Engineering Electromagnetics

Chapter 2

Coulomb's Law and Electric Field Intensity

Chapter Objectives

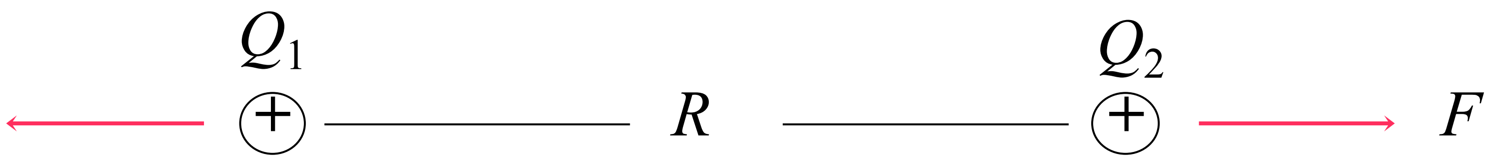
- **Introduce Coulomb's Experimental Law**
- **Understand Electric Field Intensity**
- **Explain Superposition of Fields**
- **Understand Volume Charge Density**
- **Relationship between Electric Field and Volume Charge Distributions**

Note: Sections 2.5-2.6 are omitted.

Coulomb stated that

The **force** between two very small objects **separated** in a vacuum or free space by a distance, which is large compared to their size, is **proportional** to the charge on each and **inversely proportional** to the square of the distance between them,

Coulomb's Experimental Law



Force of repulsion, *F,* occurs when charges have the same sign.

Charges attract when of opposite sign

$$F = k\frac{Q_1 Q_2}{R^2}$$

where

$$k = \frac{1}{4\pi\epsilon_0}$$

- Q_1 and Q_2 $\pm$charge, in coulombs (C).
- R is the separation, in meters (m).
- k is a proportionality constant.
- $\boldsymbol{F}$ in Newton (N).

Free Space Permittivity

The new constant ϵ_0 is called the permittivity of free space and has magnitude, measured in farads per meter (F/m).

The quantity ϵ_0 is not dimensionless, Coulomb's law shows that it has the label C2/N · m2

$$\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi} 10^{-9} \text{ F/m}$$

with which the Coulomb force becomes:

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

The coulomb is an extremely large unit of charge, for the smallest known quantity of charge is that of the electron (negative) or proton (positive), given in SI units as 1.602×10^{-19}C; hence a negative charge of one coulomb represents about 6×10^{18} electrons. (that is equivalent to **~3.74 x 10^{37}** electrons' charges combined)

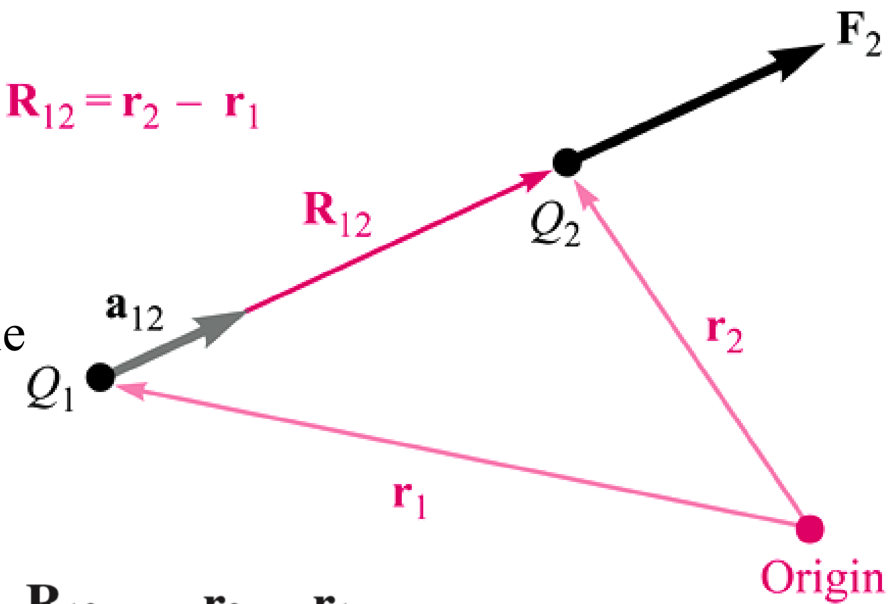
Coulomb's law shows that the force between two charges of one coulomb each, separated by one meter, is 9×10^{9} N, or about one million tons.

The electron has a rest mass of 9.109×10^{-31} kg and has a radius of the order of magnitude of 3.8×10^{-15}m.

Coulomb Force with Charges Off-Origin

In order to write the vector form of ***F***, we need the additional fact that the force acts along the line joining the two charges and is repulsive if the charges are alike in sign or attractive if they are of opposite sign.

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$



If Q1 and Q2 have like signs, the vector force F2 on Q2 is in the same direction as the vector R12.

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

Electric Field Intensity

Consider the force acting on a test charge, Q_t, arising from charge Q_1:

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t}$$

where $\mathbf{a}_{1t}$ is the unit vector directed from Q_1 to Q_t

The ***electric field intensity*** is defined as the force per unit test charge, or

$$\mathbf{E}_1 = \frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\mathbf{a}_{1t} \quad \text{N/C}$$

E1 is interpreted as the vector force, arising from charge Q_1, that acts on a unit positive test charge Q_t.

A more convenient unit for electric field is V/m, as will be shown.

More generally

$$\mathbf{E} = \frac{\mathbf{F}_t}{Q_t}$$

E is the ***electric field intensity*** evaluated at the test charge location that arises from all other charges in the vicinity—meaning the electric field arising from the test charge itself is not included in E.

The electric field of a single point charge located at the origin

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

R is the magnitude of the vector ***R***, which is the directed line segment from the point at which the point charge Q is located to the point at which ***E*** is desired, and $\boldsymbol{a_R}$ is a unit vector in the R direction

Electric Field of a Charge not located at the Origin

If we consider a charge that is not at the origin of our coordinate system, the field no longer possesses spherical symmetry, and we might as well use rectangular coordinates.

