diff : differentiable

$f'(x_0)$: f prime at x_0

$f''(x_0)$: f double prime at x_0

$f'''(x_0)$: f triple prime at x_0

$f''''(x_0) = f^{(4)}(x_0)$: f super 4 at x_0 $\quad f', f'', f''', f^{(4)}(x_0)$

$$f^4(x_0) = [f(x_0)]^4 = f(x_0) \cdot f(x_0) \cdot f(x_0) \cdot f(x_0)$$

$f^{(10)}(x_0)$: f super 10 at x_0 . . . — —

Def The derivative of $f(x)$ at point x_0 is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} \qquad \text{" limit exist "}$$

قَ(س) = نها ه ← ٠ ‏[ق(س+ه) − ق(س)] / ه

Ex Find derivative of $f(x) = 2x - 1$ at $x_0 = 2$

Find derivative of $f(x) = 2x - 1$ at $x_0 = 2$ using the definition of derivative

$f'(x) = 2$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{\left(2[2+h] - 1\right) - \left(2\boxed{2} - 1\right)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} + 2h \cancel{-1} - \cancel{(4-1)}}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = \boxed{2}$$

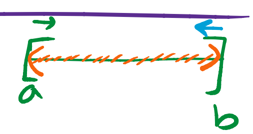
f is diff at 2

limit exists

$f'(2) = 2$

• If $f'(x_0)$ exists $\Rightarrow$ f is differentiable at x_0

Q. when f is diff on $[a, b]$



A. $\rightarrow$ f must be diff on open interval (a, b)

✓ " $f'(x)$ exists for every $x \in (a, b)$

$\rightarrow$ The right-hand derivative of f at a exists

$f'\ldots \lim \quad f(a+h) - f(a)$

$$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

→ The left - hand derivative of f at b exists

$$f'_-(b) = \lim_{h \to 0^-} \frac{f(b+h) - f(b)}{h}$$

Remark f is diff at c iff ($\Longleftrightarrow$) if and only if

① $f'_+(c)$ exists and

② $f'_-(c)$ exists and

③ $f'_+(c) = f'_-(c)$

Exp ① True / False

If f is diff at c then

f is cont. at c $\boxed{T}$

② If f is cont. at c then

f is diff at c $\boxed{F}$

Exp Find cont. function at $x=0$ but the function is not diff at $x=0$

$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

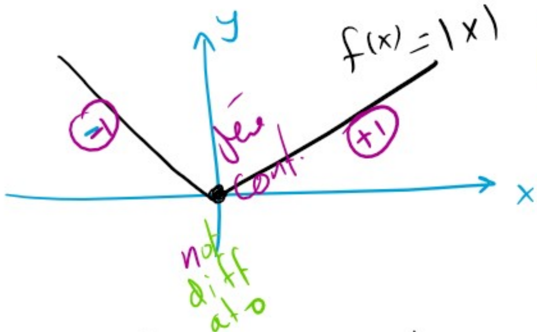
$$f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

• f cont. at $x=0$ $\Rightarrow$ $f(0) = \lim_{x \to 0} f(x)$

$0 = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x) = 0$



• But f is not diff at $x=0$ since

$$f'_+(0) \overset{??}{\neq} f'_-(0)$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} \overset{??}{=} \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \to 0^+} \frac{f(h)}{h} \overset{??}{=} \lim_{h \to 0^-} \frac{f(h)}{h}$$

