Question 1 Solution:

The expression

$$F(s) = \frac{24s}{(s+2)(s+4)(s+6)}$$

can be written in a partial fraction expansion of the form

$$\frac{24s}{(s+2)(s+4)(s+6)} = \frac{k_1}{s+2} + \frac{k_2}{s+4} + \frac{k_3}{s+6}$$

Multiplying the entire equation by the term s + 2 yields

$$\frac{24s}{(s+4)(s+6)} = k_1 + \frac{k_2(s+2)}{s+4} + \frac{k_3(s+2)}{s+6}$$

If we now evaluate each term at s = -2, we find that the last two terms on the right side of the equation vanish and we have

$$\left.\frac{24s}{(s+4)(s+6)}\right|_{s=-2} = k_1$$

$$-6 = k_1$$

Repeating this procedure for the two remaining terms in the denominator, i.e., (s + 4) and (s + 6) yields

$$\left.\frac{24s}{(s+2)(s+6)}\right|_{s=-4} = k_2$$

$$24 = k_2$$

And

$$\left.\frac{24s}{(s+2)(s+4)}\right|_{s=-6} = k_3$$

$$-18 = k_3$$

Now the function F(s) can be written in the form

$$F(s) = \frac{-6}{s+2} + \frac{24}{s+4} - \frac{18}{s+6}$$

The reader can check the validity of this expansion by recombining the terms to produce the original expression.

Once F(s) is in this latter form, we can use the transform pair

$$L\left[e^{-at}\right] = \frac{1}{s+a}$$

And hence

$$f(t) = \left[-6e^{-2t} + 24e^{-4t} - 18e^{-6t}\right]u(t)$$

Question 2 Solution:

We begin by writing the function in a partial fraction expansion. Therefore, we need to know the roots of the quadratic term. We can either employ the quadratic formula or recognize that

$$\begin{aligned} s^2 + 8s + 20 &= s^2 + 8s + 16 + 4 \\ &= (s+4)^2 + 4 \\ &= (s+4-j2)\,(s+4+j2) \end{aligned}$$

Hence, the function F(s) can be written as

$$F(s) = \frac{4(s+4)}{s(s+4-j2)(s+4+j2)} = \frac{k_0}{s} + \frac{k_1}{s+4-j2} + \frac{k_1^*}{s+4+j2}$$

Multiplying the entire equation by s and evaluating it at s = 0 yields

$$\left.\frac{4(s+4)}{s^2 + 8s + 20}\right|_{s=0} = k_0$$

$$\frac{4}{5} = k_0$$

Using the same procedure for k_1, we obtain

$$\left.\frac{4(s+4)}{s(s+4+j2)}\right|_{s=-4+j2} = k_1$$

$$\frac{1}{-2+j} = k_1$$

$$\frac{-1}{2-j} = k_1$$

$$\frac{-(2+j)}{5} = k_1$$

$$\frac{1}{\sqrt{5}} \angle 206.56° = k_1$$

Then, we know that

$$\frac{1}{\sqrt{5}} \angle -206.56° = k_1^*$$

Now using the fact that

$$L\left[\frac{|k_1|\angle\theta}{s+a-jb} + \frac{|k_1|\angle-\theta}{s+a+jb}\right] = 2|k_1|\, e^{-at} \cos(bt+\theta)$$

The function f(t) is

$$f(t) = \left[\frac{4}{5} + \frac{2}{\sqrt{5}} e^{-4t} \cos(2t + 206.56°)\right] u(t)$$

Question 3 Solution:

In order to perform a partial fraction expansion on the function F(s), we need to factor the quadratic term. We can use the quadratic formula or simply note that (s + 1) (s + 1) = s^2 + 2s + 1. Therefore, F(s) can be expressed as

$$F(s) = \frac{12(s+2)}{(s+1)^2 (s+3)}$$

or in the form

$$F(s) = \frac{12(s+2)}{(s+1)^2 (s+3)} = \frac{k_{11}}{s+1} + \frac{k_{12}}{(s+1)^2} + \frac{k_2}{s+3}$$

