Essential University Physics

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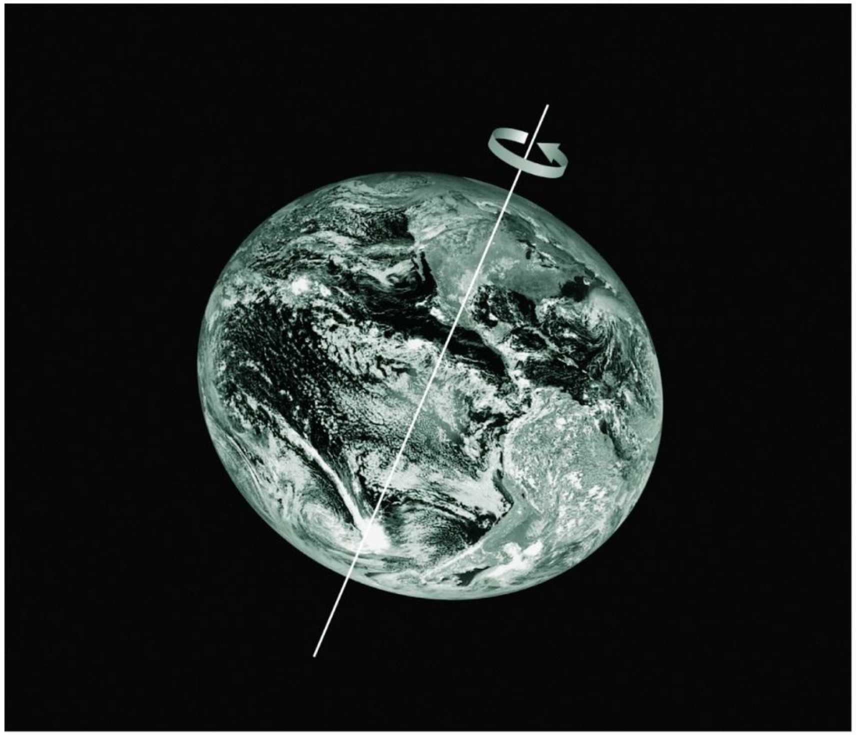
2nd Edition

