Kazem jaber

Structural analysis

introduction

A structure may be defined as a system of connected parts used to support a load

- For a structure to be designed properly, the designer should carefully consider the followins

- safety : التصميم ال members بحيث تتحمل ال loads دون failure ميكانيكيا
- serviceability : الحفاظ على انه لا يكون ال deformation كبير بحيث يؤثر على فاعلية ال structure
- Economy.
- Esthetic: تحقيق متطلبات المعماري من حيث الجمالية والمنطقية في التقيم.
- Environment: محاولة انه يكون ال structure صديق environmentally مثل مثلا ال green building وال green concrete الي بقللو من الانبعاثات

- The structural engineer must carefully study the following.
  - the structural loads that the structure would be exposed to.
  - ~~the~~ material properties used in the structures (stress strain diagram which shows how a material responds to load.
  - And type of structural system to be used (trusses, frame, arch) ..etc
    حسب الاستخدام + ال archetictur requirement

Types of structural elements

1. Bar elements: most simple structural elements

- structural members subjected to only axial force, either compression or tension force.

Tie rods (tension)
Bracing struts (compression)
Truss members.

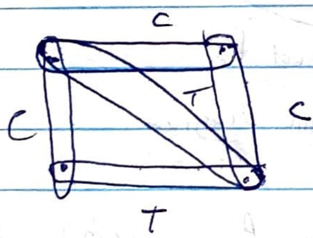
ال Bar elements يتم ~~[illegible]~~ استخدامها بحيث تتحمل axial forces، فقط عند استخدامها
تصميم ال truss

في truss system . كونها تتحمل axial forces فقط يتم استخدام pin connection بين

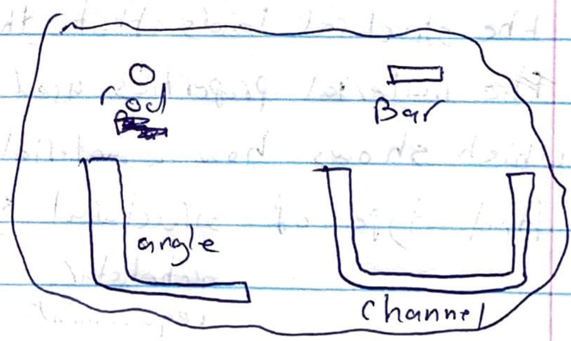
ال different bar elements ، وهاي ال pin connection بتحول ال loads

بحيث ال bar elements ل axial force مع الحفاظ على إمكانية

ال rotation لل elements .

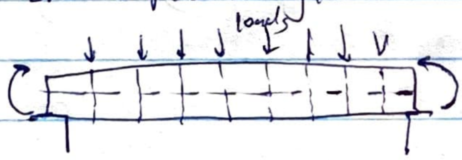


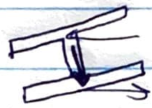
Tie rods.



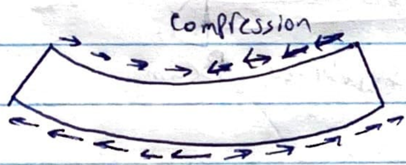
typical cross sections.

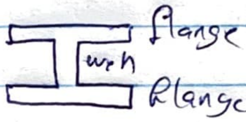
- Beams are used to support loads by bending (flexure)
- The forces developed in the top and bottom flanges of the beam from the necessary couple used to resist the applied moment M.
- The web is effective in resisting applied the shear V.
- When the beam is required to have a very large span and the loads applied are rather large, the cross section may take the form of a plate girder (built up section from steel plates)

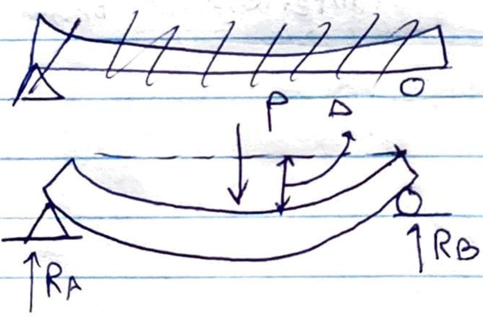


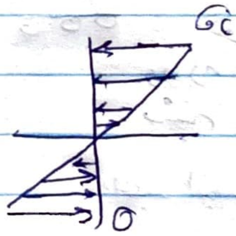


wide-flange beam

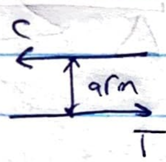








Bending stresses



internal couple

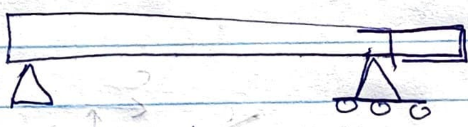
2- Beam elements:

- slender structural members that are used to support load that is applied perpendicular to their longitudinal axis.

- They are often classified according to the way they are supported

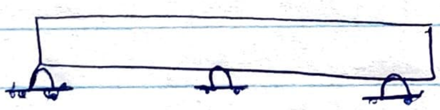
simple supported beam

fixed - supported Beam.

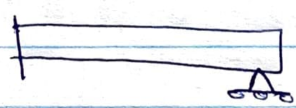


overhanging
بيشبه ال Simple بس ال Beam
بعتبر البعد ال support كأنش شرها

canteilevered beam.



continuous beam

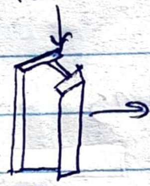


Beam fixed at one end and simply supported at the other end (proped canteilevered)

STUDENTS-HUB.com

3. Column elements:

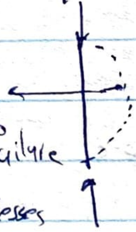
- Members that are generally vertical and resist axial compressive loads
- Columns are subjected to both an axial load and bending moment as show in the slides figure.
- These members are referres to as beam columns
- Columns ~~that~~ may fail in crushing, yielding or buckling. The slenderness L/r in compression members, where L is the length of the member is important. and r is the radius of gyration.



Columns

نفس ال ... بس لما يكون عليه moment من الجنب بنسميه
Beam columns.

هاي ال columns بتستخدم لنقل الأحمال من ال Beams وال Floors
وتنقلها للـ foundation.



هاد ال deformation بيسمى Buckling, هاد ال failure هو
elastic failure مش material failure، يعني بسبب هاد ال failure
moment stresses (يعني لما تحاول تضغط مسطرة من الجنب)

radius of gyration $\boxed{r = \sqrt{\frac{I}{A}}}$

Physical meaning.

Types of structural systems

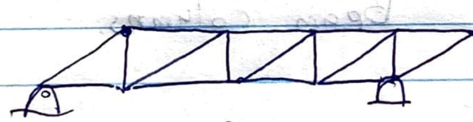
A. Trusses

- A truss is a structural system composed of slender bars whose ends are assumed to be connected by frictionless pin joints.

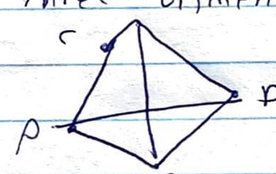
- If pin-jointed trusses are loaded at the joints only, direct or axial stress develops in all bars.

- planar trusses are composed of members that lie in the same plane and are frequently used for bridges and roofs.

- space trusses have members extending in three dimensions and are suitable for tower.



2D



3D

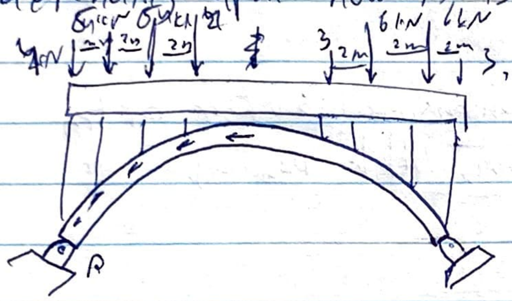
B - Cables

. usually flexible and carry their loads in tension, They are used

C- Arches

- The arch achieves its strength in compression, since it has a reverse curvature to that of the cable

- Due to arch rigidity, it may also resist Bending and shear depending upon how it is loaded and shaped (geometry)



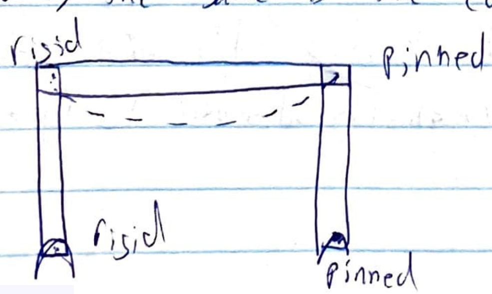
D. Frames:

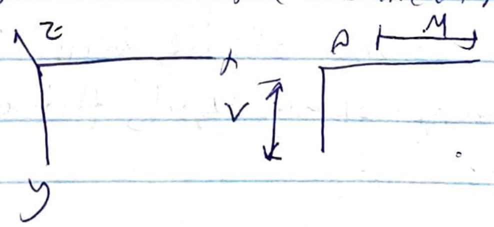
- Frames are composed of Beams and columns that are either pin or fixed connected.

- The loading on a frame (composed of slender elements) causes bending of its members

- If it has continuous (rigid) joint connections, then the internal forces (axial, shear, and moment are transferred between frame elements)

- For a joint to be continuous (rigid), the angle between the members must not change, the rotation of the Beams end is the same as the column's end at the connecting joint



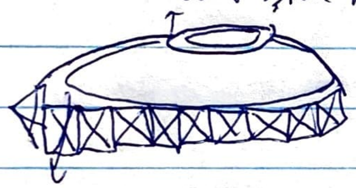


E_ Surface Structures

- A surface structure is made from a material having a very small thickness compared to its other dimensions. They are referred to as shells

- They can span large distance because of the inherent strength and stiffnes of the curved shape.

- The loading is resisted by the three-dimensional surface, often through tension and compression with very little bending.



* Loads

- In order to design a structure, it is therefore necessary to first specify the loads that act on it

- The design loading for a structure is often specified in codes.

- The ultimate responsibility for the design lies with the structural engine

- Types of loads :

1. Dead loads consist of the weight of the various structural members and the weight of any objects that are permanently attached to the structure

- Hence, for a buildings, the dead loads include the weights of the columns, beams, shear walls, floor slabs, roofing, partitions, floor finishes window, plumbing, electrical fixtures, and other miscellaneous attachments

- structural dead load is calculated using simple formulas based on the densities given by building standards or/and manufacturers.

2. Live Loads

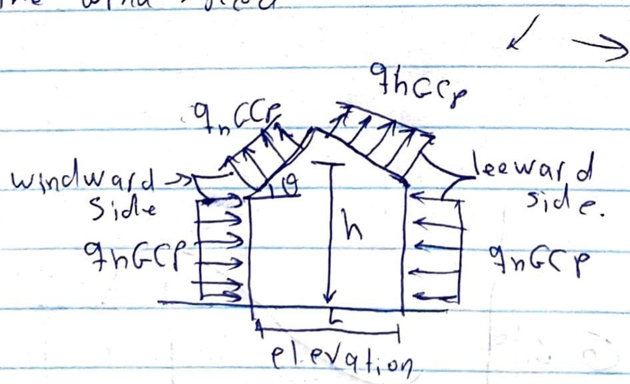
- Live loads can vary both in their magnitude and location

- They may be caused by the weights of occupants using the structure, objects temporarily placed on a structure, moving vehicles or natural forces.

- The minimum live loads are specified in standards and categorized by the occupancy or usage of the structure

3- Wind Loads.

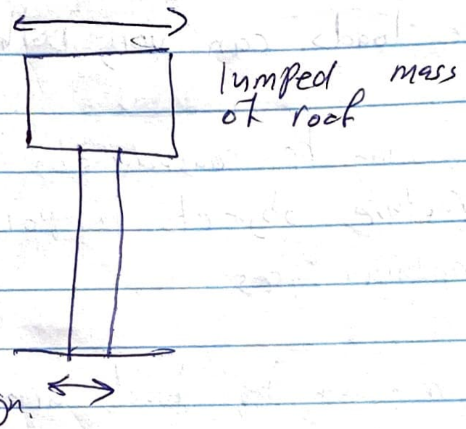
- the pressure created by the wind is proportional to the square of the wind speed



4- Earthquake loads.

- Earthquake causes inertia forces at the floor levels (mass points) of the Building.

- The magnitude of an earthquake loads (seismic loads), depends on the severity and probability of occurence of earthquake in the region and the mass, stiffness and importance of the structure



5- Hydrostatic and soil pressure loads.

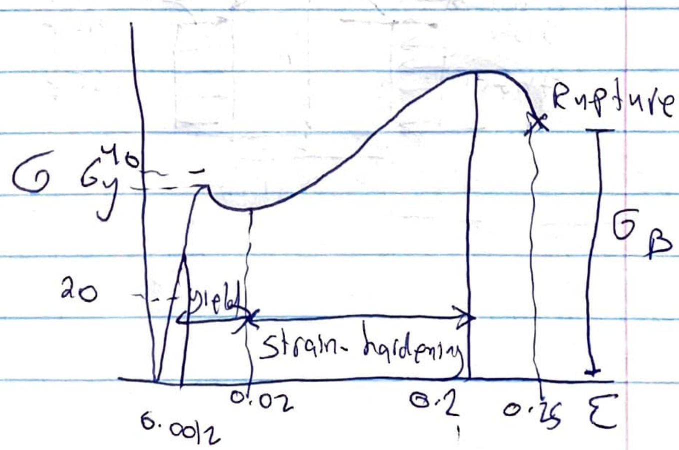
- When structures are used to retain water, soil, or granular materials, the pressure developed by these loading becomes an important criteria for their design.

- Other Natural Loads. Several other types of live loads may also have to be considered in the design of a structure, depending on its location or use. These include the effect of Blast, temperature changes, and differential settlement of the foundation.

structural Design

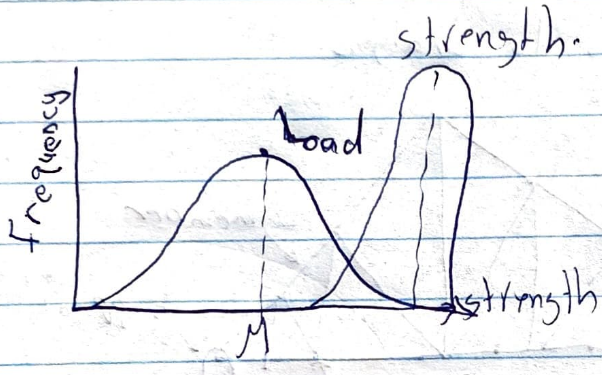
- Allowable stress design (ASD)

$$\sigma_{all} \rightarrow \frac{\sigma_y}{F.S}$$



- Load and resistance factored design (LRFD)

  - this methods considers the variability in the applied loads (internal) and component strength.



← Factors Applied Loads
Factors on component strength.

- Load factors and load combination

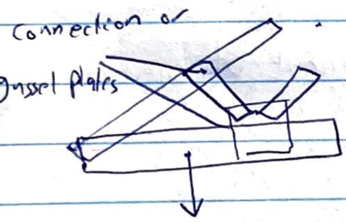
  D: Dead load　　S: snow　　W: Wind عالميك
  L: Live load　　R: Rain　　E: earthquake

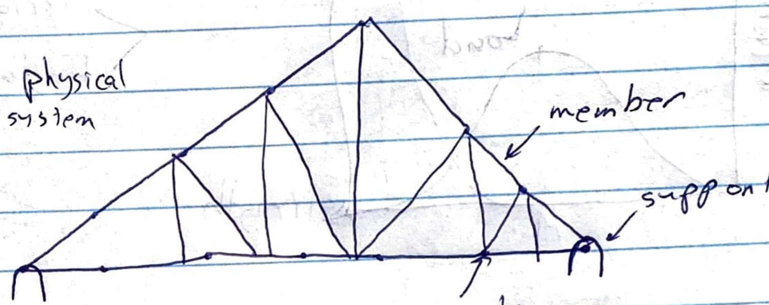
- After structural analysis we got ultimate internal forces in the member in consideration

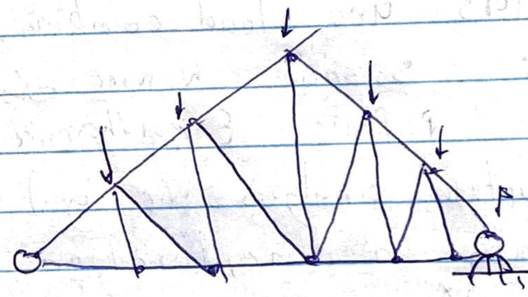
  M_u　　V_u　　P_u

Idealization of structures

- The process of replacing an actual (physical) structure with a simple (mathematical) system conductive to analysis.





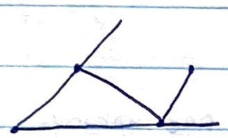


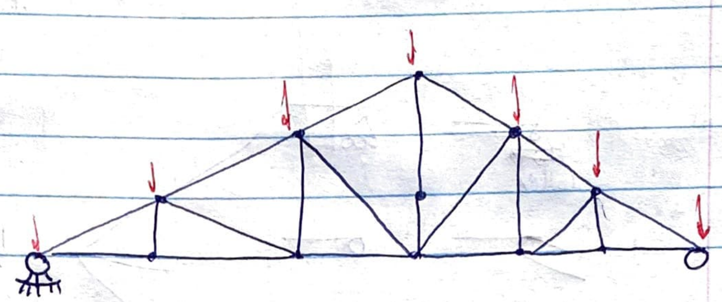
- An exact model of a structure can never be constructed.
- Connections to be modeled based on expected behaviour.

  - typical " pin-supported" connection (metal)   rotation يسمح
  - tpical " fixed supported" connection (metal)

idealization of structures

- The process of replacing an actual (physical) structure with a simple (mathematical) system conducive to analysis





بتحليل الستراكتشر، نقوم بتبسيط صورة الستراكتشر بشكل كامل عشان

نعمل analysis فمثلاً ال members بتترابط ببعض وباستخدام

gusset plates بس بال analysis ما بنوخذها بعين الاعتبار وبنعتبر ال members

ملتقيات بنقطة واحدة، + ما بيفرق من ال thickness إذ كيف طبقنا ال load

على ال structure لانو بالنهاية ما بأثر على النتائج النهائية.

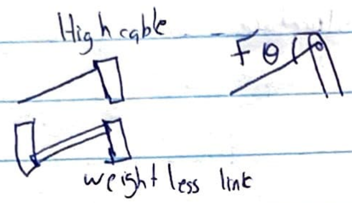
- An exact model of a structure can never be constructed.

- Connections to be modeled based on expected behavior

صور توضيحية لأنواع ال Pinconnections .

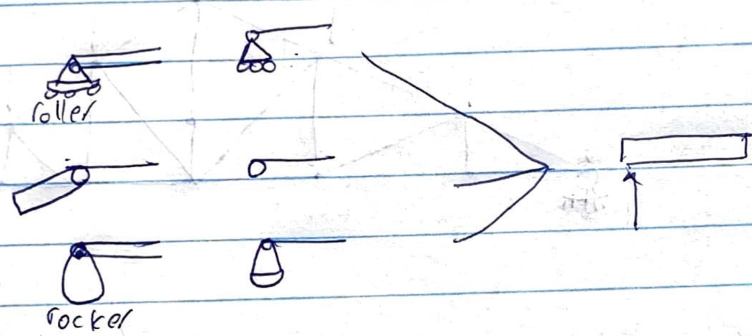
أنواع ال supports
(الجدول ع السلايدات)

(1)



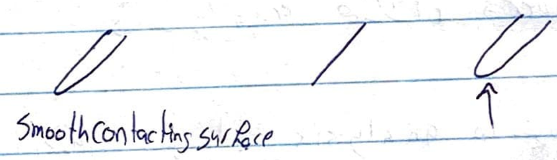
one unknows, The reaction is a force that acts in the direction of the cable or the link.

(2)



one unknown, the reaction is a force that acts perpindicular to the surface at the point of contact.