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} = \frac{Q(\mathbf{r}-\mathbf{r}')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{Q[(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z]}{4\pi\epsilon_0[(x-x')^2+(y-y')^2+(z-z')^2]^{3/2}}$$

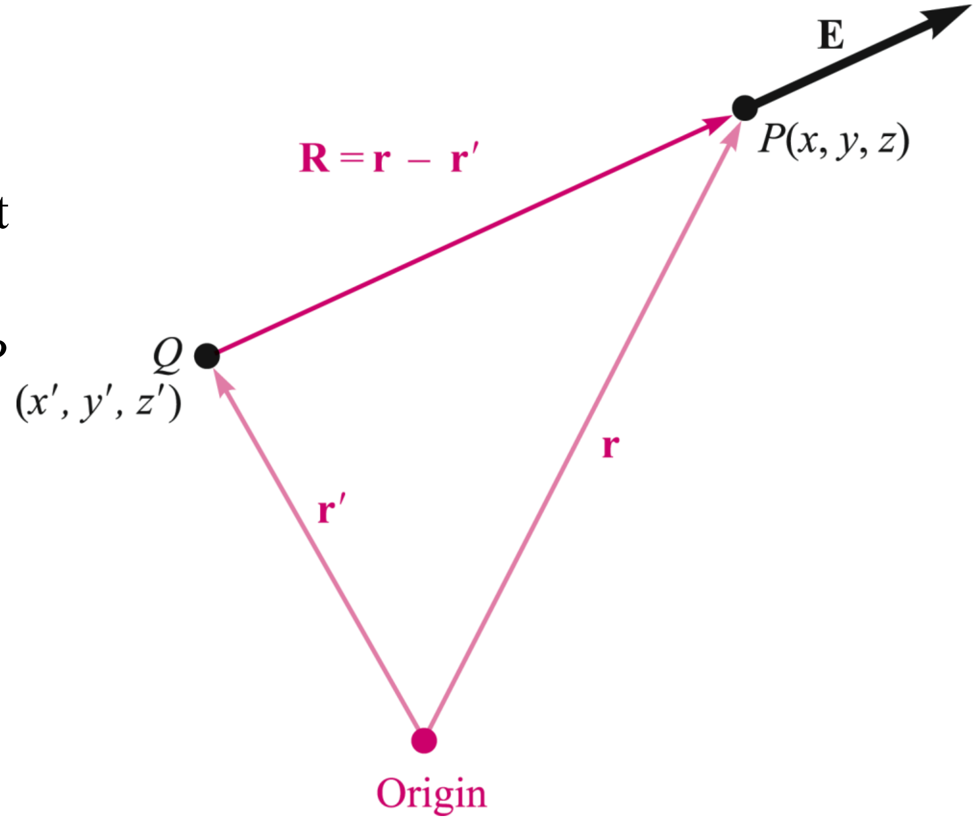
For a charge Q located at the source point

$$\mathbf{r}' = x'\mathbf{a}_x + y'\mathbf{a}_y + z'\mathbf{a}_z$$

we find the field at a general field point P

$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

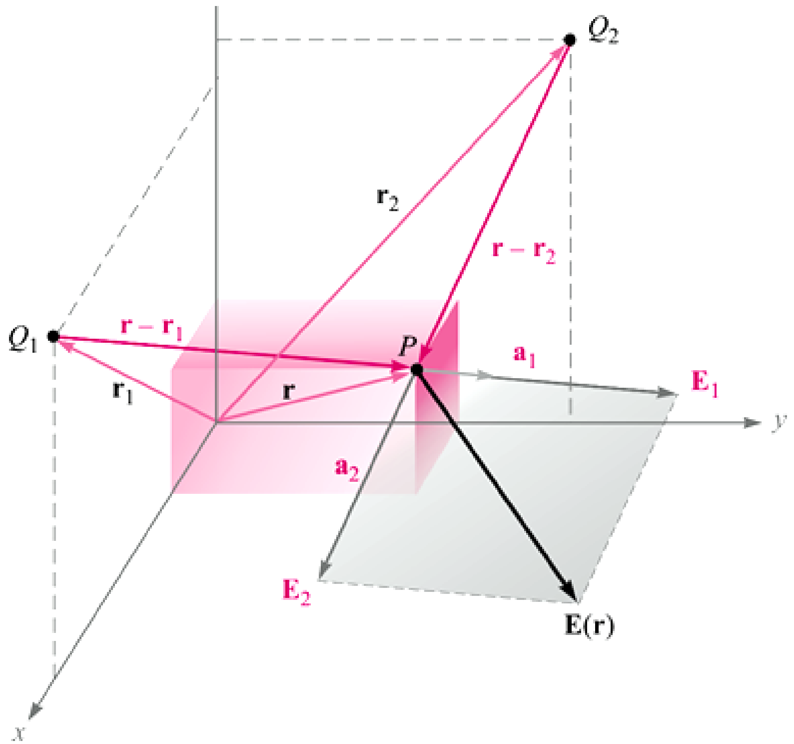
expressing $\mathbf{R}$ as $\boldsymbol{r} - \boldsymbol{r}'$



Superposition of Fields From Two Point Charges

Because the coulomb forces are linear, the ***electric field intensity*** arising from two point charges, Q_1 at $\boldsymbol{r_1}$ and Q_2 at $\boldsymbol{r_2}$, is the sum of the forces on Q_t (at point $\boldsymbol{P}$) caused by Q_1 and Q_2 acting alone, this is called **superposition of fields.**

$$\mathbf{E}(\mathbf{r}) = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$



For n charges:

