

Chapter 5

Time value of money

Present value

(القيمة الحالية)

Future value

(القيمة المستقبلية)

النقد المستقبلي \$10,000 إلى ربح أقيمهم بعد 5 سنين

قيمة اليوم أعلى

قيمة اليوم أعلى مع مرور الزمن

لأنه لو لم يكن لقيمة اليوم أعلى لكانت القيمة المستقبلية أعلى

* المصارف التي استثمرت بعد 5 سنين ← خسرت الإيراد

التي يمكن أن تحصل على مائة دولار اليوم

discounting present value لأننا نأخذ القيمة الحالية

⇒ We calculate the present value of a certain amount of money that is expected to be received at same future time.

أهم ما يتر
يكون المستحق
والقوام كمان

Future value : we calculate the future value for a certain amount of money that we have today (Compounding)

فائدة مركبة (مركبة إلى المرات)
 الفائدة بضيقها على آخر مبلغ (مستحق ال simple)

Simple interest

ما قبل
 ما بعد

$$PV = \$1000$$

$$r = 5\%$$

$$n = 2 \text{ years}$$

$$FV = ??$$

$$\Rightarrow r = 1000 \times 5\% \times 2 = \$100$$

$$FV = 1000 + 100 = \$1100$$

Compounded interest

$$PV = \$1000$$

$$r = 5\%$$

$$n = 2 \text{ years}$$

$$FV = ??$$

$$\Rightarrow r = 5\% \times 1000 = 50$$

after 1 year = $1000 + 50 = 1050$

after year 2 :

$$r = 5\% \times 1050 = \$52.5$$

من 1000 (simple) إلى القيمة إلى 1050
 مع آخر سنة إلى 1050 إلى 1102.5 (1000)

$$\Rightarrow FV = 1000 + 52.5 = \cancel{1100} \$1102.5$$

$$(1050 + \cancel{50} 52.5)$$

* Basic cash flow patterns:

- ① Single amount
- ② Annuity = equal cash flows (could be in/out) - equal amount of money.
- ③ Mixed stream = unequal cash flows.

I Calculating future value of a single amount

↓
Compounding of interest

قلم

$$FV = PV (1+r)^n$$

future value present value rate of return # of periods

$$S = P(1+r)^t$$

e.g (Page 215)

$$PV = \$800$$

$$r = 6\% \text{ compounded annually.}$$

$$n = 5 \text{ years}$$

$$FV = ??$$

The higher

(n, r)

The higher FV

$$FV = PV (1+r)^n$$

$$= 800 (1+0.06)^5$$

$$= \cancel{\$1,070.58} \quad \$1,070.58$$

2] Calculating the present value of single amount

$$FV = PV (1+r)^n \Rightarrow PV = \frac{FV}{(1+r)^n}$$

⇒ The higher the (n, r) the longer the lower the present value

eg Page 217: $FV = \$300$ $n = 1$ year

$r = 6\%$

- discount rate
- interest rate
- opportunity cost
- rate of return

$PV = ??$

$$PV = \frac{FV}{(1+r)^n}$$

Timeline diagram showing a cash flow of \$300 at time 1 (Today).

$$PV = \frac{300}{(1.06)^1} = \$283.02$$

3] Calculating future of an annuity:

Annuity (equal cash flows)

ordinary annuity.

annuity due.

في نهاية كل فترة

في بداية كل فترة

assumes that cash inflows/outflows happen at the end of each period.

assumes that cash inflows/outflows happen at the beginning of each period.

2

eg (page 221)

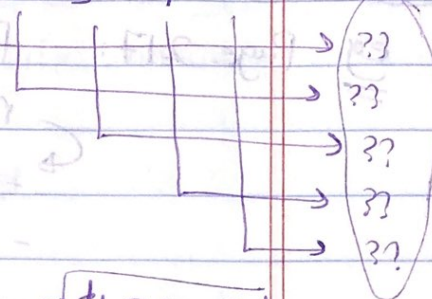
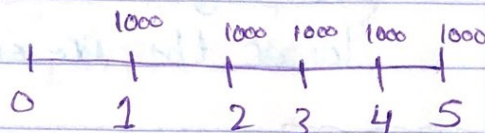
$$CF_1 = \$1000 \text{ / year.}$$

ordinary annuity

$n = 5$ years.