(3)



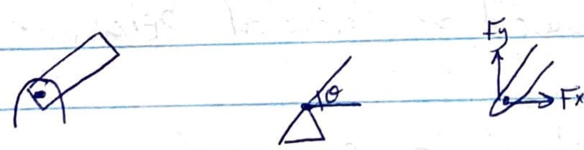
one unknow, The reaction is a force that acts perpendicular to the surface at the point of contact

(4)

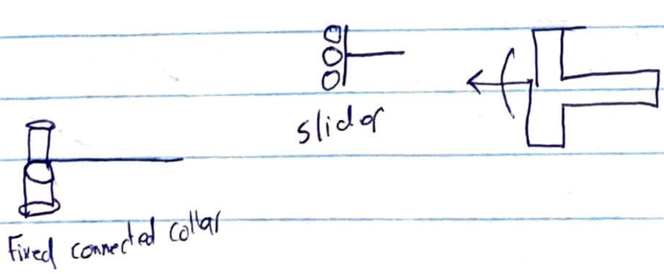
//

smooth pin connected collar

(5)

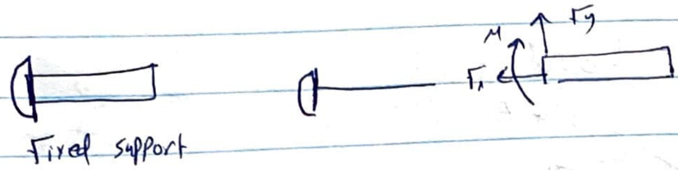


Two unknown, the reaction are two Force components.

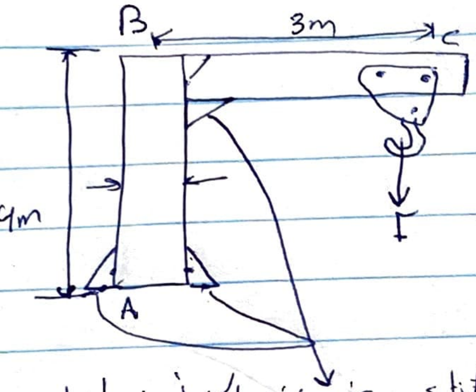


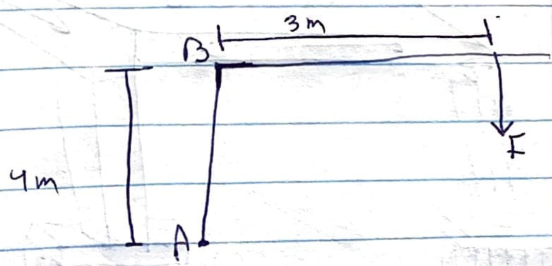
two unknowns, the reaction are a force and a moment

(7)



Three unknowns, three reactions
The reactions are the moment and two force component





وجود ال stiffners يعني إنه الجزأين إذا
داروا، راح يدوروا مع بعض، وبالتالي راح يحافظ
على الزاوية الي بين ال members مشان نضمن توصيل
ال load، وبتشتغل زي fixed

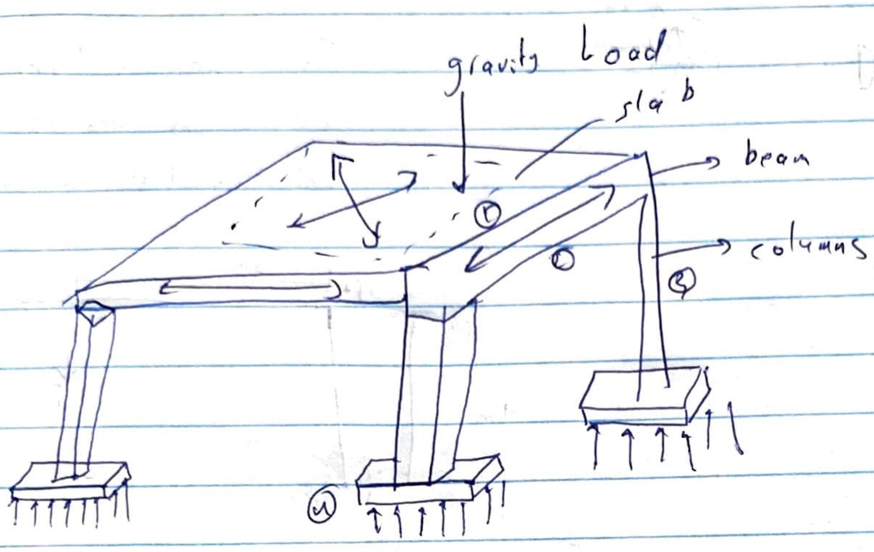
على أرض الواقع يختلف عن التحليلي، نتعامل بالتحليل مع idealized، ولكن في الواقع يختلف الأمر، حيث نتعامل
مع ال structures بشكل حيث ثلاثي 3d مع مانقوم بتحليله (على الورق).

Load Path.

- how the loads are tranmitted through various structural members from point of application to the foundation

(الهدف من ال structure هو تحمل القوى ونقلها الى نقاط أخرى.

∴ Like a chain, which is " as strong as its weakest Link", so a structure is only as strong as the weakest part along it Load path



الأسهم إتجاه انتقال القوى.

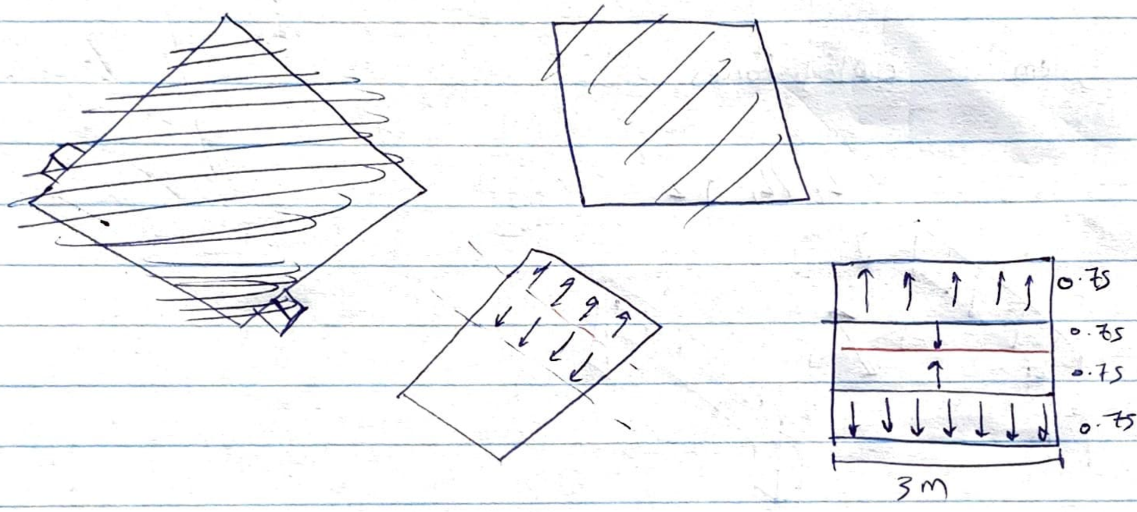
Load transfer and tributary Loadings

- Many floor system consists of a reinforced concrete slab supported on a rectangular grid of beams
- The supporting beams reduce the span of the slab and permit the designer to reduce the depth and weight of the floor system
- The distribution of dead loads to a floor beam depends on the geometric configuration of the beams forming the grid
- Determine how the load and these surfaces is transmitted to the various structural elements used for their supports
- There are generally two ways in which this can be done
  - one way system.
  - two way system

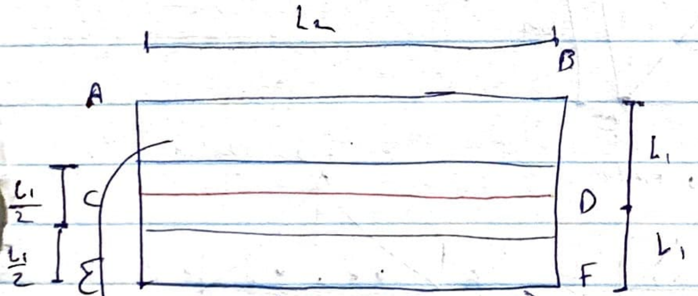
لأنه ال Load بيحث عن أقصر طريق ليرجع فلذلك في تركيب من ال system
one way, two way.

- One - way system: ينتقل ال Load باتجاه ليصل ال foundation.

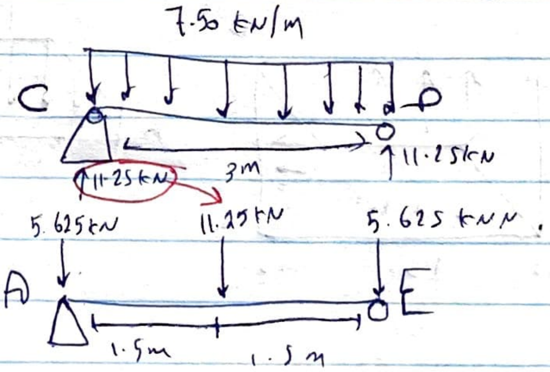
- According to the ASCE-7, if $L_2 \geq L_1$ and the support ration $(L_2/L_1) \geq 2$, then the Load is assumed to be transferred to the supporting beams and girders in one direction



لو الرسمة بالبلاطات، بالفراغات أسفل ال floor تاع ال structure لا يمكن انه ينتقل لها Load لانه لا يوجد supports في تلك المنطقة، لذلك لا يمكن انه ينتقل لها حتى لو كانت ال ratio $(L_2/L_1)<2$



توضيح لباقي المرحلة بالكامل



Concrete slab is reinforced in two directions, poured on plane forms.

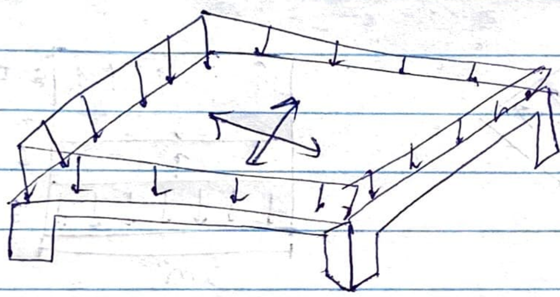
idealized framing plan for one-way slab action requires $L_2/L_1 > 2$

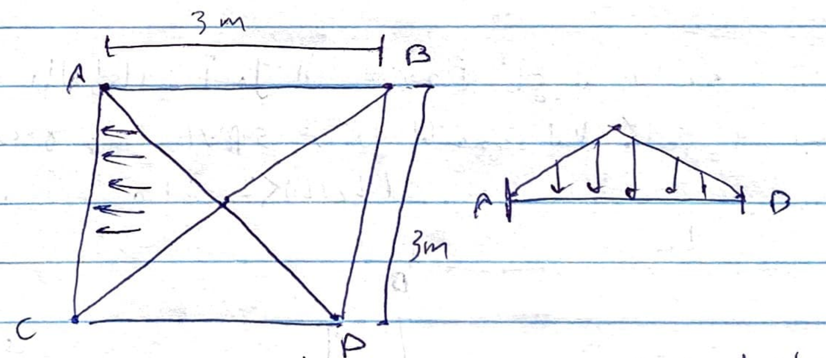
كيف نحسب ال load ع EF :

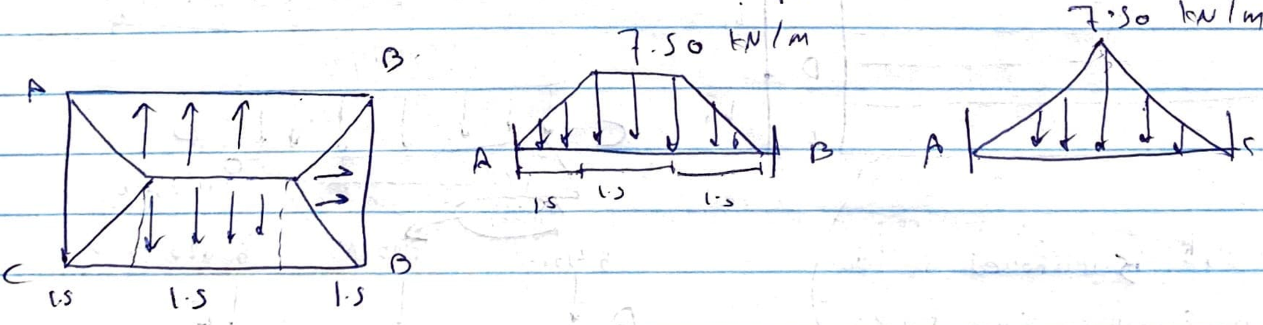
ال T_w لل EF = 0.75m ، ونحول ال Load كل طوله ، وبالتالي لازم نحول ال 5 kN/m² الى وحدة الطول ، ع لذلك نضربه بال width تاعت ال Beam. لنعرف كيف بيتوزع ال load ع كل Beam.

• two way system : تنتقل ال forces باتجاهين

$L_2/L_1 \leq 2$







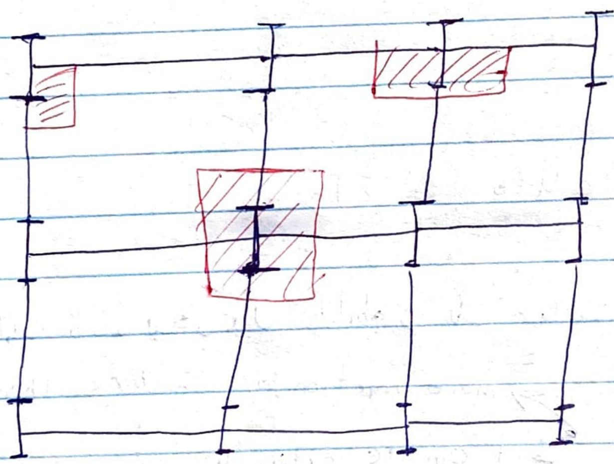
- Column Load.

- To determine the gravity loads transmitted into a column from a floor slab, the designer can either.

  - Determine the reaction of the beams framing into the colum

  - Multiply the tributary area of the floor surrounding the column by the magnitude of the load per unit area acting on the floor.

- The tributary area of a column is defined as the area surrounding the column that is bounded by the panel center lines



عشان نطلع كل عامود كم بحمل بنطلع ع ال system تاع الأعمدة، كل عامود
بحمل نفس الي بحملو العامود الثاني. ~~كل~~ ال area الناتجة عن تقاطع ال Beams
بين الأعمدة بنطلع من خلالها ال Loads، ال side columns بحملو أقل لأنو بتشاركو
ما أعمدة أقل، والوسطانيات بحملو أكبر لأنهم بشاركوهم أعمدة أكثر.

Determinancy and stability of structural system.

يجب أن يكون ال structure ثابت ومقاوم لل failure.

- Criteria for the static classification of structures can be stated as.

1- if there are more equation than there are unknown → statically unstable

2- " " " same number of equations are unknowns → statically determinate.

3- " " " fewer equations than unknowns → statically indeterminate.

- first criteria is <u>absolute</u>, the second and third are conditions.

<u>2D</u>

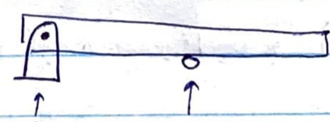
$\Sigma F_y = 0 \qquad \Sigma M = 0 \qquad \Sigma F_y = 0$

هاي المعادلات بتعني ال stability لل structure يعني لا حركة أفقية، عمودية، دورانية

- structure is indeterminat
  - → more reaction or members than necessary
  - → it can be externally or internally.

- Redundant reaction/force: reaction or force that is an excess one, in otheor words not necessary for the stability of the structure

- static indeterminacy refers the number of force that must be determined in order to render the equilibrium solution complete →



r = 3 n = 3

stable, determinate.

درجة ال indeterminacy بتحددها عدد ال reaction الزيادة.

- Kinematic indeterminacy refers to the number of displacement quantities (kinematic degree of freedom) that are necessary to define the deformational response of the structure.

2 degrees

6 degrees

حسب القوى المؤثرة على ال structure

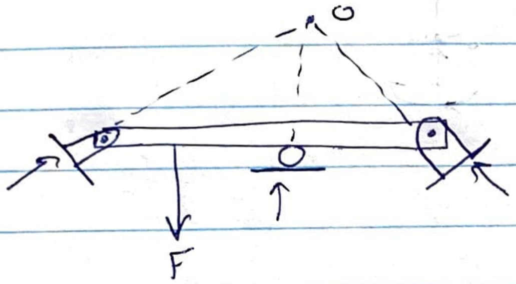
- Pros

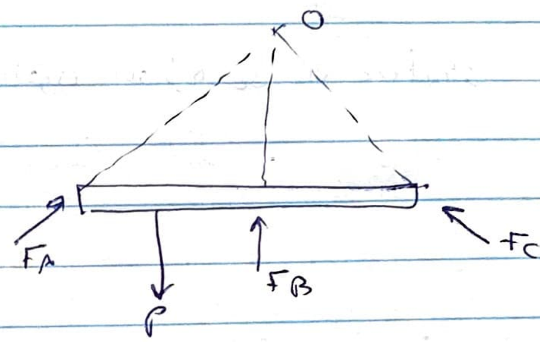
- Saving in materia in the design stage, as maximum stresses and deflection of indeterminate structures are generally smaller than those of statically determinate structures

- statically indeterminate structure tends to redistribute its Load to its redundant supports or members in cases of overloading

- To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equation of equilibrium but the member must also be properly held or constrained by their supports

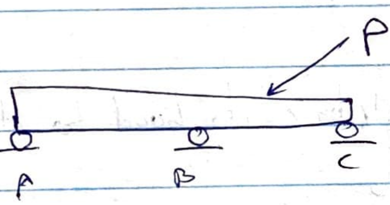
- The following situations may occur, and they are example of instability of structural system

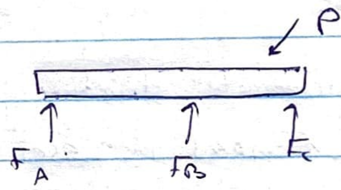




إذا التقو ال reaction بنقطة وحدة
بكون ال structure مش stable

current reactions



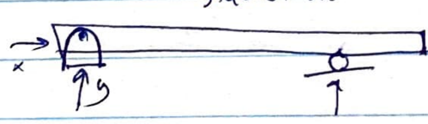


parallel reactions

- Beams

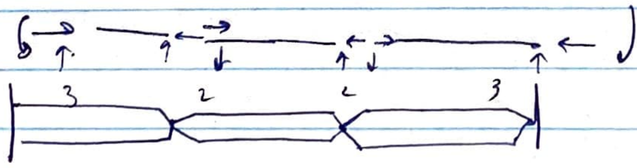
$r=3$ $n=3$

stable, det



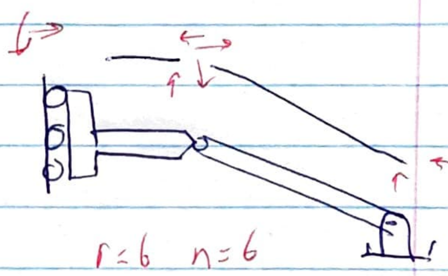
$r=5$ $n=3$

stable, indet to 2^{nd} degree



$r=10$ $n=3\times3=9$

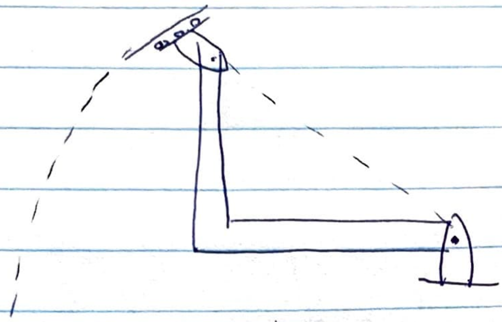
stable, indeterminate 1^{st} degree



$r=6$ $n=6$

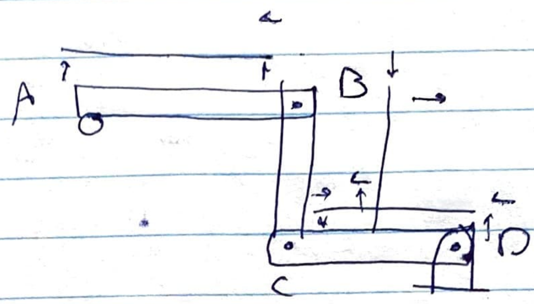
stable, determinate

. stable or unstable



unstable

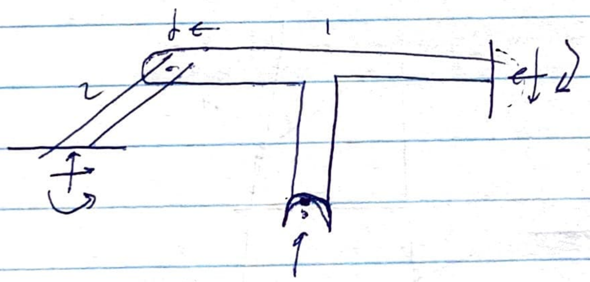
لأنه ال support مش إشي بيمنع حركته



r = 7   n = 9

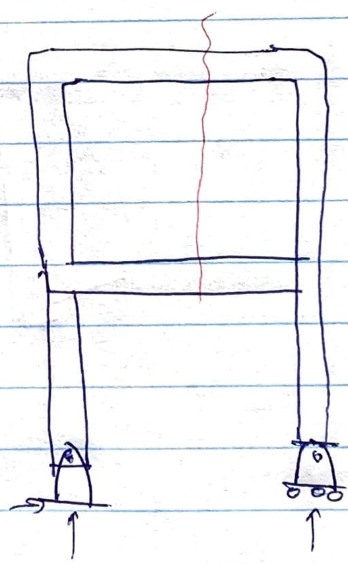
unstable

. Frames



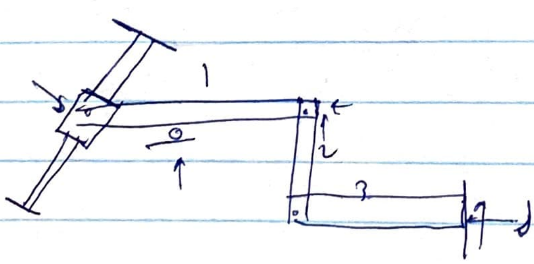
r = 10,   n = 6

indeterminat to 4th



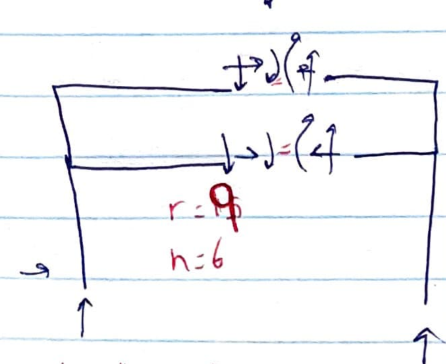
r = 3   determinat

n = 3



r = 9 ,   n = 9

stable , determinate



r = 9

n = 6

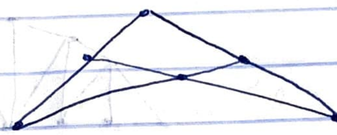
يعني مشان كل ال internal لازم ال external يكون موجود

Analysis of statically determinate trusses

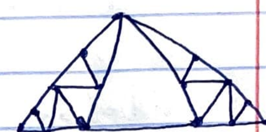
Type of trusses:

- Roof trusser

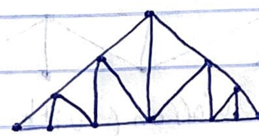
أجزاء ال truss
موجودة بالعلالي



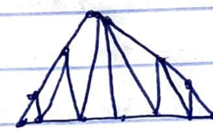
scissors



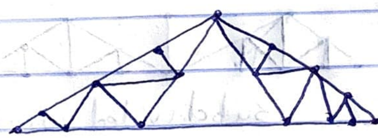
Fink.