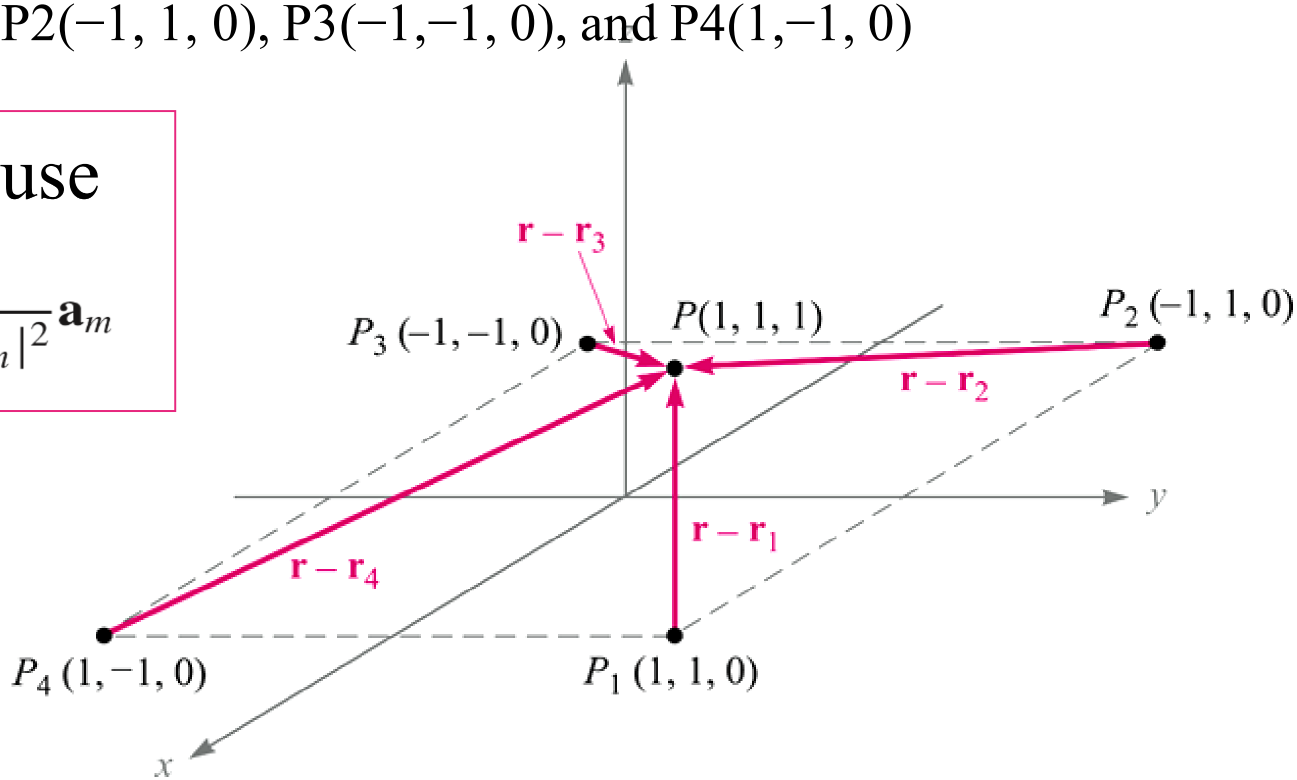
$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$

Example

Find $\boldsymbol{E}$ at P(1, 1, 1) caused by four identical 3-nC (nanocoulomb) charges located at P1(1, 1, 0), P2(−1, 1, 0), P3(−1,−1, 0), and P4(1,−1, 0)

To find $\boldsymbol{E}$ at P, use

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$



First, find the vectors:

$$\mathbf{r} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$$

$$\mathbf{r}_1 = \mathbf{a}_x + \mathbf{a}_y$$

$$\mathbf{r}_2 = -\mathbf{a}_x + \mathbf{a}_y$$

$$\mathbf{r}_3 = -\mathbf{a}_x - \mathbf{a}_y$$

$$\mathbf{r}_4 = \mathbf{a}_x - \mathbf{a}_y$$

Then:

$$|\mathbf{r}-\mathbf{r}_1| = 1$$

$$|\mathbf{r}-\mathbf{r}_2| = \sqrt{5}$$

$$|\mathbf{r}-\mathbf{r}_3| = 3$$

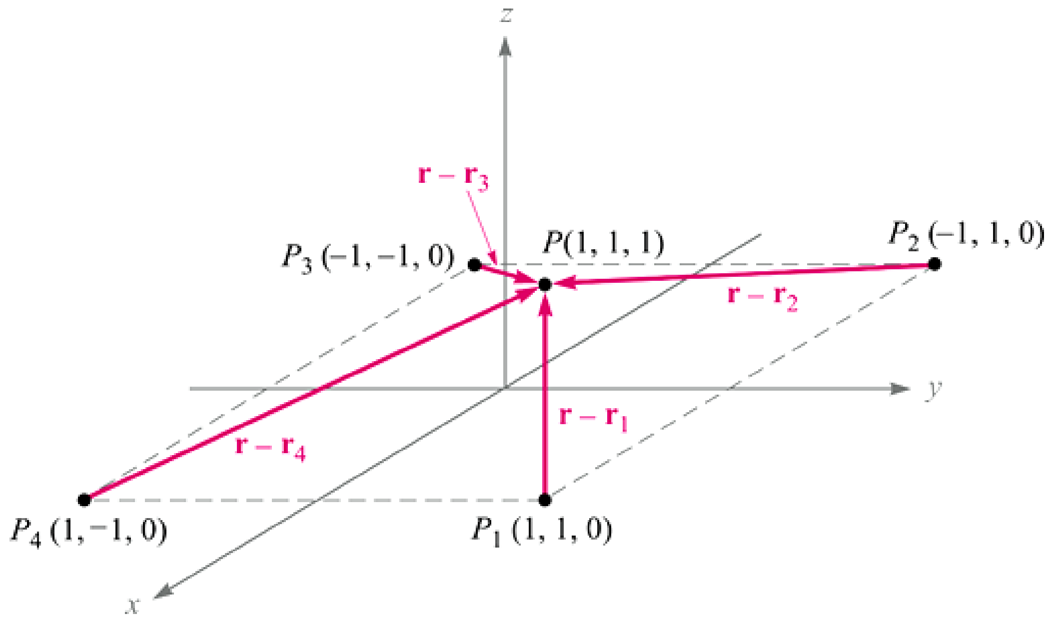
$$|\mathbf{r}-\mathbf{r}_4| = \sqrt{5}$$

Example (continued)

Find $\mathbf{E}$ at P, using

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \mathbf{a}_m$$

where $\mathbf{a}_m = \dfrac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|}$