Rotational Vectors and Angular Momentum

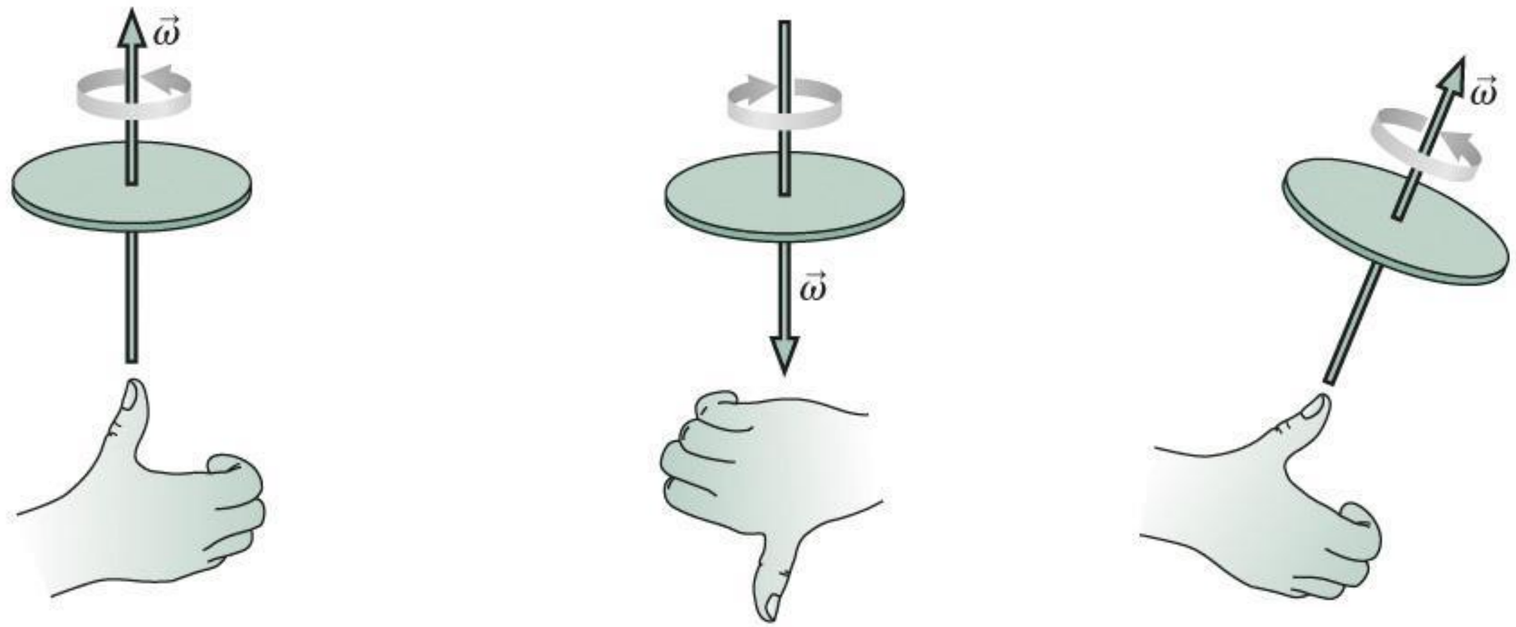
In this lecture you'll learn

- To treat angular quantities as vectors, with direction as well as magnitude
  - Angular velocity
  - Angular acceleration
  - Torque
- About angular momentum, the rotational analog of linear momentum
- To define the vector cross product, and use it in expressing torque and angular momentum
- To handle quantitative problems involving conservation of angular momentum in one direction
- To describe qualitatively the phenomenon of precession



Direction of the Angular Velocity Vector

- The direction of angular velocity is given by the **right-hand rule.**
  - Curl the fingers of your right hand in the direction of rotation, and your thumb points in the direction of the angular velocity vector $\vec{\omega}$.

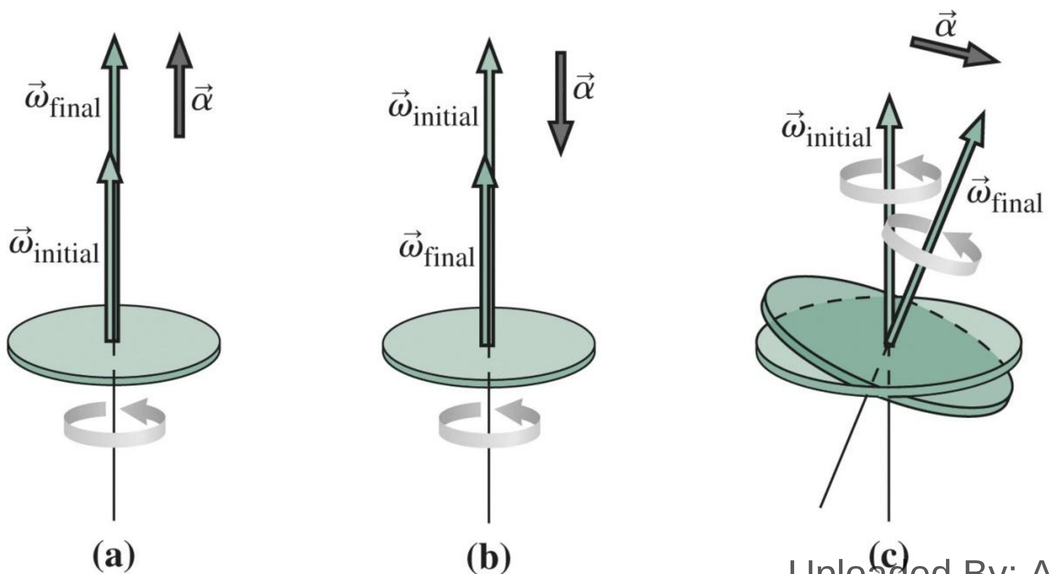


Direction of the Angular Acceleration

- Angular acceleration points in the direction of the change in the angular velocity $\Delta\vec{\omega}$:

$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

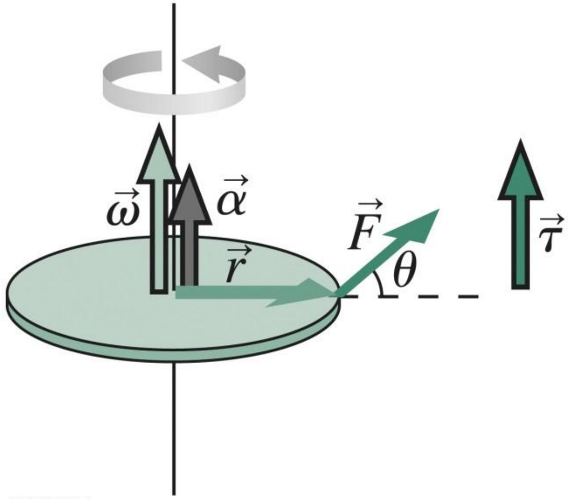
  - The change can be in the same direction as the angular velocity, increasing the angular speed.
  - The change can be opposite the angular velocity, decreasing the angular speed.
  - Or it can be in an arbitrary direction, changing the direction and speed as well.

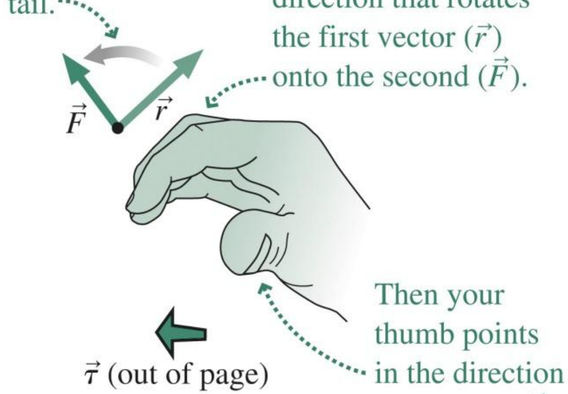


Direction of the Torque Vector

- The torque vector is perpendicular to both the force vector and the displacement vector from the rotation axis to the force application point.
  - The magnitude of the torque is $\tau = rF\sin\theta$.
  - Of the two possible directions perpendicular to $\vec{r}$ and $\vec{F}$, the correct direction is given by the right-hand rule.
  - Torque is compactly expressed using the **vector cross product**:

$$\vec{\tau} = \vec{r} \times \vec{F}$$





The Cross Product

- Forming from two vectors $\vec{A}$ and $\vec{B}$ a third vector $\vec{C}$ of magnitude $C = AB\sin\theta$ and direction given by the right-hand rule is called the **cross product**:

The cross product $\vec{C}$ of two vectors $\vec{A}$ and $\vec{B}$ is written

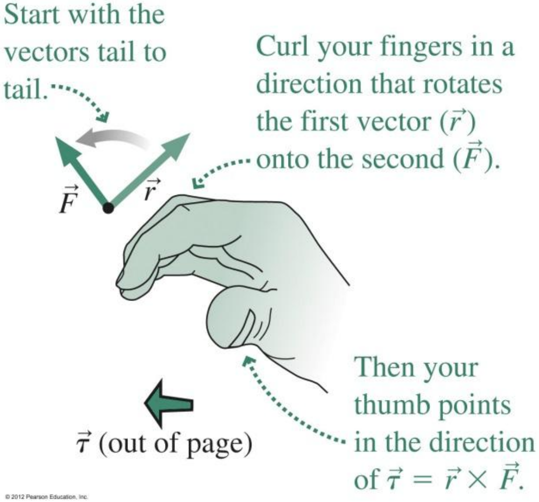
$$\vec{C} = \vec{A} \times \vec{B}$$

and is a vector with magnitude $AB\sin\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$, and where the direction of $\vec{C}$ is given by the right-hand rule of Fig. 11.4.