Pratt



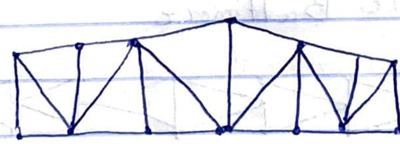
Hows



cambered fink

fan

saw tooth



warren (with verticals)

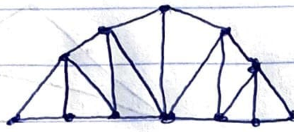
bow string

• Bridge trusses

أجزاء ال Bridge trusses
على السلايدات

Pratt

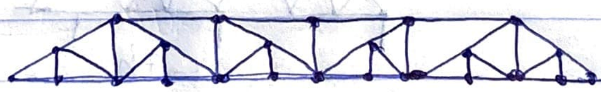
Howe.



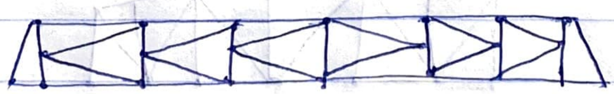
Parker

Warren (with verticals)

subdivided warren



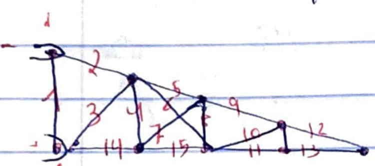
Baltimore



K- truss.

criteria for the static classifcation of structures can be stated as

- if there are more equations than there are unknowns → the structure is statically unstable
- if there is the same number of equations as unknowns → the structure is statically determinate
- if there are fewer equations than unknowns → the structure is satically indeterminate



لما يكونوا كل ال members من two force member's

⇒ عدد المعادلات = عدد ال joints 2 x

⇐ 9 x 2

= 18eq

عدد المجاهيل = unknown = عدد المعمبرز member + عدد ال Reaction

من السؤال ⇐ 15+4 = 19 unknown

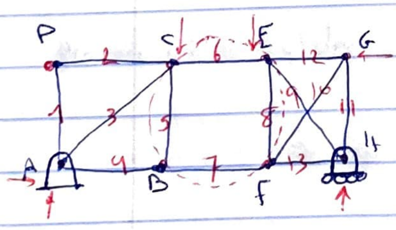
⇐ statically indeterminate

$b+r<2j$ unstable

$b+r\geq 2j$ unstable if truss support reaction are concurrent or Parallel or if the components of the truss form a collapsable mechanism

$b+r=2j$: statically determinate

$b+r>2j$ statically indeterminate

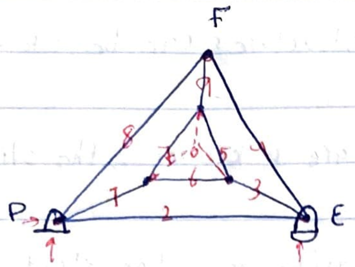


equations: $2\times 8 = 16$ eq.

unknowns: members + reaction

$= 13 + 3 = 16$ unkowns.

المنطقة الي مدورة بالأحمر
معرضة ل collapsible mechanism
يعني بالنظر لو تعرض ال joints
الي مكتوبين ل external forces
راح تتعرض لإنحراف وتخرب
بدون ما نلاقي موانع لها لإنحراف
وبالتالي الحالة رح يصير ال deflection بسبب
ال members من المنطقة
المدورة unstable



كمان مثال على ال collapsabile

mechanism

لا يكون في members بتلتقي في نقطة

وحدة جوا ال truss وبالتالي

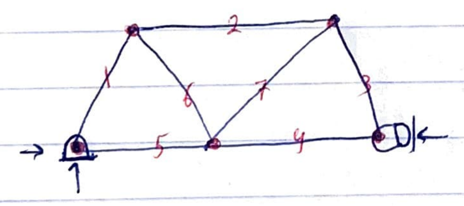
لو تعرض لقوة خارجية راح يصير

unstable

equations: $6 \times 2 = 12$ equation

unknowns: $9 + 3 = 12$ equation

The truss is unstable.



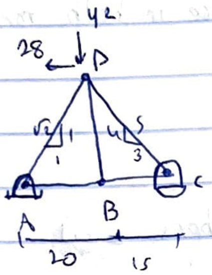
concorent reactions

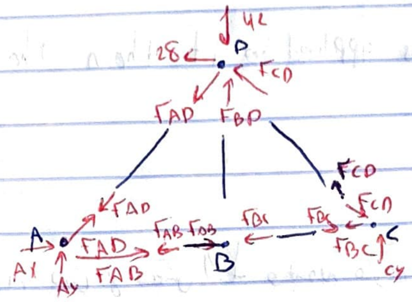
كمان نحكي إنه ال reaction ⇐ concorent لازم كل

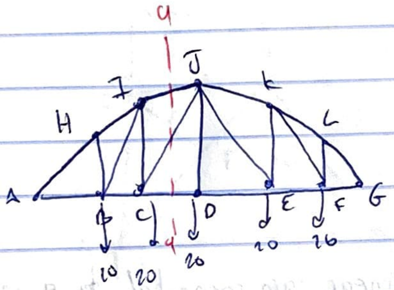
ال reaction of supports يكونوا يلتقوا أو جا ما نقطة واحدة)

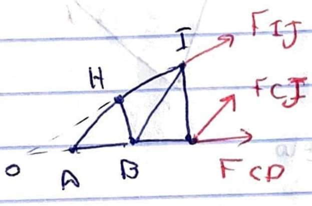
• There are two main methods to analyze statically determinate trusses

- Method of joints, helpful to identify zero force members
- Methods of sections.









- Method of joints

Process at the slides

مفيدة لتحديد ال zero force

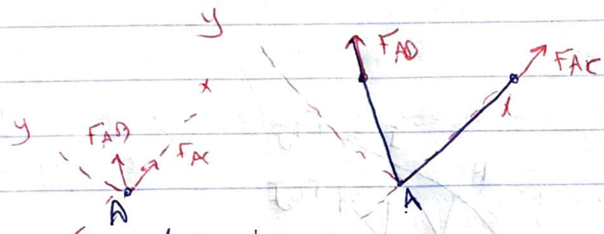
- Method of sections.

لتحديد ال internal في member معينة بدون ال كل مة
لتحديد القوى على كل ال structure

- Zero force members.

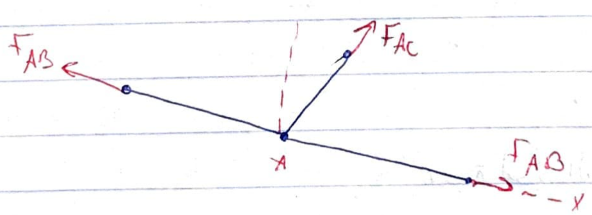
- if only two noncollinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero

Zero force هم members تتبين مش ع نفس الخط وبدون قوة خارجية



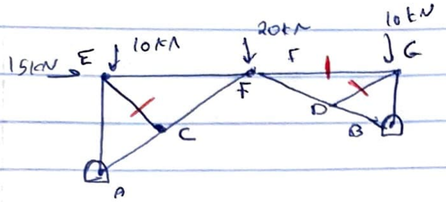
ما في قوة تعاكس F_{AD} بال y .

- if three members, two of which are collinear, are connected to a joint that has no external loads or reaction applied to it, then the force in the member that is not collinear is zero

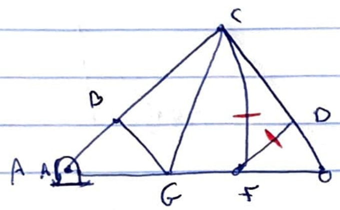


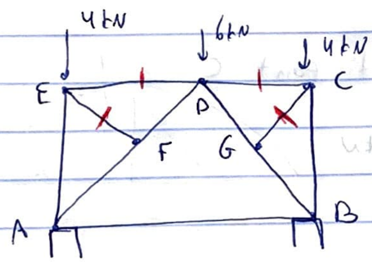
لو بدنا نحلل F_{AC} عال y بنلاقي إنه ما في قوة تعاكس F_{AC} في الاتجاه وهي مركبتها بال y وهي صفر، وبما إنه مركبتها بال y صفر، بديهياً كمانه ع x هي صفر، وبالتالي

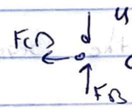
zero force member.



—: Zero Force members

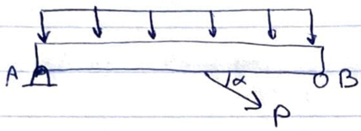




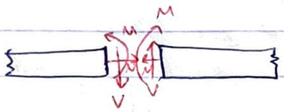


$F_{CD} = 0$

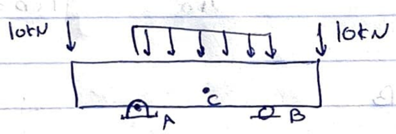
Analysis of Beams



Internal Loads in Beams?



Ex: Determine the moment in the Beam at point C



$\sum M_B = 0$

$10(3) - 30(3) + B_y(6) - 10(9) = 0$

$\Rightarrow B_y = \frac{150}{6} = 25\,kN$

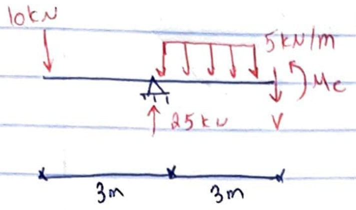
$\sum F_y = 0$.

$-10 - 30 - 10 + 25 + A_y = 0$

$\Rightarrow A_y = 25\,kN$

في symmetric في الأبعاد بال Beam وفي Symmetric بال loading وبالتالي ال reaction تكون متساوية لل supports

$\frac{1}{2} \times$ total load على ال support A والمثل ع ال support B.



$\Sigma M_c = 0$

$10(6) - 25(3) + 15(1.5) + M_c = 0$

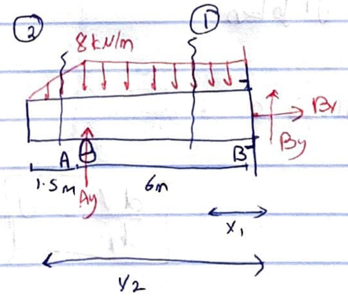
$\Rightarrow M_c = -7.5 \text{ kN.m}$

$\Sigma F_y = 0$

$-10 - 15 + 25 - V_c = 0$

$\Rightarrow V_c = 0 \text{ kN}$

Example 2 Express the internal shear and moment along the beam as a function of x.



$\Sigma M_B = 0$

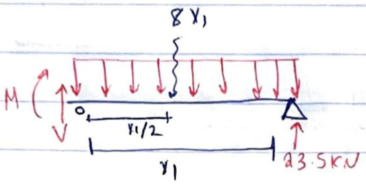
$6(6.5) - A_y(6) + 48(3) = 0.$

$A_y = 30.5 \text{ kN}$

$\Sigma F_y = 0 \Rightarrow -6 + 30.5 - 48 + B_y = 0.$

$B_y = 23.5 \text{ kN}$

Section 1



$\Sigma M_o = 0$

$-M - 8x_1\left(\frac{x_1}{2}\right) + 23.5x_1 = 0$

$\boxed{M = -4x_1^2 + 23.5x_1}$ kN.m   power = 2

$\Sigma F_y = 0$

$V - 8x_1 + 23.5 = 0$

$\boxed{V = 8x_1 - 23.5 \text{ kN}}$ power = 1

slope

$\frac{dM}{dx} = V$

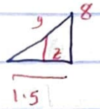
$\int V dx = M$

معادل ال shear في ال contant distributed load
(مش مثلثات واشكال)

هو قيمة ال distributed.

$\boxed{\frac{dV}{dx} = w}$

Section 2



$$\frac{8}{1.5} = \frac{y}{z-1.5}$$

$$y = 5.3z - 8$$
$$= 5.3(x_2 - 6) - 8$$
$$\Rightarrow y = 5.3x_2 - 39.8$$

$\Sigma M_o = 0$

$$M = \begin{cases} M_1(x_1) & 0 < x_1 < 6 \\ M_2(x_2) & 6 < x_2 < 7.5 \end{cases}$$

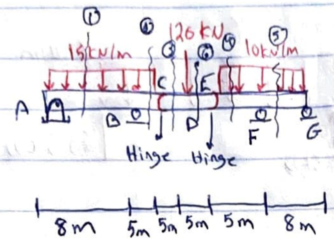
بنقسم المومنت حسب السكشن لأنو
خالف ال internal على طول ال Beam
إذا بالمثال أخذت x_1 هو ال reference
بس لو أخذنا x_2 هو ال reference بتنعكس
بصير $0 < x_1 < 1.5$ و $1.5 < x_2 < 7.5$

$\Sigma F_y = 0$

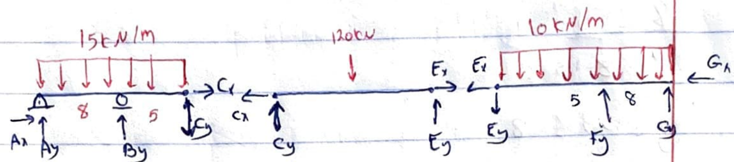
نفس
ال reference

$$V = \begin{cases} V_1(x_1) & 0 < x_1 < 6 \\ V_2(x_2) & 6 < x_2 < 7.5 \end{cases}$$

Example:



step 1: reactions



$3n = 9E \rightarrow 9$ unknowns

$\therefore$ det. Beam

$\sum F_y = 0$

$\sum F_x = 0$

$\sum M = 0$

$\sum M_E = 0$

$120(5) - 10C_y = 0$

$600 = 10C_y \Rightarrow \boxed{C_y = 60\,kN} \uparrow$

$\sum F_y = 0$

$60 - 120 + E_y = 0$

$\boxed{E_y = 60\,kN} \uparrow$

$\sum M_A = 0$

$-1267.5 + 8B_y - 13(60) = 0$

$\boxed{B_y = 256\,kN} \uparrow$

$\Sigma F_y = 0$

$A_y + B_y - C_y - 15(13) = 0.$

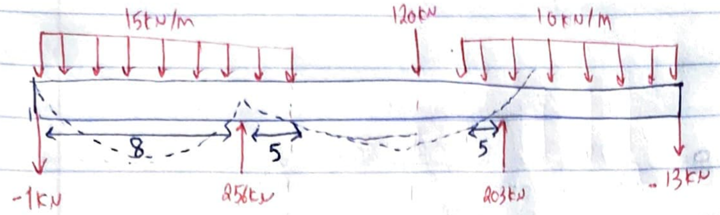
$A_y + 256 - 60 - 195 = 0$

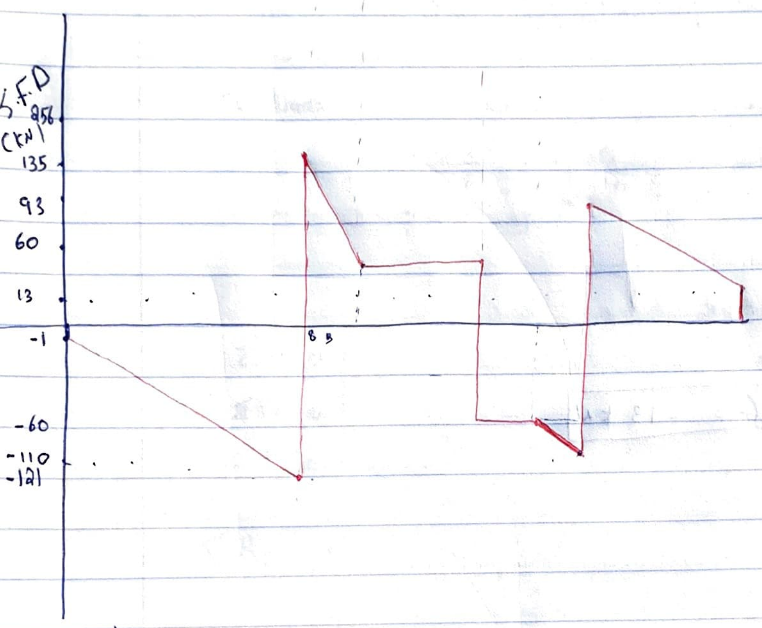
$\boxed{A_y = -1 \text{ kN}}$

$\Sigma M_G = 0.$

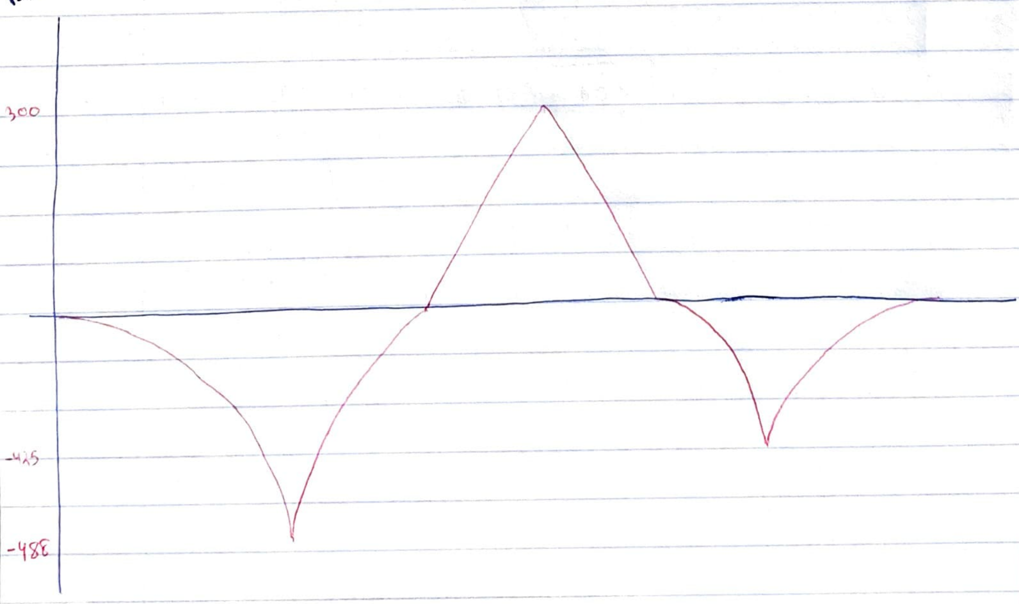
$-8 F_y + 13(60) + 845 = 0.$

$\Rightarrow \boxed{F_y = 203 \text{ kN}} \Rightarrow \boxed{G = -13 \text{ kN}}$