Now: $Q/4\pi\epsilon_0 = 3 \times 10^{-9}/(4\pi \times 8.854 \times 10^{-12}) = 26.96 \text{ V} \cdot \text{m}$

so that: $$\mathbf{E} = 26.96 \left[\frac{\mathbf{a}_z}{1} \frac{1}{1^2} + \frac{2\mathbf{a}_x + \mathbf{a}_z}{\sqrt{5}} \frac{1}{(\sqrt{5})^2} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{3} \frac{1}{3^2} + \frac{2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{5}} \frac{1}{(\sqrt{5})^2} \right]$$

$$= 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z \text{ V/m}$$

Volume Charge Density

If we visualize a region of space filled with a tremendous number of charges separated by minute distances, we see that we can replace this distribution of very small particles with a smooth continuous distribution described by a ***volume charge density***, denoted as ρ_v having the units of **coulombs per cubic meter (C/m3)**

Given a charge ΔQ within a volume Δv, the volume charge density is defined as:

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

….so that the charge contained within a volume is

$$Q = \int_{\text{vol}} \rho_v \, dv$$

Only one integral sign is customarily indicated, but the differential dv signifies integration throughout a volume, and hence a **triple** integration.

Example

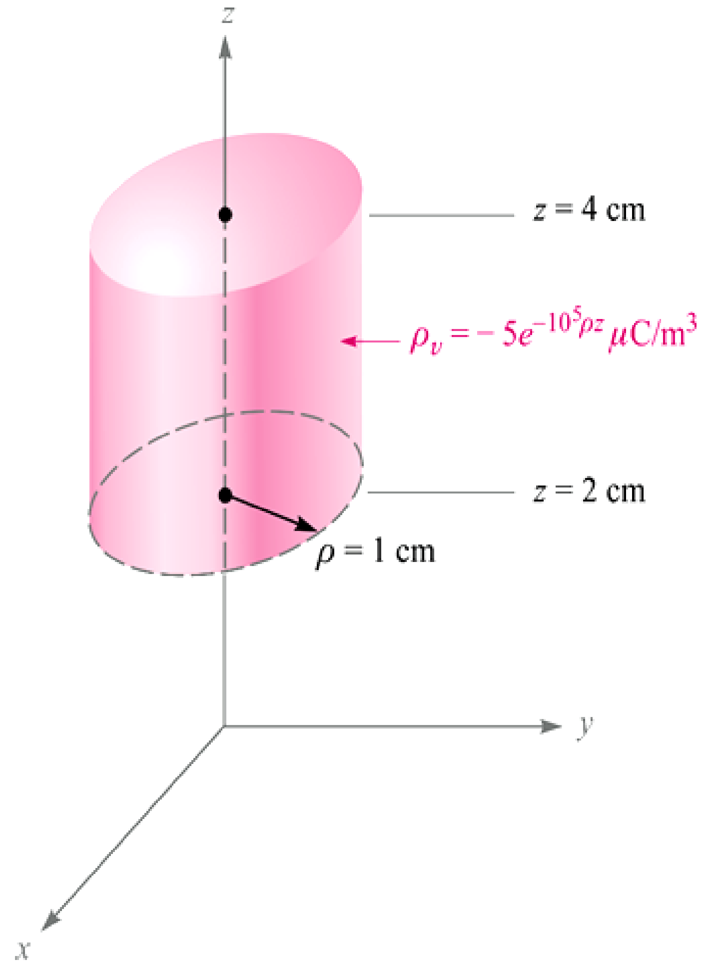
Find the charge contained within a 2-cm length of the electron beam shown below, in which the charge density is $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}$ C/m^2

$$Q = \int_{0.02}^{0.04} \int_{0}^{2\pi} \int_{0}^{0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho \, d\rho \, d\phi \, dz$$

$$= \int_{0.02}^{0.04} \int_{0}^{0.01} -10^{-5} \pi e^{-10^5 \rho z} \rho \, d\rho \, dz$$

$$= \int_{0}^{0.01} \left(\frac{-10^{-5}\pi}{-10^5 \rho} e^{-10^5 \rho z} \rho \, d\rho \right)_{z=0.02}^{z=0.04}$$

$$= \int_{0}^{0.01} -10^{-5} \pi (e^{-2000\rho} - e^{-4000\rho}) d\rho$$



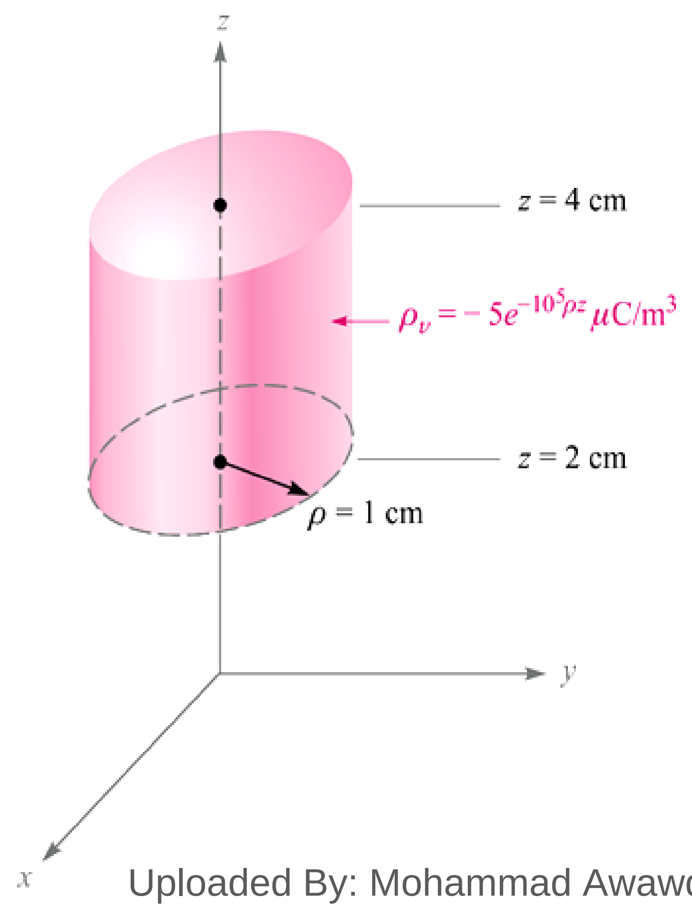
Example (continued)