If we now multiply the entire equation by $(s + 1)^2$, we obtain

$$\frac{12(s+2)}{s+3} = k_{11}(s+1) + k_{12} + \frac{k_2 (s+1)^2}{s+3}$$

Now evaluating this equation at s = -1 yields

$$\left.\frac{12(s+2)}{s+3}\right|_{s=-1} = k_{12}$$

$$6 = k_{12}$$

In order to evaluate k_{11} we differentiate each term of the equation with respect to s and evaluate all terms at s = -1. Note that the derivative of k_{12} with respect to s is zero, the derivative of the last term in the equation with respect to s will still have an (s + 1) term in the numerator that will vanish when evaluated at s = -1, and the derivative of the first term on the right side of the equation with respect to s simply yields k_{11}. Therefore,

$$\frac{d}{ds}\left[\frac{12(s+2)}{s+3}\right]_{s=-1} = k_{11}$$

$$\left.\frac{(s+3)(12) - 12(s+2)(1)}{(s+3)^2}\right|_{s=-1} = k_{11}$$

$$3 = k_{11}$$

Finally,

$$\left.\frac{12(s+2)}{(s+1)^2}\right|_{s=-3} = k_2$$

$$-3 = k_2$$

And therefore, F(s) can be expressed in the form

$$F(s) = \frac{3}{s+1} + \frac{6}{(s+1)^2} - \frac{3}{s+2}$$

Using the transform pairs, we find that

$$f(t) = \left[3e^{-t} + 6te^{-t} - 3e^{-2t}\right]u(t)$$

Question 4 Solution:

First, let us use the Theorems to evaluate the function in the s-domain.

The initial value can be derived from the Theorem

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

Therefore,

$$\begin{aligned}\lim sF(s) &= \lim_{s \to \infty}\left[\frac{24(s+10)}{(s+2)(s+4)}\right] \\ &= \lim_{s \to \infty}\left[\frac{24s+240}{s^2+6s+8}\right] \\ &= \lim_{s \to \infty}\left[\frac{\frac{24}{s}+\frac{240}{s^2}}{1+\frac{6}{s}+\frac{8}{s^2}}\right] \\ &= 0\end{aligned}$$

The final value is derived from the expression

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

Hence,

$$\begin{aligned}\lim_{s \to 0} sF(s) &= \lim_{s \to 0}\left[\frac{24(s+10)}{(s+2)(s+4)}\right] \\ &= \frac{240}{8} \\ &= 30\end{aligned}$$

The time function can be derived from a partial fraction expansion as

$$F(s) = \frac{24(s+10)}{s(s+2)(s+4)} = \frac{k_0}{s} + \frac{k_1}{s+2} + \frac{k_2}{s+4}$$

where

$$\left.\frac{24(s+10)}{(s+2)(s+4)}\right|_{s=0} = k_0 = 30$$

$$\left.\frac{24(s+10)}{s(s+4)}\right|_{s=-2} = k_1 = -48$$

$$\left.\frac{24(s+10)}{s(s+2)}\right|_{s=-4} = k_2 = 18$$

Hence,

$$F(s) = \frac{30}{s} - \frac{48}{s+2} + \frac{18}{s+4}$$

and then

$$f(t) = \left[30 - 48e^{-2t} + 18e^{-4t}\right]u(t)$$

Given this expression, we find that

$$\lim_{t \to 0} f(t) = [30 - 48 + 18] = 0$$

and

$$\lim_{t \to \infty} f(t) = [30 - 0 + 0] = 30$$

Question 5 Solution:

(a) Consider the transformed network in Fig. S14.1(a).

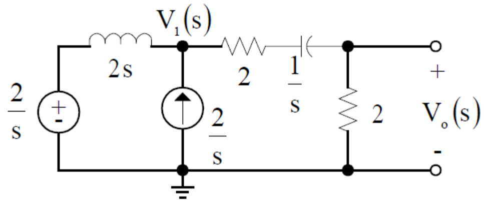

Fig. S14.1(a)

A brute force approach to this problem would be to write two nodal equations for the nodes labeled $V_1(s)$ and $V_0(s)$. Using KCL and summing the currents leaving each node yields the two linearly independent equations

$$\frac{V_1(s)-\frac{2}{s}}{2s}-\frac{2}{s}+\frac{V_1(s)-V_0(s)}{2+\frac{1}{s}}=0$$

and

$$\frac{V_0(s)-V_1(s)}{2+\frac{1}{s}}+\frac{V_0}{2}=0$$

Solving these equations for $V_0(s)$ and then performing the inverse Laplace transform would yield $v_0(t)$.