B.M.D (kN.m)



STUDENTS-HUB.com

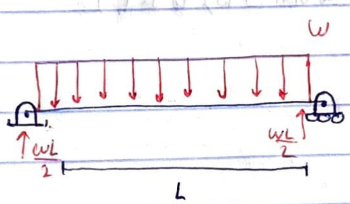
Analysis of Beam-Diagrams

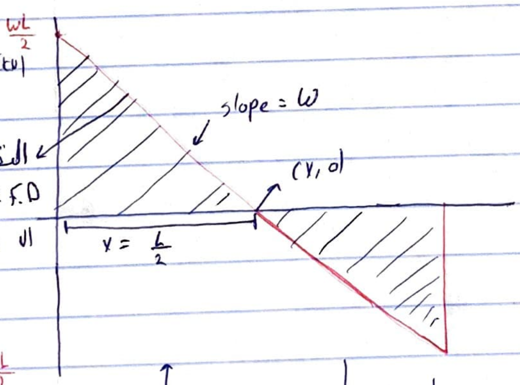
$$\frac{dV}{dx} = w$$

slope of shear diagram = intensity of Distributed Load

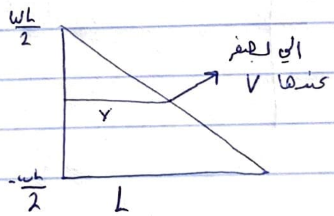
$$W \xrightarrow{\int w} V \xrightarrow{\int V} M$$

$$W \xleftarrow{\partial V} V \xleftarrow{\partial M} M$$





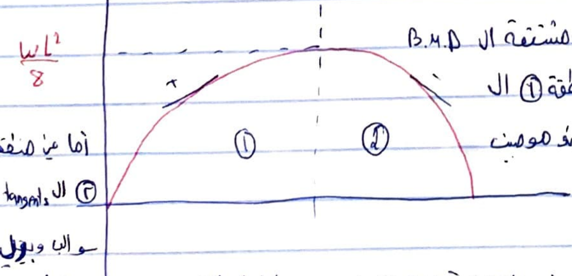
المقاطع كان S.F.D هي مشتقة ال B.M.D



derivative ↑ integration ↓

B.M.D

$$\frac{x}{L} = \frac{\frac{wL}{2}}{wL} \Rightarrow x = \frac{L}{2}$$



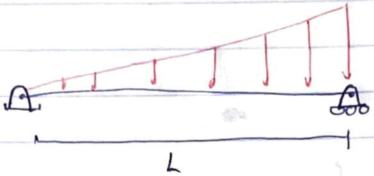
بما أنه مشتقة ال B.M.D + في منطقة ① ال tangents راح يكون موجب وبقل ويميلهم

أما في منطقة ② ال tangents سالب وبيزيد ميلهم

مساحة ① $\frac{1}{2}\left(\frac{wL}{2}\right)\left(\frac{L}{2}\right) = +\frac{wL^2}{8}$

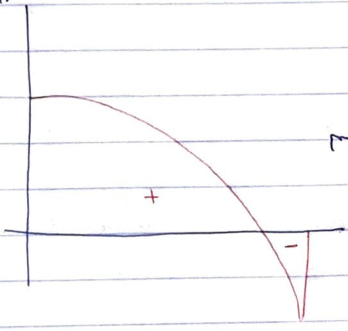
مساحة ② $\frac{1}{2}\left(-\frac{wL}{2}\right)\left(\frac{L}{2}\right) = -\frac{wL^2}{8}$

وبما أنه المشتقة الثانية لل B.M.D كلها سالبة ف المنحنى كلو concave down



2^{nd} order

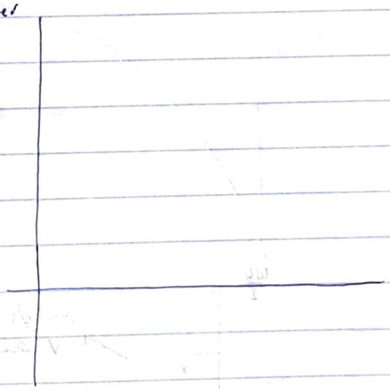
S.F.D



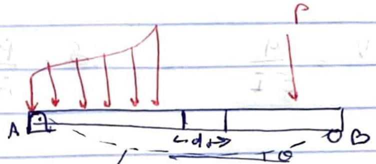
ال slopes بتزيد حسب تقسيم
ال distributed وبالتالي ال tangents بتزيد
ميلانها كلما تحركنا من الشمال لليمين.

3^{rd} order

B.M.D

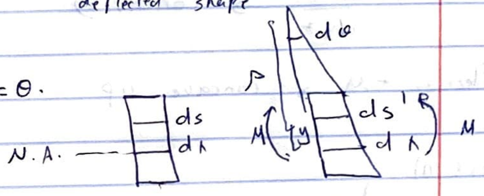


Elastic Beam theory



$$\varepsilon = \frac{(\rho - y)\,d\theta - \rho\,d\theta}{\rho\,d\theta}$$

$\tan\theta = \theta$.



$$\Rightarrow \varepsilon = \frac{-y}{\rho} \rightarrow \sigma = E\varepsilon \rightarrow \sigma = -\frac{My}{I}$$

$$-\frac{My}{I} = \frac{E * -y}{\rho}$$

$ds' = (\rho - y)\,d\theta$

$ds = \rho\,d\theta$

$$K \left\{ \frac{1}{\rho} = \frac{M}{EI} \right.$$

$$\frac{1}{\rho} = \frac{d^2V/dx^2}{[1 + (dV/dx)^2]^{\frac{3}{2}}}$$

$\frac{dV}{dx} \xrightarrow{\text{تقترب}} 0$

فلا تؤثر على المعادلة

$$\Rightarrow \frac{1}{\rho} = \frac{d^2V}{dx^2}$$

$$\Rightarrow \frac{d^2V}{dx^2} = \frac{M}{EI}$$ internal forces

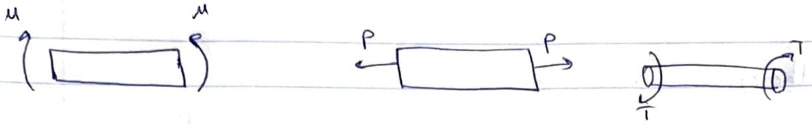
$$w \xrightarrow{\int} V \xrightarrow{\int} M \xrightarrow{\int} \theta \xrightarrow{\int} V$$

/EI نقسم

$$\xleftarrow{d} \quad \xleftarrow{d} \quad \xleftarrow{d} \quad \xleftarrow{d}$$

deflection

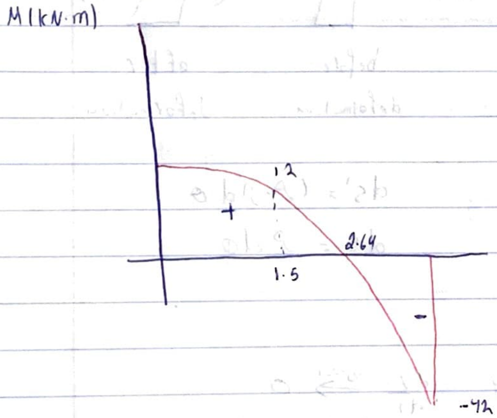
Elastic Beam theory



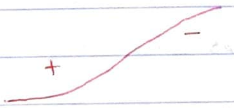
$$\frac{\partial^2 V}{\partial x^2} = \frac{M}{EI} \qquad \delta = \frac{PL}{AE} \qquad \theta = \frac{TL}{JG}$$

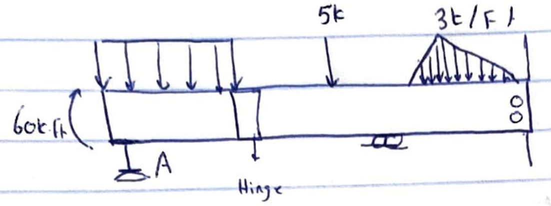
$$\frac{d^2V}{dx^2} \rightarrow \text{concavity}$$

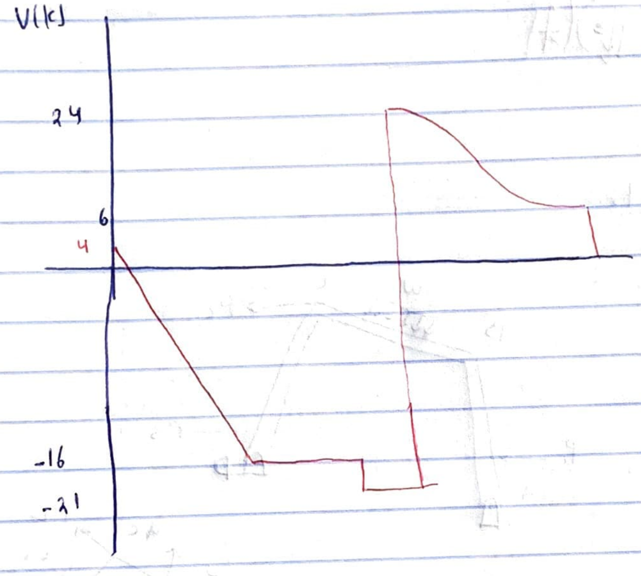
deflection: $+M \rightarrow$ concave up $\quad -M \rightarrow$ concave down.

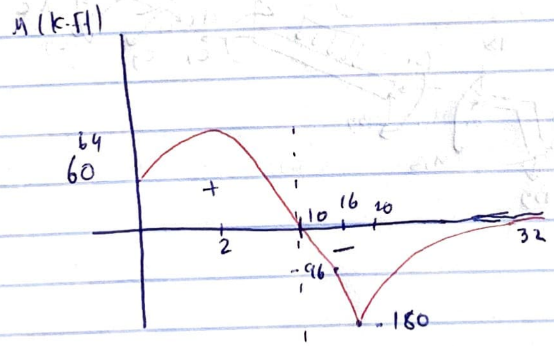


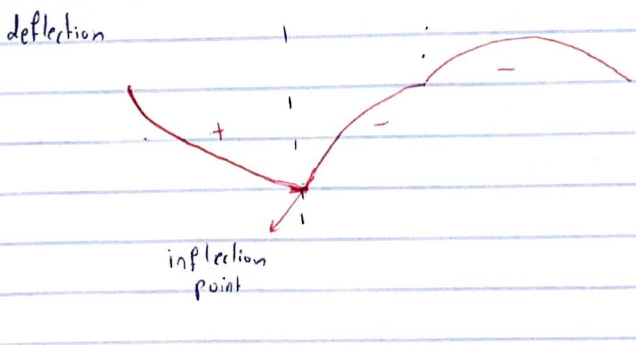
deflected shape:





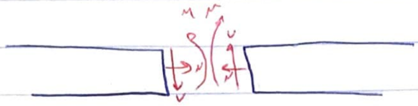




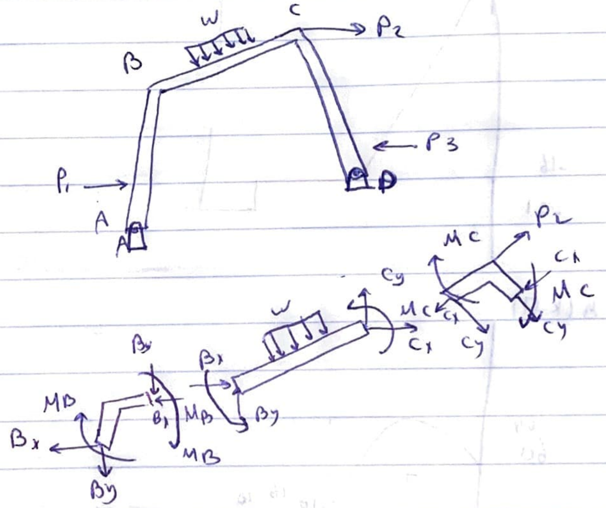


عند ال supports ال deflection بكون صفر، وبالتالي لما نعمل ال sketching لل deflection
عند ال supports بصفّرو.

Analysis of Frames :



load transfer between joints



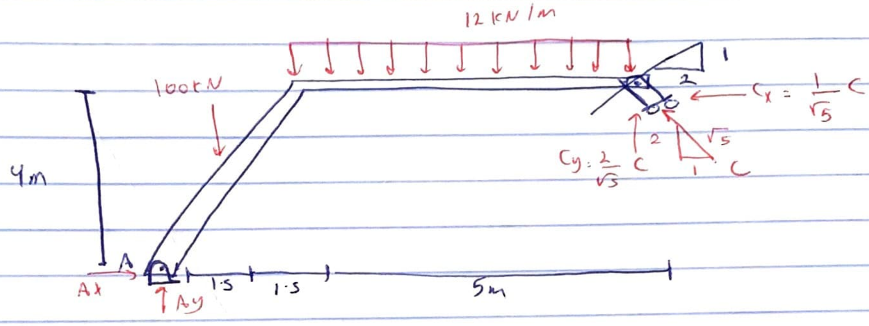
STUDENTS-HUB.com

Example:

Draw shear and moment diagrams

Develop an equation for shear and moment along the frame

Sketch the deflected shape of the beam



Reactions:

$\Sigma F_y = 0$

$A_y + \frac{2}{\sqrt{5}} C - 100 - 12(5) = 0 \Rightarrow \boxed{A_y = 112 \text{ kN}}$

$\Sigma F_x = 0$

$A_x - \frac{1}{\sqrt{5}} C = 0 \Rightarrow \boxed{A_x = 24 \text{ kN}}$

$+\circlearrowleft \Sigma M_A = 0$

$-100(1.5) - 12(5)(5.5) + \frac{2}{\sqrt{5}}(8)(C) + 4\left(\frac{1}{\sqrt{5}}\right)C = 0$

$\frac{20}{\sqrt{5}} C = 480$

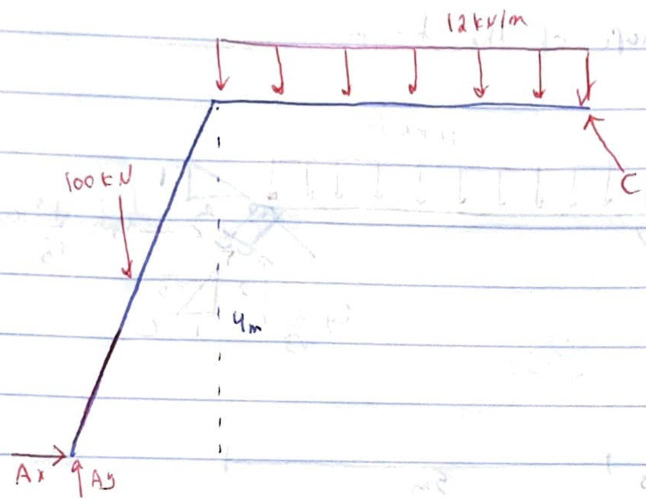
$\boxed{C = 53.66 \text{ kN}}$

$C_x = \frac{1}{\sqrt{5}}(53.66) = 24 \text{ kN}$

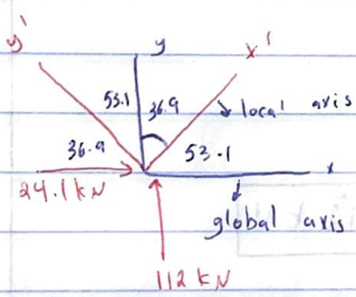
$C_y = \frac{2}{\sqrt{5}}(53.66) = 48 \text{ kN}$

* رسمة ال shear وال moment وال deflection بتكون بنفس اتجاه ال Frame
يعني المايل برسمه مايل و المستقيم مستقيم وهيك.

ال shear وال moment وال deflation تكون بنفس اتجاه ال Frame ← رسمة ال
يعني المايل برسمه مايل و المستقيم مستقيم وهيك.



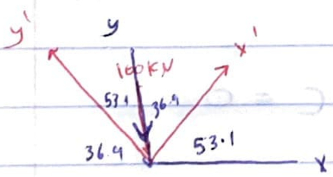
at A:



$x' \rightarrow 112 \cos 36.9 + 24.1 \cos 53.1 = 104 \text{ kN}$

$y' \rightarrow 112 \cos 53.1 - 24.1 \cos 36.9 = 47.9 \text{ kN}$

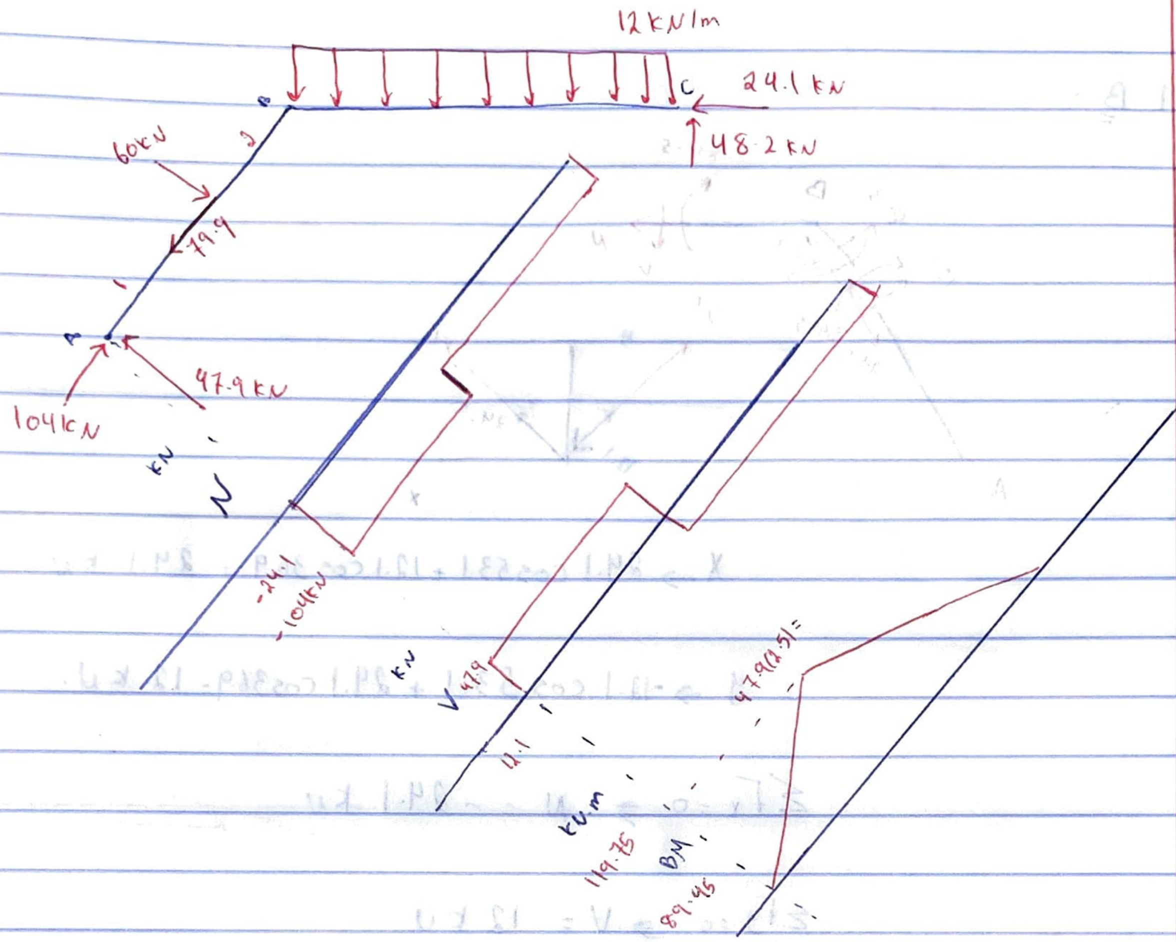
For 100 kN :