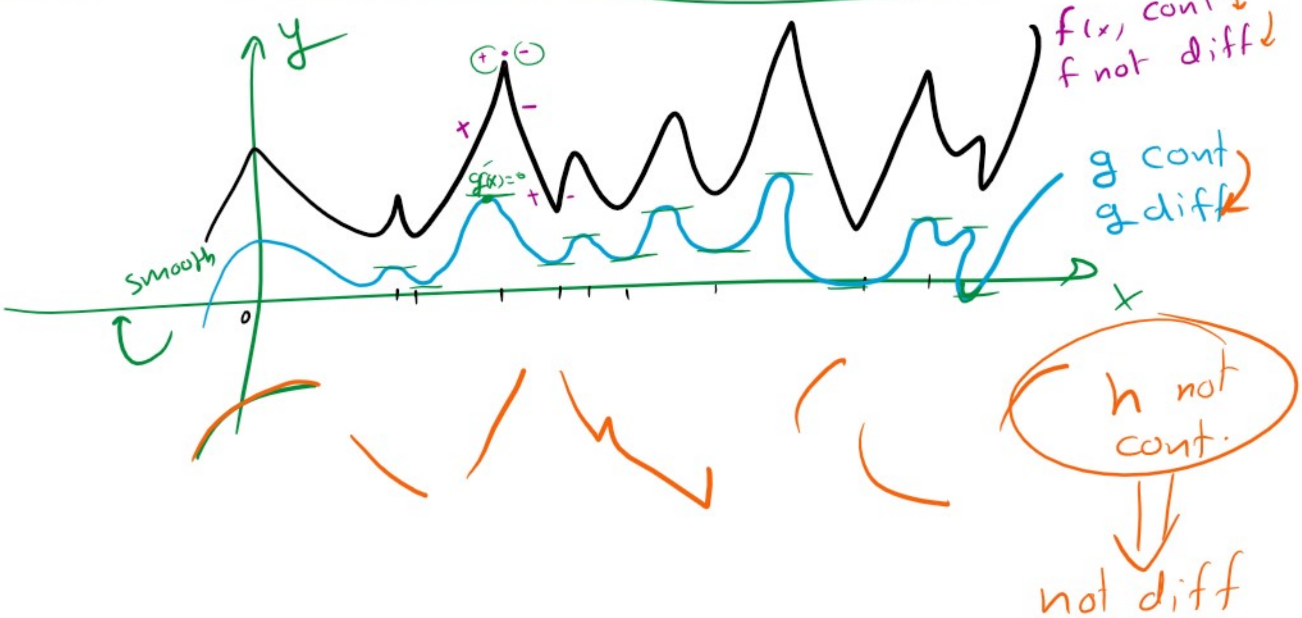
h $-h$

$h \to 0^{+}$ h $h \to 0^{-}$ h

$$1 \neq -1$$

$f'(0)$ does not exist $\underline{DNE}$

f is diff at x_0 قابل للاشتقاق عند x_0 f



$\underline{Th}$ (Differentiation Rules) قواعد الإشتقاق

Assume f and g diff at x. Then

(1) $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

1. $\left(f(x) \pm g(x) \right)' = f'(x) \pm g'(x)$

2. $\left(f(x)\, g(x) \right)' = f(x)\, g'(x) + g(x)\, f'(x)$

3. $\left(\dfrac{f(x)}{g(x)} \right)' = \dfrac{g(x)\, f'(x) - f(x)\, g'(x)}{\left[g(x) \right]^2}$, $g \neq 0$

4. $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ Chain Rule

$(f \circ g)(x) = f(g(x))$

Exp Find $\dfrac{d}{dx} \left(\dfrac{5 - x^2}{2x - 4} \right) \Big|_{x=-2}$

$f'(x) = \dfrac{df}{dx} = (f(x))' = Df = \dfrac{d}{dx} f$

$\dfrac{(2x-4)(-2x) - (5 - x^2)(2)}{(2x-4)^2}$

$x = -2$

$\dfrac{(-\cancel{8}\mathbf{4}-4)(4) - (5-4)(2)}{(-4-4)^2}$

-34

$$\frac{(-\cancel{12}\,8)(4)-(1)(2)}{(-8)^2} = \frac{-\cancel{48}\,32-2}{64} = \frac{\cancel{-50}\,-34}{64}$$