$$Q = \int_0^{0.01} -10^{-5}\pi(e^{-2000\rho} - e^{-4000\rho})d\rho$$

$$= -10^{-10}\pi\left(\frac{e^{-2000\rho}}{-2000} - \frac{e^{-4000\rho}}{-4000}\right)_0^{0.01}$$

$$= -10^{-10}\pi\left(\frac{1}{2000} - \frac{1}{4000}\right)$$

$$= \frac{-\pi}{40} = \underline{0.0785 \text{ pC}}$$



Electric Field from Volume Charge Distributions

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_m|^2}\mathbf{a}_m$$

$$\mathbf{a}_m = \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|}$$

The incremental contribution to the electric field intensity at $\mathbf{r}$ produced by an incremental charge ΔQ at $\mathbf{r}'$ is

$$\Delta\mathbf{E}(\mathbf{r}) = \frac{\Delta Q}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^2}\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$= \frac{\rho_\nu \Delta\nu}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^2}\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

Next, sum all contributions throughout a volume and take the limit as $\Delta\nu$ approaches zero, to obtain the integral:

$$\mathbf{E}(\mathbf{r}) = \int_{\text{vol}} \frac{\rho_\nu(\mathbf{r}')\,d\nu'}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^2}\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

Line Charge Electric Field

Consider a filamentlike distribution of volume charge density, such as a charged conductor of very small radius. Assume a straight-line charge extending along the z axis in a **cylindrical** coordinate system from −∞ to ∞ of **constant density ρ_L C/m** lies along the entire *z* axis.

We desire the electric field intensity *E* at any and every point resulting from a *uniform* line charge density ρ_L

At point *P*, the electric field arising from charge *dQ* on the *z* axis is:

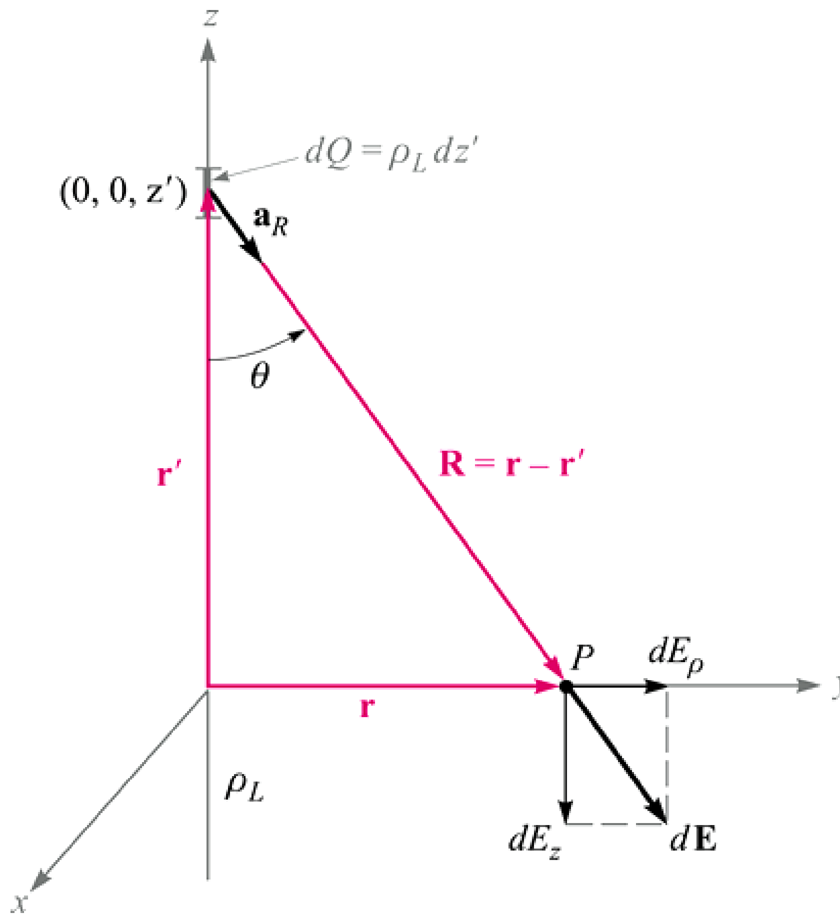
$$d\mathbf{E} = \frac{\rho_L dz'(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

where $\mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho$

and $\mathbf{r}' = z'\mathbf{a}_z$

so that $\mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z$

Therefore $d\mathbf{E} = \frac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$



Line Charge Field (continued)

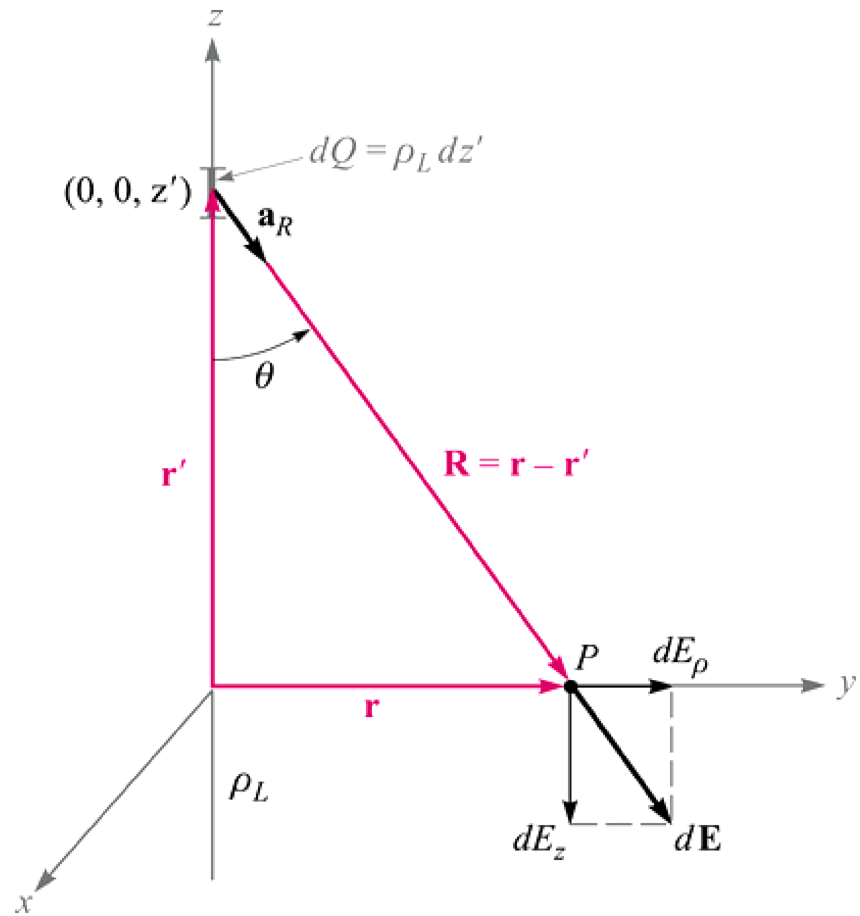
We have: $$d\mathbf{E} = \frac{\rho_L dz'(\rho \mathbf{a}_\rho - z' \mathbf{a}_z)}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