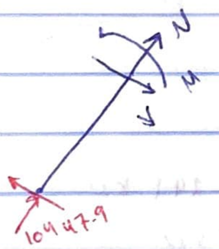
$x' \rightarrow -100 \cos 36.9 = -80 \text{ kN}$

$y' \rightarrow -100 \cos 53.1 = -60 \text{ kN}$

STUDENTS-HUB.com



section 1:

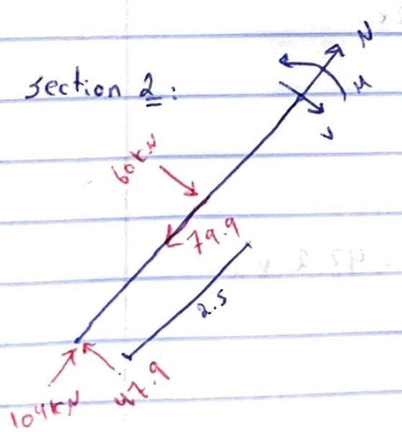


$\Sigma F_{x'} = 0 \Rightarrow N = -104\,kN$

$\Sigma F_{y'} = 0 \Rightarrow V = 47.9\,kN$

$\Sigma M_o = 0 \Rightarrow -47.9x_1 + M \Rightarrow M = 47.9x_1$

section 2:

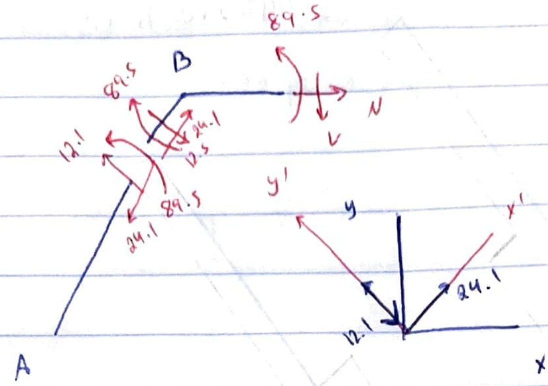


$\Sigma F_{x'} = 0 \Rightarrow N = -24.1\,kN$

$\Sigma F_{y'} = 0 \Rightarrow V = -12.1\,kN$

$\Sigma M_o = 0 \Rightarrow 47.9x_2 - 60x_2 + 150 = 150 - 12.1x_2$

At joint $\underline{\underline{B}}$:



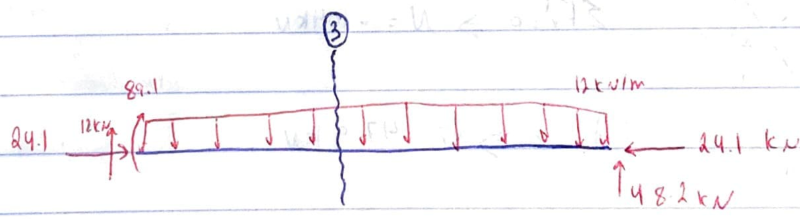
$$X \rightarrow 24.1\cos 53.1 + 12.1\cos 36.9 = 24.1 \text{ kN}$$

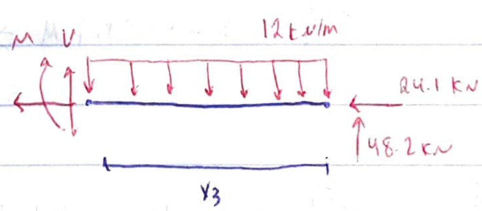
$$y \rightarrow -12.1\cos 53.1 + 24.1\cos 36.9 = 12 \text{ kN}.$$

$$\Sigma F_x = 0 \Rightarrow N = -24.1 \text{ kN}$$

$$\Sigma F_y = 0 \Rightarrow V = 12 \text{ kN}.$$

$$\Sigma M_o = 0 \Rightarrow M = 89.5 \text{ kN.m}$$



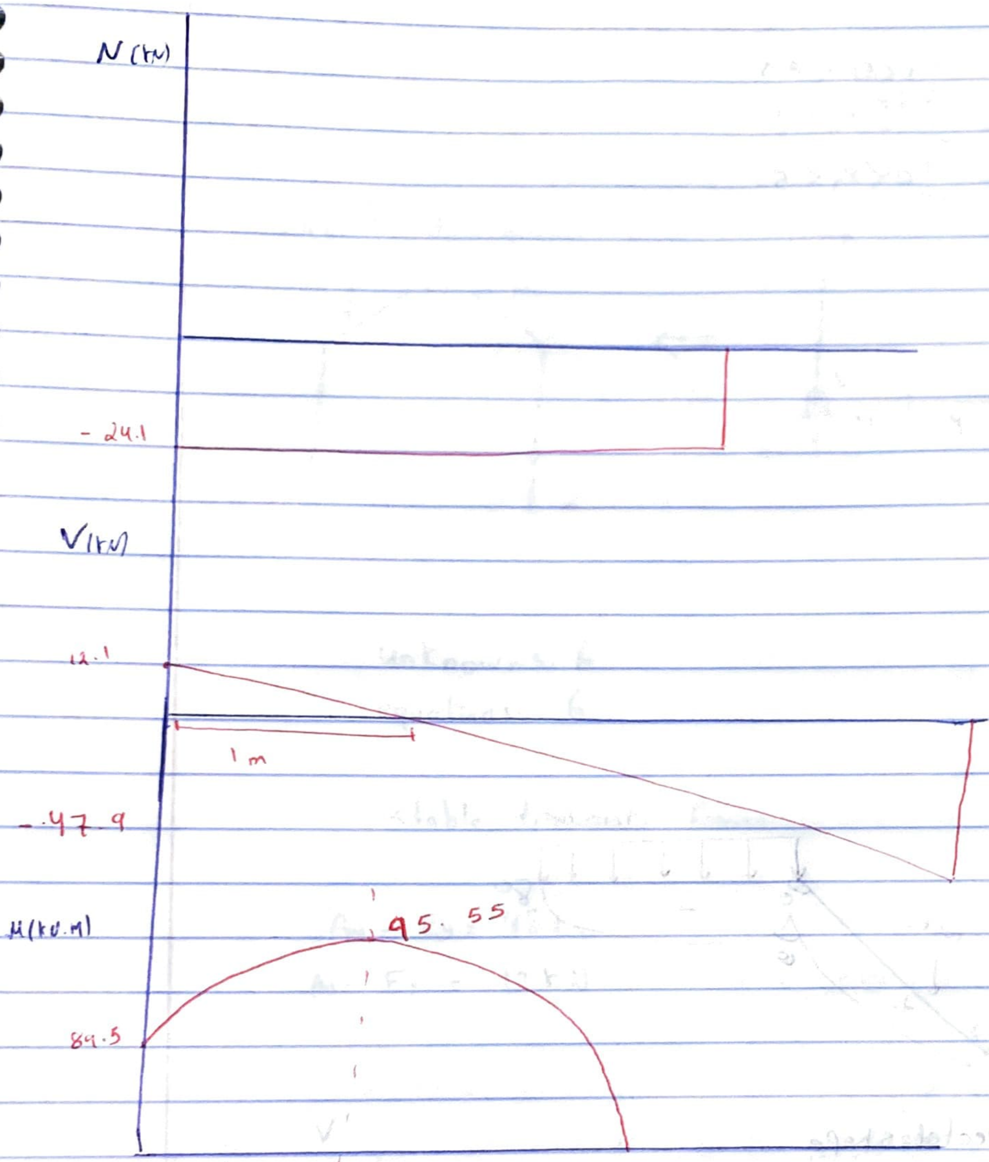


$$\Sigma F_x = 0 \rightarrow N = -24.1 \text{ kN}$$

$$\Sigma F_y = 0 \rightarrow V = -48.2 \text{ kN} + 12X_3 = 12X_3 - 48.2 \text{ kN}$$

$$\Sigma M_o = 0 \rightarrow 48.2X_3 - 12X_3\left(\frac{X_3}{2}\right) - M = 0$$

$$M = 48.2X_3 - 6X_3^2$$



N (kN)
-24.1
V (kN)
12.1
1 m
-47.9
M (kN.m)
95.55
89.5

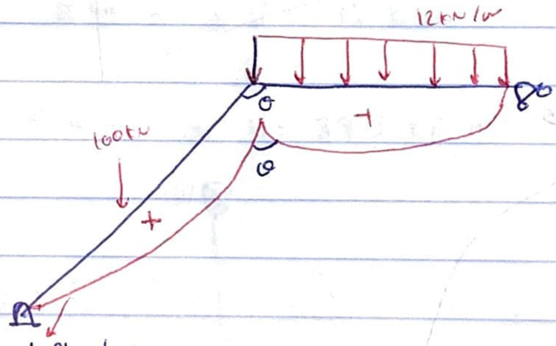
b-

$$V(x) = \begin{cases} AB: & 47.9 & 0 < x_1 < 2.5 \\ & -12.1 & 2.5 < x_2 < 5 \\ BC: & 12x_3 - 48.2 & 0 < x_4 < 5 \end{cases}$$

$N(x) =$

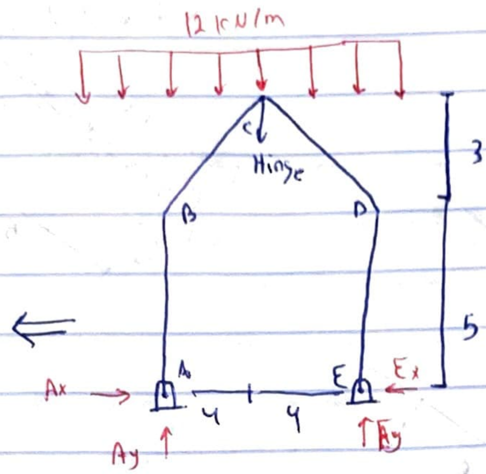
$M(x)$

c- deflected shape

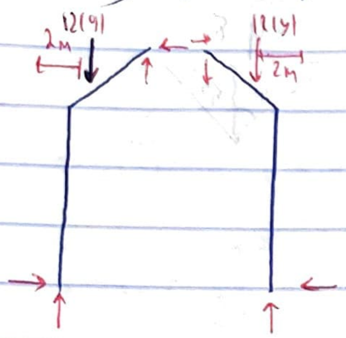


deflected shape

Example



stability, determancy



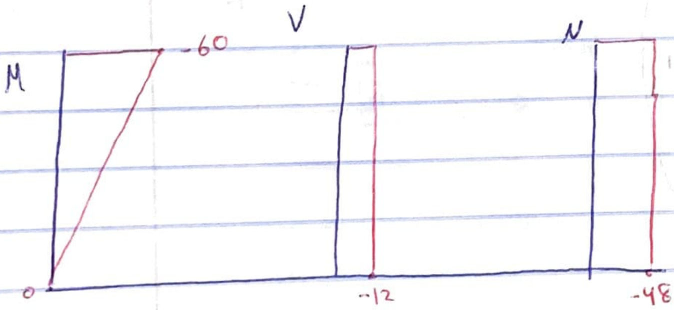
unknowns: 6

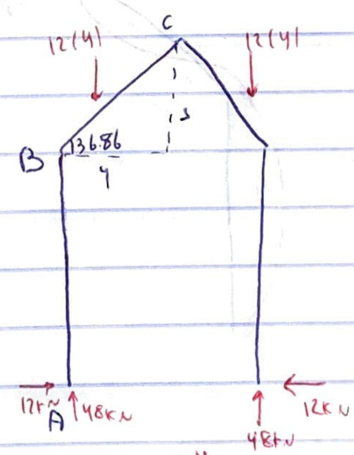
equations: 6

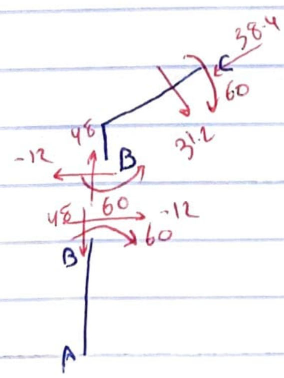
stable, determinate Frame

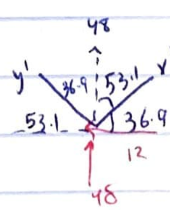
$A_y = E_y = 48\ kN$

$A_x = E_x = 12\ kN.$



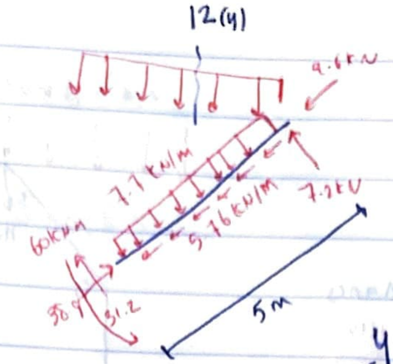


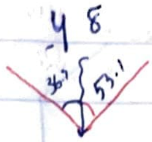


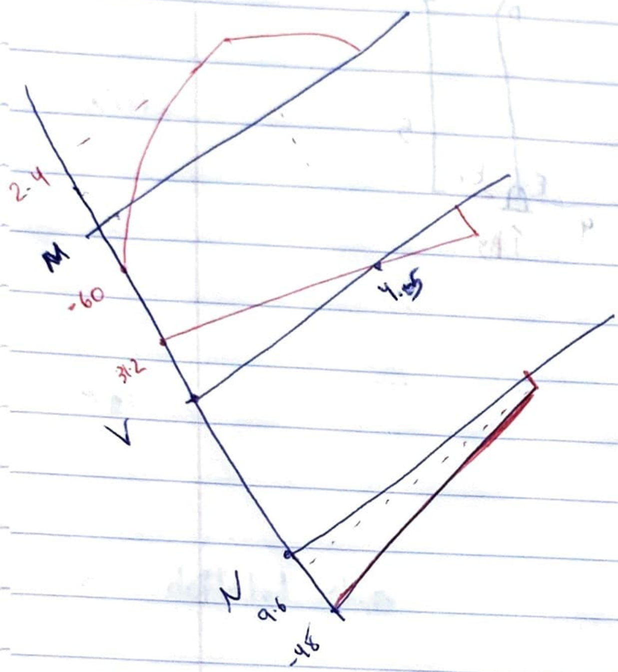


$x' = 48\cos 53.1 + 12\cos 36.9$

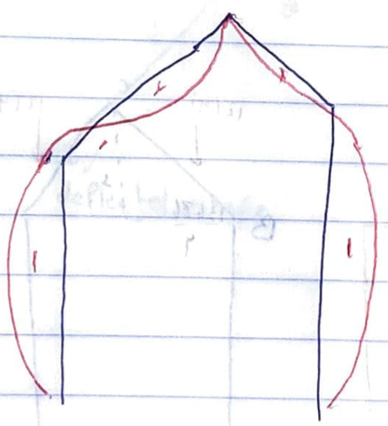
$y' = 48\cos 36.9 + 12\cos 53.1$







deflected shapes:



Deflections- double integration

$$\boxed{\frac{d^2 v}{dy^2} = \frac{M}{EI}}$$

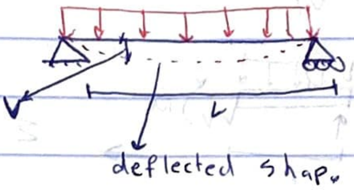
- axia load: $\frac{PL}{AE}$
- Torsion: $\frac{TL}{JG}$

Elastic Beam Theory: $\frac{d^2 v}{dy^2} = \frac{M}{EI}$

$$\int M(x) = \frac{\partial v}{\partial x} = \theta(x)$$

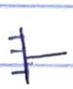
$$\iint M(x) = v(x)$$

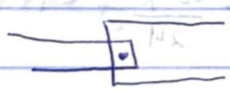
max — at specific point



constants of integration need to be determined using the bindary conditions

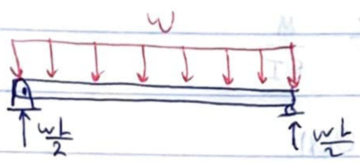
 → $\theta = ?$ $\quad v = 0$

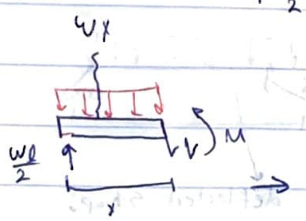
 → $\theta = 0 \quad v = 0$

 → $\bar{r} = \overset{+}{r}$

Example 1

Find the rotation and the deflection of the beam along the beam as a function of x, find the maximum deflection in of the beam





$\Sigma M_o = 0$

$-\frac{wL}{2}x + wx\left(\frac{x}{2}\right) + M = 0$

$M = \frac{wl}{2}x - \frac{wx^2}{2}$

$\theta = \int \frac{M}{EI} = \int \left(\frac{wlx}{2} - \frac{wx^2}{2}\right)dx / EI = \frac{wlx^2}{4} - \frac{wx^3}{6} + C / EI$

$EI.V = \int \theta dx = \int \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1 dx = \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$

Bondary conditions:

Pin → $V(0) = 0$ → $C_2 = 0$

roller → $V(L) = 0$ → $\frac{wl^4}{12} - \frac{wl^4}{24} + C_1 l$

$\frac{wl^4}{24} + C_1 l = 0$

$\frac{wl^4 + 24C_1 l}{24} = 0$ → $24 C_1 \cancel{l} = -wl^{\cancel{4}3}$

$C_1 = -\frac{wl}{24}$

$$EI.\ \theta(x) = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$

$$EI.\ V(x) = \frac{wlx^3}{12} - \frac{wl^4}{24} - \frac{wl^3x}{24}$$

2- $\theta(x)=0 \rightarrow V = max$

$$EI\ \theta(x) = 0 \rightarrow \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2}{24} = 0$$

$$6wlx^2 - 4wx^3 - wl^3 = 0$$

$$-4x^3 + 6lx^2 - L^3 = 0$$

$$\begin{array}{r|l} & -4x^3+4Lx+2L^2 \\ x-\frac{L}{2} & -4x^3+6Lx^2-L^3 \\ & -4x^3+2Lx^2 \\ \hline & 4Lx^2-L^3 \\ & 4Lx^2 \mp 2L^2x \\ \hline & 2L^2x - L^3 \\ & 2L^2x - \frac{2L^3}{2} \\ \hline & 0 \end{array}$$

$$(-4x^2 + 4Lx + 2L^2)(x - \frac{L}{2}) = 0$$

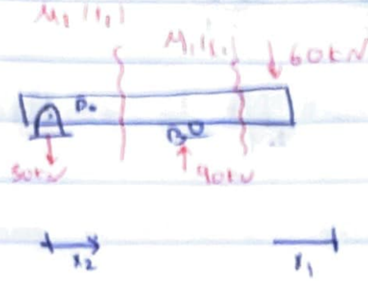
$$x_1 = \frac{-b \pm \sqrt{B^2 - 4ac}}{2a} \qquad x = \frac{L}{2}$$

$$= -L \pm \sqrt{L^2 - (L^2 2L^2)}$$

max at $x = \frac{L}{2}$

$$\frac{w}{4} l \left(\frac{l^2}{4}\right) -$$

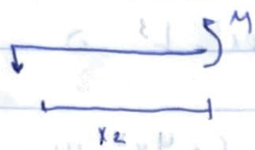
Example 2:



1- $M_1(Y_1) \rightarrow$

$M_1(Y_1) = -60Y_1$

2-



$M_2(X_2) = -30X_2$

$$U = \begin{cases} \iint \frac{M_1}{EI} & 0 < Y_1 < 3 \\ \iint \frac{M_2}{EI} & 0 < X_2 < 6 \end{cases}$$

$V(x) -$

STUDENTS-HUB.com

Deflections - double integration

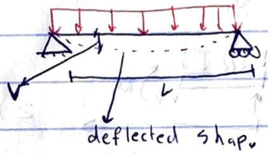
$$\boxed{\frac{d^2 v}{dy^2} = \frac{M}{EI}}$$

- axia load: $\frac{PL}{AE}$
- Torsion: $\frac{TL}{JG}$

Elastic Beam Theory : $\frac{d^2 v}{dy^2} = \frac{M}{EI}$

$$\int M(x) = \frac{\partial V}{\partial x} = \theta(x)$$

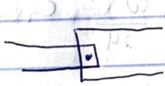
$$\iint M(V) = \underbrace{V(x)}_{\text{max} \quad \text{at specific point}}$$



constants of integration need to be determined using the bindary conditions

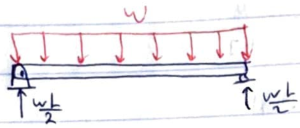
: $\rightarrow$ $\theta = ?$ $\quad V = 0$.

