

Rotation

10-1 ROTATIONAL VARIABLES

Learning Objectives

After reading this module, you should be able to . . .

- 10.01** Identify that if all parts of a body rotate around a fixed axis locked together, the body is a rigid body. (This chapter is about the motion of such bodies.)
- 10.02** Identify that the angular position of a rotating rigid body is the angle that an internal reference line makes with a fixed, external reference line.
- 10.03** Apply the relationship between angular displacement and the initial and final angular positions.
- 10.04** Apply the relationship between average angular velocity, angular displacement, and the time interval for that displacement.
- 10.05** Apply the relationship between average angular acceleration, change in angular velocity, and the time interval for that change.
- 10.06** Identify that counterclockwise motion is in the positive direction and clockwise motion is in the negative direction.
- 10.07** Given angular position as a *function of time*, calculate the instantaneous angular velocity at any particular time and the average angular velocity between any two particular times.

- 10.08** Given a *graph* of angular position versus time, determine the instantaneous angular velocity at a particular time and the average angular velocity between any two particular times.
- 10.09** Identify instantaneous angular speed as the magnitude of the instantaneous angular velocity.
- 10.10** Given angular velocity as a *function of time*, calculate the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.
- 10.11** Given a *graph* of angular velocity versus time, determine the instantaneous angular acceleration at any particular time and the average angular acceleration between any two particular times.
- 10.12** Calculate a body's change in angular velocity by integrating its angular acceleration function with respect to time.
- 10.13** Calculate a body's change in angular position by integrating its angular velocity function with respect to time.

Key Ideas

- To describe the rotation of a rigid body about a fixed axis, called the rotation axis, we assume a reference line is fixed in the body, perpendicular to that axis and rotating with the body. We measure the angular position θ of this line relative to a fixed direction. When θ is measured in radians,

$$\theta = \frac{s}{r} \quad (\text{radian measure}),$$

where s is the arc length of a circular path of radius r and angle θ .

- Radian measure is related to angle measure in revolutions and degrees by

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}.$$

- A body that rotates about a rotation axis, changing its angular position from θ_1 to θ_2 , undergoes an angular displacement

$$\Delta\theta = \theta_2 - \theta_1,$$

where $\Delta\theta$ is positive for counterclockwise rotation and negative for clockwise rotation.

- If a body rotates through an angular displacement $\Delta\theta$ in a time interval Δt , its average angular velocity ω_{avg} is

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}.$$

The (instantaneous) angular velocity ω of the body is

$$\omega = \frac{d\theta}{dt}.$$

Both ω_{avg} and ω are vectors, with directions given by a right-hand rule. They are positive for counterclockwise rotation and negative for clockwise rotation. The magnitude of the body's angular velocity is the angular speed.

- If the angular velocity of a body changes from ω_1 to ω_2 in a time interval $\Delta t = t_2 - t_1$, the average angular acceleration α_{avg} of the body is

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}.$$

The (instantaneous) angular acceleration α of the body is

$$\alpha = \frac{d\omega}{dt}.$$

Both α_{avg} and α are vectors.

What Is Physics?

As we have discussed, one focus of physics is motion. However, so far we have examined only the motion of **translation**, in which an object moves along a straight or curved line, as in Fig. 10-1*a*. We now turn to the motion of **rotation**, in which an object turns about an axis, as in Fig. 10-1*b*.

You see rotation in nearly every machine, you use it every time you open a beverage can with a pull tab, and you pay to experience it every time you go to an amusement park. Rotation is the key to many fun activities, such as hitting a long drive in golf (the ball needs to rotate in order for the air to keep it aloft longer) and throwing a curveball in baseball (the ball needs to rotate in order for the air to push it left or right). Rotation is also the key to more serious matters, such as metal failure in aging airplanes.

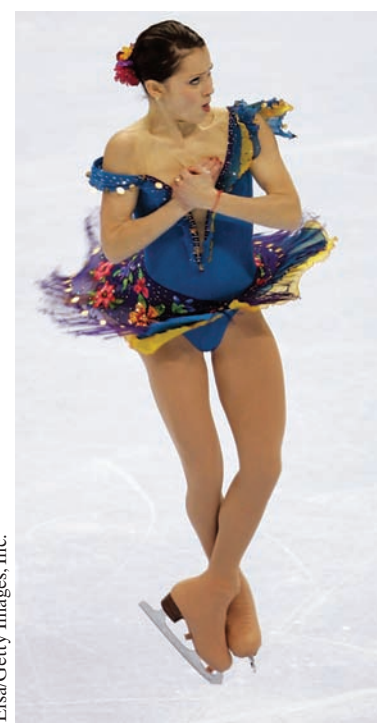
We begin our discussion of rotation by defining the variables for the motion, just as we did for translation in Chapter 2. As we shall see, the variables for rotation are analogous to those for one-dimensional motion and, as in Chapter 2, an important special situation is where the acceleration (here the rotational acceleration) is constant. We shall also see that Newton's second law can be written for rotational motion, but we must use a new quantity called *torque* instead of just force. Work and the work-kinetic energy theorem can also be applied to rotational motion, but we must use a new quantity called *rotational inertia* instead of just mass. In short, much of what we have discussed so far can be applied to rotational motion with, perhaps, a few changes.

Caution: In spite of this repetition of physics ideas, many students find this and the next chapter very challenging. Instructors have a variety of reasons as to why, but two reasons stand out: (1) There are a lot of symbols (with Greek



Mike Segar/Reuters/Landov LLC

(a)



Elsa/Getty Images, Inc.

(b)

Figure 10-1 Figure skater Sasha Cohen in motion of (a) pure translation in a fixed direction and (b) pure rotation about a vertical axis.

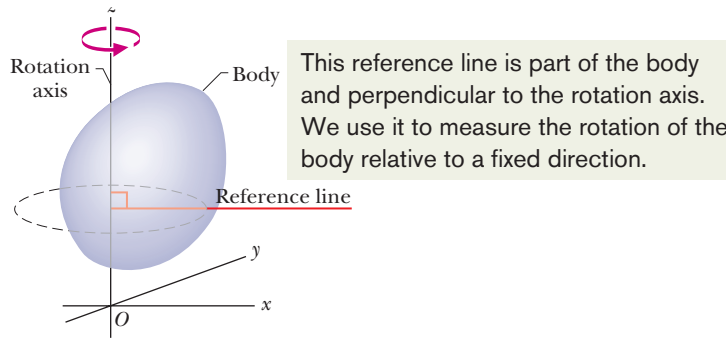


Figure 10-2 A rigid body of arbitrary shape in pure rotation about the z axis of a coordinate system. The position of the *reference line* with respect to the rigid body is arbitrary, but it is perpendicular to the rotation axis. It is fixed in the body and rotates with the body.

letters) to sort out. (2) Although you are very familiar with linear motion (you can get across the room and down the road just fine), you are probably very unfamiliar with rotation (and that is one reason why you are willing to pay so much for amusement park rides). If a homework problem looks like a foreign language to you, see if translating it into the one-dimensional linear motion of Chapter 2 helps. For example, if you are to find, say, an *angular* distance, temporarily delete the word *angular* and see if you can work the problem with the Chapter 2 notation and ideas.

Rotational Variables

We wish to examine the rotation of a rigid body about a fixed axis. A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape. A **fixed axis** means that the rotation occurs about an axis that does not move. Thus, we shall not examine an object like the Sun, because the parts of the Sun (a ball of gas) are not locked together. We also shall not examine an object like a bowling ball rolling along a lane, because the ball rotates about a moving axis (the ball's motion is a mixture of rotation and translation).

Figure 10-2 shows a rigid body of arbitrary shape in rotation about a fixed axis, called the **axis of rotation** or the **rotation axis**. In pure rotation (*angular motion*), every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval. In pure translation (*linear motion*), every point of the body moves in a straight line, and every point moves through the same *linear distance* during a particular time interval.

We deal now—one at a time—with the angular equivalents of the linear quantities position, displacement, velocity, and acceleration.

Angular Position

Figure 10-2 shows a *reference line*, fixed in the body, perpendicular to the rotation axis and rotating with the body. The **angular position** of this line is the angle of the line relative to a fixed direction, which we take as the **zero angular position**. In Fig. 10-3, the angular position θ is measured relative to the positive direction of the x axis. From geometry, we know that θ is given by

$$\theta = \frac{s}{r} \quad (\text{radian measure}). \quad (10-1)$$

Here s is the length of a circular arc that extends from the x axis (the zero angular position) to the reference line, and r is the radius of the circle.

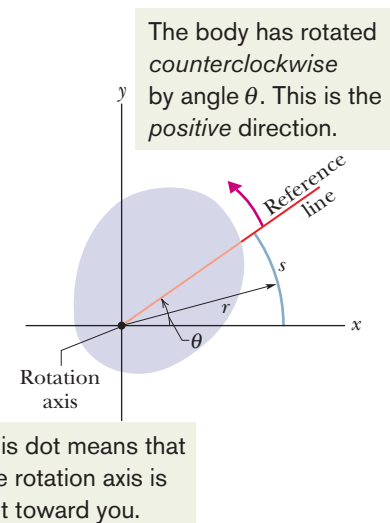


Figure 10-3 The rotating rigid body of Fig. 10-2 in cross section, viewed from above. The plane of the cross section is perpendicular to the rotation axis, which now extends out of the page, toward you. In this position of the body, the reference line makes an angle θ with the x axis.

An angle defined in this way is measured in **radians** (rad) rather than in revolutions (rev) or degrees. The radian, being the ratio of two lengths, is a pure number and thus has no dimension. Because the circumference of a circle of radius r is $2\pi r$, there are 2π radians in a complete circle:

$$1 \text{ rev} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}, \quad (10-2)$$

and thus

$$1 \text{ rad} = 57.3^\circ = 0.159 \text{ rev}. \quad (10-3)$$

We do *not* reset θ to zero with each complete rotation of the reference line about the rotation axis. If the reference line completes two revolutions from the zero angular position, then the angular position θ of the line is $\theta = 4\pi$ rad.

For pure translation along an x axis, we can know all there is to know about a moving body if we know $x(t)$, its position as a function of time. Similarly, for pure rotation, we can know all there is to know about a rotating body if we know $\theta(t)$, the angular position of the body's reference line as a function of time.

Angular Displacement

If the body of Fig. 10-3 rotates about the rotation axis as in Fig. 10-4, changing the angular position of the reference line from θ_1 to θ_2 , the body undergoes an **angular displacement** $\Delta\theta$ given by

$$\Delta\theta = \theta_2 - \theta_1. \quad (10-4)$$

This definition of angular displacement holds not only for the rigid body as a whole but also for *every particle within that body*.

Clocks Are Negative. If a body is in translational motion along an x axis, its displacement Δx is either positive or negative, depending on whether the body is moving in the positive or negative direction of the axis. Similarly, the angular displacement $\Delta\theta$ of a rotating body is either positive or negative, according to the following rule:



An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.

The phrase “*clocks are negative*” can help you remember this rule (they certainly are negative when their alarms sound off early in the morning).



Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a negative angular displacement: (a) -3 rad, $+5$ rad, (b) -3 rad, -7 rad, (c) 7 rad, -3 rad?

Angular Velocity

Suppose that our rotating body is at angular position θ_1 at time t_1 and at angular position θ_2 at time t_2 as in Fig. 10-4. We define the **average angular velocity** of the body in the time interval Δt from t_1 to t_2 to be

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \quad (10-5)$$

where $\Delta\theta$ is the angular displacement during Δt (ω is the lowercase omega).

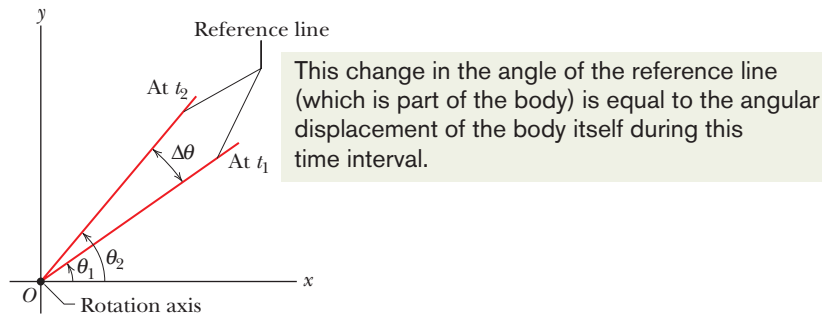


Figure 10-4 The reference line of the rigid body of Figs. 10-2 and 10-3 is at angular position θ_1 at time t_1 and at angular position θ_2 at a later time t_2 . The quantity $\Delta\theta (= \theta_2 - \theta_1)$ is the angular displacement that occurs during the interval $\Delta t (= t_2 - t_1)$. The body itself is not shown.

The **(instantaneous) angular velocity** ω , with which we shall be most concerned, is the limit of the ratio in Eq. 10-5 as Δt approaches zero. Thus,

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \quad (10-6)$$

If we know $\theta(t)$, we can find the angular velocity ω by differentiation.

Equations 10-5 and 10-6 hold not only for the rotating rigid body as a whole but also for *every particle of that body* because the particles are all locked together. The unit of angular velocity is commonly the radian per second (rad/s) or the revolution per second (rev/s). Another measure of angular velocity was used during at least the first three decades of rock: Music was produced by vinyl (phonograph) records that were played on turntables at “33 $\frac{1}{3}$ rpm” or “45 rpm,” meaning at 33 $\frac{1}{3}$ rev/min or 45 rev/min.

If a particle moves in translation along an x axis, its linear velocity v is either positive or negative, depending on its direction along the axis. Similarly, the angular velocity ω of a rotating rigid body is either positive or negative, depending on whether the body is rotating counterclockwise (positive) or clockwise (negative). (“Clocks are negative” still works.) The magnitude of an angular velocity is called the **angular speed**, which is also represented with ω .

Angular Acceleration

If the angular velocity of a rotating body is not constant, then the body has an angular acceleration. Let ω_2 and ω_1 be its angular velocities at times t_2 and t_1 , respectively. The **average angular acceleration** of the rotating body in the interval from t_1 to t_2 is defined as

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}, \quad (10-7)$$

in which $\Delta\omega$ is the change in the angular velocity that occurs during the time interval Δt . The **(instantaneous) angular acceleration** α , with which we shall be most concerned, is the limit of this quantity as Δt approaches zero. Thus,

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}. \quad (10-8)$$

As the name suggests, this is the angular acceleration of the body at a given instant. Equations 10-7 and 10-8 also hold for *every particle of that body*. The unit of angular acceleration is commonly the radian per second-squared (rad/s²) or the revolution per second-squared (rev/s²).



Sample Problem 10.01 Angular velocity derived from angular position

The disk in Fig. 10-5a is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by

$$\theta = -1.00 - 0.600t + 0.250t^2, \quad (10-9)$$

with t in seconds, θ in radians, and the zero angular position as indicated in the figure. (If you like, you can translate all this into Chapter 2 notation by momentarily dropping the word “angular” from “angular position” and replacing the symbol θ with the symbol x . What you then have is an equation that gives the position as a function of time, for the one-dimensional motion of Chapter 2.)

(a) Graph the angular position of the disk versus time from $t = -3.0$ s to $t = 5.4$ s. Sketch the disk and its angular position reference line at $t = -2.0$ s, 0 s, and 4.0 s, and when the curve crosses the t axis.

KEY IDEA

The angular position of the disk is the angular position $\theta(t)$ of its reference line, which is given by Eq. 10-9 as a function of time t . So we graph Eq. 10-9; the result is shown in Fig. 10-5b.

Calculations: To sketch the disk and its reference line at a particular time, we need to determine θ for that time. To do so, we substitute the time into Eq. 10-9. For $t = -2.0$ s, we get

$$\begin{aligned} \theta &= -1.00 - (0.600)(-2.0) + (0.250)(-2.0)^2 \\ &= 1.2 \text{ rad} = 1.2 \text{ rad} \frac{360^\circ}{2\pi \text{ rad}} = 69^\circ. \end{aligned}$$

This means that at $t = -2.0$ s the reference line on the disk is rotated counterclockwise from the zero position by angle $1.2 \text{ rad} = 69^\circ$ (counterclockwise because θ is positive). Sketch 1 in Fig. 10-5b shows this position of the reference line.

Similarly, for $t = 0$, we find $\theta = -1.00 \text{ rad} = -57^\circ$, which means that the reference line is rotated clockwise from the zero angular position by 1.0 rad , or 57° , as shown in sketch 3. For $t = 4.0$ s, we find $\theta = 0.60 \text{ rad} = 34^\circ$ (sketch 5). Drawing sketches for when the curve crosses the t axis is easy, because then $\theta = 0$ and the reference line is momentarily aligned with the zero angular position (sketches 2 and 4).

(b) At what time $t_{\min}$ does $\theta(t)$ reach the minimum value shown in Fig. 10-5b? What is that minimum value?

