarrow f \text{ is faster than } g \text{ as } x \rightarrow \infty \\ \frac{\infty}{\infty} & \Rightarrow f \text{ and } g \text{ grow at same rate as } x \rightarrow \infty \end{cases}$$

Exp $\log_3 x$, $\ln x$

$$\lim_{x \rightarrow \infty} \frac{\log_3 x}{\ln x} = \lim_{x \rightarrow \infty} \frac{\frac{\ln x}{\ln 3}}{\ln x} = \frac{1}{\ln 3}$$

$\log_3 x$ and $\ln x$ grow at same rate.

$\log_3 x$ and $\ln x$ grow at same ..

(2) $x^2, (\ln 2)^x$

$$\lim_{x \rightarrow \infty} \frac{(\ln 2)^x}{x^2} = 0$$

$\lim_{x \rightarrow \infty} \frac{1}{x^2} \rightarrow 0$
 $\lim_{x \rightarrow \infty} (\ln 2)^x \rightarrow 0$

x^2 is faster than $(\ln 2)^x$ as $x \rightarrow \infty$

$$\lim_{x \rightarrow \infty} \left(\frac{1}{2}\right)^x = 0$$

$$\lim_{x \rightarrow \infty} (2)^x = \infty$$

$$\lim_{x \rightarrow \infty} a^x = \begin{cases} 0 & 0 < a < 1 \\ \infty & a > 1 \end{cases}$$

$$1 < 2 < e \approx 2.718$$

$$\ln 1 < \ln 2 < \ln e$$

$$0 < \ln 2 < 1$$

Exp $(\ln 2)^x, e^x, x^2$

e^x faster than x^2
 x^2 faster than $(\ln 2)^x$

$$(\ln 2)^x < x^2 < e^x$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{e^x}{x^2} = \infty$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{2^x}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{2^x}$$

$$(\ln x)^x < x^2 < e^x$$

$$\lim \frac{e^x}{x}$$