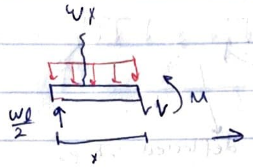
$\rightarrow$ $\theta = 0$ $\quad V = 0$

 $\rightarrow$ $\bar{r} = \overset{+}{r}$

Example 1

Find the rotation and the deflection of the beam along the beam as a function of x, find the maximum deflection in of the beam





$$\Sigma M_o = 0$$

$$-\frac{wL}{2}x + wx\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{wl}{2}x - \frac{wx^2}{2}$$

$$\theta = \int \frac{M}{EI} = \int \left(\frac{wlx}{2} - \frac{wx^2}{2}\right) dx / EI = \frac{wlx^2}{4} - \frac{wx^3}{6} + C / EI$$

$$EI \cdot V = \int \theta \, dx = \int \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1 \, dx = \frac{wlx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$

Bondary conditions:

Pin $\rightarrow V(0) = 0 \rightarrow C_2 = 0$

roller $\rightarrow V(L) = 0 \rightarrow \frac{wl^4}{12} - \frac{wl^4}{24} + C_1 l$

$$\frac{wl^4}{24} + C_1 l = 0.$$

$$\frac{wl^4 + 24C_1 l}{24} = 0$$

$$24 C_1 l = -wl^3$$

$$C_1 = -\frac{wl}{24}$$

$$EI.\ \theta(x) = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24}$$

$$EI.\ V(x) = \frac{wlx^3}{12} - \frac{wl^4}{24} - \frac{wl^3x}{24}$$

2- $\theta(x) = 0 \rightarrow V = max$

$$EI\,\theta(x) = 0 \rightarrow \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^2}{24} = 0$$

$$6wlx^2 - 4wx^3 - wl^3 = 0$$

$$-4x^3 + 6lx^2 - L^3 = 0$$

$$\begin{array}{r|l} & -4x^3 + 4Lx + 2L^2 \\ \hline x - \frac{L}{2} & -4x^3 + 6Lx^2 - L^3 \\ & -4x^3 + 2Lx^2 \\ \hline & 4Lx^2 - L^3 \\ & 4Lx^2 - 2L^2x \\ \hline & 2L^2x - L^3 \\ & 2L^2x - \frac{2L^3}{2} \\ \hline & 0 \end{array}$$

$$(-4x^2 + 4Lx + 2L^2)(x - \frac{L}{2}) = 0$$

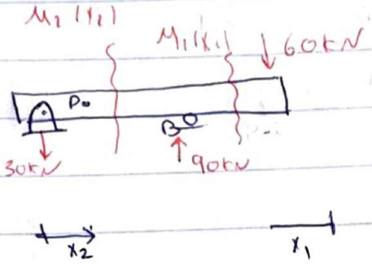
$$x_1 = \frac{-b \pm \sqrt{B^2 - 4ac}}{2a} \qquad x = \frac{L}{2}$$

$$= -L \pm \sqrt{L^2 - (L^2 + 2L^2)}$$

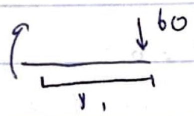
max at $x = \frac{L}{2}$

$$\frac{w}{4} l\left(\frac{l^2}{4}\right) -$$

Example 2:

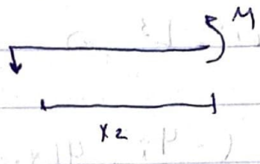


1- $M_1(x_1) \rightarrow$



$M_1(x_1) = -60x_1$

2-



$M_2(x_2) = -30x_2$

$$v = \begin{cases} \iint \frac{M_1}{EI} & 0 < x_1 < 3 \\ \iint \frac{M_2}{EI} & 0 < x_2 < 6 \end{cases}$$

$$EI\; v(x) = \begin{cases} -10x_1^3 + C_1x_1 + C_2 & 0 < x_1 < 3 \\ -5x_2^3 + C_3x_2 + C_4 & 0 < x_2 < 6 \end{cases}$$

B.C.

at A: $v_A(0) = 0 \leftarrow x_2$

at B: $v_B(4) = 0 \leftarrow x_1$

$x_1 = 3$

$v_B(x_2 = 6) = 0 \leftarrow x_2$

$\int \uparrow$

$$\theta(x) = \begin{cases} (-30x_1^2 + C_1)EI & 0 < x_1 < 3 \\ (-15x_2^2 + C_3)EI & 0 < x_2 < 6 \end{cases}$$

$\bar{\theta}_B = \overset{+}{\theta}_B$

$\bar{\theta}_B(x_1 = 3) = \overset{+}{\theta}_B(x_2 = 6)$

$V_A(x_2=0)=0 \rightarrow \boxed{C_4=0}$ — 1

$V_B(x_1=3)=0 \rightarrow -270+3C_1+C_2=0$ — 2

$V_B(x_2=6)=0 \rightarrow -1080+6C_3=0 \rightarrow \boxed{C_3=180}$ — 3

$\theta_B^-(x_2=6) = \theta_B^+(x_1=3)$

$-270+C_1 = -540+180 \rightarrow \boxed{C_1=-90}$

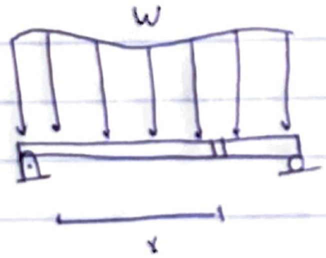
— 2 - $\quad -270-270+C_2=0 \rightarrow \boxed{C_2=540}$

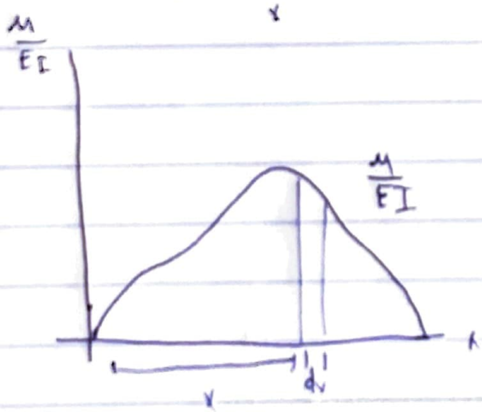
$$EI\,v(x) = \begin{cases} -10x_1^3-90x_1+540 & 0<x_1<3 \\ -5x_2^3+180x_2 & 0<x_2<6 \end{cases}$$

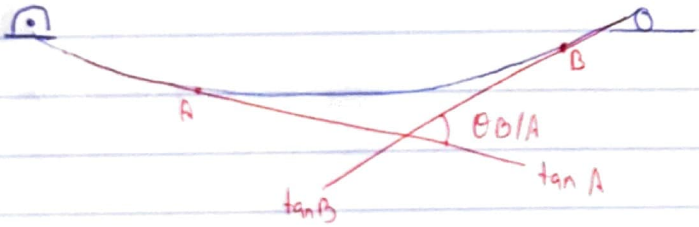
$$EI\,\theta(x) = \begin{cases} -30x_1^2-90 & 0<x_1<3 \\ -15x_2^2+180 & 0<x_2<6 \end{cases}$$

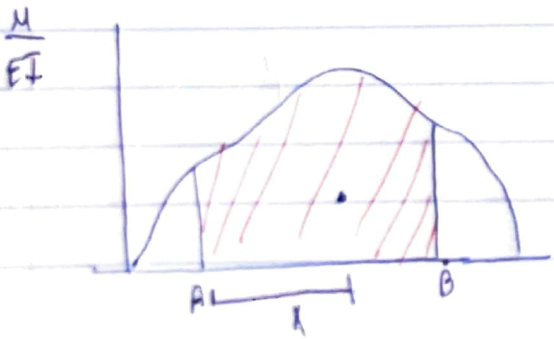
Deflection - moment area theoreom

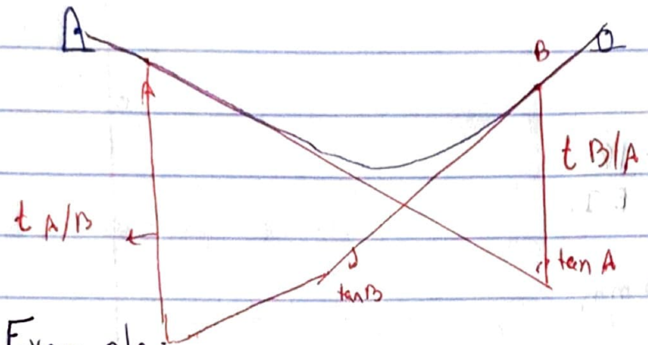
Theorem 1 : The change in slope between any two points on the elastic curve equals the area of the M/EI diagram between these two points



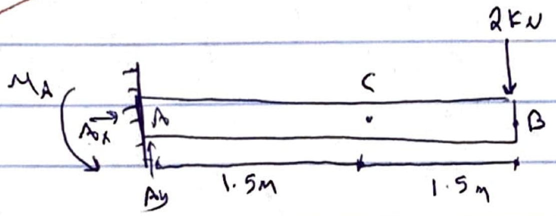








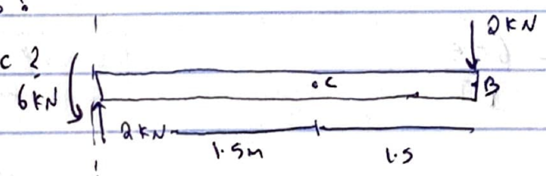
Example:

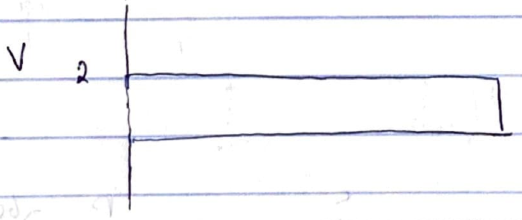


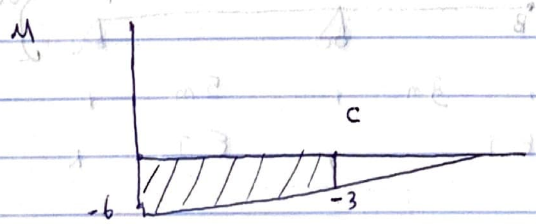
$EI = const$

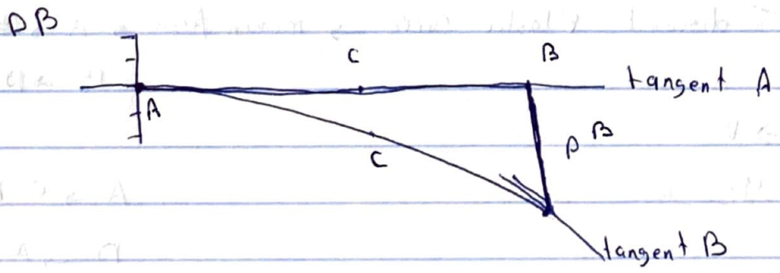
Find D_B ?

θ_C ?









$D_B =$ tan A → B

↓ moment area A → B about B

$$\text{Area of } \frac{M}{EI} = \underbrace{\frac{1}{2} \times 3 \times \frac{-6}{EI}}_{\text{area}}$$

$$D_B = \frac{1}{2} \times 3 \times \frac{-6}{EI} \times (2m) = -\frac{18}{EI}$$

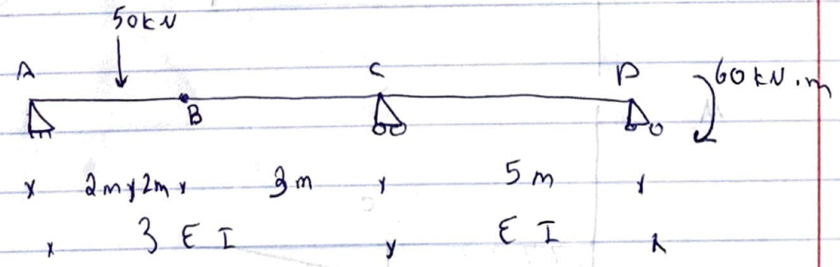
$$D_B = -0.01285\ m = -12.85\ mm$$

2. θ_c

$$\theta_c = \theta_{A/C} = \text{Area } C \rightarrow A = \frac{1}{2}\left(-\frac{6}{EI} - \frac{3}{EI}\right) \times 1.5$$

$$\theta_c = -\frac{6.75}{EI}$$

Example:



discont Elastic curve → momen Area → A → B, B → D

A → B

$$\Sigma M_B = 0$$

A → C ✗

D → A ✗

$$2(50) - 4A_y = 0$$

$$A_y = \frac{50}{2} = \boxed{25\ kN \uparrow} \rightarrow \boxed{B_y = 25\ kN \uparrow}$$

B → D

$\Sigma M_D = 0$

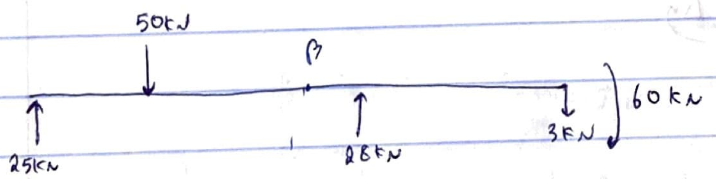
$-60 - 5C_y + 8(25) = 0$

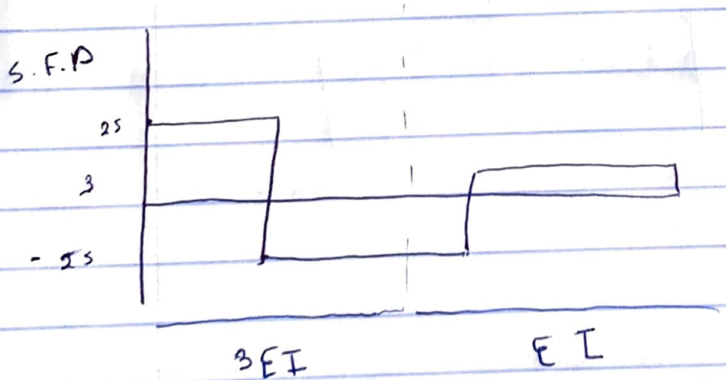
$\boxed{C_y = 28\,kN \uparrow}$

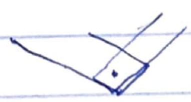
$\Sigma F_y = 0$

$28 - 25 + D_y = 0$

$\boxed{D_y = 3\,kN \downarrow}$

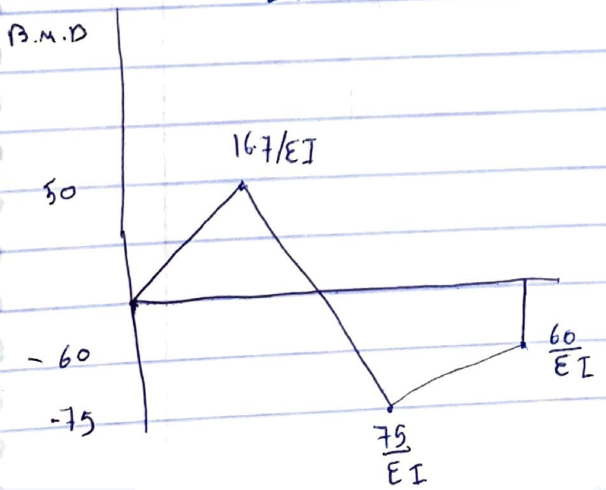


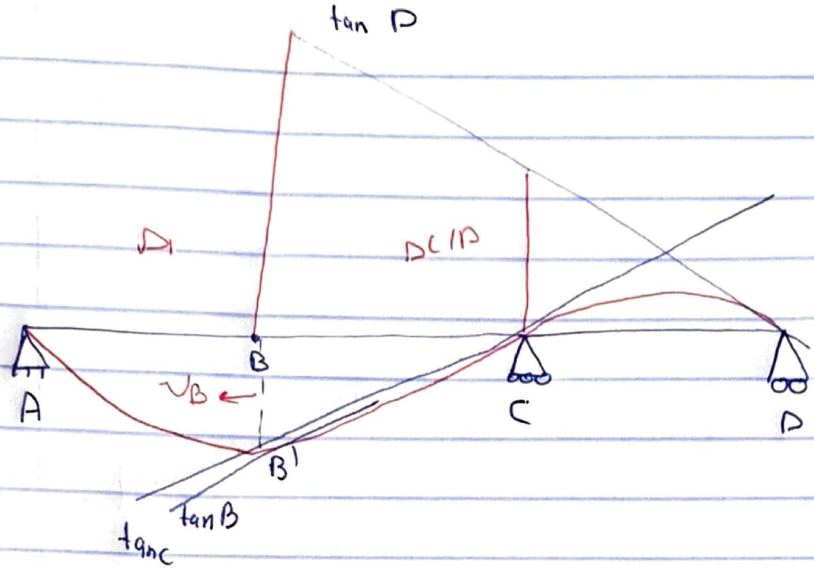




discont. elastic curve

→ Moment area → A → B
B → D





$v_B = \Delta_{B/D} - \Delta_1$

$\dfrac{\Delta_{C/D}}{5} = \dfrac{\Delta_1}{8}$

$\Delta_1 = \dfrac{8}{5}\,\Delta_{C/D}$

$v_B = \Delta_{B/D} - \dfrac{8}{5}\,\Delta_{C/B}$ عن ركة الموصت

Area

$\Delta_{B/D}$ = Moment $(B \rightarrow D)$ about $B = \frac{1}{2} \times 3 \times \frac{75}{EI} \times (2) + \frac{1}{2} \times 5 \times \frac{75-66}{EI} \times \left(3 + \frac{5}{3}\right) + 5 \times \frac{66}{EI} \times (3 + 5.5)$

$\Delta_{C/D}$ = Moment $(C \rightarrow D)$ about C

$\Rightarrow \Delta_{B/C} = \dfrac{2050}{EI}$

لازم نوخذ بالاعتبار إشارة المومنت كاعتبا
صاحات ، أحيانا ً بنزيد ، أو بنقص المساحات

$\Rightarrow \Delta_{C/D}$ = Moment $(C \rightarrow D)$ about C: $\frac{1}{2} \times 5 \times \frac{15}{EI} \times \left(\frac{5}{3}\right) + \frac{5 \times 60}{EI}(2.5) = \frac{812.5}{EI}$

$\Rightarrow \Delta_{C/D} = \dfrac{812.5}{EI}$

$v_B = \Delta_{B/D} - \dfrac{8}{5}\,\Delta_{C/D} = \dfrac{2050}{EI} - \dfrac{8}{5}\left(\dfrac{812.5}{EI}\right) = \dfrac{-750}{EI}$

الماسِن إتجاه الدفليكشن

Conjugate beam

Load W

Shear $V = \int W$

Moment $M = \int V = \iint W$

$\longleftrightarrow$

$\frac{\partial V}{\partial x} = \int \frac{M}{EI}$ slope $\theta = \int \frac{M}{EI}$

$\frac{\partial^2 V}{\partial x^2} = \frac{M}{EI}$ Deflection $\Rightarrow V = \iint \frac{M}{EI}$

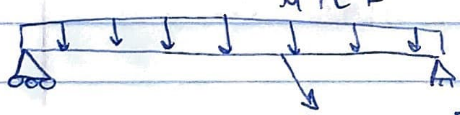
$$\frac{\partial^2 v}{\partial y^2} = \frac{M}{EI}$$

Moment area

$\theta = \int \text{area}$

$\Delta = \iint$ Moment area

M/EI



Beam

Shear $V \rightarrow 0$

$M = V$ ← B.C.