- Some properties of cross products:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$



Angular Momentum

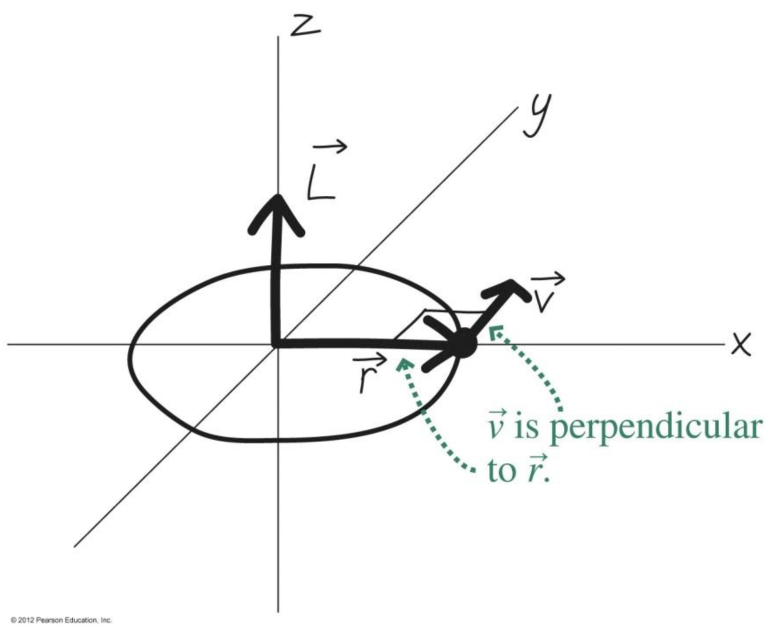
- For a single particle, angular momentum $\vec{L}$ is a vector given by the cross product of the displacement vector from the rotation axis with the linear momentum of the particle:

$$\vec{L} = \vec{r} \times \vec{p}$$

  – For the case of a particle in a circular path, $L = mvr$, and $\vec{L}$ is upward, perpendicular to the circle.

  – For sufficiently symmetric objects, $\vec{L}$ is the product of rotational inertia and angular velocity:

$$\vec{L} = I\vec{\omega}$$

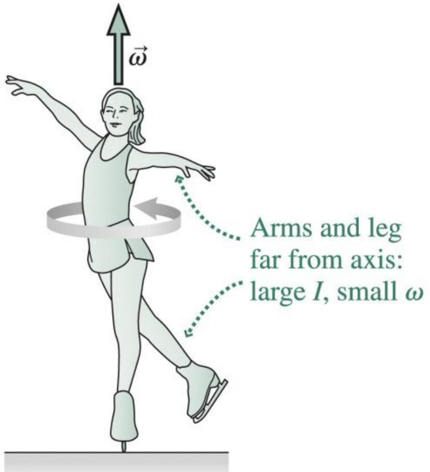


Newton's Law and Angular Momentum

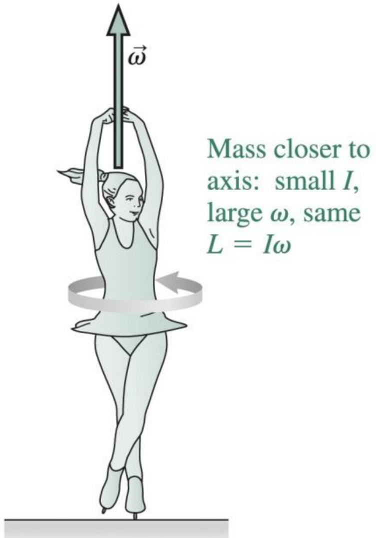
- In terms of angular momentum, the rotational analog of Newton's second law is