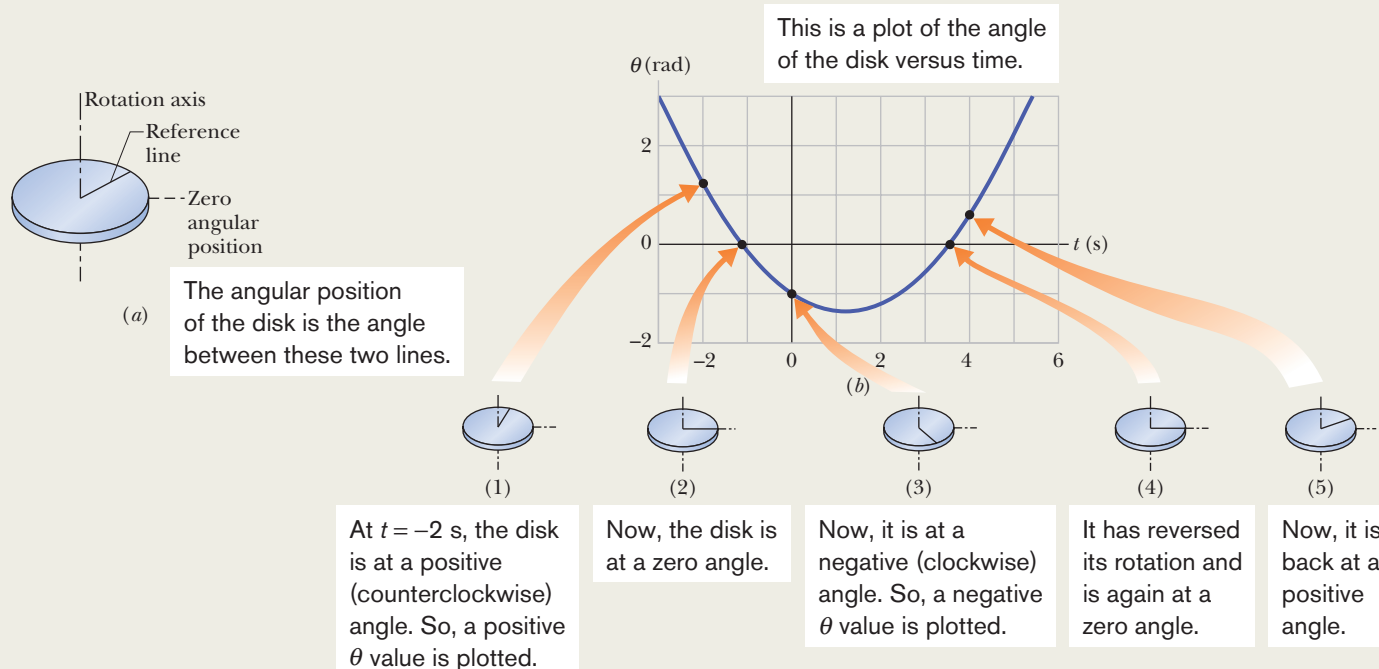


Figure 10-5 (a) A rotating disk. (b) A plot of the disk's angular position $\theta(t)$. Five sketches indicate the angular position of the reference line on the disk for five points on the curve. (c) A plot of the disk's angular velocity $\omega(t)$. Positive values of ω correspond to counterclockwise rotation, and negative values to clockwise rotation.

KEY IDEA

To find the extreme value (here the minimum) of a function, we take the first derivative of the function and set the result to zero.

Calculations: The first derivative of $\theta(t)$ is

$$\frac{d\theta}{dt} = -0.600 + 0.500t. \quad (10-10)$$

Setting this to zero and solving for t give us the time at which $\theta(t)$ is minimum:

$$t_{\min} = 1.20 \text{ s.} \quad (\text{Answer})$$

To get the minimum value of θ , we next substitute $t_{\min}$ into Eq. 10-9, finding

$$\theta = -1.36 \text{ rad} \approx -77.9^\circ. \quad (\text{Answer})$$

This *minimum* of $\theta(t)$ (the bottom of the curve in Fig. 10-5b) corresponds to the *maximum clockwise* rotation of the disk from the zero angular position, somewhat more than is shown in sketch 3.

(c) Graph the angular velocity ω of the disk versus time from

$t = -3.0 \text{ s}$ to $t = 6.0 \text{ s}$. Sketch the disk and indicate the direction of turning and the sign of ω at $t = -2.0 \text{ s}$, 4.0 s , and $t_{\min}$.

KEY IDEA

From Eq. 10-6, the angular velocity ω is equal to $d\theta/dt$ as given in Eq. 10-10. So, we have

$$\omega = -0.600 + 0.500t. \quad (10-11)$$

The graph of this function $\omega(t)$ is shown in Fig. 10-5c. Because the function is linear, the plot is a straight line. The slope is 0.500 rad/s^2 and the intercept with the vertical axis (not shown) is -0.600 rad/s .

Calculations: To sketch the disk at $t = -2.0 \text{ s}$, we substitute that value into Eq. 10-11, obtaining

$$\omega = -1.6 \text{ rad/s.} \quad (\text{Answer})$$

The minus sign here tells us that at $t = -2.0 \text{ s}$, the disk is turning clockwise (as indicated by the left-hand sketch in Fig. 10-5c).

Substituting $t = 4.0 \text{ s}$ into Eq. 10-11 gives us

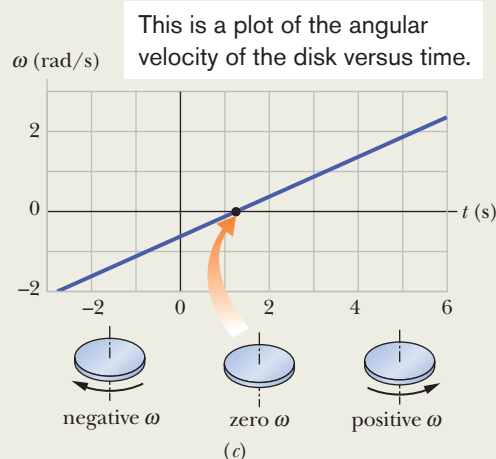
$$\omega = 1.4 \text{ rad/s.} \quad (\text{Answer})$$

The implied plus sign tells us that now the disk is turning counterclockwise (the right-hand sketch in Fig. 10-5c).

For $t_{\min}$, we already know that $d\theta/dt = 0$. So, we must also have $\omega = 0$. That is, the disk momentarily stops when the reference line reaches the minimum value of θ in Fig. 10-5b, as suggested by the center sketch in Fig. 10-5c. On the graph of ω versus t in Fig. 10-5c, this momentary stop is the zero point where the plot changes from the negative clockwise motion to the positive counterclockwise motion.

(d) Use the results in parts (a) through (c) to describe the motion of the disk from $t = -3.0 \text{ s}$ to $t = 6.0 \text{ s}$.

Description: When we first observe the disk at $t = -3.0 \text{ s}$, it has a positive angular position and is turning clockwise but slowing. It stops at angular position $\theta = -1.36 \text{ rad}$ and then begins to turn counterclockwise, with its angular position eventually becoming positive again.



The angular velocity is initially negative and slowing, then momentarily zero during reversal, and then positive and increasing.

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Sample Problem 10.02 Angular velocity derived from angular acceleration

A child's top is spun with angular acceleration

$$\alpha = 5t^3 - 4t,$$

with t in seconds and α in radians per second-squared. At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top. That is, find an expression that explicitly indicates how the angular velocity depends on time. (We can tell that there is such a dependence because the top is undergoing an angular acceleration, which means that its angular velocity is changing.)

KEY IDEA

By definition, $\alpha(t)$ is the derivative of $\omega(t)$ with respect to time. Thus, we can find $\omega(t)$ by integrating $\alpha(t)$ with respect to time.

Calculations: Equation 10-8 tells us

$$d\omega = \alpha dt,$$

so

$$\int d\omega = \int \alpha dt.$$

From this we find

$$\omega = \int (5t^3 - 4t) dt = \frac{5}{4}t^4 - \frac{4}{2}t^2 + C.$$

To evaluate the constant of integration C , we note that $\omega = 5$ rad/s at $t = 0$. Substituting these values in our expression for ω yields

$$5 \text{ rad/s} = 0 - 0 + C,$$

so $C = 5$ rad/s. Then

$$\omega = \frac{5}{4}t^4 - 2t^2 + 5. \quad (\text{Answer})$$

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

KEY IDEA

By definition, $\omega(t)$ is the derivative of $\theta(t)$ with respect to time. Therefore, we can find $\theta(t)$ by integrating $\omega(t)$ with respect to time.

Calculations: Since Eq. 10-6 tells us that

$$d\theta = \omega dt,$$

we can write

$$\begin{aligned} \theta &= \int \omega dt = \int \left(\frac{5}{4}t^4 - 2t^2 + 5 \right) dt \\ &= \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + C' \\ &= \frac{1}{4}t^5 - \frac{2}{3}t^3 + 5t + 2, \end{aligned} \quad (\text{Answer})$$

where C' has been evaluated by noting that $\theta = 2$ rad at $t = 0$.



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Are Angular Quantities Vectors?

We can describe the position, velocity, and acceleration of a single particle by means of vectors. If the particle is confined to a straight line, however, we do not really need vector notation. Such a particle has only two directions available to it, and we can indicate these directions with plus and minus signs.

In the same way, a rigid body rotating about a fixed axis can rotate only clockwise or counterclockwise as seen along the axis, and again we can select between the two directions by means of plus and minus signs. The question arises: “Can we treat the angular displacement, velocity, and acceleration of a rotating body as vectors?” The answer is a qualified “yes” (see the caution below, in connection with angular displacements).

Angular Velocities. Consider the angular velocity. Figure 10-6a shows a vinyl record rotating on a turntable. The record has a constant angular speed $\omega (= 33\frac{1}{3} \text{ rev/min})$ in the clockwise direction. We can represent its angular velocity as a vector $\vec{\omega}$ pointing along the axis of rotation, as in Fig. 10-6b. Here's how: We choose the length of this vector according to some convenient scale, for example, with 1 cm corresponding to 10 rev/min. Then we establish a direction for the vector $\vec{\omega}$ by using a **right-hand rule**, as Fig. 10-6c shows: Curl your right hand about the rotating record, your fingers pointing *in the direction of rotation*. Your extended thumb will then point in the direction of the angular velocity vector. If the record were to rotate in the opposite sense, the right-

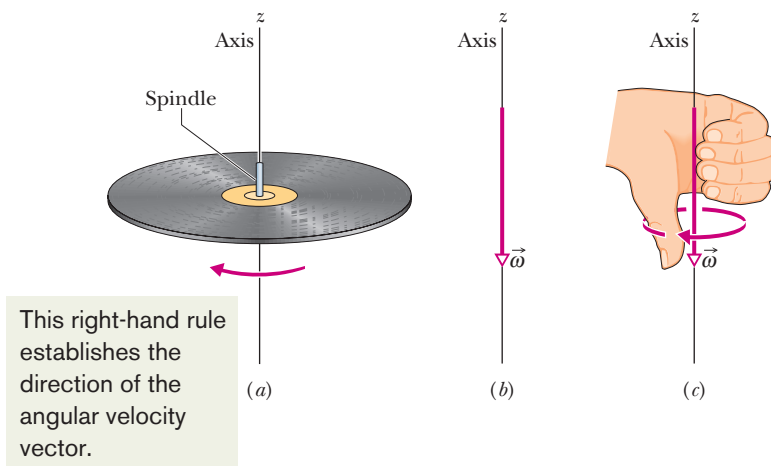


Figure 10-6 (a) A record rotating about a vertical axis that coincides with the axis of the spindle. (b) The angular velocity of the rotating record can be represented by the vector $\vec{\omega}$, lying along the axis and pointing down, as shown. (c) We establish the direction of the angular velocity vector as downward by using a right-hand rule. When the fingers of the right hand curl around the record and point the way it is moving, the extended thumb points in the direction of $\vec{\omega}$.

hand rule would tell you that the angular velocity vector then points in the opposite direction.

It is not easy to get used to representing angular quantities as vectors. We instinctively expect that something should be moving *along* the direction of a vector. That is not the case here. Instead, something (the rigid body) is rotating *around* the direction of the vector. In the world of pure rotation, a vector defines an axis of rotation, not a direction in which something moves. Nonetheless, the vector also defines the motion. Furthermore, it obeys all the rules for vector manipulation discussed in Chapter 3. The angular acceleration $\vec{\alpha}$ is another vector, and it too obeys those rules.

In this chapter we consider only rotations that are about a fixed axis. For such situations, we need not consider vectors—we can represent angular velocity with ω and angular acceleration with α , and we can indicate direction with an implied plus sign for counterclockwise or an explicit minus sign for clockwise.

Angular Displacements. Now for the caution: Angular *displacements* (unless they are very small) *cannot* be treated as vectors. Why not? We can certainly give them both magnitude and direction, as we did for the angular velocity vector in Fig. 10-6. However, to be represented as a vector, a quantity must *also* obey the rules of vector addition, one of which says that if you add two vectors, the order in which you add them does not matter. Angular displacements fail this test.

Figure 10-7 gives an example. An initially horizontal book is given two 90° angular displacements, first in the order of Fig. 10-7a and then in the order of Fig. 10-7b. Although the two angular displacements are identical, their order is not, and the book ends up with different orientations. Here's another example. Hold your right arm downward, palm toward your thigh. Keeping your wrist rigid, (1) lift the arm forward until it is horizontal, (2) move it horizontally until it points toward the right, and (3) then bring it down to your side. Your palm faces forward. If you start over, but reverse the steps, which way does your palm end up facing? From either example, we must conclude that the addition of two angular displacements depends on their order and they cannot be vectors.

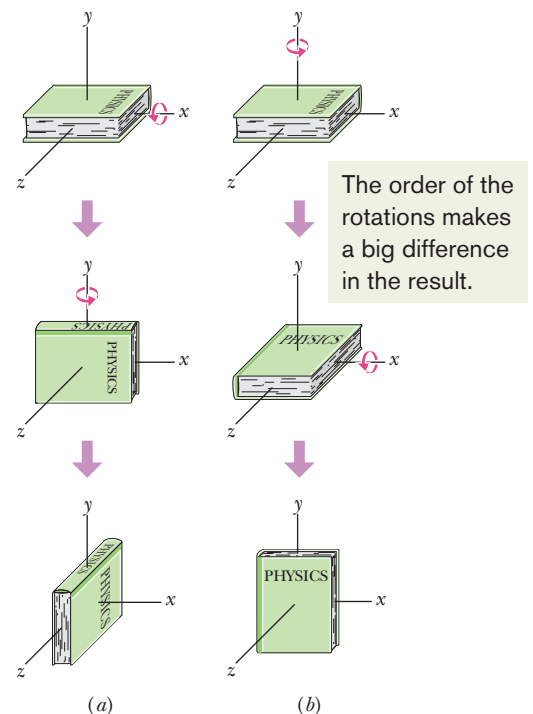


Figure 10-7 (a) From its initial position, at the top, the book is given two successive 90° rotations, first about the (horizontal) x axis and then about the (vertical) y axis. (b) The book is given the same rotations, but in the reverse order.

10-2 ROTATION WITH CONSTANT ANGULAR ACCELERATION

Learning Objective

After reading this module, you should be able to . . .

10.14 For constant angular acceleration, apply the relationships between angular position, angular displacement, angular velocity, angular acceleration, and elapsed time (Table 10-1).

Key Idea

● Constant angular acceleration ($\alpha = \text{constant}$) is an important special case of rotational motion. The appropriate kinematic equations are

$$\begin{aligned}\omega &= \omega_0 + \alpha t, \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2} \alpha t^2, \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0), \\ \theta - \theta_0 &= \frac{1}{2}(\omega_0 + \omega)t, \\ \theta - \theta_0 &= \omega t - \frac{1}{2} \alpha t^2.\end{aligned}$$

Rotation with Constant Angular Acceleration

In pure translation, motion with a *constant linear acceleration* (for example, that of a falling body) is an important special case. In Table 2-1, we displayed a series of equations that hold for such motion.

In pure rotation, the case of *constant angular acceleration* is also important, and a parallel set of equations holds for this case also. We shall not derive them here, but simply write them from the corresponding linear equations, substituting equivalent angular quantities for the linear ones. This is done in Table 10-1, which lists both sets of equations (Eqs. 2-11 and 2-15 to 2-18; 10-12 to 10-16).

Recall that Eqs. 2-11 and 2-15 are basic equations for constant linear acceleration—the other equations in the Linear list can be derived from them. Similarly, Eqs. 10-12 and 10-13 are the basic equations for constant angular acceleration, and the other equations in the Angular list can be derived from them. To solve a simple problem involving constant angular acceleration, you can usually use an equation from the Angular list (*if you have the list*). Choose an equation for which the only unknown variable will be the variable requested in the problem. A better plan is to remember only Eqs. 10-12 and 10-13, and then solve them as simultaneous equations whenever needed.



Checkpoint 2

In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = 2/t^2 - 4/t$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations of Table 10-1 apply?

Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable	Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2} at^2$	v	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2} at^2$	v_0	$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$	(10-16)

Sample Problem 10.03 Constant angular acceleration, grindstone

A grindstone (Fig. 10-8) rotates at constant angular acceleration $\alpha = 0.35 \text{ rad/s}^2$. At time $t = 0$, it has an angular velocity of $\omega_0 = -4.6 \text{ rad/s}$ and a reference line on it is horizontal, at the angular position $\theta_0 = 0$.

(a) At what time after $t = 0$ is the reference line at the angular position $\theta = 5.0 \text{ rev}$?

KEY IDEA

The angular acceleration is constant, so we can use the rotation equations of Table 10-1. We choose Eq. 10-13,

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2,$$

because the only unknown variable it contains is the desired time t .

Calculations: Substituting known values and setting $\theta_0 = 0$ and $\theta = 5.0 \text{ rev} = 10\pi \text{ rad}$ give us

$$10\pi \text{ rad} = (-4.6 \text{ rad/s})t + \frac{1}{2}(0.35 \text{ rad/s}^2)t^2.$$

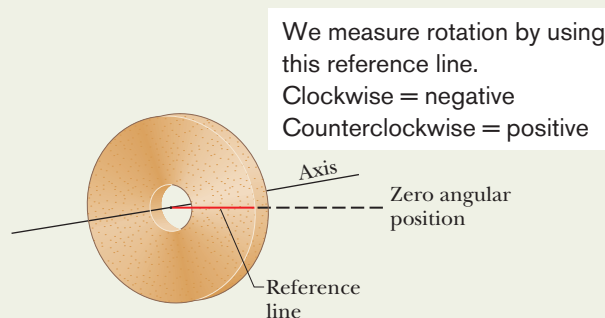


Figure 10-8 A grindstone. At $t = 0$ the reference line (which we imagine to be marked on the stone) is horizontal.