$$r = 7\%$$

Future Value of Annuity $\leftarrow FVA = ?$



$$\textcircled{2} FV = PV (1+r)^n$$
$$= 1000 (1+0.07)^4 = \$1,310.8$$

$$(2) FV = 1000 (1 + 0.07)^3 = \$1,225.04$$

③ $FV = 1000 (1 + 0.07)^2 = \$1,144.9$

④ $FV = 1000 (1 + 0.07)^1 = \1070

⑤ $FV = 1000 (1 + 0.07)^0 = \1000

مثلاً ، 1000 في نهاية السنة الخامسة ، 1000 في

$$\Rightarrow FVA = 1,310.8 + 1225.04 + 1144.9$$

$$= \$5750,74$$

Dec 10, 2019
Tuesday

$$* FVA = CF \left(\frac{(1+r)^n - 1}{r} \right)$$

future value of an ordinary annuity

 cash flow (in/out)

 required of return interest rate

eg page 221 : $CF = \$1000 / \text{year}$

$n = 5$ years

$r = 7\%$

كلمة مثال المحاضرة، الماضية

بِسْ مَخْصَرَةٍ بِهَذَا لَقَاوُنَا

(FVA)

$$FVA = CF \left(\frac{(1+r)^n - 1}{r} \right) = 1000 \left(\frac{(1+7\%)^5 - 1}{7\%} \right)$$

$$= \$5,750.74$$

لاحظْ أُخْرَى: نفس الجواب (٢)

[4] Calculating present value of an ordinary annuity :

* Example page 223: at the end of the year $\Rightarrow$ ordinary
لأننا في نهاية الفترة يسكنه

$$CF = \$700 / \text{year}$$

$n = 5$ years

$$r = 8\%$$

$\Rightarrow PVA = ?$

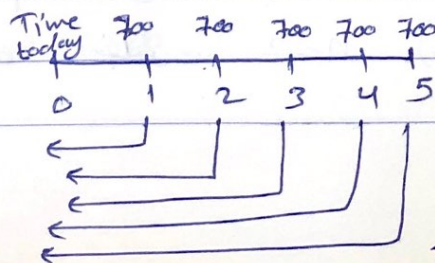
Present value of an annuity

منرجع ال time زيرو

عسکان بہارا

QV

\$ 648.15
\$ 600.14
\$ 555.68
\$ 514.52
\$ 476.41



$$P_v = \frac{F_v}{(1+r)^n}$$

(single amount)

$$PVA = \text{مجموع} = \$2,794.90$$

$$PVA = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^n} \right)$$

$$= \frac{700}{0.08} \left(1 - \frac{1}{(1+0.08)^5} \right) = \$2,794.90$$

نفس الجواب

④ Annuity ordinary → يكون بنهاية فترة معينة

Annuity due → يكون بالبداية

يكون متأخرًا

extra one period. (في حالة ال future value)

e.g : CF = \$100 \setminus \text{year (at the beginning of each year)}

n = 3 years

r = 5%

FVA due = ?? : (extra one period)

$$① FV = PV (1+r)^n = 100 (1+5\%)^3 = \$115.76$$

$$② FV = 100 (1+5\%)^2 = \$110.25$$

$$③ FV = 100 (1+5\%)^1 = ~~\$105~~ \$105$$

$$FVA_{\text{due}} = ① + ② + ③ = \$331.01$$

(n=3 سنة) n=2 سنة ordinary

$$FVA_{due} = CF \left(\frac{(1+r)^n - 1}{r} \right) (1+r)$$

FVA Ordinary مادي مضاف

$$= 100 \left(\frac{(1+5\%)^5 - 1}{5\%} \right) (1+5\%) = \$331.01$$

(نفس الجواب)

6 Calculating present value of annuity due :

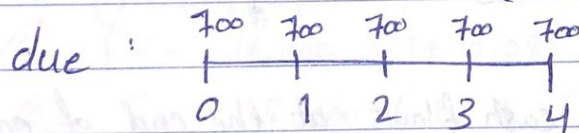
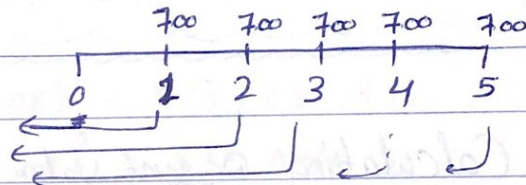
eg page 226 : CF = \$700/year (at the beginning of each year)

$$r = 8\%$$

$$n = 5 \text{ years}$$

PVA_{due} ?

