STAT2361

ملاحظات المحاضرات

من إعداد موقع

BZU-HUB

Chapter 6

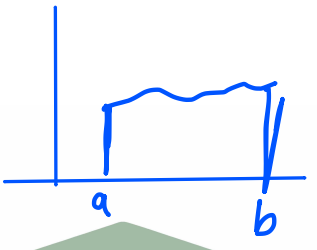
ملاحظة: المحاضرات من شرح الدكتور محمد مضية

Chapter (6): Continuous Random Variables

مجال القيم $[a,b]$

X = C.R.V ⟹ Probability = Area

$\int_a^x f(t)\,dt$



$P(X \geq 1) = P(X > 1)$

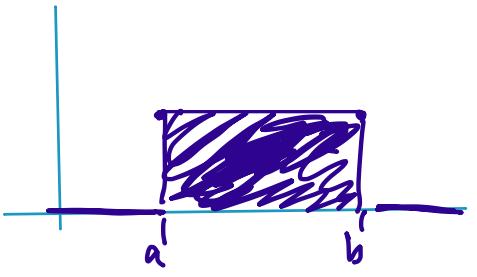
لأن المتغير مستمر

* Probability desity function

6.1: The Uniform Distribution

* **Def:** A uniform random variable X over an interval $[a,b]$ ⇒ $X = U[a,b]$ is a continous random variable with probability desity function given by:

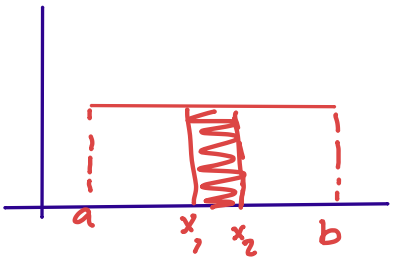
$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$



* Probability between : $f(x) \geq 0$
* Total Area = 1

* If $X = U[a,b]$, then:

1. $P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b-a}$
2. $E(X) = \frac{a+b}{2}$
3. $Var(X) = \frac{(b-a)^2}{12}$



*Ex: The time needed to finish a Stat final exam is uniformly distribution between 40 to 90 minutes.

$X = U[40, 90]$

1 Write and graph the probability desity function

$$f(x) = \begin{cases} \frac{1}{50}, & 40 \le x \le 90 \\ 0, & \text{elsewhere} \end{cases}$$

2 Find the average time (expected time) for the exam

$E(x) = \frac{40+90}{2} = 65$

3 Find $P(65 < x < 80) = \frac{80-65}{50} = 0.3$

4 more than 1 hour? $= P(x > 60) = \frac{90-60}{50} = 0.6$

5 $P(30 < x < 100)$

$= 0 + 1 + 0 = 1$

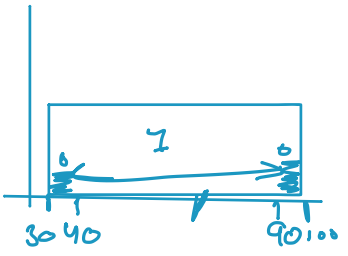
6 $P(x = 80) \Rightarrow$ السؤال ما بينسأل

$= 0$

7 Find (a) such that $P(x < a) = 0.4$

$P(x < a) = 0.4$

$P(x > a) = 0.6$



$\frac{a-40}{50} = 0.4 \Rightarrow a - 40 = 20$

$a = 60$

8 Find P_{40}

$= P(x < a) = 40\%$

6.2: The normal Distribution

↳ bell-Shaped distribution

1. Symmetric
2. total area (probability) = 1
3. Continous
4. $P(a<x<b) = \int_a^b f(x)\,dx$ → probability desity function
5. Table
   ↳ Standard normal distribution

Normal μ σ ⇒ Different Normal Distribution

⇓

Standard Normal Distribution

If X normal with mean μ and S.D σ $X = N(\mu, \sigma)$

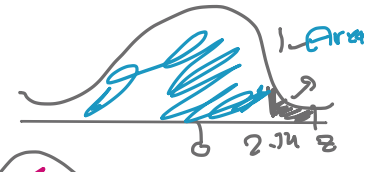
Standard Normal Distribution: Normal distribution with mean $\mu = 0$ and S.D $\sigma = 1$

$X = N(\mu, \sigma) \longrightarrow$ Z-Score $\longrightarrow Z_\mu = \frac{\mu - \mu}{\sigma} = 0$