(We converted 5.0 rev to $10\pi \text{ rad}$ to keep the units consistent.) Solving this quadratic equation for t , we find

$$t = 32 \text{ s.} \quad (\text{Answer})$$

Now notice something a bit strange. We first see the wheel when it is rotating in the negative direction and through the $\theta = 0$ orientation. Yet, we just found out that 32 s later it is at the positive orientation of $\theta = 5.0 \text{ rev}$. What happened in that time interval so that it could be at a positive orientation?

(b) Describe the grindstone's rotation between $t = 0$ and $t = 32 \text{ s}$.

Description: The wheel is initially rotating in the negative (clockwise) direction with angular velocity $\omega_0 = -4.6 \text{ rad/s}$, but its angular acceleration α is positive. This initial opposition of the signs of angular velocity and angular acceleration means that the wheel slows in its rotation in the negative direction, stops, and then reverses to rotate in the positive direction. After the reference line comes back through its initial orientation of $\theta = 0$, the wheel turns an additional 5.0 rev by time $t = 32 \text{ s}$.

(c) At what time t does the grindstone momentarily stop?

Calculation: We again go to the table of equations for constant angular acceleration, and again we need an equation that contains only the desired unknown variable t . However, now the equation must also contain the variable ω , so that we can set it to 0 and then solve for the corresponding time t . We choose Eq. 10-12, which yields

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6 \text{ rad/s})}{0.35 \text{ rad/s}^2} = 13 \text{ s.} \quad (\text{Answer})$$

Sample Problem 10.04 Constant angular acceleration, riding a Rotor

While you are operating a Rotor (a large, vertical, rotating cylinder found in amusement parks), you spot a passenger in acute distress and decrease the angular velocity of the cylinder from 3.40 rad/s to 2.00 rad/s in 20.0 rev , at constant angular acceleration. (The passenger is obviously more of a “translation person” than a “rotation person.”)

(a) What is the constant angular acceleration during this decrease in angular speed?

KEY IDEA

Because the cylinder's angular acceleration is constant, we can relate it to the angular velocity and angular displacement via the basic equations for constant angular acceleration (Eqs. 10-12 and 10-13).

Calculations: Let's first do a quick check to see if we can solve the basic equations. The initial angular velocity is $\omega_0 = 3.40$

rad/s , the angular displacement is $\theta - \theta_0 = 20.0 \text{ rev}$, and the angular velocity at the end of that displacement is $\omega = 2.00 \text{ rad/s}$. In addition to the angular acceleration α that we want, both basic equations also contain time t , which we do not necessarily want.

To eliminate the unknown t , we use Eq. 10-12 to write

$$t = \frac{\omega - \omega_0}{\alpha},$$

which we then substitute into Eq. 10-13 to write

$$\theta - \theta_0 = \omega_0 \left(\frac{\omega - \omega_0}{\alpha} \right) + \frac{1}{2} \alpha \left(\frac{\omega - \omega_0}{\alpha} \right)^2.$$

Solving for α , substituting known data, and converting 20 rev to 125.7 rad , we find

$$\begin{aligned} \alpha &= \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} = \frac{(2.00 \text{ rad/s})^2 - (3.40 \text{ rad/s})^2}{2(125.7 \text{ rad})} \\ &= -0.0301 \text{ rad/s}^2. \end{aligned} \quad (\text{Answer})$$

(b) How much time did the speed decrease take?

Calculation: Now that we know α , we can use Eq. 10-12 to solve for t :

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{2.00 \text{ rad/s} - 3.40 \text{ rad/s}}{-0.0301 \text{ rad/s}^2} = 46.5 \text{ s.} \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS

10-3 RELATING THE LINEAR AND ANGULAR VARIABLES

Learning Objectives

After reading this module, you should be able to . . .

10.15 For a rigid body rotating about a fixed axis, relate the angular variables of the body (angular position, angular velocity, and angular acceleration) and the linear variables of a particle on the body (position, velocity, and acceleration) at any given radius.

10.16 Distinguish between tangential acceleration and radial acceleration, and draw a vector for each in a sketch of a particle on a body rotating about an axis, for both an increase in angular speed and a decrease.

Key Ideas

● A point in a rigid rotating body, at a perpendicular distance r from the rotation axis, moves in a circle with radius r . If the body rotates through an angle θ , the point moves along an arc with length s given by

$$s = \theta r \quad (\text{radian measure}),$$

where θ is in radians.

● The linear velocity $\vec{v}$ of the point is tangent to the circle; the point's linear speed v is given by

$$v = \omega r \quad (\text{radian measure}),$$

where ω is the angular speed (in radians per second) of the body, and thus also the point.

● The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The tangential component is

$$a_t = \alpha r \quad (\text{radian measure}),$$

where α is the magnitude of the angular acceleration (in radians per second-squared) of the body. The radial component of $\vec{a}$ is

$$a_r = \frac{v^2}{r} = \omega^2 r \quad (\text{radian measure}).$$

● If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \quad (\text{radian measure}).$$

Relating the Linear and Angular Variables

In Module 4-5, we discussed uniform circular motion, in which a particle travels at constant linear speed v along a circle and around an axis of rotation. When a rigid body, such as a merry-go-round, rotates around an axis, each particle in the body moves in its own circle around that axis. Since the body is rigid, all the particles make one revolution in the same amount of time; that is, they all have the same angular speed ω .

However, the farther a particle is from the axis, the greater the circumference of its circle is, and so the faster its linear speed v must be. You can notice this on a merry-go-round. You turn with the same angular speed ω regardless of your distance from the center, but your linear speed v increases noticeably if you move to the outside edge of the merry-go-round.

We often need to relate the linear variables s , v , and a for a particular point in a rotating body to the angular variables θ , ω , and α for that body. The two sets of variables are related by r , the *perpendicular distance* of the point from the rotation axis. This perpendicular distance is the distance between the point and the rotation axis, measured along a perpendicular to the axis. It is also the radius r of the circle traveled by the point around the axis of rotation.

The Position

If a reference line on a rigid body rotates through an angle θ , a point within the body at a position r from the rotation axis moves a distance s along a circular arc, where s is given by Eq. 10-1:

$$s = \theta r \quad (\text{radian measure}). \quad (10-17)$$

This is the first of our linear–angular relations. **Caution:** The angle θ here must be measured in radians because Eq. 10-17 is itself the definition of angular measure in radians.

The Speed

Differentiating Eq. 10-17 with respect to time—with r held constant—leads to

$$\frac{ds}{dt} = \frac{d\theta}{dt} r.$$

However, ds/dt is the linear speed (the magnitude of the linear velocity) of the point in question, and $d\theta/dt$ is the angular speed ω of the rotating body. So

$$v = \omega r \quad (\text{radian measure}). \quad (10-18)$$

Caution: The angular speed ω must be expressed in radian measure.

Equation 10-18 tells us that since all points within the rigid body have the same angular speed ω , points with greater radius r have greater linear speed v . Figure 10-9a reminds us that the linear velocity is always tangent to the circular path of the point in question.

If the angular speed ω of the rigid body is constant, then Eq. 10-18 tells us that the linear speed v of any point within it is also constant. Thus, each point within the body undergoes uniform circular motion. The period of revolution T for the motion of each point and for the rigid body itself is given by Eq. 4-35:

$$T = \frac{2\pi r}{v}. \quad (10-19)$$

This equation tells us that the time for one revolution is the distance $2\pi r$ traveled in one revolution divided by the speed at which that distance is traveled. Substituting for v from Eq. 10-18 and canceling r , we find also that

$$T = \frac{2\pi}{\omega} \quad (\text{radian measure}). \quad (10-20)$$

This equivalent equation says that the time for one revolution is the angular distance 2π rad traveled in one revolution divided by the angular speed (or rate) at which that angle is traveled.

The Acceleration

Differentiating Eq. 10-18 with respect to time—again with r held constant—leads to

$$\frac{dv}{dt} = \frac{d\omega}{dt} r. \quad (10-21)$$

Here we run up against a complication. In Eq. 10-21, dv/dt represents only the part of the linear acceleration that is responsible for changes in the *magnitude* v of the linear velocity $\vec{v}$. Like $\vec{v}$, that part of the linear acceleration is tangent to the path of the point in question. We call it the *tangential component* a_t of the linear acceleration of the point, and we write

$$a_t = \alpha r \quad (\text{radian measure}), \quad (10-22)$$

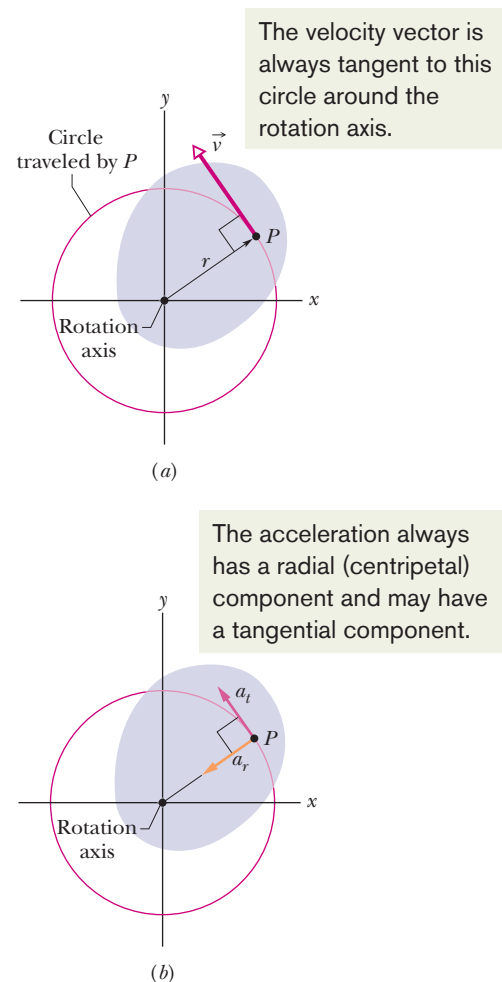


Figure 10-9 The rotating rigid body of Fig. 10-2, shown in cross section viewed from above. Every point of the body (such as P) moves in a circle around the rotation axis. (a) The linear velocity $\vec{v}$ of every point is tangent to the circle in which the point moves. (b) The linear acceleration $\vec{a}$ of the point has (in general) two components: tangential a_t and radial a_r .

where $\alpha = d\omega/dt$. *Caution:* The angular acceleration α in Eq. 10-22 must be expressed in radian measure.

In addition, as Eq. 4-34 tells us, a particle (or point) moving in a circular path has a *radial component* of linear acceleration, $a_r = v^2/r$ (directed radially inward), that is responsible for changes in the *direction* of the linear velocity $\vec{v}$. By substituting for v from Eq. 10-18, we can write this component as

$$a_r = \frac{v^2}{r} = \omega^2 r \quad (\text{radian measure}). \quad (10-23)$$

Thus, as Fig. 10-9b shows, the linear acceleration of a point on a rotating rigid body has, in general, two components. The radially inward component a_r (given by Eq. 10-23) is present whenever the angular velocity of the body is not zero. The tangential component a_t (given by Eq. 10-22) is present whenever the angular acceleration is not zero.



Checkpoint 3

A cockroach rides the rim of a rotating merry-go-round. If the angular speed of this system (*merry-go-round + cockroach*) is constant, does the cockroach have (a) radial acceleration and (b) tangential acceleration? If ω is decreasing, does the cockroach have (c) radial acceleration and (d) tangential acceleration?



Sample Problem 10.05 Designing The Giant Ring, a large-scale amusement park ride

We are given the job of designing a large horizontal ring that will rotate around a vertical axis and that will have a radius of $r = 33.1$ m (matching that of Beijing's The Great Observation Wheel, the largest Ferris wheel in the world). Passengers will enter through a door in the outer wall of the ring and then stand next to that wall (Fig. 10-10a). We decide that for the time interval $t = 0$ to $t = 2.30$ s, the angular position $\theta(t)$ of a reference line on the ring will be given by

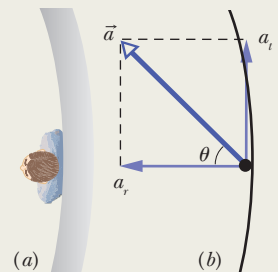
$$\theta = ct^3, \quad (10-24)$$

with $c = 6.39 \times 10^{-2} \text{ rad/s}^3$. After $t = 2.30$ s, the angular speed will be held constant until the end of the ride. Once the ring begins to rotate, the floor of the ring will drop away from the riders but the riders will not fall—indeed, they feel as though they are pinned to the wall. For the time $t = 2.20$ s, let's determine a rider's angular speed ω , linear speed v , angular acceleration α , tangential acceleration a_t , radial acceleration a_r , and acceleration $\vec{a}$.

KEY IDEAS

(1) The angular speed ω is given by Eq. 10-6 ($\omega = d\theta/dt$). (2) The linear speed v (along the circular path) is related to the angular speed (around the rotation axis) by Eq. 10-18 ($v = \omega r$). (3) The angular acceleration α is given by Eq. 10-8 ($\alpha = d\omega/dt$). (4) The tangential acceleration a_t (along the circular path) is related to the angular acceleration (around the rotation axis) by Eq. 10-22 ($a_t = \alpha r$). (5) The radial acceleration a_r is given Eq. 10-23 ($a_r = \omega^2 r$). (6) The tangential

Figure 10-10 (a) Overhead view of a passenger ready to ride The Giant Ring. (b) The radial and tangential acceleration components of the (full) acceleration.



and radial accelerations are the (perpendicular) components of the (full) acceleration $\vec{a}$.

Calculations: Let's go through the steps. We first find the angular velocity by taking the time derivative of the given angular position function and then substituting the given time of $t = 2.20$ s:

$$\begin{aligned} \omega &= \frac{d\theta}{dt} = \frac{d}{dt}(ct^3) = 3ct^2 \\ &= 3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^2 \\ &= 0.928 \text{ rad/s.} \end{aligned} \quad (10-25) \quad (\text{Answer})$$

From Eq. 10-18, the linear speed just then is

$$\begin{aligned} v &= \omega r = 3ct^2 r \\ &= 3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^2(33.1 \text{ m}) \\ &= 30.7 \text{ m/s.} \end{aligned} \quad (10-26) \quad (\text{Answer})$$

Although this is fast (111 km/h or 68.7 mi/h), such speeds are common in amusement parks and not alarming because (as mentioned in Chapter 2) your body reacts to accelerations but not to velocities. (It is an accelerometer, not a speedometer.) From Eq. 10-26 we see that the linear speed is increasing as the square of the time (but this increase will cut off at $t = 2.30$ s).

Next, let's tackle the angular acceleration by taking the time derivative of Eq. 10-25:

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(3ct^2) = 6ct$$

$$= 6(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s}) = 0.843 \text{ rad/s}^2. \quad (\text{Answer})$$

The tangential acceleration then follows from Eq. 10-22:

$$a_t = ar = 6ctr \quad (10-27)$$

$$= 6(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})(33.1 \text{ m})$$

$$= 27.91 \text{ m/s}^2 \approx 27.9 \text{ m/s}^2, \quad (\text{Answer})$$

or 2.8g (which is reasonable and a bit exciting). Equation 10-27 tells us that the tangential acceleration is increasing with time (but it will cut off at $t = 2.30$ s). From Eq. 10-23, we write the radial acceleration as

$$a_r = \omega^2 r.$$

Substituting from Eq. 10-25 leads us to

$$a_r = (3ct^2)^2 r = 9c^2 t^4 r \quad (10-28)$$

$$= 9(6.39 \times 10^{-2} \text{ rad/s}^3)^2 (2.20 \text{ s})^4 (33.1 \text{ m})$$

$$= 28.49 \text{ m/s}^2 \approx 28.5 \text{ m/s}^2, \quad (\text{Answer})$$

or 2.9g (which is also reasonable and a bit exciting).

The radial and tangential accelerations are perpendicular to each other and form the components of the rider's acceleration $\vec{a}$ (Fig. 10-10b). The magnitude of $\vec{a}$ is given by

$$a = \sqrt{a_r^2 + a_t^2} \quad (10-29)$$

$$= \sqrt{(28.49 \text{ m/s}^2)^2 + (27.91 \text{ m/s}^2)^2}$$

$$\approx 39.9 \text{ m/s}^2, \quad (\text{Answer})$$

or 4.1g (which is really exciting!). All these values are acceptable.

To find the orientation of $\vec{a}$, we can calculate the angle θ shown in Fig. 10-10b:

$$\tan \theta = \frac{a_t}{a_r}.$$

However, instead of substituting our numerical results, let's use the algebraic results from Eqs. 10-27 and 10-28:

$$\theta = \tan^{-1} \left(\frac{6ctr}{9c^2 t^4 r} \right) = \tan^{-1} \left(\frac{2}{3ct^3} \right). \quad (10-30)$$

The big advantage of solving for the angle algebraically is that we can then see that the angle (1) does not depend on the ring's radius and (2) decreases as t goes from 0 to 2.20 s. That is, the acceleration vector $\vec{a}$ swings toward being radially inward because the radial acceleration (which depends on t^4) quickly dominates over the tangential acceleration (which depends on only t). At our given time $t = 2.20$ s, we have

$$\theta = \tan^{-1} \frac{2}{3(6.39 \times 10^{-2} \text{ rad/s}^3)(2.20 \text{ s})^3} = 44.4^\circ. \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS



10-4 KINETIC ENERGY OF ROTATION

Learning Objectives

After reading this module, you should be able to . . .

- 10.17** Find the rotational inertia of a particle about a point.
10.18 Find the total rotational inertia of many particles moving around the same fixed axis.

- 10.19** Calculate the rotational kinetic energy of a body in terms of its rotational inertia and its angular speed.