Another approach that might be simpler would be to write a node equation for $V_1(s)$, ignoring $V_0(s)$, and then use voltage division to derive $V_0(s)$ once $V_1(s)$ is known. Applying KCL at $V_1(s)$ yields

$$\frac{V_1(s)-\frac{2}{s}}{2s}-\frac{2}{s}+\frac{V_1(s)}{4+\frac{1}{s}}=0$$

Rearranging terms we obtain

$$V_1(s)\left[\frac{1}{2s}+\frac{s}{4s+1}\right]=\frac{1}{s^2}+\frac{2}{s}$$

or

$$V_1(s)\left[\frac{2s^2+4s+1}{2s(4s+1)}\right]=\frac{2s+1}{s^2}$$

Solving for $V_1(s)$ yields

$$V_1(s)=\frac{2(2s+1)(4s+1)}{s(2s^2+4s+1)}$$

Now applying voltage division

$$V_0(s)=V_1(s)\left(\frac{2}{4+\frac{1}{s}}\right)$$
$$=\frac{4(2s+1)}{2s^2+4s+1}$$

This function can be written in partial fraction expansion form as

$$\frac{4s+2}{s^2+2s+\frac{1}{2}}=\frac{A}{s+0.29}+\frac{B}{s+1.71}$$

where

$$A=\left.\frac{4s+2}{s+1.71}\right|_{s=-0.29}=0.59$$

and

$$B=\left.\frac{4s+2}{s+0.29}\right|_{s=-1.71}=3.41$$

Therefore,

$$v_0(t)=\left[0.59e^{-0.29t}+3.41e^{-1.71t}\right]u(t)\text{ V}$$

(b) Using source transformation we can convert the voltage source in series with the inductor to a current source in parallel with the inductor yielding the network in Fig. S14.1(b).

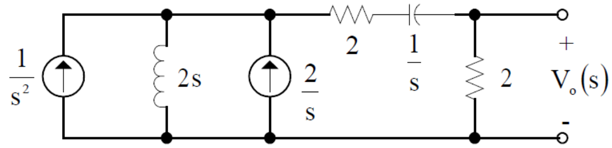

Fig. S14.1(b)

Adding the current sources that are in parallel produces an equivalent source of

$$I_{EQ}(s) = \frac{1}{s^2} + \frac{2}{s} = \frac{2s+1}{s^2}$$

The network is then reduced to that shown in Fig. S14.1(c).

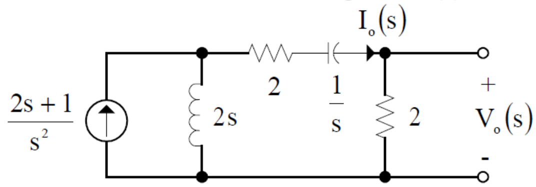

Fig. S14.1(c)

We could, at this point, transform the current source and inductor back to a voltage source in series with the inductor. However, we can simply apply current division at this point with Ohm's Law and derive the answer immediately.

$$I_o(s) = \frac{2s+1}{s^2}\left(\frac{2s}{2s+2+\frac{1}{s}+2}\right)$$

$$= \frac{4s+2}{2s^2+4s+1}$$

And

$$V_o(s) = 2I_o(s) = \frac{4s+2}{s^2+2s+\frac{1}{2}}$$

which is identical to the expression obtained earlier.

(c) To apply Norton's Theorem we will break the network to the right of the current source and form a Norton equivalent circuit for the elements to the left of the break as shown in Fig. S14.1(d).

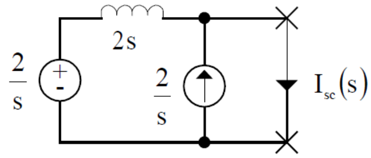

Fig. S14.1(d)

The short-circuit current is

$$I_{sc}(s) = \frac{\frac{2}{s}}{2s} + \frac{2}{s}$$
$$= \frac{2s+1}{s^2}$$

And the Thevenin equivalent impedance is derived from the network in Fig. S14.1(e) as

$$Z_{TH}(s) = 2s$$

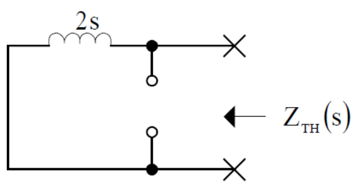

Fig. S14.1(e)

Therefore, attaching the Norton equivalent circuit to the remainder of the network yields the circuit in Fig. S14.1(f) which is the same as that in Fig. S14.1(c).

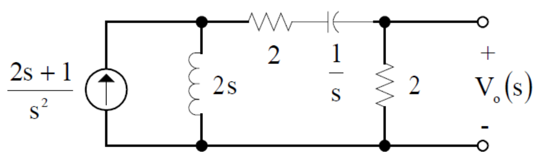

Fig. S14.1(f)

Question 6 Solution:

(a) the transformed network is shown in Fig. S14.2(a).