⇒ Ordinary annuity :



PV : PV of annuity due

less one period

FV: extra 1 period

$$① PV = \frac{FV}{(1+r)^n} = \frac{700}{(1+8\%)^0} = \$700$$

$$② PV = \frac{700}{(1+8\%)^1} = \$648.15$$

$$③ PV = \frac{700}{(1+8\%)^2} = \$600.14$$

$$④ \quad PV = \frac{700}{(1+8\%)^3} = \$555.68$$

$$⑤ \quad PV = \frac{700}{(1+8\%)^4} = \$514.52$$

$$PVA_{due} = ① + ② + ③ + ④ = \$3,018.49$$

بالقانون : $PVA_{due} = \frac{CF}{r} \left(1+r \right) \left(1 - \frac{1}{(1+r)^n} \right)$

$$= \frac{700}{0.08} (1+0.08) \left(1 - \frac{1}{(1+8\%)^5} \right) = \$3,018.49$$

ن
يعودونها
زي الى
السؤال
(ما بعد)

إذا كانت $n = \infty$ بقدر أعيد ال PV بين ما بقدر أعيد FV

7 Calculating present value of a perpetuity:

* perpetuity = equal cash flows at the end of each year. $n = \infty$

$$PV = \frac{CF}{r}$$

e.g. Preferred stock

$$CF = D = \$100$$

$$r = 15\%$$

$$n = \infty$$

stock price today = ??

لما استنجا

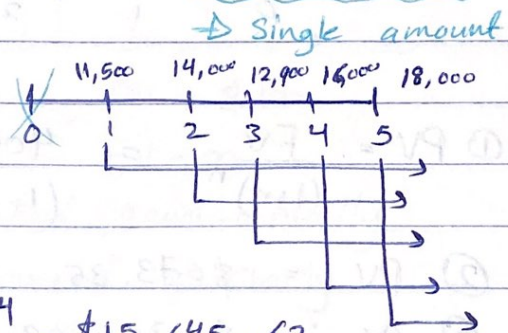
$$PV = ??$$

$$\Rightarrow PV = \frac{CF}{r} = \frac{100}{0.15} = \$666.67$$

التفسير (من التوزيع (القيمة 50)) :

8 Calculating future value of a mixed stream:

eg / p.229 : ~~XXXX~~



$$\textcircled{1} FV = PV (1+r)^n$$

(n = 4)

$$= 11,500 (1 + 8\%)^4 = \$15,645.62$$

$$\textcircled{2} FV = 14,000 (1 + 0.08)^3 = \$17,635.97$$

$$\textcircled{3} FV = 12,900 (1 + 0.08)^2 = \$15,048.56$$

$$\textcircled{4} FV = 16,000 (1 + 0.08)^1 = \$17,280$$

$$\textcircled{5} FV = 18,000 (1 + 0.08)^0 = \$18,000$$

$$\Rightarrow FV = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5} = \boxed{\$83,602.15}$$

End of the year $\rightarrow$ المكان السؤال $\rightarrow$ الأرقام إلى معطيات لينا : كذا ال

بداية السنة $\rightarrow$ ال 0

إذا مكاننا بال Beginning of the year $\rightarrow$ مرجع رجوع (منه) $n=5$ لهذا $n=1$

9 Calculating Present value of a mixed stream:

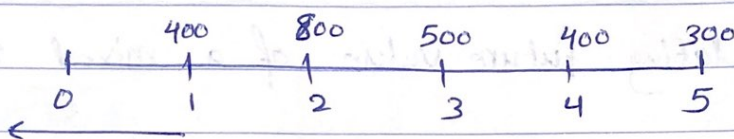
eg: Page 230

$r = 9\%$

$n = 5 \text{ years}$

End of the year

PV = ??



$$\textcircled{1} PV = \frac{FV}{(1+r)^n} = \frac{400}{(1+9\%)^1} = \$366.97$$

$$\textcircled{2} PV = \$673.35$$

$$\textcircled{3} PV = \$386.09 \Rightarrow PV = \textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5}$$

$$\textcircled{4} PV = \$283.37 = \$1904.76$$

$$\textcircled{5} PV = \$194.98$$

مراجعة
القوانين
Dec 10, 19

* Compounding interest more frequently than annually:

Before calculating future value or present value or present value some modifications are needed to be made:

② If interest was compounded quarterly, then ~~then~~

$$\frac{r}{4}, n \times 4 \Rightarrow (4n)$$

① If interest was compounded semi-annually, then

$$\frac{r}{2}, n \times 2 \Rightarrow (2n)$$

③ If interest was compounded monthly, then:

$$\frac{r}{12}, n \times 12 \Rightarrow (12n)$$

④ If interest was compounded weekly, then:

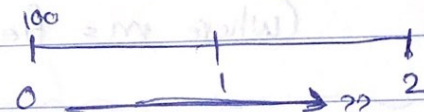
$$\frac{r}{52}, n \times 52 \Rightarrow (52n)$$

* Note: Started = r .

eg page 233: $n = 2$ year . $PV = \$100$
 $r = 8\%$. $FV = ??$

a- If interest was compounded semi-annually

b- If interest was compounded quarterly.

