7.7 2+6+10+15+20+22+46+52+54+57+60

(1)

(2) $\sinh x = \frac{4}{3}$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 1 + \frac{16}{9}$

$\cosh x = \sqrt{\frac{25}{9}} = \frac{5}{3}$

$\tanh x = \frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}$

$\operatorname{sech} x = \frac{3}{5}$

$\operatorname{csch} x = \frac{3}{4}$

$\coth x = \frac{5}{4}$

(6) $\sinh(2\ln x)$

$$\sinh(2\ln x) = \sinh(\ln x^2) = \frac{e^{\ln x^2} - e^{-\ln x^2}}{2}$$

$$= \frac{x^2 - \frac{1}{x^2}}{2} = \frac{x^4 - 1}{2x^2}$$

(10) $\ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$

2

$= \ln[(\cosh x + \sinh x)(\cosh x - \sinh x)]$

$= \ln[\cosh^2 x - \sinh^2 x]$

$= \ln 1 = 0$

(15) $y = 2\sqrt{t} \tanh\sqrt{t}$

$= 2t^{\frac{1}{2}} \tanh(t^{\frac{1}{2}})$

$\frac{dy}{dt} = 2t^{\frac{1}{2}} \cdot \operatorname{sech}^2(t^{\frac{1}{2}}) \cdot \frac{1}{2} t^{-\frac{1}{2}} + \tanh t^{\frac{1}{2}} \cdot 2 \cdot \frac{1}{2} t^{-\frac{1}{2}}$

$= \operatorname{sech}^2\sqrt{t} + \frac{\tanh\sqrt{t}}{\sqrt{t}}$

(20) $y = \operatorname{csch}\theta(1 - \ln \operatorname{csch}\theta)$

$y' = \operatorname{csch}\theta\left[-\frac{-\operatorname{csch}\theta \coth\theta}{\operatorname{csch}\theta}\right] + (1 - \ln\operatorname{csch}\theta) \cdot -\operatorname{csch}\theta \coth\theta$

$= \operatorname{csch}\theta \coth\theta - \operatorname{csch}\theta \coth\theta + (\operatorname{csch}\theta)(\coth\theta) \ln \operatorname{csch}\theta$

$y' = (\operatorname{csch}\theta)(\coth\theta) \ln \operatorname{csch}\theta$

22 $y = \ln \sinh x - \frac{1}{2} \coth^2 x$

$$\bar{y} = \frac{\cosh x}{\sinh x} - \frac{1}{2} \cdot 2 \coth x \cdot -\operatorname{csch}^2 x$$

$$= \coth x + \coth x \operatorname{csch}^2 x$$

$$= \coth x \left[1 + \operatorname{csch}^2 x\right] = \coth x \coth^2 x = \coth^3 x$$

46 $\int \coth \frac{\theta}{\sqrt{3}} \, d\theta$

$$\int \frac{\cosh \frac{\theta}{\sqrt{3}}}{\sinh \frac{\theta}{\sqrt{3}}} \, d\theta$$

$\boxed{u = \sinh \frac{\theta}{\sqrt{3}}}$, $du = \cosh \frac{\theta}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \cdot d\theta \rightarrow \boxed{d\theta = \frac{\sqrt{3}\, du}{\cosh \frac{\theta}{\sqrt{3}}}}$

$$I = \int \frac{\sqrt{3}}{u} \, du = \sqrt{3} \ln|u| + C$$

$$= \sqrt{3} \ln\left|\sinh \frac{\theta}{\sqrt{3}}\right| + C$$

$$= \sqrt{3} \ln\left|\frac{e^{\frac{\theta}{\sqrt{3}}} - e^{-\frac{\theta}{\sqrt{3}}}}{2}\right| + C$$

(53) $\int_0^{\ln 2} \tanh 2x \, dx$ (4)

$\int \frac{\sinh 2x}{\cosh 2x} \, dx$

$u = \cosh 2x$
$du = \sinh 2x \cdot 2 \, dx$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|$

$I = \frac{1}{2}\ln|\cosh 2x| \Big|_0^{\ln 2}$

$= \frac{1}{2}\ln|\cosh 2\ln 2| - \frac{1}{2}\ln|\cosh 0|$

$= \frac{1}{2}\ln|\cosh \ln 4| - \frac{1}{2}\ln|\cosh 0|$

$\cosh x = \frac{e^{x} + e^{-x}}{2} \rightarrow \cosh(\ln 4) = \frac{e^{\ln 4} + e^{-\ln 4}}{2} = \frac{4 + \frac{1}{4}}{2} = \frac{17}{8}$

(54) $\cancel{\int_0^{\ln 2} 4e^{-\theta} \sinh\theta \, d\theta}$

$\cosh 0 = \frac{e^{0} + e^{0}}{2} = 1$

So $I = \frac{1}{2}\left(\ln\frac{17}{8} - \ln 1\right) = \frac{1}{2}\left(\ln\frac{17}{8} - 0\right) = \cancel{\frac{17}{}}\ \frac{1}{2}\ln\frac{17}{8}$

54) $$\int_0^{\ln 2} 4e^{-\theta}\sinh\theta \, d\theta$$

$$\sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$$

$$4e^{-\theta}\sinh\theta = 2\left[1-e^{-2\theta}\right]$$

$$2\int_0^{\ln 2} 1-e^{-2\theta}\, d\theta = 2\left[\theta - \frac{e^{-2\theta}}{-2}\right]\Bigg|_0^{\ln 2}$$

$$= 2\left[\theta + \frac{e^{-2\theta}}{2}\right]_0^{\ln 2}$$

$$= 2\left[\ln 2 + \frac{e^{-2\ln 2}}{2}\right] - 2\left[0+\frac{1}{2}\right]$$

$$= 2\left[\ln 2 + \frac{1}{8}\right] - 1$$

$$= 2\ln 2 + \frac{1}{4} - 1 = 2\ln 2 - \frac{3}{4} = \boxed{\ln 4 - \frac{3}{4}}$$

57) $$\int_1^2 \frac{\cosh(\ln t)}{t}\,dt$$

$$\cosh \ln t = \frac{e^{\ln t} + e^{-\ln t}}{2} = \frac{t + \frac{1}{t}}{2}$$

Let $u = \ln t$, $t = 1 \to u = \ln 1 = 0$

$$\frac{du}{dt} = \frac{1}{t} \qquad t = 2 \to u = \ln 2$$

$$\int_0^{\ln 2} \frac{\cosh u}{t} \cdot t\,du = \int_0^{\ln 2} \cosh u\,du$$

$$= \sinh u \Big|_0^{\ln 2}$$

$$= \sinh \ln 2 - \sinh 0$$

$$= \left(\frac{e^{\ln 2} - e^{-\ln 2}}{2}\right) - \left(\frac{e^{0} - e^{0}}{2}\right)$$

$$= \frac{2 - \frac{1}{2}}{2} - 0 = \frac{3}{4}$$

(60) $\int_0^{\ln 10} 4\sinh^2\left(\frac{X}{2}\right) dX$

$= 4\int_0^{\ln 10} \frac{\cosh X - 1}{2} \, dX$

$= 2\int_0^{\ln 10} \cosh X - 1 \, dX$

$= 2\left[\sinh X - X\right]\Big|_0^{\ln 10}$

$= 2\left[\sinh \ln 10 - \ln 10\right] - 2\left[\sinh 0 - 0\right]$

$= \frac{99}{10} - 2\ln 10$