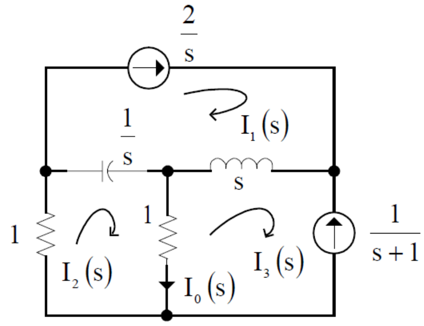

Fig. S14.2(a)

Since there are three "window panes" we will need three linearly independent simultaneous equations to calculate the loop currents. Two of the currents go directly through the current sources and therefore two of the three equations are

$$I_1(s) = \frac{2}{s}$$

$$I_3(s) = \frac{-1}{s+1}$$

The remaining equation is obtained by using KVL around the loop defined by the current $I_2(s)$. That equations is

$$1I_2(s) + \frac{1}{s}\left[I_2(s) - I_1(s)\right] + 1\left[I_2(s) - I_3(s)\right] = 0$$

Substituting the first two equations into the last equation yields

$$I_2(s)\left[1 + \frac{1}{s} + 1\right] = \frac{2}{s^2} - \frac{1}{s+1}$$

or

$$I_2(s) = \frac{-s^2 + 2s + 2}{s(s+1)(2s+1)}$$

Then

$$\begin{aligned} I_0(s) &= I_2(s) - I_3(s) \\ &= \frac{-s^2 + 2s + 2}{s(s+1)(2s+1)} + \frac{1}{s+1} \\ &= \frac{s^2 + 3s + 2}{s(s+1)(2s+1)} \\ &= \frac{s+2}{s(2s+1)} \\ &= \frac{\frac{1}{2}(s+2)}{s\left(s+\frac{1}{2}\right)} \end{aligned}$$

Expressing this function in partial fraction expansion form we obtain

$$I_0(s) = \frac{\frac{1}{2}(s+2)}{s\left(s+\frac{1}{2}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{2}}$$

where

$$A = \left.\frac{\frac{1}{2}(s+2)}{s+\frac{1}{2}}\right|_{s=0} = 2$$

$$B = \left.\frac{\frac{1}{2}(s+2)}{s}\right|_{s=-\frac{1}{2}} = -\frac{3}{2}$$

Therefore,

$$i_0(t) = \left[2 - \frac{3}{2}e^{-\frac{t}{2}}\right]u(t)\,A$$

(b) In order to apply Thevenin's Theorem, we first break the circuit between the points where the current $I_0(s)$ is located as shown in Fig. S14.2(b).

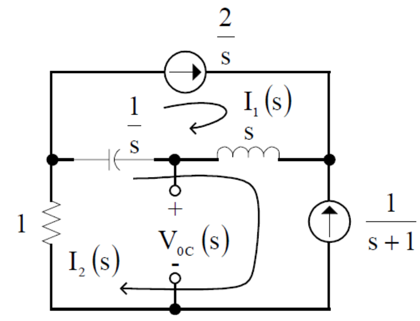

Fig. S14.2(b)

Applying KVL to the closed path in the lower left-hand corner of the network yields

$$1I_2(s) + \frac{1}{s}[I_2(s) - I_1(s)] + V_{0C}(s) = 0$$

where

$$I_1(s) = \frac{2}{s}$$
$$I_2(s) = \frac{-1}{s+1}$$

Combining these equations we obtain

$$V_{0C}(s) = \frac{1}{s+1} + \frac{1}{s(s+1)} + \frac{2}{s^2}$$
$$= \frac{s+2}{s^2}$$

The Thevenin equivalent impedance obtained by looking into the open circuit terminals with all sources made zero (current sources open-circuited) is derived from the network in Fig. S14.2(c).

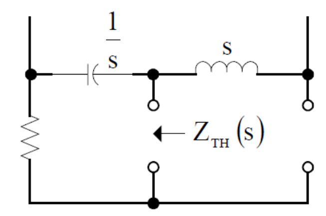

Fig. S14.2(c)

Clearly,

$$Z_{TH}(s) = \frac{1}{s} + 1 = \frac{s+1}{s}$$

If the resistor containing the $I_0(s)$ is now attached to the Thevenin equivalent circuit we obtain the network in Fig. S14.2(d).

