AF

0.7 Algebraic Fractions 1

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AF

بسط / مقام = numerator / denominator

special case when Poly / Poly = Rational function (RF)

Poly: كثيرة حدود

n: عدد صحيح موجب أو صفر

Exp ①

$\frac{x^1 - 1}{2x^2 - 5}$ → AF (also RF)

②

$\frac{x^3 - 5}{1 - \sqrt{x}}$ ($\sqrt{x} = x^{\frac{1}{2}}$) → AF (but not RF)

③

$\frac{5}{2x^1}$ ($5x^0$) → AF (also RF)

④

$\frac{5x^2 - 3x + 7}{1 - x^{\frac{2}{3}}}$ → AF (but not RF)

Poly

not poly

not poly

Exp Simplify the following AF's

① $\frac{x-2y}{2x-4y} = \frac{x-2y}{2(x-2y)} = \frac{1}{2}$

② $\frac{x^2-5x+6}{9-x^2} = \frac{(x-3)(x-2)}{(3-x)(3+x)}$

$ab = 6$

$a+b = -5$

$= \frac{-(-x+3)(x-2)}{(3-x)(3+x)} = \frac{-(x-2)}{x+3} = \frac{-x+2}{x+3}$ أبسط صورة

$3-x = -x+3$

$2-4y = -[-2+4y]$

$2x+4y = -[-2x-4y]$

$-x^3-z^3 = -[x^3+z^3]$

③ $\frac{1-4x^2}{(1-2x)^2} = \frac{(1-2x)(1+2x)}{(1-2x)(1-2x)} = \frac{1+2x}{1-2x}$ أبسط صورة

$x^2-4 = (x+2)(x-2)$

$1-9x^2 = (1-3x)(1+3x)$

$= \frac{1+2x}{-(1+2x)}$

$y^2 = y \cdot y$

$$(1-2x)^2 = (1-2x)(1-2x)$$

Remark $\quad \frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D}$

Exp (Multiply AF's) Simplify

① $\frac{2x^2}{y} \cdot \frac{5x^3}{3y^3} \cdot \frac{x}{y^2} = \frac{2x^2 \cdot 5x^3 \cdot x}{y \cdot 3y^3 \cdot y^2} = \frac{10x^6}{3y^6}$

② $\frac{2x}{5y} \cdot \frac{4x^2}{y^2} \cdot \frac{y}{3x^2} = \frac{2x \cdot 4x^2 \cdot y}{5y \cdot y^2 \cdot 3x^2} = \frac{8x^3y}{15y^3x^2}$

$= \frac{8x^{3-2}}{15y^{3-1}} = \frac{8x}{15y^2}$

③ $\frac{-4x+8}{3x+6} \cdot \frac{2x+4}{4x+12} = \frac{(-4x+8)(2x+4)}{(3x+6)(4x+12)}$

$= \frac{-4x \cdot 2x + \overset{-16x}{-4x \cdot 4} + \overset{16x}{8 \cdot 2x} + 8 \cdot 4}{3x \cdot 4x + 3x \cdot 12 + 6 \cdot 4x + 6 \cdot 12}$

$= \frac{-8x^2 + 32}{12x^2 + 36x + 24x + 72} = \frac{-8x^2 + 32}{12x^2 + 60x + 72}$

$12x^2 + 36x + 24x + \dots$

$$= \frac{-8(x^2-4)}{12[x^2+5x+6]} = \frac{-2(x-2)(x+2)}{3(x+2)(x+3)}$$

$ab = 6$, $a+b = 5$

$$= \frac{-2(x-2)}{3(x+3)}$$

مع آخر

$$\frac{-4x+8}{3x+6} \cdot \frac{2x+4}{4x+12} = \frac{(-4x+8)(2x+4)}{(3x+6)(4x+12)}$$

$$= \frac{-4(x-2)(2)(x+2)}{3(x+2)(4)(x+3)}$$

$$= \frac{-2(x-2)}{3(x+3)}$$

(4) $$(x^2-9) \cdot \frac{2x+1}{x-3} = \frac{(x^2-9)(2x+1)}{(x-3)} = \frac{(x-3)(x+3)(2x+1)}{(x-3)}$$

$$= (x+3)(2x+1)$$

Remark

$$\frac{\frac{A}{B}}{\frac{C}{D}} = \frac{A}{B} \cdot \frac{D}{C}$$

$$\frac{\frac{3}{2}}{\frac{4}{5}} = \frac{3}{2} \cdot \frac{5}{4} = \frac{15}{8}$$

Exp (Dividing Fractions)

Simplify

① $\dfrac{16ac^2}{7bd} \div \dfrac{4a}{14b^2d}$ $=$ (or) $\dfrac{\frac{16ac^2}{7bd}}{\frac{4a}{14b^2d}}$

$$\frac{16ac^2}{7bd} \cdot \frac{14b^2d}{4a} = \frac{16ac^2 \cdot 14b^2d}{7bd \cdot 4a} = 8b^{2-1}c^2$$

$$= \frac{16 \cdot 14}{7 \cdot 2} = 8bc^2 \checkmark$$