[a]: $\frac{r}{2} = \frac{8\%}{2} = 4\%$ 
 $m = 2$ (عدد الفترات)

$$n = 2 \times 2 = 4 \text{ periods}$$

$$\Rightarrow FV = 100 (1 + 4\%)^4 = \$116.99$$

[b]: $\frac{r}{4} = \frac{8\%}{4} = 2\%$

$$n = 4 \times 2 = 8 \text{ periods}$$

$$\Rightarrow FV = 100 (1 + 2\%)^8 = \$117.7$$

كلما زاد الـ Frequency $\Rightarrow$ زاد الـ FV
 ($117.7 > 116.99$)
 quarterly $\downarrow$ semi-annually $\downarrow$
 لأن تراكب الفوائد يزداد

* Nominal interest rate versus effective annual rate:

① Nominal interest rate = stated interest rate
= annual interest rate

$r = 8\%$ = annual interest rate
= stated interest rate

② Effective annual rate = interest rate
↓ actually paid

$$\Rightarrow \text{EAR} = \left(1 + \frac{r}{m}\right)^m - 1$$

(where m = frequency).

e.g. page 238: $\text{EAR} = ??$

$r = 8\%$ = nominal interest rate.

a - when interest is compounded annually?

b - " " " " semi-annually?

c - " " " " quarterly?

$$\textcircled{a} \text{ EAR} = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{8\%}{1}\right)^1 - 1 = \boxed{8\%}$$

$\therefore \Rightarrow$ If interest was compounded annually ($m=1$), then
 $\text{EAR} = \text{nominal interest rate}.$

- (also 8% $\Rightarrow$ is 8%)

$$b) \text{ EAR} = \left(1 + \frac{8\%}{2}\right)^2 - 1 = \boxed{8.16\%}$$

$$c) \text{ EAR} = \left(1 + \frac{8\%}{4}\right)^4 - 1 = \boxed{8.24\%}$$

* Special applications of time value of money:

* Loan ~~amortization~~ amortization schedule.

eg Page 241

Loan amount = \$6,000

$r = 10\%$

$n = 4$ years

$\text{PMT} = ??$

in equal payment

→ Payment cash flow.

(دفعة سنوية) : annuity

$$\text{PVA} = \frac{\text{CF}}{r} \left(1 - \frac{1}{(1+r)^n}\right)$$

$$\$6,000 = \frac{\text{CF}}{0.1} \left(1 - \frac{1}{(1+0.1)^4}\right) \Rightarrow \text{CF} = \$1892.82$$

PMT

Loan PMT = interest + principal

المبلغ الذي سددته = Loan PMT - interest

Year	Beg. Bal	loan Payment	interest (10%)	Principal	End. Balance
1	\$6,000	\$18,92.82	\$600	\$1292.82	\$4,707.18
2	\$4,707.18	\$1892.82	\$470.72	\$1,422.1	\$3,285.08
3	\$3,285.08	\$1892.82	\$328.51	\$1,564.31	\$1,720.77
4	\$1,720.77	\$1892.82	\$172.08	\$1,720.74	- 0 -

دفعة سنوية

لا يتبقى شيء ولا يتبقى

ملاحظة: ~~ملاحظة~~ مقاسين و ~~ملاحظة~~ سابق :

① Interest = Beg. Balance $\times$ interest rate.

② principal = $\frac{(\text{loan}) - \text{interest}}{\text{PMT}}$

③ End. Balance = Beg. Balance - principal.

④ ال End. Bal. ينقل ال Beg. Bal في السنة التالية .

* أي زيادة في ال Assets تعبر cash outflow

تفسير P4-18 :

⑤ expenses = fixed + variable
exp. are not all variable

⑥ \$180,000 \$230,625 \$285,500
overstated understated of
of Profit profit.

بالإضافة
التي
التي
التي

\$167,325 \$295,400 \$383,640