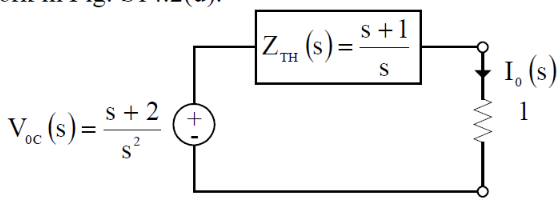

Fig. S14.2(d)

Then

$$I_0(s) = \frac{\frac{s+2}{s^2}}{\frac{s+1}{s} + 1}$$

$$= \frac{s+2}{s(2s+1)}$$

which is identical to the result obtained earlier.

Question 7 Solution:

To begin, we first determine the initial conditions in the network prior to switch action. In the steady-state period prior to switch action, the capacitor looks like an open-circuit and the inductor acts like a short-circuit. Therefore, in this time interval the circuit appears as that shown in Fig. S14.3(a).

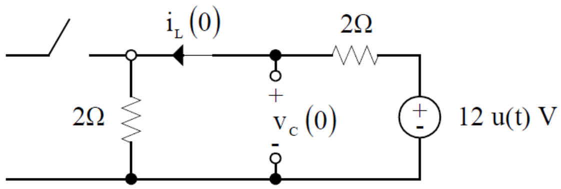

Fig. S14.3(a)

This network indicates that in the steady-state condition for $t < 0$

$$i_L(0) = \frac{12}{2+2} = 3A$$

and

$$v_c(0) = 12\left(\frac{2}{2+2}\right) = 6V$$

These conditions cannot change instantaneously and hence the network for t > 0 is shown in Fig. S14.3(b).

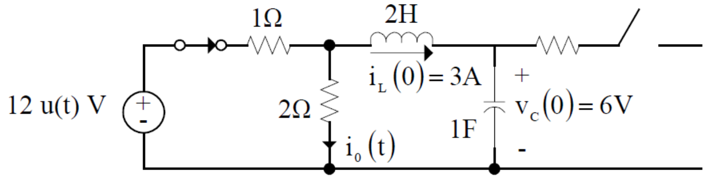

Fig. S14.3(b)

The corresponding transformed network is shown in Fig. S14.3(c).

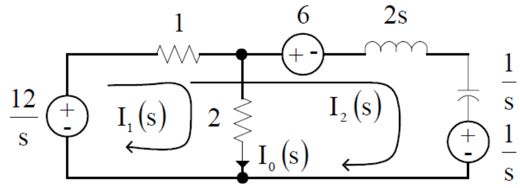

Fig. S14.3(c)

Since the current $I_0(s)$ is located in the center leg of the circuit, we will employ loop equations and specify them such that one of the loops is the same as $I_0(s)$. The two equations for the loop currents specified in the network are

$$\frac{-12}{s} + 1(I_1(s) + I_2(s)) + 2I_1(s) = 0$$

$$\frac{-12}{s} + 1(I_1(s) + I_2(s)) + 6 + 2sI_2(s) + \frac{1}{s}I_2(s) + \frac{1}{s} = 0$$

Solving the second equation for $I_2(s)$ yields

$$I_2(s) = \frac{11 - 6s - sI_1(s)}{2s^2 + s + 1}$$

Substituting this value into the first equation we obtain

$$I_1(s) = I_0(s) = \frac{\frac{1}{6}(30s^2 + s + 12)}{s\left(s^2 + \frac{1}{3}s + \frac{1}{2}\right)}$$

The roots of the quadratic term in the denominator, obtained using the quadratic formula, are

$$s_1, s_2 = -\frac{1}{6} \pm j\frac{\sqrt{17}}{6}$$

The expression for the desired current can now be written in partial fraction expansion form as

$$\frac{\frac{1}{6}\left(30s^2 + s + 12\right)}{s\left(s + \frac{1}{6} \pm j\frac{\sqrt{17}}{6}\right)} = \frac{A}{s} + \frac{B}{s - \frac{1}{6} + j\frac{\sqrt{17}}{6}} + \frac{B^*}{s + \frac{1}{6} + j\frac{\sqrt{17}}{6}}$$

where

$$\left.\frac{\frac{1}{6}\left(30s^2 + s + 12\right)}{s^2 + \frac{1}{3}s + \frac{1}{2}}\right|_{s=0} = A$$

$$4 = A$$

and

$$\left.\frac{\frac{1}{6}\left(30s^2 + s + 12\right)}{s\left(s + \frac{1}{6} + j\frac{\sqrt{17}}{6}\right)}\right|_{s=-\frac{1}{6}+j\frac{\sqrt{17}}{6}} = B$$