Key Idea

- The kinetic energy K of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2} I \omega^2 \quad (\text{radian measure}),$$

in which I is the rotational inertia of the body, defined as

$$I = \sum m_i r_i^2$$

for a system of discrete particles.

Kinetic Energy of Rotation

The rapidly rotating blade of a table saw certainly has kinetic energy due to that rotation. How can we express the energy? We cannot apply the familiar formula $K = \frac{1}{2}mv^2$ to the saw as a whole because that would give us the kinetic energy only of the saw's center of mass, which is zero.

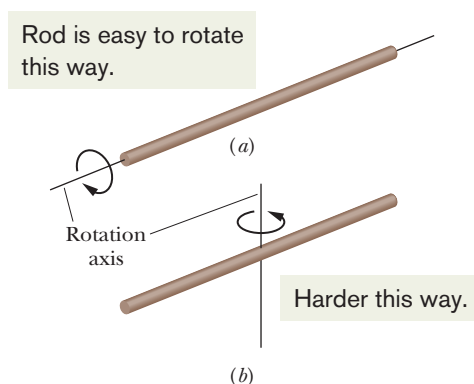


Figure 10-11 A long rod is much easier to rotate about (a) its central (longitudinal) axis than about (b) an axis through its center and perpendicular to its length. The reason for the difference is that the mass is distributed closer to the rotation axis in (a) than in (b).

Instead, we shall treat the table saw (and any other rotating rigid body) as a collection of particles with different speeds. We can then add up the kinetic energies of all the particles to find the kinetic energy of the body as a whole. In this way we obtain, for the kinetic energy of a rotating body,

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \cdots = \sum \frac{1}{2}m_i v_i^2, \quad (10-31)$$

in which m_i is the mass of the i th particle and v_i is its speed. The sum is taken over all the particles in the body.

The problem with Eq. 10-31 is that v_i is not the same for all particles. We solve this problem by substituting for v from Eq. 10-18 ($v = \omega r$), so that we have

$$K = \sum \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2, \quad (10-32)$$

in which ω is the same for all particles.

The quantity in parentheses on the right side of Eq. 10-32 tells us how the mass of the rotating body is distributed about its axis of rotation. We call that quantity the **rotational inertia** (or **moment of inertia**) I of the body with respect to the axis of rotation. It is a constant for a particular rigid body and a particular rotation axis. (*Caution:* That axis must always be specified if the value of I is to be meaningful.)

We may now write

$$I = \sum m_i r_i^2 \quad (\text{rotational inertia}) \quad (10-33)$$

and substitute into Eq. 10-32, obtaining

$$K = \frac{1}{2}I\omega^2 \quad (\text{radian measure}) \quad (10-34)$$

as the expression we seek. Because we have used the relation $v = \omega r$ in deriving Eq. 10-34, ω must be expressed in radian measure. The SI unit for I is the kilogram-square meter ($\text{kg} \cdot \text{m}^2$).

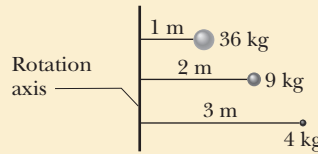
The Plan. If we have a few particles and a specified rotation axis, we find mr^2 for each particle and then add the results as in Eq. 10-33 to get the total rotational inertia I . If we want the total rotational kinetic energy, we can then substitute that I into Eq. 10-34. That is the plan for a few particles, but suppose we have a huge number of particles such as in a rod. In the next module we shall see how to handle such *continuous bodies* and do the calculation in only a few minutes.

Equation 10-34, which gives the kinetic energy of a rigid body in pure rotation, is the angular equivalent of the formula $K = \frac{1}{2}Mv_{\text{com}}^2$, which gives the kinetic energy of a rigid body in pure translation. In both formulas there is a factor of $\frac{1}{2}$. Where mass M appears in one equation, I (which involves both mass and its distribution) appears in the other. Finally, each equation contains as a factor the square of a speed—translational or rotational as appropriate. The kinetic energies of translation and of rotation are not different kinds of energy. They are both kinetic energy, expressed in ways that are appropriate to the motion at hand.

We noted previously that the rotational inertia of a rotating body involves not only its mass but also how that mass is distributed. Here is an example that you can literally feel. Rotate a long, fairly heavy rod (a pole, a length of lumber, or something similar), first around its central (longitudinal) axis (Fig. 10-11a) and then around an axis perpendicular to the rod and through the center (Fig. 10-11b). Both rotations involve the very same mass, but the first rotation is much easier than the second. The reason is that the mass is distributed much closer to the rotation axis in the first rotation. As a result, the rotational inertia of the rod is much smaller in Fig. 10-11a than in Fig. 10-11b. In general, smaller rotational inertia means easier rotation.

**Checkpoint 4**

The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.



10-5 CALCULATING THE ROTATIONAL INERTIA

Learning Objectives

After reading this module, you should be able to . . .

10.20 Determine the rotational inertia of a body if it is given in Table 10-2.

10.21 Calculate the rotational inertia of a body by integration over the mass elements of the body.

10.22 Apply the parallel-axis theorem for a rotation axis that is displaced from a parallel axis through the center of mass of a body.

Key Ideas

- I is the rotational inertia of the body, defined as

$$I = \sum m_i r_i^2$$

for a system of discrete particles and defined as

$$I = \int r^2 dm$$

for a body with continuously distributed mass. The r and r_i in these expressions represent the perpendicular distance from the axis of rotation to each mass element in the body, and the integration is carried out over the entire body so as to include every mass element.

- The parallel-axis theorem relates the rotational inertia I of a body about any axis to that of the same body about a parallel axis through the center of mass:

$$I = I_{\text{com}} + Mh^2.$$

Here h is the perpendicular distance between the two axes, and I_{com} is the rotational inertia of the body about the axis through the com. We can describe h as being the distance the actual rotation axis has been shifted from the rotation axis through the com.

Calculating the Rotational Inertia

If a rigid body consists of a few particles, we can calculate its rotational inertia about a given rotation axis with Eq. 10-33 ($I = \sum m_i r_i^2$); that is, we can find the product mr^2 for each particle and then sum the products. (Recall that r is the perpendicular distance a particle is from the given rotation axis.)

If a rigid body consists of a great many adjacent particles (it is *continuous*, like a Frisbee), using Eq. 10-33 would require a computer. Thus, instead, we replace the sum in Eq. 10-33 with an integral and define the rotational inertia of the body as

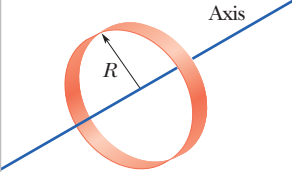
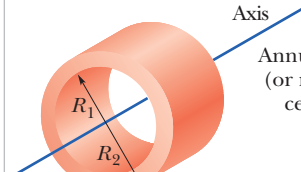
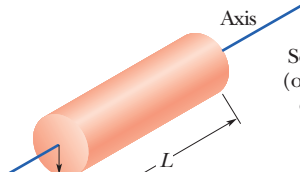
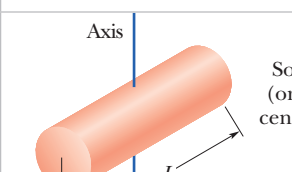
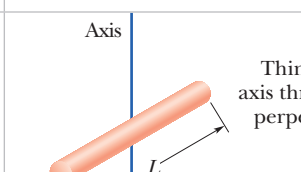
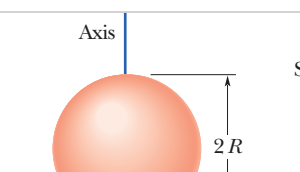
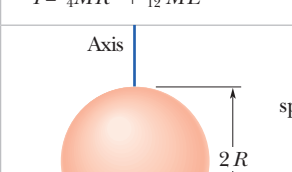
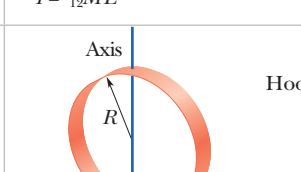
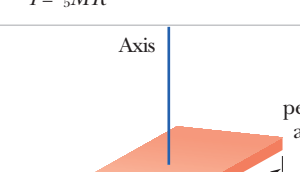
$$I = \int r^2 dm \quad (\text{rotational inertia, continuous body}). \quad (10-35)$$

Table 10-2 gives the results of such integration for nine common body shapes and the indicated axes of rotation.

Parallel-Axis Theorem

Suppose we want to find the rotational inertia I of a body of mass M about a given axis. In principle, we can always find I with the integration of Eq. 10-35. However, there is a neat shortcut if we happen to already know the rotational inertia I_{com} of the body about a *parallel* axis that extends through the body's center of mass. Let h be the perpendicular distance between the given axis and the axis

Table 10-2 Some Rotational Inertias

 Hoop about central axis $I = MR^2$ (a)	 Annular cylinder (or ring) about central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)	 Solid cylinder (or disk) about central axis $I = \frac{1}{2}MR^2$ (c)
 Solid cylinder (or disk) about central diameter $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)	 Thin rod about axis through center perpendicular to length $I = \frac{1}{12}ML^2$ (e)	 Solid sphere about any diameter $I = \frac{2}{5}MR^2$ (f)
 Thin spherical shell about any diameter $I = \frac{2}{3}MR^2$ (g)	 Hoop about any diameter $I = \frac{1}{2}MR^2$ (h)	 Slab about perpendicular axis through center $I = \frac{1}{12}M(a^2 + b^2)$ (i)

We need to relate the rotational inertia around the axis at P to that around the axis at the com.

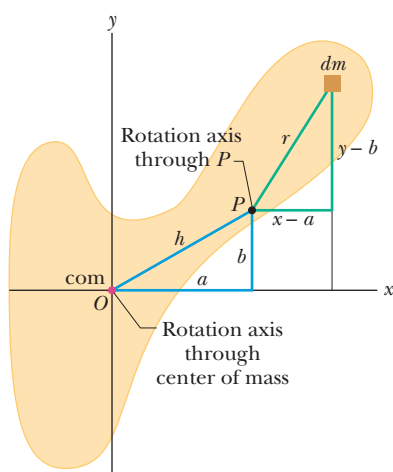


Figure 10-12 A rigid body in cross section, with its center of mass at O . The parallel-axis theorem (Eq. 10-36) relates the rotational inertia of the body about an axis through O to that about a parallel axis through a point such as P , a distance h from the body's center of mass.

through the center of mass (remember these two axes must be parallel). Then the rotational inertia I about the given axis is

$$I = I_{\text{com}} + Mh^2 \quad (\text{parallel-axis theorem}). \quad (10-36)$$

Think of the distance h as being the distance we have shifted the rotation axis from being through the com. This equation is known as the **parallel-axis theorem**. We shall now prove it.

Proof of the Parallel-Axis Theorem

Let O be the center of mass of the arbitrarily shaped body shown in cross section in Fig. 10-12. Place the origin of the coordinates at O . Consider an axis through O perpendicular to the plane of the figure, and another axis through point P parallel to the first axis. Let the x and y coordinates of P be a and b .

Let dm be a mass element with the general coordinates x and y . The rotational inertia of the body about the axis through P is then, from Eq. 10-35,

$$I = \int r^2 dm = \int [(x - a)^2 + (y - b)^2] dm,$$

which we can rearrange as

$$I = \int (x^2 + y^2) dm - 2a \int x dm - 2b \int y dm + \int (a^2 + b^2) dm. \quad (10-37)$$

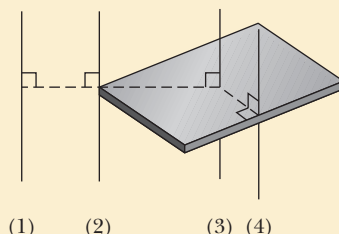
From the definition of the center of mass (Eq. 9-9), the middle two integrals of Eq. 10-37 give the coordinates of the center of mass (multiplied by a constant)

and thus must each be zero. Because $x^2 + y^2$ is equal to R^2 , where R is the distance from O to dm , the first integral is simply I_{com} , the rotational inertia of the body about an axis through its center of mass. Inspection of Fig. 10-12 shows that the last term in Eq. 10-37 is Mh^2 , where M is the body's total mass. Thus, Eq. 10-37 reduces to Eq. 10-36, which is the relation that we set out to prove.



Checkpoint 5

The figure shows a book-like object (one side is longer than the other) and four choices of rotation axes, all perpendicular to the face of the object. Rank the choices according to the rotational inertia of the object about the axis, greatest first.



Sample Problem 10.06 Rotational inertia of a two-particle system

Figure 10-13a shows a rigid body consisting of two particles of mass m connected by a rod of length L and negligible mass.

(a) What is the rotational inertia I_{com} about an axis through the center of mass, perpendicular to the rod as shown?

KEY IDEA

Because we have only two particles with mass, we can find the body's rotational inertia I_{com} by using Eq. 10-33 rather than by integration. That is, we find the rotational inertia of each particle and then just add the results.

Calculations: For the two particles, each at perpendicular distance $\frac{1}{2}L$ from the rotation axis, we have

$$I = \sum m_i r_i^2 = (m)\left(\frac{1}{2}L\right)^2 + (m)\left(\frac{1}{2}L\right)^2 = \frac{1}{2}mL^2. \quad (\text{Answer})$$

(b) What is the rotational inertia I of the body about an axis through the left end of the rod and parallel to the first axis (Fig. 10-13b)?

KEY IDEAS

This situation is simple enough that we can find I using either of two techniques. The first is similar to the one used in part (a). The other, more powerful one is to apply the parallel-axis theorem.

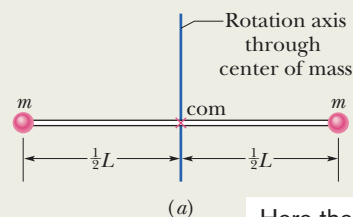
First technique: We calculate I as in part (a), except here the perpendicular distance r_i is zero for the particle on the

left and L for the particle on the right. Now Eq. 10-33 gives us

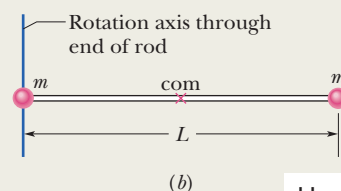
$$I = m(0)^2 + mL^2 = mL^2. \quad (\text{Answer})$$

Second technique: Because we already know I_{com} about an axis through the center of mass and because the axis here is parallel to that “com axis,” we can apply the parallel-axis theorem (Eq. 10-36). We find

$$I = I_{\text{com}} + Mh^2 = \frac{1}{2}mL^2 + (2m)\left(\frac{1}{2}L\right)^2 = mL^2. \quad (\text{Answer})$$



Here the rotation axis is through the com.



Here it has been shifted from the com without changing the orientation. We can use the parallel-axis theorem.

Figure 10-13 A rigid body consisting of two particles of mass m joined by a rod of negligible mass.



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Sample Problem 10.07 Rotational inertia of a uniform rod, integration

Figure 10-14 shows a thin, uniform rod of mass M and length L , on an x axis with the origin at the rod's center.

(a) What is the rotational inertia of the rod about the perpendicular rotation axis through the center?

KEY IDEAS

(1) The rod consists of a huge number of particles at a great many different distances from the rotation axis. We certainly don't want to sum their rotational inertias individually. So, we first write a general expression for the rotational inertia of a mass element dm at distance r from the rotation axis: $r^2 dm$.
 (2) Then we sum all such rotational inertias by integrating the expression (rather than adding them up one by one). From Eq. 10-35, we write

$$I = \int r^2 dm. \quad (10-38)$$

(3) Because the rod is uniform and the rotation axis is at the center, we are actually calculating the rotational inertia I_{com} about the center of mass.

Calculations: We want to integrate with respect to coordinate x (not mass m as indicated in the integral), so we must relate the mass dm of an element of the rod to its length dx along the rod. (Such an element is shown in Fig. 10-14.) Because the rod is uniform, the ratio of mass to length is the same for all the elements and for the rod as a whole. Thus, we can write

$$\frac{\text{element's mass } dm}{\text{element's length } dx} = \frac{\text{rod's mass } M}{\text{rod's length } L}$$

or
$$dm = \frac{M}{L} dx.$$

We can now substitute this result for dm and x for r in Eq. 10-38. Then we integrate from end to end of the rod (from $x = -L/2$ to $x = L/2$) to include all the elements. We find

$$\begin{aligned} I &= \int_{x=-L/2}^{x=+L/2} x^2 \left(\frac{M}{L} \right) dx \\ &= \frac{M}{3L} \left[x^3 \right]_{-L/2}^{+L/2} = \frac{M}{3L} \left[\left(\frac{L}{2} \right)^3 - \left(-\frac{L}{2} \right)^3 \right] \\ &= \frac{1}{12} ML^2. \end{aligned} \quad (\text{Answer})$$

(b) What is the rod's rotational inertia I about a new rotation axis that is perpendicular to the rod and through the left end?

KEY IDEAS

We can find I by shifting the origin of the x axis to the left end of the rod and then integrating from $x = 0$ to $x = L$. However, here we shall use a more powerful (and easier) technique by applying the parallel-axis theorem (Eq. 10-36), in which we shift the rotation axis without changing its orientation.

Calculations: If we place the axis at the rod's end so that it is parallel to the axis through the center of mass, then we can use the parallel-axis theorem (Eq. 10-36). We know from part (a) that I_{com} is $\frac{1}{12} ML^2$. From Fig. 10-14, the perpendicular distance h between the new rotation axis and the center of mass is $\frac{1}{2} L$. Equation 10-36 then gives us

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{1}{12} ML^2 + (M)\left(\frac{1}{2} L\right)^2 \\ &= \frac{1}{3} ML^2. \end{aligned} \quad (\text{Answer})$$

Actually, this result holds for any axis through the left or right end that is perpendicular to the rod.