Derivatives of Trigometric functions

① $(\sin x)' = \cos x$

② $(\cos x)' = -\sin x$

③ $(\tan x)' = \sec^2 x$ | $f = \tan x = \frac{\sin x}{\cos x} \Rightarrow f' = \frac{1}{\cos^2 x} = \sec^2 x$

④ $(\cot x)' = -\csc^2 x$

⑤ $(\sec x)' = \sec x \tan x$

⑥ $(\csc x)' = -\csc x \cot x$

Exp $f(x) = \tan\sqrt{x}$ Find tangent line at $x_0 = \pi^2$ $\boxed{x>0}$

Tangent line

$y - y_0 = m(x - x_0)$

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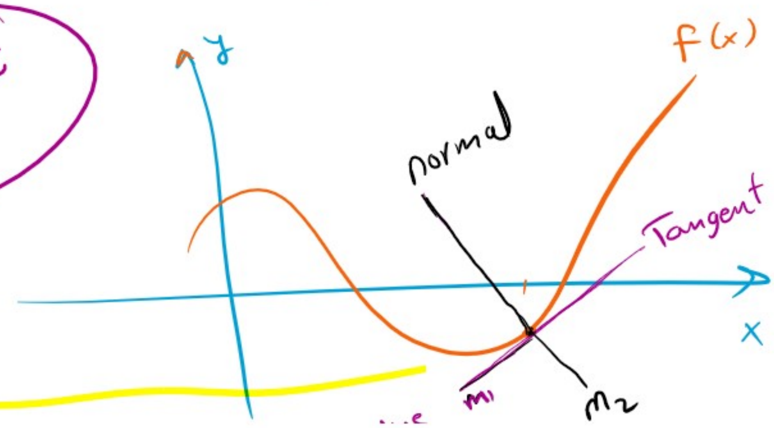
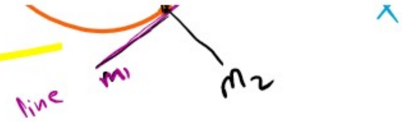


Point $(x_0, y_0) = (\pi^2, f(\pi^2))$

Point $(x_0, y_0) = (\pi^2, f(\pi^2))$



Exp

$= (\pi^2, \tan\sqrt{\pi^2})$ $\qquad \sqrt{\pi^2} = |\pi|$

$= (\pi^2, \tan \pm\pi)$

$= (\pi^2, 0)$

$x_0 \quad y_0$

$\boxed{m_1 m_2 = -1}$

$$f(x) = \tan\sqrt{x}$$

$$f'(x) = \sec^2\sqrt{x} \quad \frac{1}{2\sqrt{x}}$$

$$m = f'(x_0) = f'(\pi^2)$$

$$= \sec^2\sqrt{\pi^2} \quad \frac{1}{2\sqrt{\pi^2}}$$

$$\sec^2\pi = \frac{1}{\cos^2\pi} = \frac{1}{(-1)^2} = 1$$

$$= \sec^2\pi \quad \frac{1}{2|\pi|}$$

$$= (1) \quad \frac{1}{2\pi} = \frac{1}{2|\pi|}$$

Tangent line

$$y - y_0 = m(x - x_0)$$

$$y - 0 = \frac{1}{2\pi}(x - \pi^2)$$

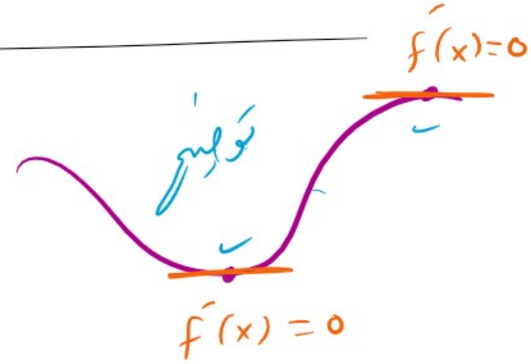
$$\boxed{y = \frac{1}{2\pi}x - \frac{\pi}{2}}$$