By symmetry, only a radial component is present:

$$dE_\rho = \frac{\rho_L \rho dz'}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho dz'}{4\pi\epsilon_0(\rho^2 + z'^2)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\rho \left(\frac{1}{\rho^2} \frac{z'}{\sqrt{\rho^2 + z'^2}} \right)_{-\infty}^{\infty}$$

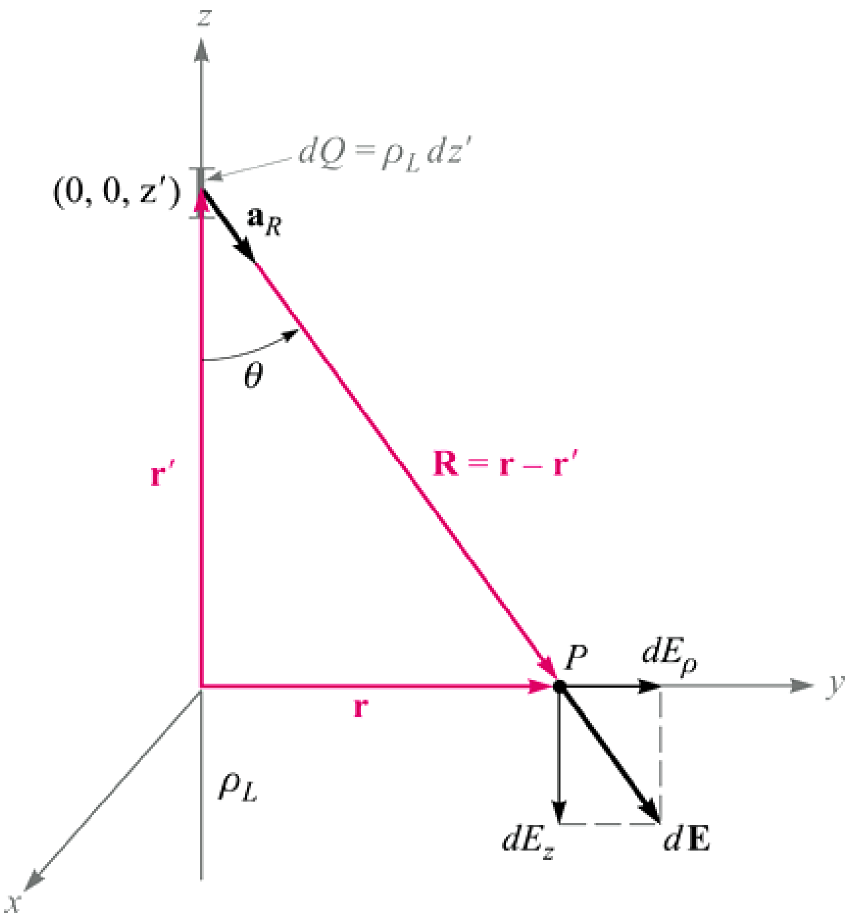


Line Charge Field Results

$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0}\rho\left(\frac{1}{\rho^2}\frac{z'}{\sqrt{\rho^2+z'^2}}\right)_{-\infty}^{\infty}$$

$$= \frac{\rho_L}{2\pi\epsilon_0\rho}$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\mathbf{a}_\rho$$



Example: Off-Axis Line Charge

With the line displaced to (6,8), the field becomes:

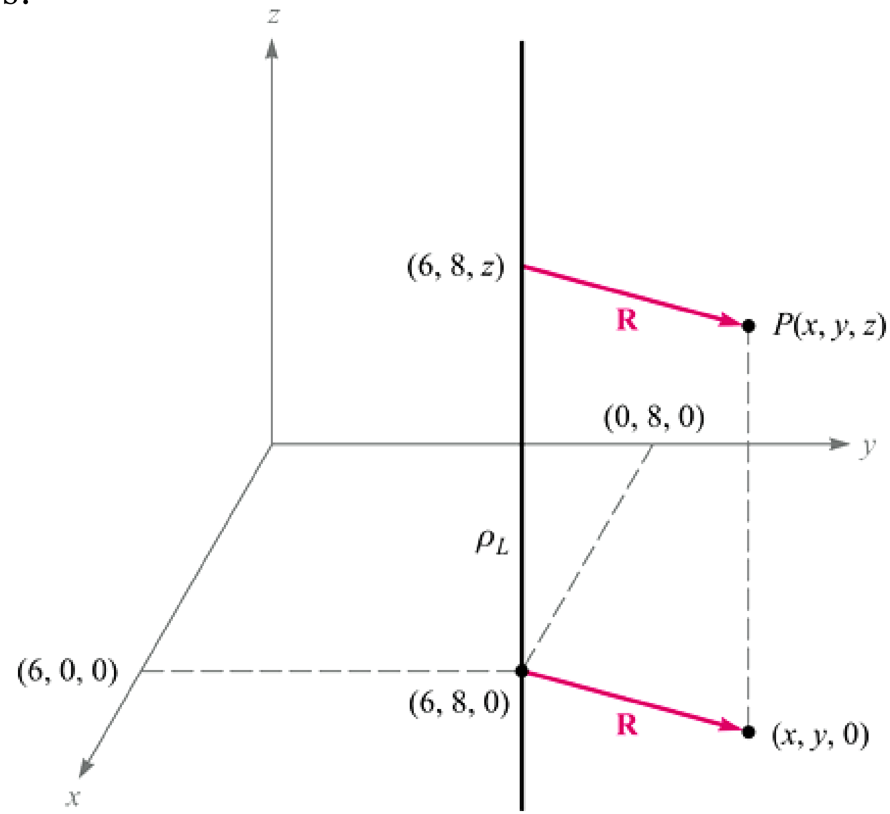
$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0\sqrt{(x-6)^2+(y-8)^2}}\mathbf{a}_R$$