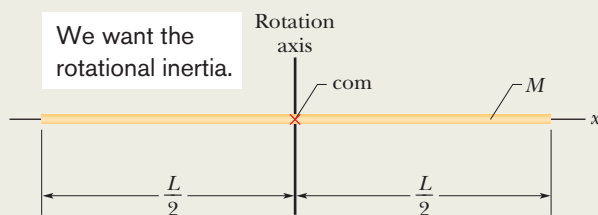
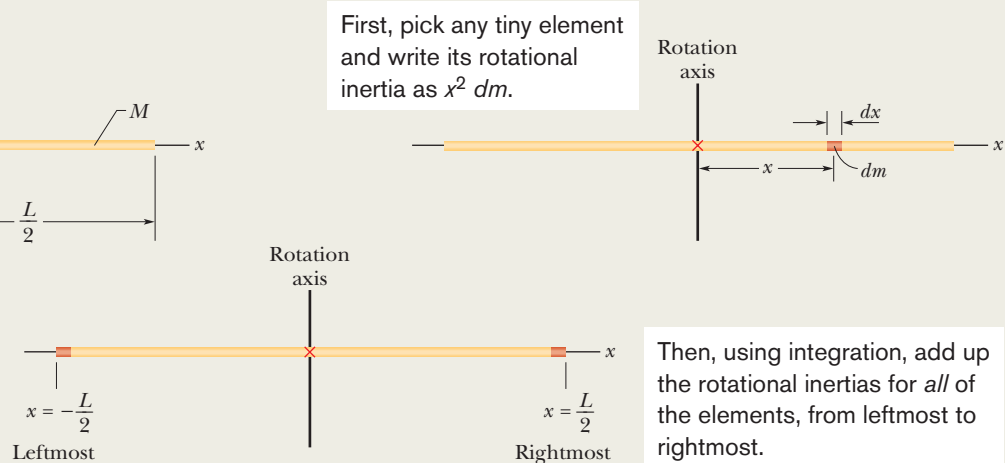


Figure 10-14 A uniform rod of length L and mass M . An element of mass dm and length dx is represented.



Sample Problem 10.08 Rotational kinetic energy, spin test explosion

Large machine components that undergo prolonged, high-speed rotation are first examined for the possibility of failure in a *spin test system*. In this system, a component is *spun up* (brought up to high speed) while inside a cylindrical arrangement of lead bricks and containment liner, all within a steel shell that is closed by a lid clamped into place. If the rotation causes the component to shatter, the soft lead bricks are supposed to catch the pieces for later analysis.

In 1985, Test Devices, Inc. (www.testdevices.com) was spin testing a sample of a solid steel rotor (a disk) of mass $M = 272$ kg and radius $R = 38.0$ cm. When the sample reached an angular speed ω of 14 000 rev/min, the test engineers heard a dull thump from the test system, which was located one floor down and one room over from them. Investigating, they found that lead bricks had been thrown out in the hallway leading to the test room, a door to the room had been hurled into the adjacent parking lot, one lead brick had shot from the test site through the wall of a neighbor's kitchen, the structural beams of the test building had been damaged, the concrete floor beneath the spin chamber had been shoved downward by about 0.5 cm, and the 900 kg lid had been blown upward through the ceiling and had then crashed back onto the test equipment (Fig. 10-15). The exploding pieces had not penetrated the room of the test engineers only by luck.

How much energy was released in the explosion of the rotor?

Figure 10-15 Some of the destruction caused by the explosion of a rapidly rotating steel disk.



Courtesy Test Devices, Inc.

KEY IDEA

The released energy was equal to the rotational kinetic energy K of the rotor just as it reached the angular speed of 14 000 rev/min.

Calculations: We can find K with Eq. 10-34 ($K = \frac{1}{2}I\omega^2$), but first we need an expression for the rotational inertia I . Because the rotor was a disk that rotated like a merry-go-round, I is given in Table 10-2c ($I = \frac{1}{2}MR^2$). Thus,

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(272 \text{ kg})(0.38 \text{ m})^2 = 19.64 \text{ kg} \cdot \text{m}^2.$$

The angular speed of the rotor was

$$\begin{aligned}\omega &= (14\,000 \text{ rev/min})(2\pi \text{ rad/rev})\left(\frac{1 \text{ min}}{60 \text{ s}}\right) \\ &= 1.466 \times 10^3 \text{ rad/s}.\end{aligned}$$

Then, with Eq. 10-34, we find the (huge) energy release:

$$\begin{aligned}K &= \frac{1}{2}I\omega^2 = \frac{1}{2}(19.64 \text{ kg} \cdot \text{m}^2)(1.466 \times 10^3 \text{ rad/s})^2 \\ &= 2.1 \times 10^7 \text{ J}.\end{aligned}\quad \text{(Answer)}$$



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10-6 TORQUE

Learning Objectives

After reading this module, you should be able to . . .

- 10.23** Identify that a torque on a body involves a force and a position vector, which extends from a rotation axis to the point where the force is applied.
- 10.24** Calculate the torque by using (a) the angle between the position vector and the force vector, (b) the line of action and the moment arm of the force, and (c) the force component perpendicular to the position vector.

Key Ideas

● Torque is a turning or twisting action on a body about a rotation axis due to a force $\vec{F}$. If $\vec{F}$ is exerted at a point given by the position vector $\vec{r}$ relative to the axis, then the magnitude of the torque is

$$\tau = rF_t = r_{\perp}F = rF \sin \phi,$$

where F_t is the component of $\vec{F}$ perpendicular to $\vec{r}$ and ϕ is the angle between $\vec{r}$ and $\vec{F}$. The quantity $r_{\perp}$ is the

- 10.25** Identify that a rotation axis must always be specified to calculate a torque.
- 10.26** Identify that a torque is assigned a positive or negative sign depending on the direction it tends to make the body rotate about a specified rotation axis: “clocks are negative.”
- 10.27** When more than one torque acts on a body about a rotation axis, calculate the net torque.

perpendicular distance between the rotation axis and an extended line running through the $\vec{F}$ vector. This line is called the line of action of $\vec{F}$, and $r_{\perp}$ is called the moment arm of $\vec{F}$. Similarly, r is the moment arm of F_t .

● The SI unit of torque is the newton-meter ($\text{N} \cdot \text{m}$). A torque τ is positive if it tends to rotate a body at rest counterclockwise and negative if it tends to rotate the body clockwise.

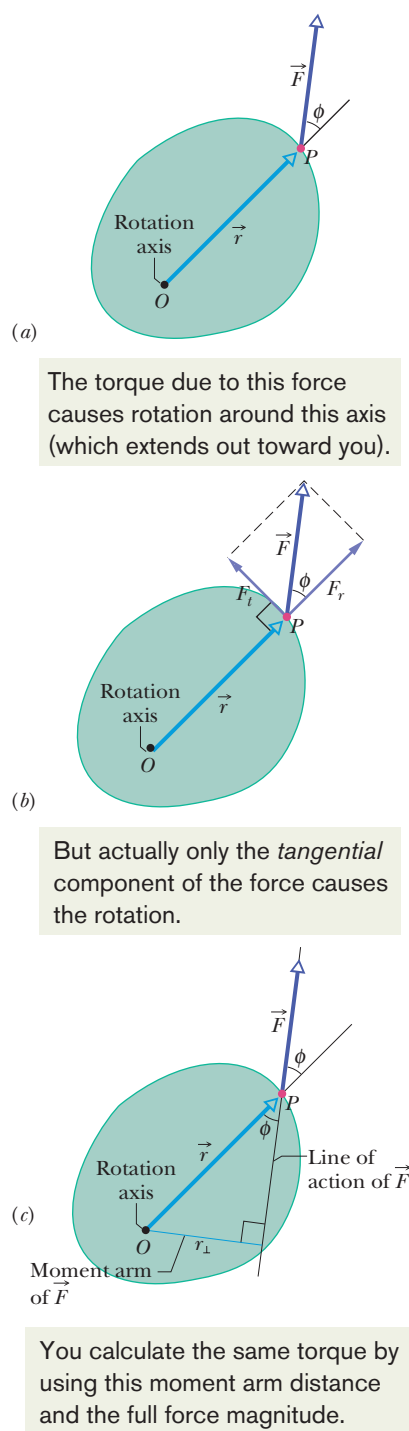
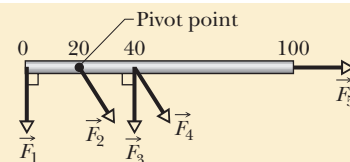


Figure 10-16 (a) A force $\vec{F}$ acts on a rigid body, with a rotation axis perpendicular to the page. The torque can be found with (a) angle ϕ , (b) tangential force component F_t , or (c) moment arm $r_\perp$.



Checkpoint 6

The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five forces on the stick are horizontal and have the same magnitude. Rank the forces according to the magnitude of the torque they produce, greatest first.



Torque

A doorknob is located as far as possible from the door's hinge line for a good reason. If you want to open a heavy door, you must certainly apply a force, but that is not enough. Where you apply that force and in what direction you push are also important. If you apply your force nearer to the hinge line than the knob, or at any angle other than 90° to the plane of the door, you must use a greater force than if you apply the force at the knob and perpendicular to the door's plane.

Figure 10-16a shows a cross section of a body that is free to rotate about an axis passing through O and perpendicular to the cross section. A force $\vec{F}$ is applied at point P , whose position relative to O is defined by a position vector $\vec{r}$. The directions of vectors $\vec{F}$ and $\vec{r}$ make an angle ϕ with each other. (For simplicity, we consider only forces that have no component parallel to the rotation axis; thus, $\vec{F}$ is in the plane of the page.)

To determine how $\vec{F}$ results in a rotation of the body around the rotation axis, we resolve $\vec{F}$ into two components (Fig. 10-16b). One component, called the *radial component* F_r , points along $\vec{r}$. This component does not cause rotation, because it acts along a line that extends through O . (If you pull on a door parallel to the plane of the door, you do not rotate the door.) The other component of $\vec{F}$, called the *tangential component* F_t , is perpendicular to $\vec{r}$ and has magnitude $F_t = F \sin \phi$. This component *does* cause rotation.

Calculating Torques. The ability of $\vec{F}$ to rotate the body depends not only on the magnitude of its tangential component F_t , but also on just how far from O the force is applied. To include both these factors, we define a quantity called **torque** τ as the product of the two factors and write it as

$$\tau = (r)(F \sin \phi). \quad (10-39)$$

Two equivalent ways of computing the torque are

$$\tau = (r)(F \sin \phi) = rF_t \quad (10-40)$$

and

$$\tau = (r \sin \phi)(F) = r_\perp F, \quad (10-41)$$

where $r_\perp$ is the perpendicular distance between the rotation axis at O and an extended line running through the vector $\vec{F}$ (Fig. 10-16c). This extended line is called the **line of action** of $\vec{F}$, and $r_\perp$ is called the **moment arm** of $\vec{F}$. Figure 10-16b shows that we can describe r , the magnitude of $\vec{r}$, as being the moment arm of the force component F_t .

Torque, which comes from the Latin word meaning “to twist,” may be loosely identified as the turning or twisting action of the force $\vec{F}$. When you apply a force to an object—such as a screwdriver or torque wrench—with the purpose of turning that object, you are applying a torque. The SI unit of torque is the newton-meter ($\text{N} \cdot \text{m}$). **Caution:** The newton-meter is also the unit of work. Torque and work, however, are quite different quantities and must not be confused. Work is often expressed in joules ($1 \text{ J} = 1 \text{ N} \cdot \text{m}$), but torque never is.

Clocks Are Negative. In Chapter 11 we shall use vector notation for torques, but here, with rotation around a single axis, we use only an algebraic sign. If a torque would cause counterclockwise rotation, it is positive. If it would cause clockwise rotation, it is negative. (The phrase “clocks are negative” from Module 10-1 still works.)

Torques obey the superposition principle that we discussed in Chapter 5 for forces: When several torques act on a body, the **net torque** (or **resultant torque**) is the sum of the individual torques. The symbol for net torque is τ_{net} .

10-7 NEWTON'S SECOND LAW FOR ROTATION

Learning Objective

After reading this module, you should be able to . . .

10.28 Apply Newton's second law for rotation to relate the net torque on a body to the body's rotational inertia and

rotational acceleration, all calculated relative to a specified rotation axis.

Key Idea

- The rotational analog of Newton's second law is

$$\tau_{\text{net}} = I\alpha,$$

where τ_{net} is the net torque acting on a particle or rigid body,

I is the rotational inertia of the particle or body about the rotation axis, and α is the resulting angular acceleration about that axis.

Newton's Second Law for Rotation

A torque can cause rotation of a rigid body, as when you use a torque to rotate a door. Here we want to relate the net torque τ_{net} on a rigid body to the angular acceleration α that torque causes about a rotation axis. We do so by analogy with Newton's second law ($F_{\text{net}} = ma$) for the acceleration a of a body of mass m due to a net force F_{net} along a coordinate axis. We replace F_{net} with τ_{net} , m with I , and a with α in radian measure, writing

$$\tau_{\text{net}} = I\alpha \quad (\text{Newton's second law for rotation}). \quad (10-42)$$

Proof of Equation 10-42

We prove Eq. 10-42 by first considering the simple situation shown in Fig. 10-17. The rigid body there consists of a particle of mass m on one end of a massless rod of length r . The rod can move only by rotating about its other end, around a rotation axis (an axle) that is perpendicular to the plane of the page. Thus, the particle can move only in a circular path that has the rotation axis at its center.

A force $\vec{F}$ acts on the particle. However, because the particle can move only along the circular path, only the tangential component F_t of the force (the component that is tangent to the circular path) can accelerate the particle along the path. We can relate F_t to the particle's tangential acceleration a_t along the path with Newton's second law, writing

$$F_t = ma_t.$$

The torque acting on the particle is, from Eq. 10-40,

$$\tau = F_t r = ma_t r.$$

From Eq. 10-22 ($a_t = \alpha r$) we can write this as

$$\tau = m(\alpha r)r = (mr^2)\alpha. \quad (10-43)$$

The quantity in parentheses on the right is the rotational inertia of the particle about the rotation axis (see Eq. 10-33, but here we have only a single particle). Thus, using I for the rotational inertia, Eq. 10-43 reduces to

$$\tau = I\alpha \quad (\text{radian measure}). \quad (10-44)$$

If more than one force is applied to the particle, Eq. 10-44 becomes

$$\tau_{\text{net}} = I\alpha \quad (\text{radian measure}), \quad (10-45)$$

which we set out to prove. We can extend this equation to any rigid body rotating about a fixed axis, because any such body can always be analyzed as an assembly of single particles.

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

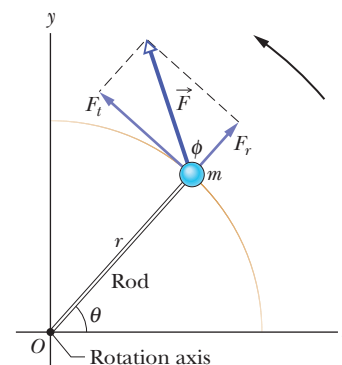
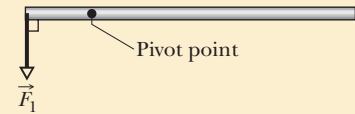


Figure 10-17 A simple rigid body, free to rotate about an axis through O , consists of a particle of mass m fastened to the end of a rod of length r and negligible mass. An applied force $\vec{F}$ causes the body to rotate.



Checkpoint 7

The figure shows an overhead view of a meter stick that can pivot about the point indicated, which is to the left of the stick's midpoint. Two horizontal forces, $\vec{F}_1$ and $\vec{F}_2$, are applied to the stick. Only $\vec{F}_1$ is shown. Force $\vec{F}_2$ is perpendicular to the stick and is applied at the right end. If the stick is not to turn, (a) what should be the direction of $\vec{F}_2$, and (b) should F_2 be greater than, less than, or equal to F_1 ?



Sample Problem 10.09 Using Newton's second law for rotation in a basic judo hip throw

To throw an 80 kg opponent with a basic judo hip throw, you intend to pull his uniform with a force $\vec{F}$ and a moment arm $d_1 = 0.30$ m from a pivot point (rotation axis) on your right hip (Fig. 10-18). You wish to rotate him about the pivot point with an angular acceleration α of -6.0 rad/s²—that is, with an angular acceleration that is *clockwise* in the figure. Assume that his rotational inertia I relative to the pivot point is 15 kg·m².

(a) What must the magnitude of $\vec{F}$ be if, before you throw him, you bend your opponent forward to bring his center of mass to your hip (Fig. 10-18a)?

KEY IDEA

We can relate your pull $\vec{F}$ on your opponent to the given angular acceleration α via Newton's second law for rotation ($\tau_{\text{net}} = I\alpha$).

Calculations: As his feet leave the floor, we can assume that only three forces act on him: your pull $\vec{F}$, a force $\vec{N}$ on him from you at the pivot point (this force is not indicated in Fig. 10-18), and the gravitational force $\vec{F}_g$. To use $\tau_{\text{net}} = I\alpha$, we need the corresponding three torques, each about the pivot point.

From Eq. 10-41 ($\tau = r_{\perp}F$), the torque due to your pull $\vec{F}$ is equal to $-d_1F$, where d_1 is the moment arm $r_{\perp}$ and the sign indicates the clockwise rotation this torque tends to cause. The torque due to $\vec{N}$ is zero, because $\vec{N}$ acts at the pivot point and thus has moment arm $r_{\perp} = 0$.

To evaluate the torque due to $\vec{F}_g$, we can assume that $\vec{F}_g$ acts at your opponent's center of mass. With the center of mass at the pivot point, $\vec{F}_g$ has moment arm $r_{\perp} = 0$ and thus the torque due to $\vec{F}_g$ is zero. So, the only torque on your opponent is due to your pull $\vec{F}$, and we can write $\tau_{\text{net}} = I\alpha$ as

$$-d_1F = I\alpha.$$

We then find

$$F = \frac{-I\alpha}{d_1} = \frac{-(15 \text{ kg} \cdot \text{m}^2)(-6.0 \text{ rad/s}^2)}{0.30 \text{ m}} = 300 \text{ N.} \quad (\text{Answer})$$

(b) What must the magnitude of $\vec{F}$ be if your opponent remains upright before you throw him, so that $\vec{F}_g$ has a moment arm $d_2 = 0.12$ m (Fig. 10-18b)?