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

  – Therefore a system's angular momentum changes only if there's a non-zero net torque acting on the system.
  – If the net torque is zero, then angular momentum is conserved.
    - Changes in rotational inertia then result in changes in angular speed:

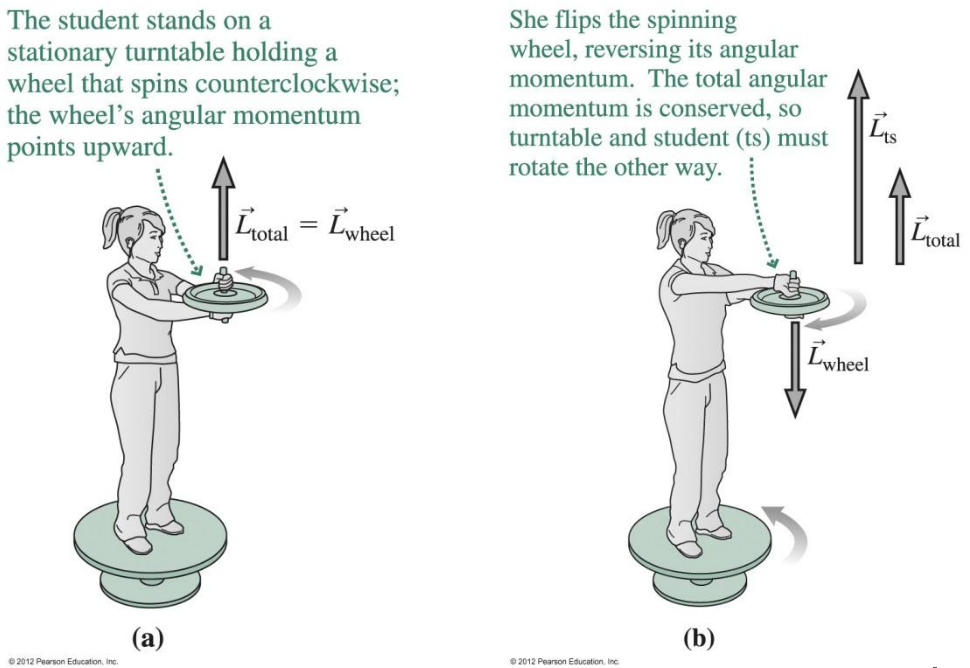


The skater's angular momentum is conserved, so her angular speed increases when she reduces her rotational inertia.



Conservation of Angular Momentum

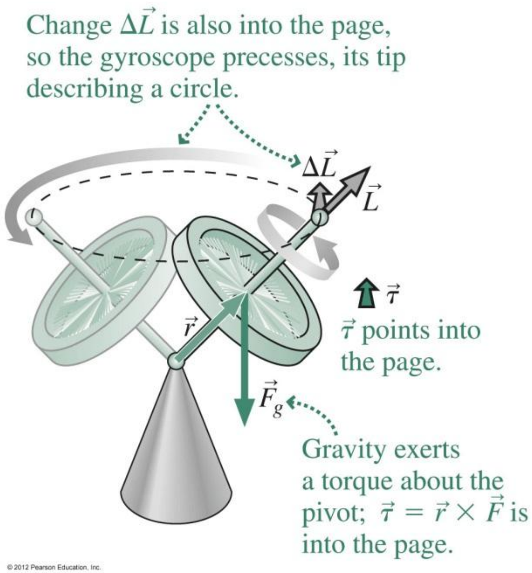
- The spinning wheel initially contains all the system's angular momentum.
- When the student turns the wheel upside down, she changes the direction of its angular momentum vector.
- Student and turntable rotate the other way to keep the total angular momentum unchanged.