where

$$\mathbf{a}_R = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{(x-6)\mathbf{a}_x + (y-8)\mathbf{a}_y}{\sqrt{(x-6)^2+(y-8)^2}}$$

Finally:

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0}\frac{(x-6)\mathbf{a}_x + (y-8)\mathbf{a}_y}{(x-6)^2+(y-8)^2}$$



Sample Questions

2.1. Three point charges are positioned in the x-y plane as follows: -5nC at $y = 5$ cm, -10 nC at $y = -5$ cm, 15 nC at $x = -5$ cm. Find the required x-y coordinates of a 20-nC fourth charge that will produce a zero electric field at the origin.

With the charges thus configured, the electric field at the origin will be the superposition of the individual charge fields:

$$\mathbf{E}_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{15}{(5)^2}\,\mathbf{a}_x - \frac{5}{(5)^2}\,\mathbf{a}_y - \frac{10}{(5)^2}\,\mathbf{a}_y\right] = \frac{1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[\mathbf{a}_x - \mathbf{a}_y] \quad \text{nC/m}$$

The field, $\mathbf{E}_{20}$, associated with the 20-nC charge (evaluated at the origin) must exactly cancel this field, so we write:

$$\mathbf{E}_{20} = \frac{-1}{4\pi\epsilon_0}\left(\frac{3}{5}\right)[\mathbf{a}_x - \mathbf{a}_y] = \frac{-20}{4\pi\epsilon_0 R^2}\left(\frac{1}{\sqrt{2}}\right)[\mathbf{a}_x - \mathbf{a}_y]$$

From this, we identify the distance from the origin: $R = \sqrt{100/(3\sqrt{2})} = 4.85$. The x and y coordinates of the 20-nC charge will both be equal in magnitude to $4.85/\sqrt{2} = 3.43$. The coodinates of the 20-nC charge are then $\underline{(3.43, -3.43)}$.

2.2. Point charges of 1nC and -2nC are located at (0,0,0) and (1,1,1), respectively, in free space. Determine the vector force acting on each charge.

First, the electric field intensity associated with the 1nC charge, evalutated at the -2nC charge location is:

$$\mathbf{E}_{12} = \frac{1}{4\pi\epsilon_0(3)}\left(\frac{1}{\sqrt{3}}\right)(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nC/m}$$

in which the distance between charges is $\sqrt{3}$ m. The force on the -2nC charge is then

$$\mathbf{F}_{12} = q_2\mathbf{E}_{12} = \frac{-2}{12\sqrt{3}\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) = \underline{\frac{-1}{10.4\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nN}}$$

The force on the 1nC charge at the origin is just the opposite of this result, or

$$\mathbf{F}_{21} = \underline{\frac{+1}{10.4\,\pi\epsilon_0}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z) \quad \text{nN}}$$

2.3. Point charges of 50nC each are located at $A(1,0,0)$, $B(-1,0,0)$, $C(0,1,0)$, and $D(0,-1,0)$ in free space. Find the total force on the charge at A.

The force will be:

$$\mathbf{F} = \frac{(50 \times 10^{-9})^2}{4\pi\epsilon_0}\left[\frac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|^3} + \frac{\mathbf{R}_{DA}}{|\mathbf{R}_{DA}|^3} + \frac{\mathbf{R}_{BA}}{|\mathbf{R}_{BA}|^3}\right]$$

where $\mathbf{R}_{CA} = \mathbf{a}_x - \mathbf{a}_y$, $\mathbf{R}_{DA} = \mathbf{a}_x + \mathbf{a}_y$, and $\mathbf{R}_{BA} = 2\mathbf{a}_x$. The magnitudes are $|\mathbf{R}_{CA}| = |\mathbf{R}_{DA}| = \sqrt{2}$, and $|\mathbf{R}_{BA}| = 2$. Substituting these leads to

$$\mathbf{F} = \frac{(50 \times 10^{-9})^2}{4\pi\epsilon_0}\left[\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{2}{8}\right]\mathbf{a}_x = \underline{21.5\mathbf{a}_x \ \mu\text{N}}$$

where distances are in meters.

2.6. Two point charges of equal magnitude q are positioned at $z = \pm d/2$.

a) find the electric field everywhere on the z axis: For a point charge at any location, we have

$$\mathbf{E} = \frac{q(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'|^3}$$

In the case of two charges, we would therefore have

$$\mathbf{E}_T = \frac{q_1(\mathbf{r} - \mathbf{r}'_1)}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'_1|^3} + \frac{q_2(\mathbf{r} - \mathbf{r}'_2)}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}'_2|^3} \tag{1}$$

In the present case, we assign $q_1 = q_2 = q$, the observation point position vector as $\mathbf{r} = z\mathbf{a}_z$, and the charge position vectors as $\mathbf{r}'_1 = (d/2)\mathbf{a}_z$, and $\mathbf{r}'_2 = -(d/2)\mathbf{a}_z$ Therefore

$$\mathbf{r} - \mathbf{r}'_1 = [z - (d/2)]\mathbf{a}_z, \quad \mathbf{r} - \mathbf{r}'_2 = [z + (d/2)]\mathbf{a}_z,$$

then

$$|\mathbf{r} - \mathbf{r}_1|^3 = [z - (d/2)]^3 \quad \text{and} \quad |\mathbf{r} - \mathbf{r}_2|^3 = [z + (d/2)]^3$$