The evaluation of this last term involves a lot of tedious, but straight forward, complex algebra. The result is

$$1.09\angle 62.74° = B$$

Therefore, knowing the values for A and B we can write the final expression for the current in the time domain as

$$i_0(t) = \left[4 + 2(1.09)e^{-\frac{t}{6}}\cos\left(\frac{\sqrt{17}}{6}t + 62.74°\right)\right]u(t)\text{ A}$$

Question 8 Solution:

(a) The transformed network is shown in Fig. S14.4.

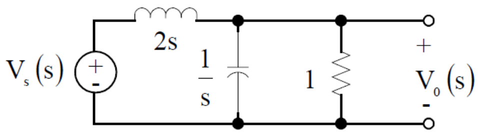

Fig. S14.4

Using voltage division, the voltage transfer function can be expressed as

$$G(s) = \frac{V_0(s)}{V_s(s)} = \frac{\dfrac{2\left(\dfrac{1}{s}\right)}{2+\dfrac{1}{s}}}{2s + \dfrac{2\left(\dfrac{1}{s}\right)}{2+\dfrac{1}{s}}}$$

$$= \frac{2}{4s^2 + 2s + 2}$$

$$= \frac{\frac{1}{2}}{s^2 + \frac{1}{2}s + \frac{1}{2}}$$

(b) + (c) easy to solve, solve these two parts!

(d) If the input to the network is a unit step function then

$$V_o(s) = \frac{\frac{1}{2}}{s\left(s^2 + \frac{1}{2}s + \frac{1}{2}\right)}$$

By employing the quadratic formula, we can write this expression in the form

$$V_o(s) = \frac{\frac{1}{2}}{s\left(s + \frac{1}{4} \pm j\frac{\sqrt{7}}{4}\right)}$$

and therefore the general form of the response is

$$v_o(t) = \left[A + B\,e^{-\frac{1}{4}t}\cos\left(\frac{\sqrt{7}}{4}t + \theta\right)\right]u(t)\ \text{v}$$

Question 9 Solution:

The transformed circuit is shown in Fig. S14.5.

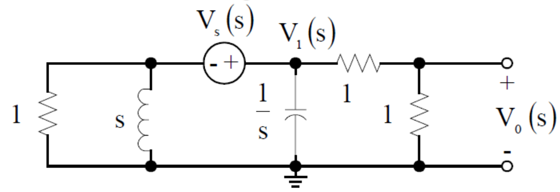

Fig. S14.5

Although the network contains three non-reference nodes, we will try to simplify the analysis by first using a supernode to find $V_1(s)$ and then employing voltage division to determine $V_o(s)$.

KCL for the supernode containing the voltage source is

$$\frac{V_1(s) - V_s(s)}{1} + \frac{V_1(s) - V_s(s)}{s} + \frac{V_1(s)}{\frac{1}{s}} + \frac{V_1(s)}{2} = 0$$

Solving this equation for $V_1(s)$ yields

$$V_1(s) = \left(\frac{s+1}{s^2 + \frac{3}{2}s + 1} \right) V_s(s)$$

And then using voltage division

$$V_o(s) = V_1(s) \left(\frac{1}{1+1} \right)$$

so that

$$V_o(s) = \left[\frac{\frac{1}{2}(s+1)}{s^2 + \frac{3}{2}s + 1} \right] V_s(s)$$

Therefore,

$$H(s) = \frac{\frac{1}{2}(s+1)}{s^2 + \frac{3}{2}s + 1}$$

Since $v_s(t) = 6 \cos 2t\, u(t)$ V, then $V_M = 6$ and $\omega_0 = 2$. Hence,

$$H(j2) = \frac{\frac{1}{2}(j2+1)}{(j2)^2 + \frac{3}{2}(j2) + 1}$$

$$= \frac{-\frac{1}{2}(2.236 \angle 63.43°)}{4.24 \angle -45°}$$

$$= 0.264 \angle -71.57°$$

and

$$|H(j2)| = 0.264$$

$$\phi(j2) = -71.57°$$

Therefore,

$$v_{0ss}(t) = V_M \left| H(j2) \right| \cos\left(2t + \phi(j2)\right)$$
$$= 1.58 \cos\left(2t - 71.57°\right) \text{ V}$$