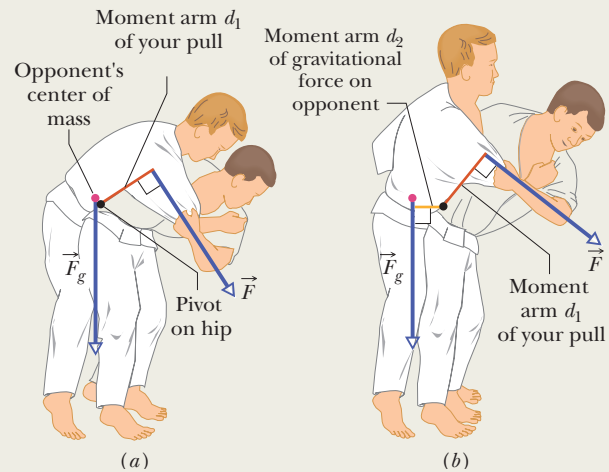


Figure 10-18 A judo hip throw (a) correctly executed and (b) incorrectly executed.

KEY IDEA

Because the moment arm for $\vec{F}_g$ is no longer zero, the torque due to $\vec{F}_g$ is now equal to d_2mg and is positive because the torque attempts counterclockwise rotation.

Calculations: Now we write $\tau_{\text{net}} = I\alpha$ as

$$-d_1F + d_2mg = I\alpha,$$

which gives

$$F = -\frac{I\alpha}{d_1} + \frac{d_2mg}{d_1}.$$

From (a), we know that the first term on the right is equal to 300 N. Substituting this and the given data, we have

$$F = 300 \text{ N} + \frac{(0.12 \text{ m})(80 \text{ kg})(9.8 \text{ m/s}^2)}{0.30 \text{ m}} = 613.6 \text{ N} \approx 610 \text{ N.} \quad (\text{Answer})$$

The results indicate that you will have to pull much harder if you do not initially bend your opponent to bring his center of mass to your hip. A good judo fighter knows this lesson from physics. Indeed, physics is the basis of most of the martial arts, figured out after countless hours of trial and error over the centuries.



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Sample Problem 10.10 Newton's second law, rotation, torque, disk

Figure 10-19a shows a uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.

KEY IDEAS

(1) Taking the block as a system, we can relate its acceleration a to the forces acting on it with Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$). (2) Taking the disk as a system, we can relate its angular acceleration α to the torque acting on it with Newton's second law for rotation ($\tau_{\text{net}} = I\alpha$). (3) To combine the motions of block and disk, we use the fact that the linear acceleration a of the block and the (tangential) linear acceleration a_t of the disk rim are equal. (To avoid confusion about signs, let's work with acceleration magnitudes and explicit algebraic signs.)

Forces on block: The forces are shown in the block's free-body diagram in Fig. 10-19b: The force from the cord is $\vec{T}$, and the gravitational force is $\vec{F}_g$, of magnitude mg . We can now write Newton's second law for components along a vertical y axis ($F_{\text{net},y} = ma_y$) as

$$T - mg = m(-a), \quad (10-46)$$

where a is the magnitude of the acceleration (down the y axis). However, we cannot solve this equation for a because it also contains the unknown T .

Torque on disk: Previously, when we got stuck on the y axis, we switched to the x axis. Here, we switch to the rotation of the disk and use Newton's second law in angular form. To calculate the torques and the rotational inertia I , we take the rotation axis to be perpendicular to the disk and through its center, at point O in Fig. 10-19c.

The torques are then given by Eq. 10-40 ($\tau = rF_t$). The gravitational force on the disk and the force on the disk from the axle both act at the center of the disk and thus at distance $r = 0$, so their torques are zero. The force $\vec{T}$ on the disk due to the cord acts at distance $r = R$ and is tangent to the rim of the disk. Therefore, its torque is $-RT$, negative because the torque rotates the disk clockwise from rest. Let α be the magnitude of the negative (clockwise) angular acceleration. From Table 10-2c, the rotational inertia I of the disk is $\frac{1}{2}MR^2$. Thus we can write the general equation $\tau_{\text{net}} = I\alpha$ as

$$-RT = \frac{1}{2}MR^2(-\alpha). \quad (10-47)$$

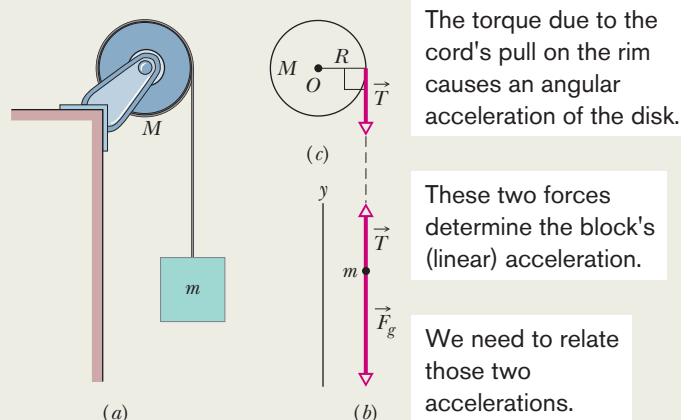


Figure 10-19 (a) The falling block causes the disk to rotate. (b) A free-body diagram for the block. (c) An incomplete free-body diagram for the disk.

This equation seems useless because it has two unknowns, α and T , neither of which is the desired a . However, mustering physics courage, we can make it useful with this fact: Because the cord does not slip, the magnitude a of the block's linear acceleration and the magnitude a_t of the (tangential) linear acceleration of the rim of the disk are equal. Then, by Eq. 10-22 ($a_t = \alpha r$) we see that here $\alpha = a/R$. Substituting this in Eq. 10-47 yields

$$T = \frac{1}{2}Ma. \quad (10-48)$$

Combining results: Combining Eqs. 10-46 and 10-48 leads to

$$a = g \frac{2m}{M + 2m} = (9.8 \text{ m/s}^2) \frac{(2)(1.2 \text{ kg})}{2.5 \text{ kg} + (2)(1.2 \text{ kg})} = 4.8 \text{ m/s}^2. \quad (\text{Answer})$$

We then use Eq. 10-48 to find T :

$$T = \frac{1}{2}Ma = \frac{1}{2}(2.5 \text{ kg})(4.8 \text{ m/s}^2) = 6.0 \text{ N}. \quad (\text{Answer})$$

As we should expect, acceleration a of the falling block is less than g , and tension T in the cord ($= 6.0$ N) is less than the gravitational force on the hanging block ($= mg = 11.8$ N). We see also that a and T depend on the mass of the disk but not on its radius.

As a check, we note that the formulas derived above predict $a = g$ and $T = 0$ for the case of a massless disk ($M = 0$). This is what we would expect; the block simply falls as a free body. From Eq. 10-22, the magnitude of the angular acceleration of the disk is

$$\alpha = \frac{a}{R} = \frac{4.8 \text{ m/s}^2}{0.20 \text{ m}} = 24 \text{ rad/s}^2. \quad (\text{Answer})$$



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10-8 WORK AND ROTATIONAL KINETIC ENERGY

Learning Objectives

After reading this module, you should be able to . . .

- 10.29** Calculate the work done by a torque acting on a rotating body by integrating the torque with respect to the angle of rotation.
- 10.30** Apply the work–kinetic energy theorem to relate the work done by a torque to the resulting change in the rotational kinetic energy of the body.

- 10.31** Calculate the work done by a *constant* torque by relating the work to the angle through which the body rotates.
- 10.32** Calculate the power of a torque by finding the rate at which work is done.
- 10.33** Calculate the power of a torque at any given instant by relating it to the torque and the angular velocity at that instant.

Key Ideas

- The equations used for calculating work and power in rotational motion correspond to equations used for translational motion and are

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

$$P = \frac{dW}{dt} = \tau\omega.$$

and

- When τ is constant, the integral reduces to

$$W = \tau(\theta_f - \theta_i).$$

- The form of the work–kinetic energy theorem used for rotating bodies is

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W.$$

Work and Rotational Kinetic Energy

As we discussed in Chapter 7, when a force F causes a rigid body of mass m to accelerate along a coordinate axis, the force does work W on the body. Thus, the body's kinetic energy ($K = \frac{1}{2}mv^2$) can change. Suppose it is the only energy of the body that changes. Then we relate the change ΔK in kinetic energy to the work W with the work–kinetic energy theorem (Eq. 7-10), writing

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W \quad (\text{work–kinetic energy theorem}). \quad (10-49)$$

For motion confined to an x axis, we can calculate the work with Eq. 7-32,

$$W = \int_{x_i}^{x_f} F dx \quad (\text{work, one-dimensional motion}). \quad (10-50)$$

This reduces to $W = Fd$ when F is constant and the body's displacement is d . The rate at which the work is done is the power, which we can find with Eqs. 7-43 and 7-48,

$$P = \frac{dW}{dt} = Fv \quad (\text{power, one-dimensional motion}). \quad (10-51)$$

Now let us consider a rotational situation that is similar. When a torque accelerates a rigid body in rotation about a fixed axis, the torque does work W on the body. Therefore, the body's rotational kinetic energy ($K = \frac{1}{2}I\omega^2$) can change. Suppose that it is the only energy of the body that changes. Then we can still relate the change ΔK in kinetic energy to the work W with the work–kinetic energy theorem, except now the kinetic energy is a rotational kinetic energy:

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W \quad (\text{work–kinetic energy theorem}). \quad (10-52)$$

Here, I is the rotational inertia of the body about the fixed axis and ω_i and ω_f are the angular speeds of the body before and after the work is done.

Also, we can calculate the work with a rotational equivalent of Eq. 10-50,

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta \quad (\text{work, rotation about fixed axis}), \quad (10-53)$$

where τ is the torque doing the work W , and θ_i and θ_f are the body's angular positions before and after the work is done, respectively. When τ is constant, Eq. 10-53 reduces to

$$W = \tau(\theta_f - \theta_i) \quad (\text{work, constant torque}). \quad (10-54)$$

The rate at which the work is done is the power, which we can find with the rotational equivalent of Eq. 10-51,

$$P = \frac{dW}{dt} = \tau\omega \quad (\text{power, rotation about fixed axis}). \quad (10-55)$$

Table 10-3 summarizes the equations that apply to the rotation of a rigid body about a fixed axis and the corresponding equations for translational motion.

Proof of Eqs. 10-52 through 10-55

Let us again consider the situation of Fig. 10-17, in which force $\vec{F}$ rotates a rigid body consisting of a single particle of mass m fastened to the end of a massless rod. During the rotation, force $\vec{F}$ does work on the body. Let us assume that the only energy of the body that is changed by $\vec{F}$ is the kinetic energy. Then we can apply the work–kinetic energy theorem of Eq. 10-49:

$$\Delta K = K_f - K_i = W. \quad (10-56)$$

Using $K = \frac{1}{2}mv^2$ and Eq. 10-18 ($v = \omega r$), we can rewrite Eq. 10-56 as

$$\Delta K = \frac{1}{2}mr^2\omega_f^2 - \frac{1}{2}mr^2\omega_i^2 = W. \quad (10-57)$$

From Eq. 10-33, the rotational inertia for this one-particle body is $I = mr^2$. Substituting this into Eq. 10-57 yields

$$\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W,$$

which is Eq. 10-52. We derived it for a rigid body with one particle, but it holds for any rigid body rotated about a fixed axis.

We next relate the work W done on the body in Fig. 10-17 to the torque τ on the body due to force $\vec{F}$. When the particle moves a distance ds along its

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{\text{net}} = ma$	Newton's second law	$\tau_{\text{net}} = I\alpha$
Work	$W = \int F dx$	Work	$W = \int \tau d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

circular path, only the tangential component F_t of the force accelerates the particle along the path. Therefore, only F_t does work on the particle. We write that work dW as $F_t ds$. However, we can replace ds with $r d\theta$, where $d\theta$ is the angle through which the particle moves. Thus we have

$$dW = F_t r d\theta. \quad (10-58)$$

From Eq. 10-40, we see that the product $F_t r$ is equal to the torque τ , so we can rewrite Eq. 10-58 as

$$dW = \tau d\theta. \quad (10-59)$$

The work done during a finite angular displacement from θ_i to θ_f is then

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta,$$

which is Eq. 10-53. It holds for any rigid body rotating about a fixed axis. Equation 10-54 comes directly from Eq. 10-53.

We can find the power P for rotational motion from Eq. 10-59:

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega,$$

which is Eq. 10-55.



Sample Problem 10.11 Work, rotational kinetic energy, torque, disk

Let the disk in Fig. 10-19 start from rest at time $t = 0$ and also let the tension in the massless cord be 6.0 N and the angular acceleration of the disk be -24 rad/s^2 . What is its rotational kinetic energy K at $t = 2.5 \text{ s}$?

KEY IDEA

We can find K with Eq. 10-34 ($K = \frac{1}{2} I \omega^2$). We already know that $I = \frac{1}{2} MR^2$, but we do not yet know ω at $t = 2.5 \text{ s}$. However, because the angular acceleration α has the constant value of -24 rad/s^2 , we can apply the equations for constant angular acceleration in Table 10-1.

Calculations: Because we want ω and know α and $\omega_0 (= 0)$, we use Eq. 10-12:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t.$$

Substituting $\omega = \alpha t$ and $I = \frac{1}{2} MR^2$ into Eq. 10-34, we find

$$\begin{aligned} K &= \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{1}{2} MR^2 \right) (\alpha t)^2 = \frac{1}{4} M (R \alpha t)^2 \\ &= \frac{1}{4} (2.5 \text{ kg}) [(0.20 \text{ m}) (-24 \text{ rad/s}^2) (2.5 \text{ s})]^2 \\ &= 90 \text{ J.} \end{aligned} \quad (\text{Answer})$$

KEY IDEA

We can also get this answer by finding the disk's kinetic energy from the work done on the disk.

Calculations: First, we relate the *change* in the kinetic energy of the disk to the net work W done on the disk, using the work–kinetic energy theorem of Eq. 10-52 ($K_f - K_i = W$). With K substituted for K_f and 0 for K_i , we get

$$K = K_i + W = 0 + W = W. \quad (10-60)$$

Next we want to find the work W . We can relate W to the torques acting on the disk with Eq. 10-53 or 10-54. The only torque causing angular acceleration and doing work is the torque due to force $\vec{T}$ on the disk from the cord, which is equal to $-TR$. Because α is constant, this torque also must be constant. Thus, we can use Eq. 10-54 to write

$$W = \tau(\theta_f - \theta_i) = -TR(\theta_f - \theta_i). \quad (10-61)$$

Because α is constant, we can use Eq. 10-13 to find $\theta_f - \theta_i$. With $\omega_i = 0$, we have

$$\theta_f - \theta_i = \omega_i t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2.$$

Now we substitute this into Eq. 10-61 and then substitute the result into Eq. 10-60. Inserting the given values $T = 6.0 \text{ N}$ and $\alpha = -24 \text{ rad/s}^2$, we have

$$\begin{aligned} K &= W = -TR(\theta_f - \theta_i) = -TR \left(\frac{1}{2} \alpha t^2 \right) = -\frac{1}{2} TR \alpha t^2 \\ &= -\frac{1}{2} (6.0 \text{ N}) (0.20 \text{ m}) (-24 \text{ rad/s}^2) (2.5 \text{ s})^2 \\ &= 90 \text{ J.} \end{aligned} \quad (\text{Answer})$$



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Review & Summary

Angular Position To describe the rotation of a rigid body about a fixed axis, called the **rotation axis**, we assume a **reference line** is fixed in the body, perpendicular to that axis and rotating with the body. We measure the **angular position** θ of this line relative to a fixed direction. When θ is measured in **radians**,

$$\theta = \frac{s}{r} \quad (\text{radian measure}), \quad (10-1)$$

where s is the arc length of a circular path of radius r and angle θ . Radian measure is related to angle measure in revolutions and degrees by

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}. \quad (10-2)$$

Angular Displacement A body that rotates about a rotation axis, changing its angular position from θ_1 to θ_2 , undergoes an **angular displacement**

$$\Delta\theta = \theta_2 - \theta_1, \quad (10-4)$$

where $\Delta\theta$ is positive for counterclockwise rotation and negative for clockwise rotation.

Angular Velocity and Speed If a body rotates through an angular displacement $\Delta\theta$ in a time interval Δt , its **average angular velocity** ω_{avg} is

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t}. \quad (10-5)$$

The **(instantaneous) angular velocity** ω of the body is

$$\omega = \frac{d\theta}{dt}. \quad (10-6)$$

Both ω_{avg} and ω are vectors, with directions given by the **right-hand rule** of Fig. 10-6. They are positive for counterclockwise rotation and negative for clockwise rotation. The magnitude of the body's angular velocity is the **angular speed**.

Angular Acceleration If the angular velocity of a body changes from ω_1 to ω_2 in a time interval $\Delta t = t_2 - t_1$, the **average angular acceleration** α_{avg} of the body is

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}. \quad (10-7)$$

The **(instantaneous) angular acceleration** α of the body is

$$\alpha = \frac{d\omega}{dt}. \quad (10-8)$$

Both α_{avg} and α are vectors.