* Z-table = Standard Normal Distribution table (Area (s) under a normal Curves)

∴ * منا الجدول:

1. $P(Z > 2.14) = 1 - P(Z < 2.14)$
   $= 0.0162$



2. $P(Z < -2.14) = P(Z > 2.14)$
   $= 0.0162$

3. $P(Z > -2.14) = 0.9838$
   $= P(Z < 2.14)$

4. $P(1.65 < Z < 2.14) = P(Z < 2.14) - P(Z < 1.65)$ (الكبير − الصغير)
   $= 0.9838 - 0.9505$

5. $P(-1.56 < Z < 1.98) = P(Z < 1.98) - P(Z \leq -1.56)$
   $= P(Z < 1.98) - (1 - P(Z < 1.56))$
   $= P(Z < 1.98) + P(Z < 1.56) - 1$
   $= \sim$

6 $P(-2<z<2) = 2P(z<2) - 1$
$= 2(0.9772) - 1$
$= 0.9544$

From Area ⟶ Z

Find A such that:

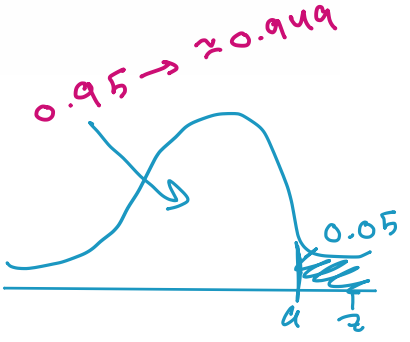
1 $P(z<a) = 0.9929$

2.45 ← من الجدول مباشرة

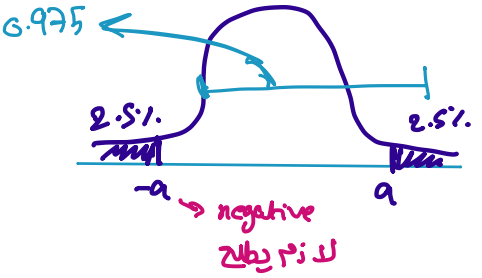
2 $P(z<a) = 0.9757$

a = 1.97 ← مش بالزبط بس تقريب (لأقرب رقم)

3 $P(z>a) = 0.05$
$= 0.999 - 0.05$
$= 0.949$
$= 1.64, 1.65$ ⟹ average
$= 1.645$



4 $P(z<a) = 0.025$
$= -1.96$



لازم يطلع

***Example:** The Salaries of BZU employees are normally distributed with mean of $2,000 and S.D of $400

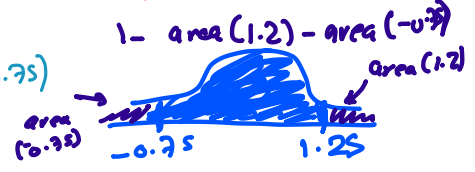
$$X = N(2{,}000, 400)$$

1 Find the % of employees with Salaries more than $2,700

$P(x > 2{,}700) \longrightarrow P\left(z > \frac{2{,}700 - 2000}{400}\right) = P(z > 1.75)$
$= 1 - P(z < 1.75)$
$= 1 - 0.9599$
$= 0.0401$
percentage $= 0.04\%$

2 Find the % of employees with Salaries between 1,700 and 2,500

$P(1{,}700 < X < 2{,}500) = P(-0.75 < z < 1.25)$
$= 1 - (1 - P(z<1.25)) - (1 - P(z<0.75))$
$= 1 - (0.1056) - (0.2266)$
$= 0.6678 \longrightarrow 66.78\%$.



[3] If the number of BZU employees is 900. Find the number of employees with salaries less than $1,000

$$P(X<1{,}000) = P(Z<-2.5)$$
$$= P(Z>2.5)$$
$$= 1 - P(Z<2.5)$$
$$= 1 - 0.9938$$
$$= 0.0062$$

# of employees $= (900)(0.0062) \cong \boxed{6}$

[4] Find the salary for which 3% of the employees salaries exceed?

Find x such that $P(X>x) = 0.03$

salary $x \longrightarrow z \longrightarrow$ area $\longrightarrow$ %.

$0.97 \Rightarrow z = 1.88$

$$z = \frac{x - \mu}{\sigma}$$

$$1.88 = \frac{x - 2{,}000}{400} \quad \therefore x = \$2752$$