$= 2x^3 - 3x^2 - 12x + 20$

Exp Find points on curve y = ... where the tangent is parallel to x-axis

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 0$

$\boxed{x^2 - x - 2 = 0}$



$(x-2)(x+1) = 0$

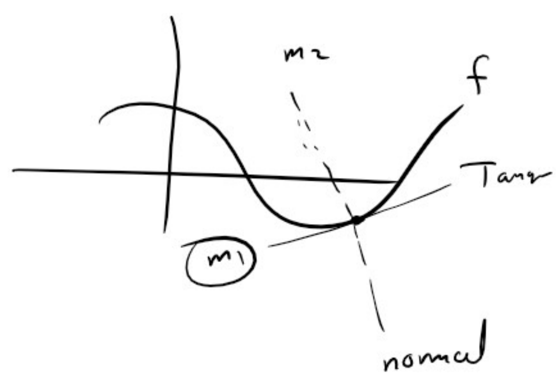
$x = 2$, $x = -1$ $\Rightarrow$ $y(-1) = 2(-1) - 3(1) + 12 + 20 = 27$

$\Downarrow$

$y(2) = 2(8) - 3(4) - 12(2) + 20 = 0$

$(2,0)$, $(-1,27)$

Exp Find normal line for $f(x) = \sec x \tan x$ at $x=0$



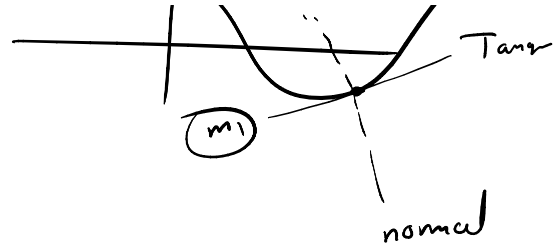
$f'(x) = \sec x \sec^2 x + \tan x \sec x \tan x$

$= \sec^3 x + \tan^2 x \sec x$

$m_1 m_2 = -1$

$m_1 = f'(0) = \sec^3 0 + \tan^2 0 \sec 0$

$$f'(x) = \sec x \; \sec^2 x + \tan x \; \sec x \; \tan x$$
$$= \sec^3 x + \tan^2 x \; \sec x$$



$m_1 m_2 = -1$

$(1)\; m_2 = -1$

$\boxed{m_2 = -1}$

$$m_1 = f'(0) = \sec^3 0 + \tan^2 0 \; \sec 0$$
$$= 1 + (0)(1)$$
$$= 1 + 0$$
$$= 1$$

normal eq. $\Rightarrow \quad y - y_0 = m_2 (x - x_0)$

$$y_0 = f(x_0) = f(0) = \sec 0 \; \tan 0 = (1)(0) = 0$$

$$y - 0 = -1 (x - 0)$$

العمودي $\boxed{y = -x}$ normal line / orthogonal line / perpendicular line

Find tangent line

$$y - y_0 = m_1 (x - x_0)$$

$$y - 0 = (1)(x - 0) \Rightarrow \underset{}{\overset{\text{Tangent}}{\boxed{y = 1x}}}$$

Implicit differential

Implicit differential

Exp Find $\frac{dy}{dx}$ if

"Assuming $y = f(x)$"

① $x^3 + y^2 = 7$

$$3x^2 + 2y\,y' = 0$$

$$\boxed{y' = -\frac{3x^2}{2y}}$$

② $xy = \cos(xy)$

$$xy' + y = -\sin(xy)\left[xy' + y\right]$$

$$= -xy'\sin(xy) - y\sin(xy)$$

$$xy' + xy'\sin(xy) = -y - y\sin(xy)$$

$$xy'\left[1 + \sin(xy)\right] = -y\left[1 + \sin(xy)\right]$$

$$-xy\left[1+\sin(xy)\right] = -y\left[1+\sin(xy)\right]$$

$$y' = -\frac{y\,\cancel{(1+\sin(xy))}}{x\,\cancel{(1+\sin(xy))}} = -\frac{y}{x}$$