15.7 : Qualitative Independent variables.

exp on Book (table 15.5).

Y : Repair time in hours.

X_1 : Months since last service.

X_2 : Type of Repair.

$$X_2 = \begin{cases} 0, & \text{if type of repair Mechanical.} \\ 1, & \text{if type of repair Electrical.} \end{cases}$$

X_2 : Dummy variable / indicator variable.

هنا بغير X_2 حسب ما عرفنا .

X_1	X_2	Y
نفسها	1	
	0	
	1	
	0	
	1	
	1	
	0	
	0	
	1	
	1	

بنعطي على الكل وبنكمل المحاضرة الجاي :)

←

exp on Excel :

Model : • $\hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ (MLRM)

• $X_2 = \begin{cases} 1 & , \text{Electrical repair.} \\ 0 & , \text{Mechanical repair} \end{cases}$

• $\hat{y} = 0.93 + 0.39 X_1 + 1.26 X_2$ (EMLRE).

• $R^2 = 0.86$
• $adj R^2 = 0.82$ } → The model has goodnes of fit.

→ $H_0 : \beta_1 = \beta_2 = 0$ (The Model is Not significant)

H_1 : Not all β_i are zero. (The Model is significant).

p-value = 0.001 , $\alpha = 0.01$

p-value $< \alpha \Rightarrow$ Reject H_0 ($\alpha = 0.01$)

$\Rightarrow$ The Model is significance ($\alpha = 0.01$).

→ $H_0^{(1)} : \beta_1 = 0$ (The variable X_1 is not significant)

$H_1^{(1)} : \beta_1 \neq 0$ (The variable X_1 is significant).

p-value = 0.0004 , $\alpha = 0.01$

→ p-value $< \alpha \Rightarrow$ Reject H_0' ($\alpha = 0.01$)

$\Rightarrow$ X_1 is a significant variable. ($\alpha = 0.01$).

→

$H_0^{(1)}$: $\beta_2 = 0$ (X_2 not significant variable)

$H_1^{(1)}$: $\beta_2 \neq 0$ (X_2 significant variable).

p-value = 0.0005 , $\alpha = 0.01$

p-value $< \alpha \Rightarrow$ Reject $H_0^{(2)}$ ($\alpha = 0.01$)

$\Rightarrow$ X_2 significant variable ($\alpha = 0.01$).

→ Validaty of Model Assumptions :

	X_1	X_2
A_1	✓	✓
A_2	✓	✓
A_3	✓	✓
A_4	✓	

Remark :

1. We can study the multicolinearity in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ if it exists.

2. We can also study the model $Y = \beta_0 + \beta_1 X_1 + \varepsilon$ and The Model $Y = \beta_0 + \beta_2 X_2 + \varepsilon$.

Model :

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \quad (MLRM).$$

$$X_2 = \begin{cases} 1, & \text{Electrical repair.} \\ 0, & \text{Mechanical repair.} \end{cases}$$

$$\hat{y} = 0.93 + 0.39 X_1 + 1.26 X_2 \quad (EMLRE).$$

- X_1 : months since last repair.
- X_2 : type of repair.
- y : Repair time (hours).

★ Estimate the repair time if the last repair took place 3 months ago and All type of repair is Mechanical.

$$X_1 = 3 \qquad X_2 = 0$$

$$\hat{y} = 0.93 + 0.39(3) + 1.26(0).$$

$$= 2.1 \text{ hours}.$$

★ Estimate the repair time if the last repair took place 3 months ago and all type of repair is Electrical.

$$X_1 = 3 \qquad X_2 = 1$$

$$\hat{y} = 0.93 + 0.39(3) + 1.26(1)$$

$$= 3.36 \text{ hours}.$$

Remark:

$\hat{y}(3,0) = 2.1$ hours.

$\hat{y}(3,1) = 3.36$ hours.

$E(y/\text{Mech.}) = \beta_0 + \beta_1 x_1$

$E(y/\text{Elect.}) = \beta_0 + \beta_1 x_1 + \beta_2$.

Exercises:

2: Consider a regression study involving a dependent variable y, a quantitative independent variable x_1 and a qualitative variable with two le... (level 1, level 2).

a. Write a multiple regression equation relating x_1 and qualitative variable to y.

$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

Where $x_2 = \begin{cases} 0, & \text{level 1} \\ 1, & \text{level 2.} \end{cases}$

b. What is the expected value of y corresponding to level 1 of the qualitative variable?

$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (0)$

$= \beta_0 + \beta_1 x_1$.

c. What is the expected value of y corresponding to level 2 of ... ?

$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 (1)$

$= \beta_0 + \beta_1 x_1 + \beta_2$.