The Kinematic Equations for Constant Angular Acceleration *Constant angular acceleration* ($\alpha = \text{constant}$) is an important special case of rotational motion. The appropriate kinematic equations, given in Table 10-1, are

$$\omega = \omega_0 + \alpha t, \quad (10-12)$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2, \quad (10-13)$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0), \quad (10-14)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t, \quad (10-15)$$

$$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2. \quad (10-16)$$

Linear and Angular Variables Related A point in a rigid rotating body, at a **perpendicular distance** r from the rotation axis,

moves in a circle with radius r . If the body rotates through an angle θ , the point moves along an arc with length s given by

$$s = \theta r \quad (\text{radian measure}), \quad (10-17)$$

where θ is in radians.

The linear velocity $\vec{v}$ of the point is tangent to the circle; the point's linear speed v is given by

$$v = \omega r \quad (\text{radian measure}), \quad (10-18)$$

where ω is the angular speed (in radians per second) of the body.

The linear acceleration $\vec{a}$ of the point has both *tangential* and *radial* components. The tangential component is

$$a_t = \alpha r \quad (\text{radian measure}), \quad (10-22)$$

where α is the magnitude of the angular acceleration (in radians per second-squared) of the body. The radial component of $\vec{a}$ is

$$a_r = \frac{v^2}{r} = \omega^2 r \quad (\text{radian measure}). \quad (10-23)$$

If the point moves in uniform circular motion, the period T of the motion for the point and the body is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \quad (\text{radian measure}). \quad (10-19, 10-20)$$

Rotational Kinetic Energy and Rotational Inertia The kinetic energy K of a rigid body rotating about a fixed axis is given by

$$K = \frac{1}{2}I\omega^2 \quad (\text{radian measure}), \quad (10-34)$$

in which I is the **rotational inertia** of the body, defined as

$$I = \sum m_i r_i^2 \quad (10-33)$$

for a system of discrete particles and defined as

$$I = \int r^2 dm \quad (10-35)$$

for a body with continuously distributed mass. The r and r_i in these expressions represent the perpendicular distance from the axis of rotation to each mass element in the body, and the integration is carried out over the entire body so as to include every mass element.

The Parallel-Axis Theorem The *parallel-axis theorem* relates the rotational inertia I of a body about any axis to that of the same body about a parallel axis through the center of mass:

$$I = I_{\text{com}} + Mh^2. \quad (10-36)$$

Here h is the perpendicular distance between the two axes, and I_{com} is the rotational inertia of the body about the axis through the com. We can describe h as being the distance the actual rotation axis has been shifted from the rotation axis through the com.

Torque *Torque* is a turning or twisting action on a body about a rotation axis due to a force $\vec{F}$. If $\vec{F}$ is exerted at a point given by the position vector $\vec{r}$ relative to the axis, then the magnitude of the torque is

$$\tau = rF_t = r_{\perp}F = rF \sin \phi, \quad (10-40, 10-41, 10-39)$$

where F_t is the component of $\vec{F}$ perpendicular to $\vec{r}$ and ϕ is the angle between $\vec{r}$ and $\vec{F}$. The quantity $r_{\perp}$ is the perpendicular distance between the rotation axis and an extended line running through the $\vec{F}$ vector. This line is called the **line of action** of $\vec{F}$, and $r_{\perp}$ is called the **moment arm** of $\vec{F}$. Similarly, r is the moment arm of F_t .

The SI unit of torque is the newton-meter ($\text{N} \cdot \text{m}$). A torque τ is positive if it tends to rotate a body at rest counterclockwise and negative if it tends to rotate the body clockwise.

Newton's Second Law in Angular Form The rotational analog of Newton's second law is

$$\tau_{\text{net}} = I\alpha, \quad (10-45)$$

where τ_{net} is the net torque acting on a particle or rigid body, I is the rotational inertia of the particle or body about the rotation axis, and α is the resulting angular acceleration about that axis.

Work and Rotational Kinetic Energy The equations used for calculating work and power in rotational motion correspond to

equations used for translational motion and are

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta \quad (10-53)$$

and

$$P = \frac{dW}{dt} = \tau\omega. \quad (10-55)$$

When τ is constant, Eq. 10-53 reduces to

$$W = \tau(\theta_f - \theta_i). \quad (10-54)$$

The form of the work–kinetic energy theorem used for rotating bodies is

$$\Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = W. \quad (10-52)$$

Problems

1 A tractor has rear wheels with a radius of 1.00 m and front wheels with a radius of 0.250 m. The rear wheels are rotating at 100 rev/min. Find (a) the angular speed of the front wheels in revolutions per minute and (b) the distance covered by the tractor in 10.0 min.

2 Just as a helicopter is landing, its blades are turning at 30.0 rev/s and slowing at a constant rate. In the 2.00 min required for them to stop, how many revolutions do they make?

3 When a slice of buttered toast is accidentally pushed over the edge of a counter, it rotates as it falls. If the distance to the floor is 76 cm and for rotation less than 1 rev, what are the (a) smallest and (b) largest angular speeds that cause the toast to hit and then topple to be butter-side down?

4 The angular position of a point on a rotating wheel is given by $\theta = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t is in seconds. At $t = 0$, what are (a) the point's angular position and (b) its angular velocity? (c) What is its angular velocity at $t = 3.0$ s? (d) Calculate its angular acceleration at $t = 4.0$ s. (e) Is its angular acceleration constant?

5 At time $t = 0$, a rotating bicycle wheel is thrown horizontally from a rooftop with a speed of 49 m/s. By the time its vertical speed is also 49 m/s, it has completed 40 revolutions. What has been its average angular speed to that point in the fall?

6 A horizontal pottery wheel (a horizontal disk) with a radius of 30.0 cm can rotate about a vertical axis with negligible friction but is initially stationary. A horizontal rubber wheel of radius 2.00 cm is placed against its rim. That wheel is mounted on a motor. When the motor is switched on at time $t = 0$, the rubber wheel undergoes a constant angular acceleration of 5.00 rad/s^2 . Its contact with the pottery wheel causes the pottery wheel to undergo an angular acceleration. When the pottery wheel reaches an angular speed of 5.00 rev/s, the rubber wheel is pulled away from contact and thereafter the pottery wheel rotates at 5.00 rev/s. From $t = 0$ to $t = 2.00$ min, how many full rotations does the pottery wheel make?

7 The wheel in Fig. 10-20 has eight equally spaced spokes and a radius of 30 cm. It is mounted on a fixed axle and is spinning at 2.5 rev/s. You want to shoot a 20 cm long arrow parallel to this axle and through the wheel without hitting any of the spokes. Assume that the arrow and the spokes are very thin. (a) What minimum speed must the arrow have? (b) Does it matter where between the axle and rim of the wheel you aim? If so, what is the best location?

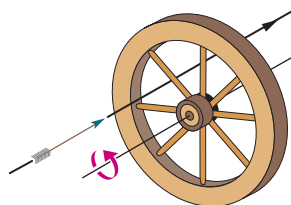


Figure 10-20 Problem 7.

8 The angular acceleration of a wheel is $\alpha = 6.0t^4 - 4.0t^2$, with α in radians per second-squared and t in seconds. At time $t = 0$, the wheel has an angular velocity of $+2.5 \text{ rad/s}$ and an angular position of $+1.5 \text{ rad}$. Write expressions for (a) the angular velocity (rad/s) and (b) the angular position (rad) as functions of time (s).

9 In 5.00 s, a 2.00 kg stone moves in a horizontal circle of radius 2.00 m from rest to an angular speed of 4.00 rev/s. What are the stone's (a) average angular acceleration and (b) rotational inertia around the circle's center?

10 Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 20 rad. During that time, what are the magnitudes of (a) the angular acceleration and (b) the average angular velocity? (c) What is the instantaneous angular velocity of the disk at the end of the 5.0 s? (d) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?

11 Two identical disks A and B can spin around vertical axes. Disk A is spinning with an initial angular speed of 40 rev/s when its rim touches initially stationary disk B , causing that disk to begin to spin. The rubbing at the contact point slows A while speeding up B . The rate at which both disks change their angular speeds is 2.0 rev/s^2 . Find the time required for the two disks to reach the same angular speed.

12 The angular speed of an automobile engine is increased at a constant rate from 1200 rev/min to 3200 rev/min in 12 s. (a) What is its angular acceleration in revolutions per minute-squared? (b) How many revolutions does the engine make during this 12 s interval?

13 A flywheel turns through 40 rev as it slows from an angular speed of 1.5 rad/s to a stop. (a) Assuming a constant angular acceleration, find the time for it to come to rest. (b) What is its angular acceleration? (c) How much time is required for it to complete the first 20 of the 40 revolutions?

14 A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one time it is rotating at 10 rev/s; 60 revolutions later, its angular speed is 15 rev/s. Calculate (a) the angular acceleration, (b) the time required to complete the 60 revolutions, (c) the time required to reach the 10 rev/s angular speed, and (d) the number of revolutions from rest until the time the disk reaches the 10 rev/s angular speed.

15 Starting from rest, a wheel has constant $\alpha = 3.0 \text{ rad/s}^2$. During a certain 4.0 s interval, it turns through 120 rad. How much time did it take to reach that 4.0 s interval?

16 A merry-go-round rotates from rest with an angular acceleration of 1.20 rad/s^2 . How long does it take to rotate through (a) the first 2.00 rev and (b) the next 2.00 rev?

17 At $t = 0$, a flywheel has an angular velocity of 4.7 rad/s , a constant angular acceleration of -0.25 rad/s^2 , and a reference line at $\theta_0 = 0$. (a) Through what maximum angle θ_{max} will the reference line turn in the positive direction? What are the (b) first and (c) second times the reference line will be at $\theta = \frac{1}{2}\theta_{\text{max}}$? At what (d) negative time and (e) positive time will the reference line be at $\theta = 10.5 \text{ rad}$? (f) Graph θ versus t , and indicate your answers.

18 A pulsar is a rapidly rotating neutron star that emits a radio beam the way a lighthouse emits a light beam. We receive a radio pulse for each rotation of the star. The period T of rotation is found by measuring the time between pulses. The pulsar in the Crab nebula has a period of rotation of $T = 0.033 \text{ s}$ that is increasing at the rate of $1.26 \times 10^{-5} \text{ s/y}$. (a) What is the pulsar's angular acceleration α ? (b) If α is constant, how many years from now will the pulsar stop rotating? (c) The pulsar originated in a supernova explosion seen in the year 1054. Assuming constant α , find the initial T .

19 An open jar of water moves in a vertical circle of radius 0.50 m with a frequency that is small enough to put the water on the verge of falling out of the jar at the top of the circle. If the same demonstration were repeated on Mars, where the gravitational acceleration is only 3.7 m/s^2 , what is the change in the circling frequency to again put the water on the verge of falling out at the top point?

20 An object rotates about a fixed axis, and the angular position of a reference line on the object is given by $\theta = 0.40e^{2t}$, where θ is in radians and t is in seconds. Consider a point on the object that is 6.0 cm from the axis of rotation. At $t = 0$, what are the magnitudes of the point's (a) tangential component of acceleration and (b) radial component of acceleration?

21 Between 1911 and 1990, the top of the leaning bell tower at Pisa, Italy, moved toward the south at an average rate of 1.2 mm/y . The tower is 55 m tall. In radians per second, what is the average angular speed of the tower's top about its base?

22 Starting from rest at time $t = 0$, a circus stunt man drives a motorbike on a horizontal circular track of radius 10.0 m . His speed is given by $v = ct^2$, where $c = 1.00 \text{ m/s}^3$. At $t = 2.00 \text{ s}$, what is the angle between his (total) acceleration vector and his radial acceleration vector?

23 A flywheel with a diameter of 1.20 m is rotating at an angular speed of 200 rev/min . (a) What is the angular speed of the flywheel in radians per second? (b) What is the linear speed of a point on the rim of the flywheel? (c) What constant angular acceleration (in revolutions per minute-squared) will increase the wheel's angular speed to 1000 rev/min in 60.0 s ? (d) How many revolutions does the wheel make during that 60.0 s ?

24 A vinyl record is played by rotating the record so that an approximately circular groove in the vinyl slides under a stylus. Bumps in the groove run into the stylus, causing it to oscillate. The equipment converts those oscillations to electrical signals and then to sound. Suppose that a record turns at the rate of $33\frac{1}{3} \text{ rev/min}$, the groove being played is at a radius of 10.0 cm , and the bumps in the groove are uniformly separated by 1.85 mm . At what rate (hits per second) do the bumps hit the stylus?

25 (a) What is the angular speed ω about the polar axis of a point on Earth's surface at latitude 40° N ? (Earth rotates about that axis.) (b) What is the linear speed v of the point? What are (c) ω and (d) v for a point at the equator?

26 The flywheel of a steam engine runs with a constant angular velocity of 160 rev/min . When steam is shut off, the friction of the bearings and of the air stops the wheel in 2.2 h . (a) What is the constant angular acceleration, in revolutions per minute-squared, of the wheel during the slowdown? (b) How many revolutions does the wheel make before stopping? (c) At the instant the flywheel is turning at 75 rev/min , what is the tangential component of the linear acceleration of a flywheel particle that is 50 cm from the axis of rotation? (d) What is the magnitude of the net linear acceleration of the particle in (c)?

27 A seed is on a turntable rotating at $33\frac{1}{3} \text{ rev/min}$, 6.0 cm from the rotation axis. What are (a) the seed's acceleration and (b) the least coefficient of static friction to avoid slippage? (c) If the turntable had undergone constant angular acceleration from rest in 0.25 s , what is the least coefficient to avoid slippage?

28 In Fig. 10-21, wheel A of radius $r_A = 10 \text{ cm}$ is coupled by belt B to wheel C of radius $r_C = 25 \text{ cm}$. The angular speed of wheel A is increased from rest at a constant rate of 2.0 rad/s^2 . Find the time needed for wheel C to reach an angular speed of 100 rev/min , assuming the belt does not slip. (Hint: If the belt does not slip, the linear speeds at the two rims must be equal.)

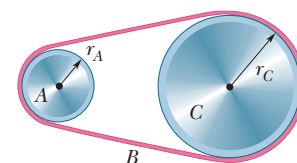


Figure 10-21 Problem 28.

29 Figure 10-22 shows an early method of measuring the speed of light that makes use of a rotating slotted wheel. A beam of light passes through one of the slots at the outside edge of the wheel, travels to a distant mirror, and returns to the wheel just in time to pass through the next slot in the wheel. One such slotted wheel has a radius of 5.0 cm and 500 slots around its edge. Measurements taken when the mirror is $L = 500 \text{ m}$ from the wheel indicate a speed of light of $3.0 \times 10^5 \text{ km/s}$. (a) What is the (constant) angular speed of the wheel? (b) What is the linear speed of a point on the edge of the wheel?

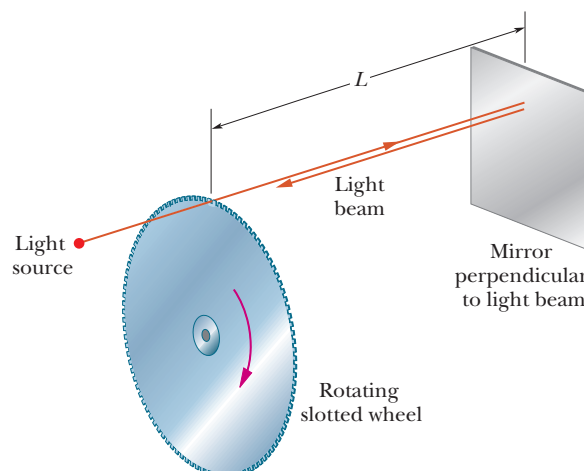


Figure 10-22 Problem 29.

30 A gyroscope flywheel of radius 2.62 cm is accelerated from rest at 14.2 rad/s^2 until its angular speed is 2760 rev/min . (a) What is the tangential acceleration of a point on the rim of the flywheel during this spin-up process? (b) What is the radial acceleration of this point when the flywheel is spinning at full speed? (c) Through what distance does a point on the rim move during the spin-up?

31 A disk, with a radius of 0.25 m, is to be rotated like a merry-go-round through 800 rad, starting from rest, gaining angular speed at the constant rate α_1 through the first 400 rad and then losing angular speed at the constant rate $-\alpha_1$ until it is again at rest. The magnitude of the centripetal acceleration of any portion of the disk is not to exceed 400 m/s². (a) What is the least time required for the rotation? (b) What is the corresponding value of α_1 ?

32 A car starts from rest and moves around a circular track of radius 32.0 m. Its speed increases at the constant rate of 0.600 m/s². (a) What is the magnitude of its *net* linear acceleration 15.0 s later? (b) What angle does this net acceleration vector make with the car's velocity at this time?

33 (a) A uniform 2.00 kg disk of radius 0.300 m can rotate around its central axis like a merry-go-round. Beginning from rest at time $t = 0$, it undergoes a constant angular acceleration of 30.0 rad/s². When is the rotational kinetic energy equal to 2000 J? (b) Repeat the calculation but substitute a ring of the same mass and radius and assume that the spokes have negligible mass.

34 Figure 10-23 gives angular speed versus time for a thin rod that rotates around one end. The scale on the ω axis is set by $\omega_s = 6.0$ rad/s. (a) What is the magnitude of the rod's angular acceleration? (b) At $t = 4.0$ s, the rod has a rotational kinetic energy of 1.60 J. What is its kinetic energy at $t = 0$?

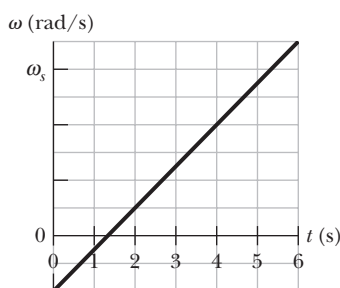


Figure 10-23 Problem 34.

35 A meter stick of negligible mass can rotate about a vertical axis through a point at distance x from the point marked "0". A small block of mass 0.100 kg is glued at the mark of "0" and a small block of mass 0.500 kg is glued at the opposite end, at the mark of "1." The stick and blocks are to rotate with an angular speed of 5.00 rad/min. (a) For what choice of x is the associated rotational kinetic energy least and (b) what is that least energy?