Substitute these results into (1) to obtain:

$$\mathbf{E}_T(z) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{[z - (d/2)]^2} + \frac{1}{[z + (d/2)]^2}\right]\mathbf{a}_z \quad \text{V/m} \tag{2}$$

b) find the electric field everywhere on the x axis: We proceed as in part a, except that now $\mathbf{r} = x\mathbf{a}_x$. Eq. (1) becomes

$$\mathbf{E}_T(x) = \frac{q}{4\pi\epsilon_0}\left[\frac{x\mathbf{a}_x - (d/2)\mathbf{a}_z}{|x\mathbf{a}_x - (d/2)\mathbf{a}_z|^3} + \frac{x\mathbf{a}_x + (d/2)\mathbf{a}_z}{|x\mathbf{a}_x + (d/2)\mathbf{a}_z|^3}\right] \qquad (3)$$

where

$$|x\mathbf{a}_x - (d/2)\mathbf{a}_z| = |x\mathbf{a}_x + (d/2)\mathbf{a}_z| = \left[x^2 + (d/2)^2\right]^{1/2}$$

Therefore (3) becomes

$$\mathbf{E}_T(x) = \frac{2qx\,\mathbf{a}_x}{4\pi\epsilon_0\left[x^2 + (d/2)^2\right]^{3/2}}$$

c) repeat parts a and b if the charge at $z = -d/2$ is $-q$ instead of $+q$: The field along the z axis is quickly found by changing the sign of the second term in (2):

$$\mathbf{E}_T(z) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{[z-(d/2)]^2} - \frac{1}{[z+(d/2)]^2}\right]\mathbf{a}_z \quad \text{V/m}$$

In like manner, the field along the x axis is found from (3) by again changing the sign of the second term. The result is

$$\frac{-2qd\,\mathbf{a}_z}{4\pi\epsilon_0\left[x^2 + (d/2)^2\right]^{3/2}}$$

2.7. A 2 μC point charge is located at $A(4,3,5)$ in free space. Find E_ρ, E_ϕ, and E_z at $P(8,12,2)$. Have

$$\mathbf{E}_P = \frac{2\times10^{-6}}{4\pi\epsilon_0}\frac{\mathbf{R}_{AP}}{|\mathbf{R}_{AP}|^3} = \frac{2\times10^{-6}}{4\pi\epsilon_0}\left[\frac{4\mathbf{a}_x + 9\mathbf{a}_y - 3\mathbf{a}_z}{(106)^{1.5}}\right] = 65.9\mathbf{a}_x + 148.3\mathbf{a}_y - 49.4\mathbf{a}_z$$

Then, at point P, $\rho = \sqrt{8^2+12^2} = 14.4$, $\phi = \tan^{-1}(12/8) = 56.3^\circ$, and $z = z$. Now,

$$E_\rho = \mathbf{E}_p\cdot\mathbf{a}_\rho = 65.9(\mathbf{a}_x\cdot\mathbf{a}_\rho) + 148.3(\mathbf{a}_y\cdot\mathbf{a}_\rho) = 65.9\cos(56.3^\circ) + 148.3\sin(56.3^\circ) = \underline{159.7}$$

and

$$E_\phi = \mathbf{E}_p\cdot\mathbf{a}_\phi = 65.9(\mathbf{a}_x\cdot\mathbf{a}_\phi) + 148.3(\mathbf{a}_y\cdot\mathbf{a}_\phi) = -65.9\sin(56.3^\circ) + 148.3\cos(56.3^\circ) = \underline{27.4}$$

Finally, $E_z = \underline{-49.4 \text{ V/m}}$

2.10. A charge of -1 nC is located at the origin in free space. What charge must be located at (2,0,0) to cause E_x to be zero at (3,1,1)?

The field from two point charges is given generally by

$$\mathbf{E}_T = \frac{q_1(\mathbf{r}-\mathbf{r}_1')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_1'|^3} + \frac{q_2(\mathbf{r}-\mathbf{r}_2')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}_2'|^3} \qquad (1)$$

where we let $q_1 = -1\text{nC}$ and q_2 is to be found. With q_1 at the origin, $\mathbf{r}_1' = 0$. The position vector for q_2 is then $\mathbf{r}_2' = 2\mathbf{a}_x$. The observation point at (3,1,1) gives $\mathbf{r} = 3\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$. Eq. (1) becomes

$$\frac{1}{4\pi\epsilon_0}\left[\frac{-1(3\mathbf{a}_x+\mathbf{a}_y+\mathbf{a}_z)}{(3^2+1+1)^{3/2}} + \frac{q_2[(3-2)\mathbf{a}_x+\mathbf{a}_y+\mathbf{a}_z]}{(1+1+1)^{3/2}}\right]$$

Requiring the x component to be zero leads to

$$q_2 = \frac{3^{5/2}}{11^{3/2}} = \underline{0.43\text{ nC}}$$

2.13. A uniform volume charge density of 0.2 μC/m^3 is present throughout the spherical shell extending from $r = 3$ cm to $r = 5$ cm. If $\rho_v = 0$ elsewhere:

a) find the total charge present throughout the shell: This will be

$$Q = \int_0^{2\pi} \int_0^{\pi} \int_{.03}^{.05} 0.2\, r^2 \sin\theta \; dr\, d\theta\, d\phi = \left[4\pi(0.2)\frac{r^3}{3} \right]_{.03}^{.05} = 8.21 \times 10^{-5} \;\mu\text{C} = \underline{82.1 \text{ pC}}$$

b) find r_1 if half the total charge is located in the region $3\,\text{cm} < r < r_1$: If the integral over r in part a is taken to r_1, we would obtain

$$\left[4\pi(0.2)\frac{r^3}{3} \right]_{.03}^{r_1} = 4.105 \times 10^{-5}$$

Thus

$$r_1 = \left[\frac{3 \times 4.105 \times 10^{-5}}{0.2 \times 4\pi} + (.03)^3 \right]^{1/3} = \underline{4.24 \text{ cm}}$$