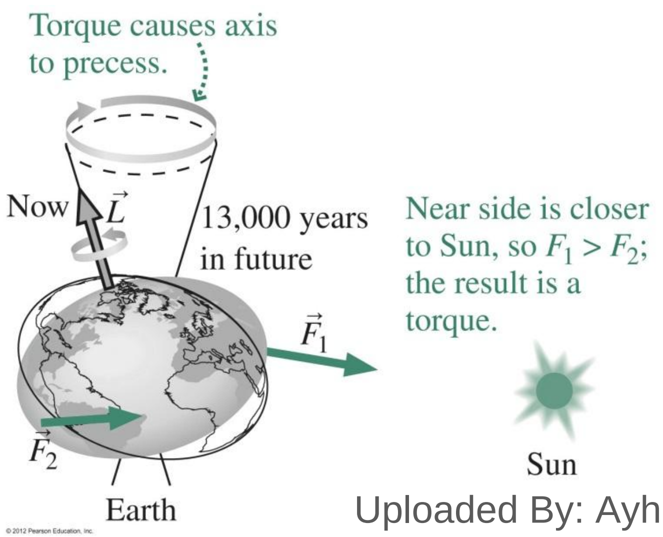
Precession

- Precession is a three-dimensional phenomenon involving rotational motion.
  - Precession occurs when a torque acts on a rotating object, changing the direction but not the magnitude of its angular momentum vector.
  - As a result the rotation axis undergoes circular motion:

Precession of a gyroscope

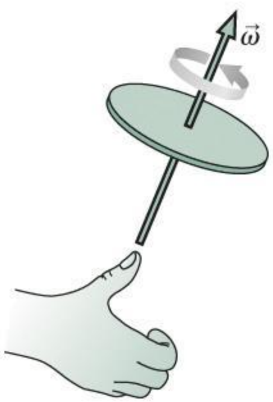


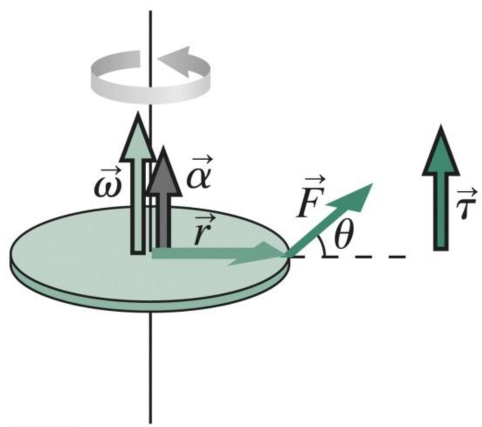
Precession slowly changes the direction of Earth's rotation axis

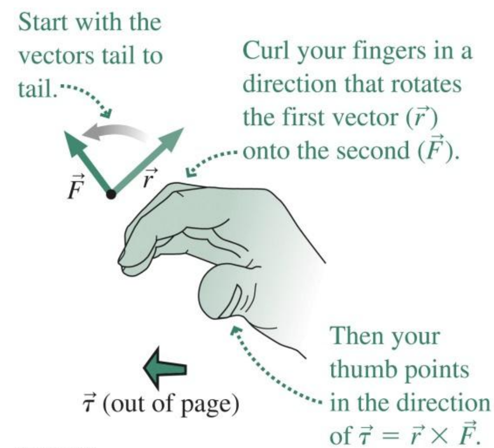


Summary

- Angular quantities are vectors whose direction is generally associated with the direction of the rotation axis.
  - Specifically, direction is given by the right-hand rule.
  - The vector cross product provides a compact representation for torque and angular momentum.







  - Angular momentum is the rotational analog of linear momentum: $\vec{L} = \vec{r} \times \vec{p}$; with symmetry, $\vec{L} = I\vec{\omega}$.
  - In the absence of a net external torque, a system's angular momentum is conserved.