36 Figure 10-24a shows a disk that can rotate about an axis at a radial distance h from the center of the disk. Figure 10-24b gives the rotational inertia I of the disk about the axis as a function of that distance h , from the center out to the edge of the disk. The scale on the I axis is set by $I_A = 0.050$ kg·m² and $I_B = 0.150$ kg·m². What is the mass of the disk?

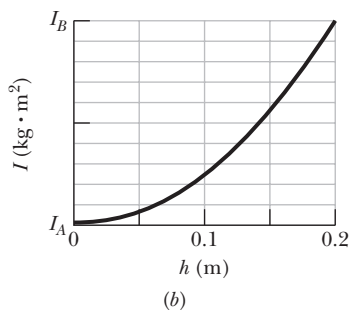
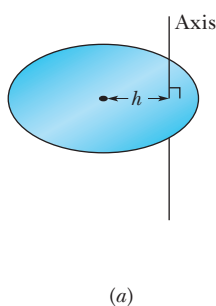


Figure 10-24 Problem 36.

37 A 0.50 kg meter stick can rotate around an axis perpendicular to the stick. Find the difference in the stick's rotational inertia about the rotation axis if that axis is initially at the point marked "40 cm" and then at the point marked "10 cm."

38 Figure 10-25 shows three 0.0100 kg particles that have been glued to a rod of length $L = 8.00$ cm and negligible mass. The assembly can rotate around a perpendicular axis through point O at the left end. If we remove one particle (that is, 33% of the mass), by what percentage does the rotational inertia of the assembly around the rotation axis decrease when that removed particle is (a) the innermost one and (b) the outermost one?

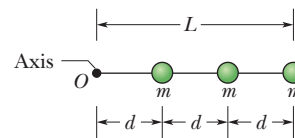


Figure 10-25 Problems 38 and 62.

39 A wheel with a rotational inertia of 0.50 kg·m² about its central axis is initially rotating at an angular speed of 15 rad/s. At time $t = 0$, a man begins to slow it at a uniform rate until it stops at $t = 5.0$ s. (a) By time $t = 3.0$ s, how much work had the man done? (b) For the full 5.0 s, at what average rate did the man do work?

40 Figure 10-26 shows an arrangement of 15 identical disks that have been glued together in a rod-like shape of length $L = 1.0000$ m and (total) mass $M = 100.0$ mg. The disks are uniform, and the disk arrangement can rotate about a perpendicular axis through its central disk at point O . (a) What is the rotational inertia of the arrangement about that axis? (b) If we approximated the arrangement as being a uniform rod of mass M and length L , what percentage error would we make in using the formula in Table 10-2e to calculate the rotational inertia?

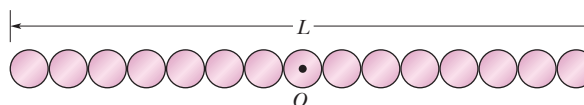


Figure 10-26 Problem 40.

41 In Fig. 10-27, two particles, each with mass $m = 0.85$ kg, are fastened to each other, and to a rotation axis at O , by two thin rods, each with length $d = 5.6$ cm and mass $M = 1.2$ kg. The combination rotates around the rotation axis with the angular speed $\omega = 0.30$ rad/s. Measured about O , what are the combination's (a) rotational inertia and (b) kinetic energy?

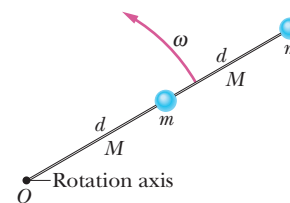


Figure 10-27 Problem 41.

42 Figure 10-28 is an overhead view of a rod of length 1.0 m and mass 1.0 kg that is lying stationary on a frictionless surface when three bullets hit it simultaneously. The bullets move along paths that are in the plane of the rod and perpendicular to the rod. Bullet 1 has mass 10 g and speed 2.0 m/s. Bullet 2 has mass 20 g and speed 3.0 m/s. Bullet 3 has mass 30 g and speed 5.0 m/s. The labelled distances are $a = 10$ cm, $b = 60$ cm, and $c = 80$ cm. As a result of the impacts, the rod-bullets system rotates around its center of mass while the center of mass moves in a straight line over the frictionless surface. (a) What is the linear speed of the system's center of mass? (b) What is the distance between the rod's center and the system's center of mass? (c) What is the rotational inertia of the system about the system's center of mass?

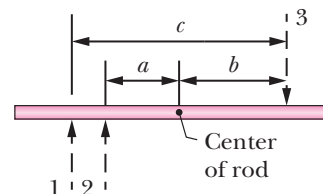


Figure 10-28 Problem 42.

43 The uniform solid block in Fig. 10-29 has mass 0.172 kg and edge lengths $a = 3.5 \text{ cm}$, $b = 8.4 \text{ cm}$, and $c = 1.4 \text{ cm}$. Calculate its rotational inertia about an axis through one corner and perpendicular to the large faces.

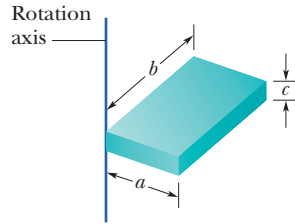


Figure 10-29 Problem 43.

44 Four identical particles of mass 0.75 kg each are placed at the vertices of a $2.0 \text{ m} \times 2.0 \text{ m}$ square and held there by four massless rods, which form the sides of the square. What is the rotational inertia of this rigid body about an axis that (a) passes through the midpoints of opposite sides and lies in the plane of the square, (b) passes through the midpoint of one of the sides and is perpendicular to the plane of the square, and (c) lies in the plane of the square and passes through two diagonally opposite particles?

45 The body in Fig. 10-30 is pivoted at O , and two forces act on it as shown. If $r_1 = 1.30 \text{ m}$, $r_2 = 2.15 \text{ m}$, $F_1 = 4.20 \text{ N}$, $F_2 = 4.90 \text{ N}$, $\theta_1 = 75.0^\circ$, and $\theta_2 = 60.0^\circ$, what is the net torque about the pivot?

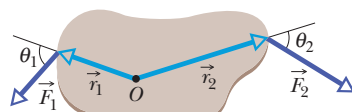


Figure 10-30 Problem 45.

46 The body in Fig. 10-31 is pivoted at O . Three forces act on it: $F_A = 12 \text{ N}$ at point A , 8.0 m from O ; $F_B = 14 \text{ N}$ at point B , 4.0 m from O ; and $F_C = 23 \text{ N}$ at point C , 3.0 m from O . What is the net torque about O ?

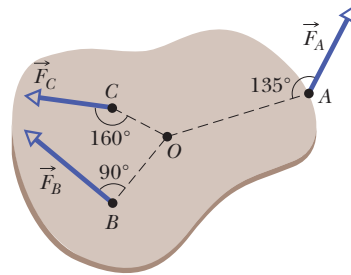


Figure 10-31 Problem 46.

47 A 60 kg father and 20 kg child sit on opposite ends of a seesaw consisting of a board of length 4.0 m and negligible mass. The pivot can be placed anywhere between the father and the child. At what distance from the child should it be placed so that the seesaw is balanced when the father and child are stationary?

48 A 100 kg cubical box lies on a floor. A child pushes horizontally at a top edge. What force magnitude puts the box on the verge of tipping over if there is sufficient friction between it and the floor to prevent sliding?

49 A cord with negligible mass is wrapped around a pulley that is a uniform disk of mass 5.00 kg and radius 0.300 m and that can rotate without friction about its central axis. A 1.0 kg bucket is attached at the free end of the cord hanging down from the pulley and then released at time $t = 0$. The cord begins to unwrap from the pulley as the bucket descends. At $t = 5.00 \text{ s}$, through how many rotations has the pulley turned (the bucket is still descending)?

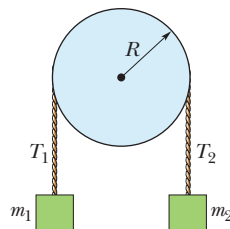


Figure 10-32 Problem 51.

50 If a $42.0 \text{ N} \cdot \text{m}$ torque on a wheel causes angular acceleration 25.0 rad/s^2 , what is the wheel's rotational inertia?

51 In Fig. 10-32, block 1 has mass $m_1 = 460 \text{ g}$, block 2 has mass $m_2 = 500 \text{ g}$, and the pulley, which is mounted on a horizontal axle with negligible friction,

has radius $R = 5.00 \text{ cm}$. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2 and (c) tension T_1 ? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

52 In Fig. 10-33, a cylinder having a mass of 3.0 kg can rotate about its central axis through point O . Forces are applied as shown: $F_1 = 6.0 \text{ N}$, $F_2 = 4.0 \text{ N}$, $F_3 = 2.0 \text{ N}$, and $F_4 = 5.0 \text{ N}$. Also, $r = 5.0 \text{ cm}$ and $R = 12 \text{ cm}$. Find the (a) magnitude and (b) direction of the angular acceleration of the cylinder. (During the rotation, the forces maintain their same angles relative to the cylinder.)

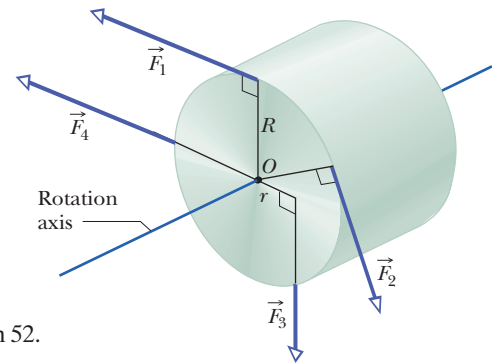


Figure 10-33 Problem 52.

53 Figure 10-34 shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.00 cm and a mass of 20.0 grams and is initially at rest. Starting at time $t = 0$, two forces are to be applied tangentially to the rim as indicated, so that at time $t = 1.25 \text{ s}$ the disk has an angular velocity of 250 rad/s counterclockwise. Force $\vec{F}_1$ has a magnitude of 0.100 N . What is magnitude F_2 ?

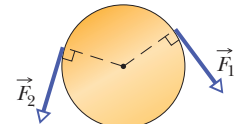


Figure 10-34 Problem 53.

54 In a judo foot-sweep move, you sweep your opponent's left foot out from under him while pulling on his gi (uniform) toward that side. As a result, your opponent rotates around his right foot and onto the mat. Figure 10-35 shows a simplified diagram of your opponent as you face him, with his left foot swept out. The rotational axis is through point O . The gravitational force $\vec{F}_g$ on him effectively acts at his center of mass, which is a horizontal distance $d = 28 \text{ cm}$ from point O . His mass is 75 kg , and his rotational inertia about point O is $65 \text{ kg} \cdot \text{m}^2$. What is the magnitude of his initial angular acceleration about point O if your pull $\vec{F}_a$ on his gi is (a) negligible and (b) horizontal with a magnitude of 300 N and applied at height $h = 1.4 \text{ m}$?

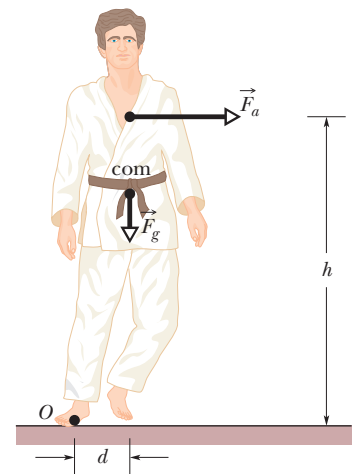


Figure 10-35 Problem 54.

55 In Fig. 10-36a, an irregularly shaped plastic plate with uniform thickness and density (mass per unit volume) is to be rotated around an axle that is perpendicular to the plate face and through point O . The rotational inertia of the plate about that axle is measured with the following method. A circular disk of mass 0.500 kg and radius 2.00 cm is glued to the plate, with its center aligned with point O (Fig. 10-36b). A string is wrapped around the edge of the disk the way a string is wrapped around a top. Then the string is pulled for 5.00 s . As a result, the disk and plate are rotated by a constant force of 0.400 N that is applied by the string tangentially to the edge of the disk. The resulting angular speed is 114 rad/s . What is the rotational inertia of the plate about the axle?

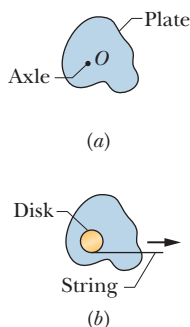


Figure 10-36
Problem 55.

56 Figure 10-37 shows particles 1 and 2, each of mass m , fixed to the ends of a rigid massless rod of length $L_1 + L_2$, with $L_1 = 20\text{ cm}$ and $L_2 = 80\text{ cm}$. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2?

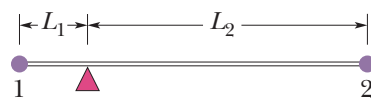


Figure 10-37 Problem 56.

57 A pulley, with a rotational inertia of $1.0 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ about its axle and a radius of 10 cm , is acted on by a force applied tangentially at its rim. The force magnitude varies in time as $F = 0.50t + 0.30t^2$, with F in newtons and t in seconds. The pulley is initially at rest. At $t = 3.0\text{ s}$ what are its (a) angular acceleration and (b) angular speed?

58 (a) If $R = 15\text{ cm}$, $M = 350\text{ g}$, and $m = 50\text{ g}$ in Fig. 10-19, find the speed of the block after it has descended 50 cm starting from rest. Solve the problem using energy conservation principles. (b) Repeat (a) with $R = 5.0\text{ cm}$.

59 A uniform metal pole of height 30.0 m and mass 100 kg is initially standing upright but then falls over to one side without its lower end sliding or losing contact with the ground. What is the linear speed of the pole's upper end just before impact?

60 A thin rod of length 0.75 m and mass 0.42 kg is suspended freely from one end. It is pulled to one side and then allowed to swing like a pendulum, passing through its lowest position with angular speed 3.5 rad/s . Neglecting friction and air resistance, find (a) the rod's kinetic energy at its lowest position and (b) how far above that position the center of mass rises.

61 Disks A and B each have a rotational inertia of $0.300\text{ kg}\cdot\text{m}^2$ about the central axis and a radius of 20.0 cm and are free to rotate on a central rod through both of them. To set them spinning around the rod in the same direction, each is wrapped with a string that is then pulled for 10.0 s (the string detaches at the end). The magnitudes of the forces pulling the strings are 30.0 N for disk A and 20.0 N for disk B . After the strings detach, the disks happen to collide and the frictional force between them brings them to the same final angular speed in 6.00 s . What are (a) magnitude of the average frictional torque that brings them to the final angular speed and (b) the loss in kinetic energy as that torque acts on them? (c) Where did the "lost energy" go?

62 In Fig. 10-25, three 0.0100 kg particles have been glued to a rod of length $L = 6.00\text{ cm}$ and negligible mass and can rotate

around a perpendicular axis through point O at one end. How much work is required to change the rotational rate (a) from 0 to 20.0 rad/s , (b) from 20.0 rad/s to 40.0 rad/s , and (c) from 40.0 rad/s to 60.0 rad/s ? (d) What is the slope of a plot of the assembly's kinetic energy (in joules) versus the square of its rotation rate (in radians-squared per second-squared)?

63 A meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end just before it hits the floor, assuming that the end on the floor does not slip. (Hint: Consider the stick to be a thin rod and use the conservation of energy principle.)

64 A uniform cylinder of radius 12 cm and mass 25 kg is mounted so as to rotate freely about a horizontal axis that is parallel to and 5.0 cm from the central longitudinal axis of the cylinder. (a) What is the rotational inertia of the cylinder about the axis of rotation? (b) If the cylinder is released from rest with its central longitudinal axis at the same height as the axis about which the cylinder rotates, what is the angular speed of the cylinder as it passes through its lowest position?

65 A tall, cylindrical chimney falls over when its base is ruptured. Treat the chimney as a thin rod of length 55.0 m . At the instant it makes an angle of 35.0° with the vertical as it falls, what are (a) the radial acceleration of the top, and (b) the tangential acceleration of the top. (Hint: Use energy considerations, not a torque.) (c) At what angle θ is the tangential acceleration equal to g ?

66 A uniform spherical shell of mass $M = 4.5\text{ kg}$ and radius $R = 8.5\text{ cm}$ can rotate about a vertical axis on frictionless bearings (Fig. 10-38). A massless cord passes around the equator of the shell, over a pulley of rotational inertia $I = 3.0 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ and radius $r = 5.0\text{ cm}$, and is attached to a small object of mass $m = 0.60\text{ kg}$. There is no friction on the pulley's axle; the cord does not slip on the pulley. What is the speed of the object when it has fallen 82 cm after being released from rest? Use energy considerations.

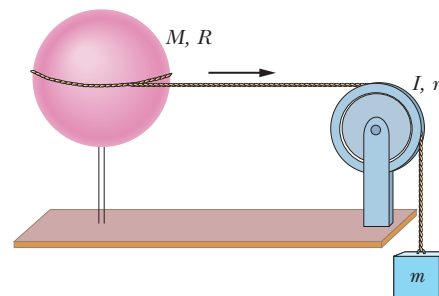


Figure 10-38 Problem 66.

67 Figure 10-39 shows a rigid assembly of a thin hoop (of mass m and radius $R = 0.150\text{ m}$) and a thin radial rod (of mass m and length $L = 2.00R$). The assembly is upright, but if we give it a slight nudge, it will rotate around a horizontal axis in the plane of the rod and hoop, through the lower end of the rod. Assuming that the energy given to the assembly in such a nudge is negligible, what would be the assembly's angular speed about the rotation axis when it passes through the upside-down (inverted) orientation?

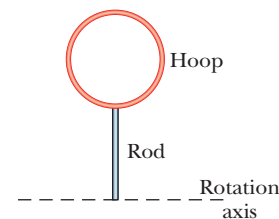


Figure 10-39 Problem 67.