Ax →

Ay

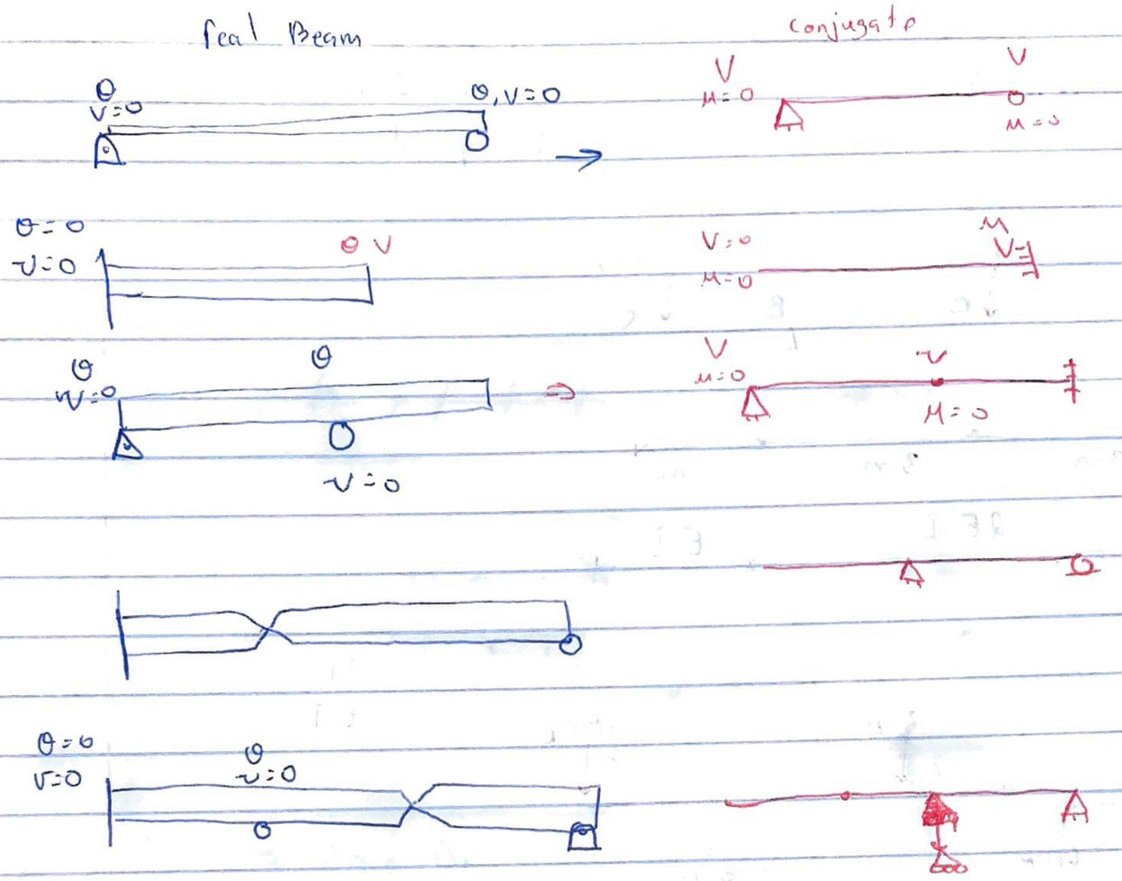
$\theta\,??\xrightarrow{V}$

$V = 0$ → $M = 0$

جدول ال conjugate Beam موجود بالكتاب

الي فوق بحكي عن similarity ، المقصود ال Deflection similar ، وال shear similar θ

ف لما نحل ع conjugate Beam وكو



Theorem 1: The slope at a point in the real Beam is numerically equal to the shear at the corresponding point in the conjugate beam

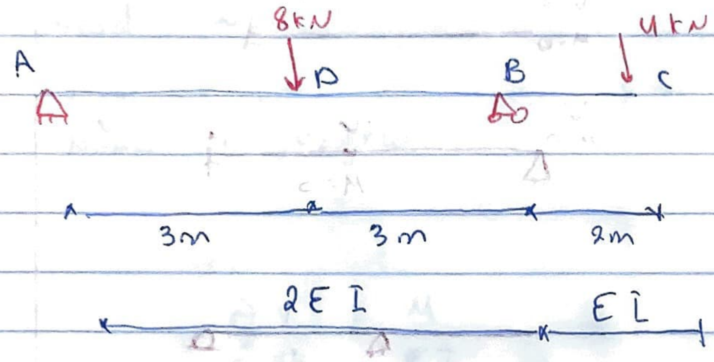
Theorem 2: The displacement of a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam

Real → conjugate → M, V
↓
θ V

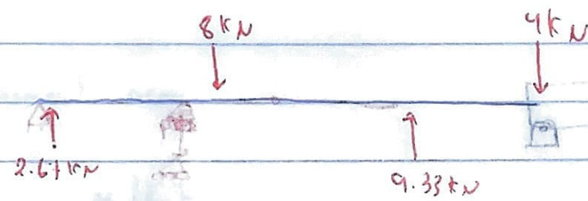
find v_C, v_D, θ_D

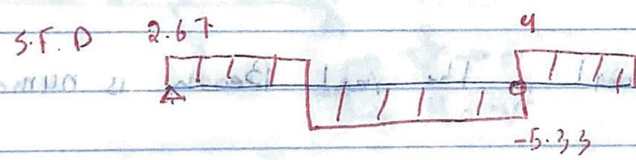
$E = 200\ GPa$

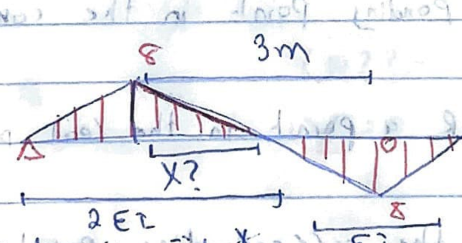
$I = 7 \times 10^{7}\ mm^{4}$



$\frac{M}{EI}$







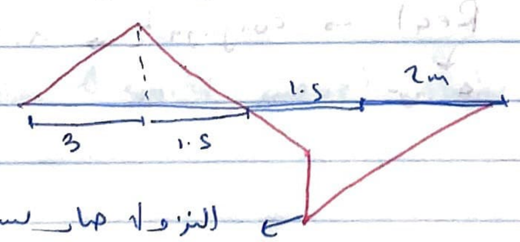
X بنقدر نوجدها من تشابه المثلثات

أو من مفهوم إنه الـ slope للمومنت = shear

ف بنحكي إنه من الـ 8 للصفر الـ slope هو 5.33-

ونعوض $\Delta y = 8$, $\Delta x = x$.

$\frac{M}{EI}$

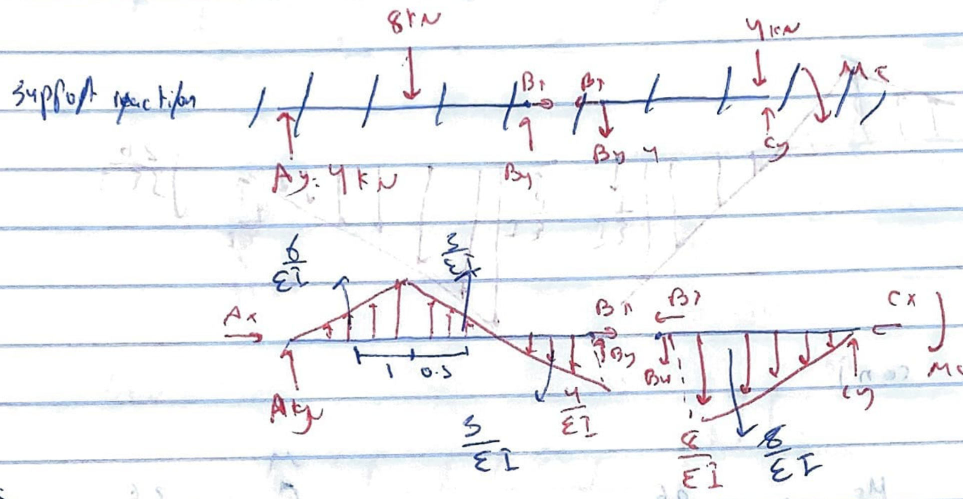


النزول صار بسبب تغير قيمة EI

لأنه إحنا بنعمل الـ sketch للـ $\frac{M}{EI}$ فبتتغير بهاي

المنطقة

Conjugate



1st sec

$\sum F_x = 0 \Rightarrow A_x = -B_x$

$\sum F_y = 0 \Rightarrow A_y + \frac{6}{EI} + \frac{3}{EI} - \frac{3}{EI} + B_y = 0$

$A_y + B_y = -\frac{6}{EI}$

$\sum M = 0 \Rightarrow \frac{6}{EI}(2) + \frac{3}{EI}(3.5) - \frac{3}{EI}(5.5) + 6B_y = 0$

$B_y = -\frac{1}{EI}$

$\Rightarrow A_y = -\frac{5}{EI}$

2nd sec $\sum F_x = 0 \Rightarrow B_x = -C_x$

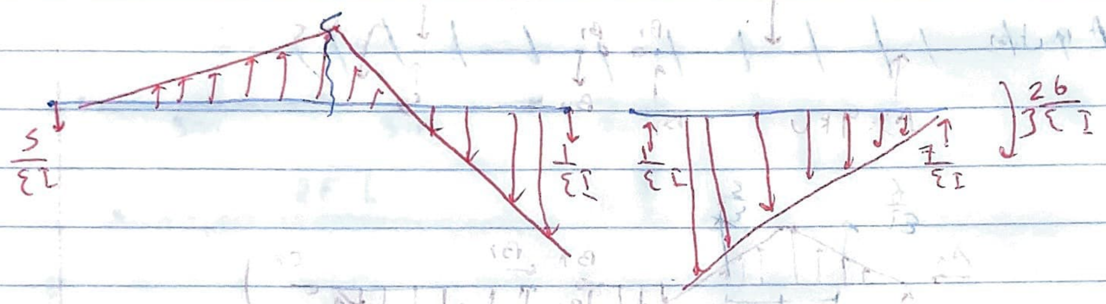
$\sum F_y = 0 \Rightarrow -B_y - \frac{8}{EI} + C_y = 0$

$\Rightarrow C_y = \frac{7}{EI}$

$\sum M_c = 0 \Rightarrow B_y(7) + \frac{8}{EI}\left(\frac{4}{3}\right) - M_c = 0$

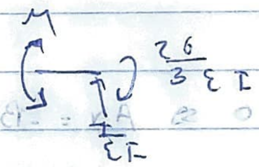
$$\frac{-2}{EI} + \frac{32}{3EI} - M_c = 0$$

$$M_c = \frac{-6+32}{3EI} = \frac{26}{3EI}$$



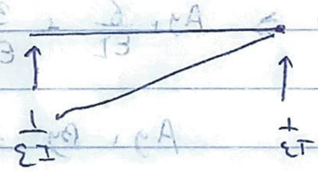
Real conj

$V_c = M_c = -\frac{26}{3EI}$

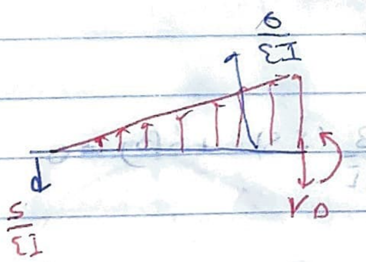


$$= \frac{-26}{3 \times 200 \times 10^6 \times 7 \times 10^7 \times 10^{-12}}$$

$V_c = -6.2 \times 10^{-4}\ m = 0.62\ mm$



$V_D =$



real conj

$V_D = M_D$

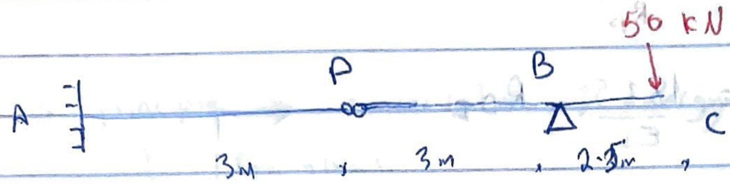
$$\Sigma M_D = 0 \Rightarrow \frac{5}{EI}(3) - \frac{6}{EI}(1) + M_D = 0$$

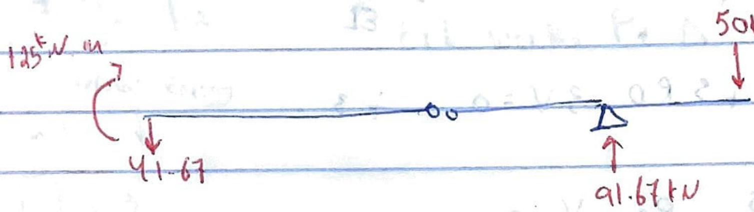
$$M_D = \frac{-9}{EI} = V_D = \frac{-9}{200 \times 10^6 \times 7 \times 10^7 \times 10^{-12}} = 0.642\ mm$$

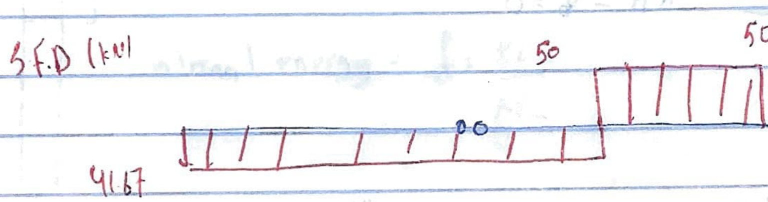
$$\theta_D = V_D \Rightarrow -\frac{5}{EI} + \frac{6}{EI} - V_D = 0 \Rightarrow V_D = \frac{1}{EI}$$

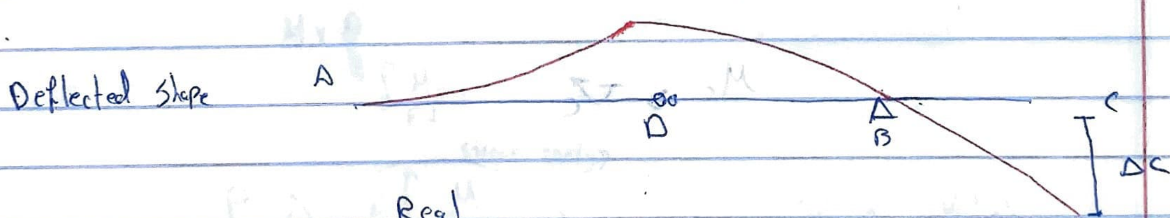
STUDENTS-HUB.com

Example 2





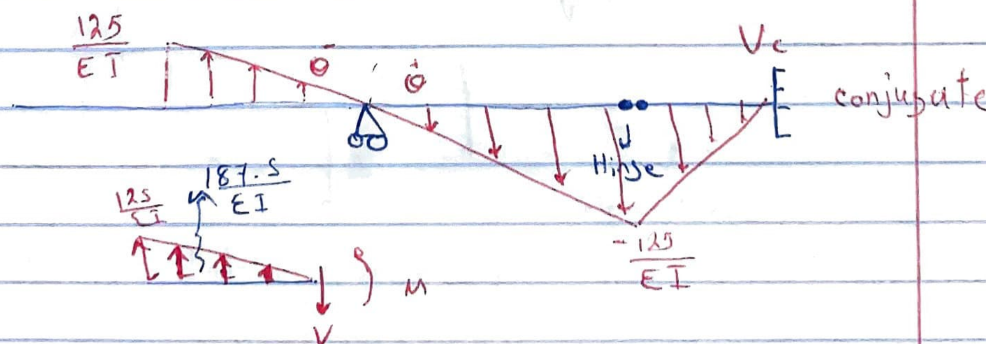




Real
↓
conjugate

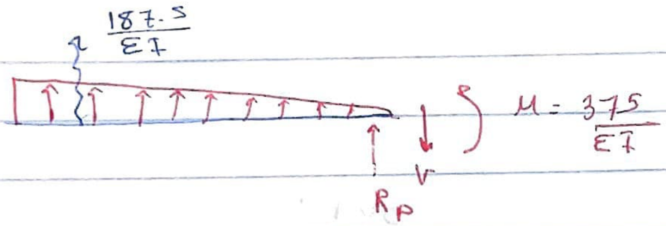
$V \rightarrow \theta_{real}$

$M \rightarrow v_{real}$



$$V_D = +\frac{187.5}{EI} = \bar{\theta}_D$$

$$M_D = +\frac{375}{EI} = v_D$$



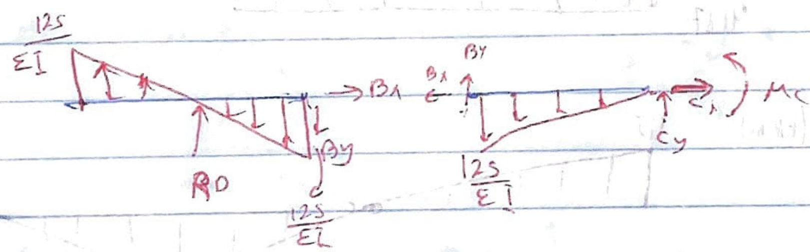
$\Sigma F_y = 0 \Rightarrow V_D = \frac{187.5}{EI} + R_D$

$\Sigma M_A = 0 \Rightarrow \frac{187.5}{EI}(1) + R_D(3) - 3V + \frac{375}{EI} = 0$

$\Sigma M_A = 0 \Rightarrow \frac{562.5}{EI} + 3R_D - 3V = 0 \quad \div 3$

$\frac{187.5}{EI} + R_D - V_D = 0$

$N_C \rightarrow 6.8$

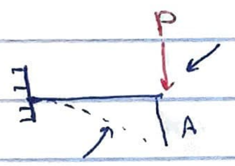


$M_C \Rightarrow V_C$

Deflection - Energy methods

Energymethod → principle → conservation of energy

Energy (work)



Ext. work = $P * \Delta$

internal Energy

internal energy = $f \times \frac{fL}{AE}$

$M \times \theta$

$\int \frac{M}{EI}$

strain energy

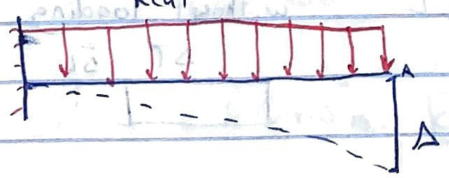
$\underbrace{P \cdot \Delta}_{Ext.} = \underbrace{M \int \frac{M}{EI}}_{int} \rightarrow$

إذا كانت M ثابت

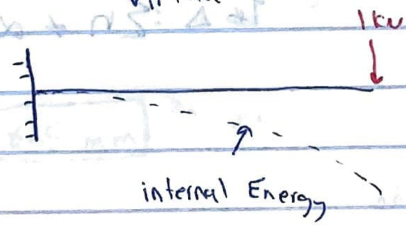
$P \cdot \Delta = \int \frac{M^2}{EI}$

* Virtual work method

Real



Virtual



External virtual work = Internal virtual work

$\underset{\substack{\text{virtual} \\ \uparrow \\ \text{Ext. force}}}{1} \times \underset{\text{real}}{\Delta} = \underset{\substack{\text{virtual} \\ \uparrow \\ \text{int. force}}}{u} \times \underset{\text{real}}{\delta L}$

U

virtual

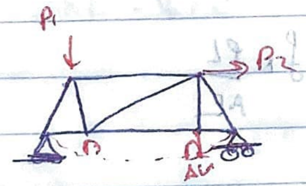
truss | Beam | Frame

$n \times \delta_x \frac{NL}{AE}$ | $m \times \int \frac{M}{EI}$ | $m \times \int \frac{M}{EI}$

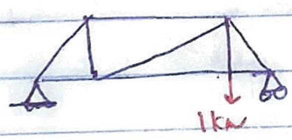
Truss

Main internal force → Axial force (N)

Real → N



Virtual → n



Ext. virtual work = Int. virtual work

$$1 * \Delta = \sum n * \frac{NL}{AE}$$

Temp change

$$1 * \Delta = \sum n * \alpha \Delta T L$$

without loading

خطأ تصنيع

Fabrication error

$$1 * \Delta = \sum n * \underbrace{\Delta L}_{error}$$

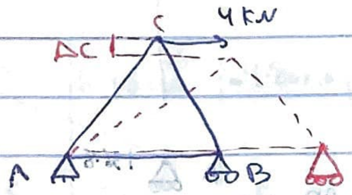
Example:

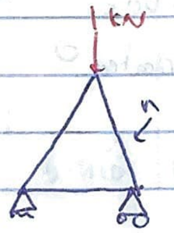
find virtual disp. at C

all members:

$A = 400 mm^2$

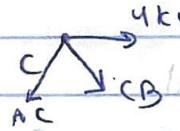
$E = 70 GPa$



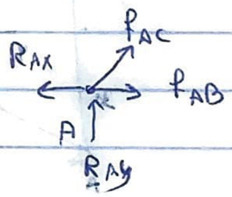


$1 * \Delta C = \sum n * \frac{NL}{AE}$

	n	N	L	nNL
AC	-0.83	2.5	5	-10.375
AB	0.67	2	8	10.72
CB	-0.83	-2.5	5	10.375
Σ				10.72

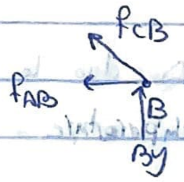


$\Sigma F_x = 0 \quad \Sigma F_y = 0$



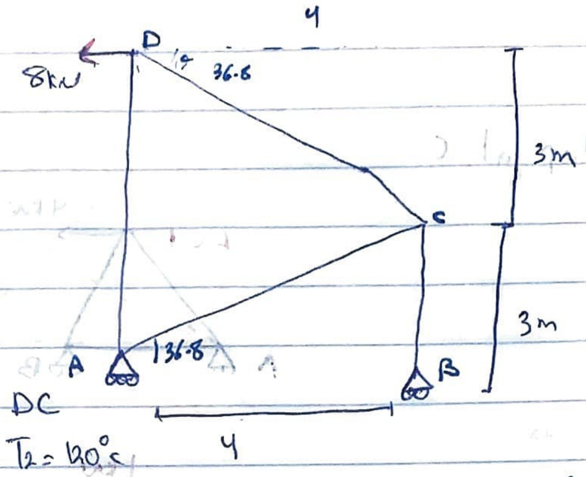
$1 * \Delta C = \frac{10.72}{AE}$

$= \frac{10.72 \times 10^3}{400 \times 10^{-6} \times 70 \times 10^9}$



$\Delta C = 3.825 \times 10^{-4} m = \boxed{0.3825 mm}$

Example 2



find the horizontal disp at D

due to: ① loading

② Temp. change in AD, DC

$T_1 = 30°c \rightarrow T_2 = 120°c$

③ member DC was fabricated

$L = 5.05\ m$

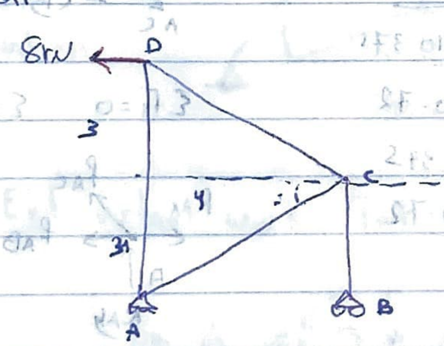
④ all

$A = 200\ mm^2$

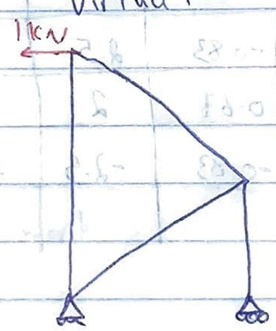
$E = 200\ GPa$

$\alpha = 1.2 \times 10^{-5}$

Real

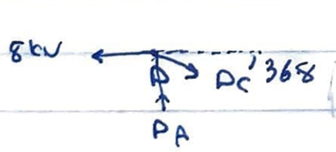


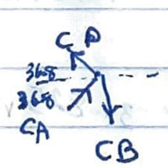
Virtual



① Δ due to loading

② Temperature





$8 - DC \cos 36.8 = 0$

$\Rightarrow DC = 10\ kN$

$-10 \cos 36.8 + CA \cos 36.8 = 0$

$\Rightarrow CA = 10\ kN$

$DA - 10 \sin 36.8 = 0$

$\Rightarrow DA = 6\ kN$

$\Rightarrow -CB + 10 \sin 36.8 + 10 \sin 36.8 = 0$

$\Rightarrow CB = 12\ kN$

① ΔD due to loading

	N	n	L	nNL	αΔTL	n×αΔTL	ΔL	n×ΔL
AD	-6	-0.75	6	27	6.48×10^{-3}	-4.86×10^{-3}	0.05	-0.0375
AC	-10	-1.25	5	6.25	0	0	0	0
BC	12	1.5	3	54	0	0	0	0
CD	10	1.25	5	6.25	5.4×10^{-3}	6.75×10^{-3}	0	0

$$1\times\Delta_D = \frac{206\ kN^2.m}{200\times10^{-6}\times200\times10^{6}} \Rightarrow \Delta_D = 5.15\ mm$$

② Temp. change ← real int disp = αΔTL×n

$$1\times\Delta_D = 1.89\times10^{-3} \Rightarrow \Delta_D = 1.89\ mm$$

③ fab. error (AD = 5.05 mm) → cont. int. disp ⇒ $\Delta_{AD} = 0.05\ m$

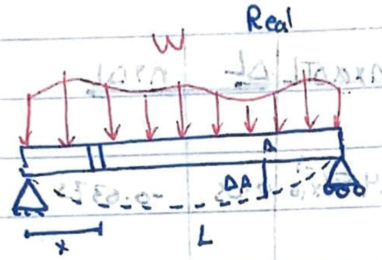
$$1\times\Delta_D = -0.0375 \Rightarrow \Delta_D = 3.75\ cm \longrightarrow$$

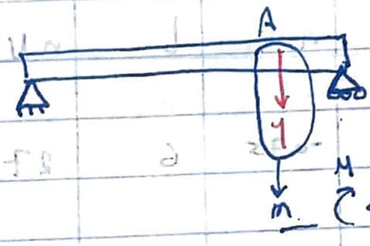
إشارة ال displacement عكس إشارة الفرض لإنها طالعة
سالبة

④ all

$$\Delta_{fab} + \Delta_{Temp} + \Delta_{load} = \overrightarrow{3.75} - \overleftarrow{1.89} - \overleftarrow{5.15} = \overrightarrow{30.46}\ mm$$

Beams

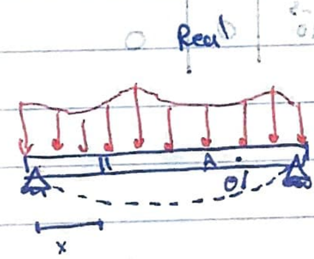


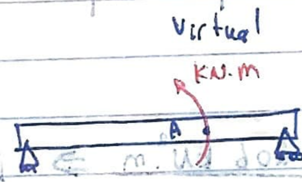


rotation: M ↔ $\int \frac{M}{EI}$

B.M

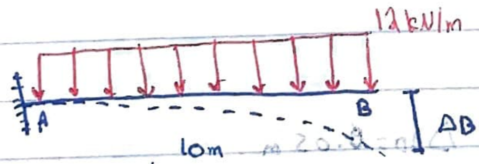
$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}\,dx$$



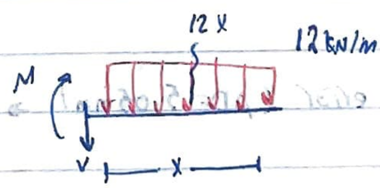


$$1 \cdot \theta = \int_0^L \frac{m_\theta M}{EI}\,dx$$

Example



find ΔB



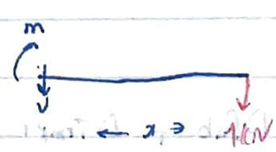
$E = 200\,GPa$, $I = 500 \times 10^6\ mm^4$

$M = -6x^2$

sol:

m: virtual int. moment

$$1 \times \Delta_B = \int_0^{10} \frac{mM}{EI}\,dx$$



$$\Delta_B = \int_0^{10} \frac{-6x^2(-x)}{EI}\,dx$$

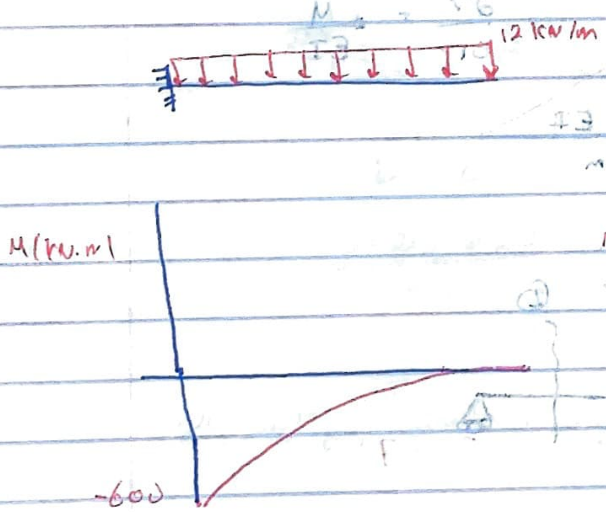
$m = -x$

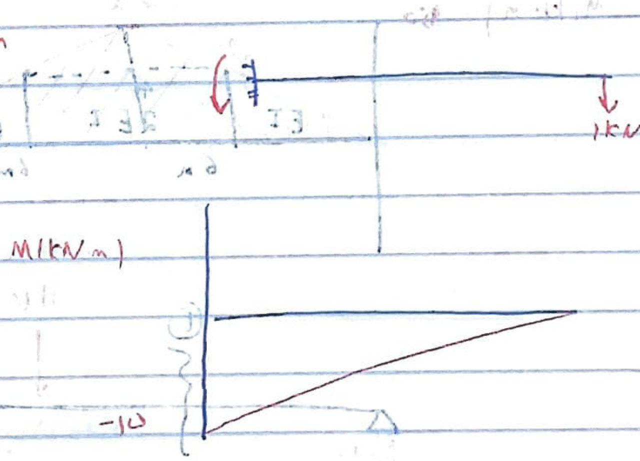
$$\int_0^{10} \frac{6x^3}{EI}\,dx$$

$$\Delta B = \frac{1}{EI} \times \frac{6}{4}(x^4) \Big|_0^{10}$$

$$\Delta B = \frac{1}{EI} \cdot \frac{6(10^4)}{4} = \frac{15000}{EI} = \frac{15000 \times 10^3}{200\times10^9 \times 500\times10^6\times10^{-12}} = 0.015\ m \downarrow$$

There is another way to solve Beams by virtual work, is by using the table (بجانب الدراكتر) that used the moment diagram shapes for virtual and real Beams to solve for the deflection

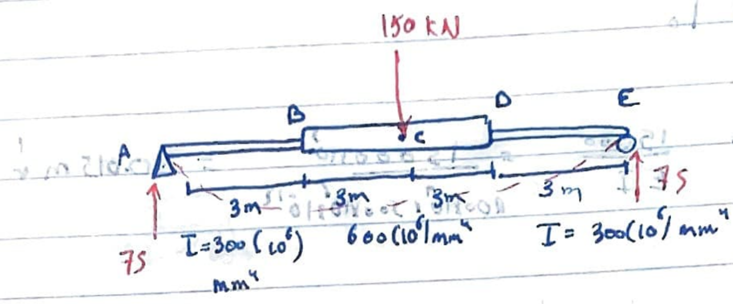




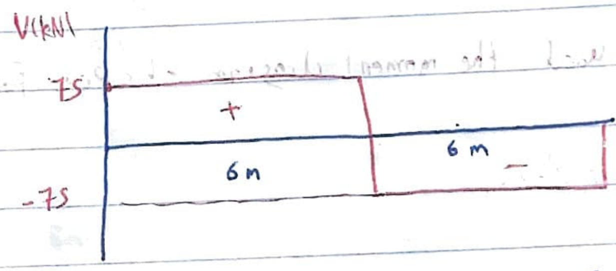
using the table

$$\int_0^{10} \frac{mM}{EI}dx = \frac{\frac{L\,M\,Q}{4}}{EI} = \frac{10(600)(10)}{4EI} = 0.015\ m\downarrow$$

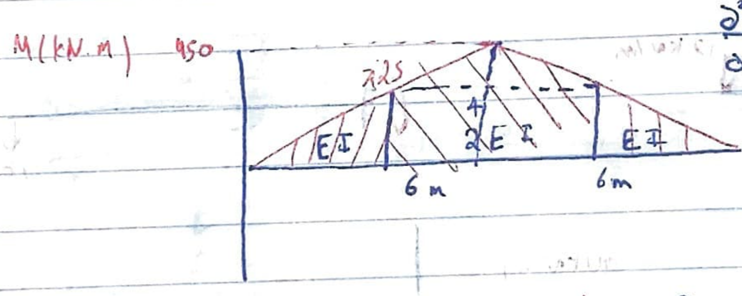
Example.

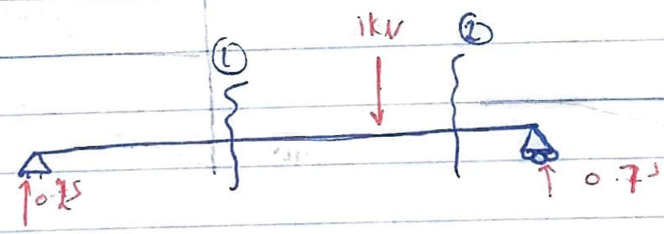


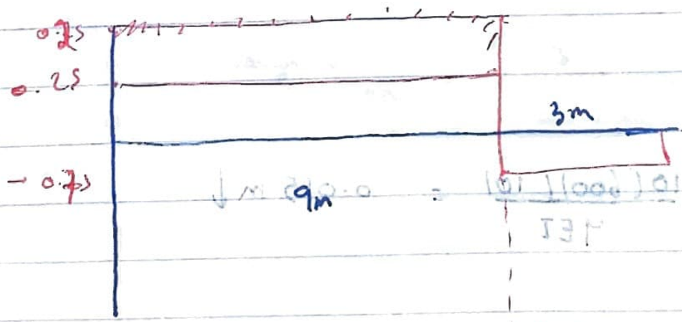
E = 200 GPa

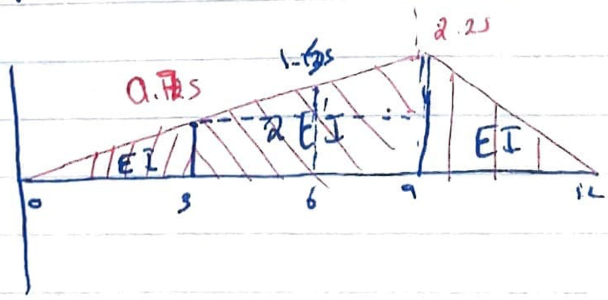


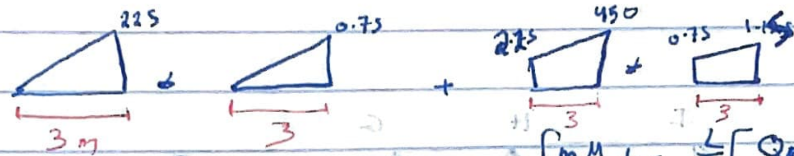
$$\frac{\partial^2 v}{\partial x^2} = \frac{M}{EI}$$









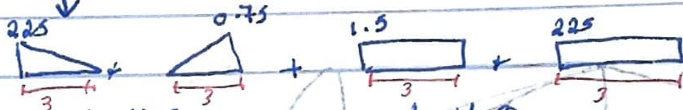


$$\int \frac{mM}{EI}\,dx = \frac{L}{6}\left[Q_a\left[2M_a+M_b\right] + Q_b\left(M_a+2M_b\right)\right]$$

$$\frac{LMQ}{3EI} = \boxed{\frac{3\times 0.75\times 225}{3EI}}$$

$$= \boxed{\frac{\frac{3}{6}\left[0.75(2\times225+450)+1.5(225+2(450))\right]}{2EI}}$$

$+$ 



$$\frac{LMQ}{6} + LMQ$$

$$\boxed{= \frac{3\times225\times0.75}{6\times2EI} + \frac{3(1.5)(225)}{2EI}}$$

$+$ 

$$LMQ$$

$$= \boxed{\frac{3\times225\times2.25}{EI}}$$

عشان نطلع قيمة ال deflection بنجمع المربعات الحمر

$$\int \frac{Mm}{EI}dx = \frac{168.75}{EI} + \cancel{\ldots} + \frac{759.375}{EI} + \frac{548.44}{EI} + \frac{506.24}{EI}$$

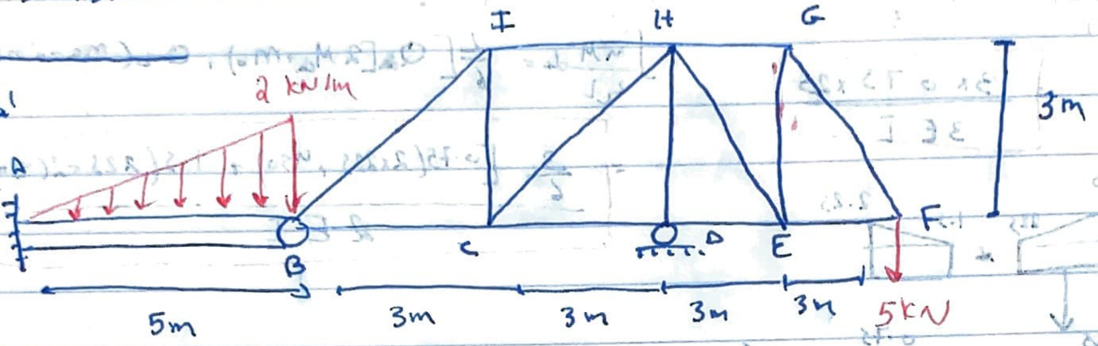
$$\int \frac{Mm}{EI}dx = \frac{1982.815}{EI}$$

$$\underset{\text{virtual}}{1} \times \underset{\text{real}}{\Delta D} = \frac{1982.815}{EI} = \frac{1982.815\times10^{3}}{200\times10^{9}\times300\times10^{6}\times10^{-12}}$$

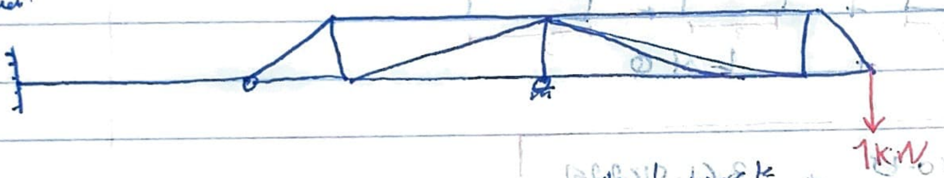
$$\Rightarrow \boxed{\Delta D = 0.033\,m = 33\,mm}$$

Discussion:

Real



virtual



virtual work

External = internal

$$1 \times \Delta F = \underbrace{\int \frac{mM}{EI}}_{\text{rotation}} + \underbrace{\sum \frac{nN}{AE}}_{\text{elongation}}$$

Frame and other types of strain energy

frames → internal → Forces → N ← int disp = dL/AE

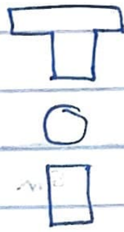
→ V ←

→ M ← $\theta = \int \frac{M}{EI}$

$\tau = \frac{VQ}{It} \Rightarrow$ rec. $\tau_{avg} = \frac{V}{A}$

$\gamma = k\frac{V}{GA}$

modulus of rigidity



rect, $k = 1.2$

circle, $k = \frac{10}{9}$

T, $k = 1$, A# web

real → virtual

$\frac{NL}{AE} \rightarrow n\frac{NL}{AE}$

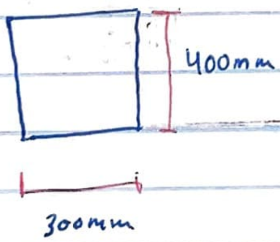
$\frac{kV}{GA} \rightarrow v$

$\int \frac{M}{EI} \rightarrow \int \frac{mM}{EI}$

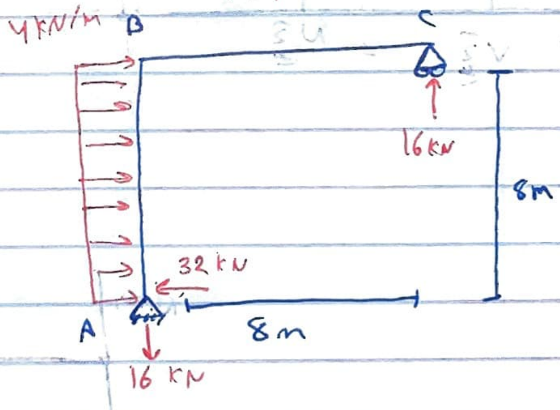
Example:

rect. section:

$E = 25\,GPa$



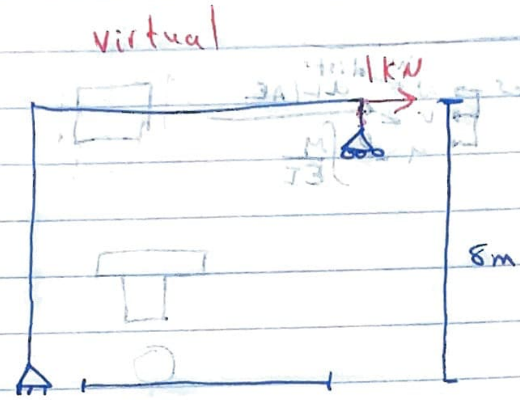
real



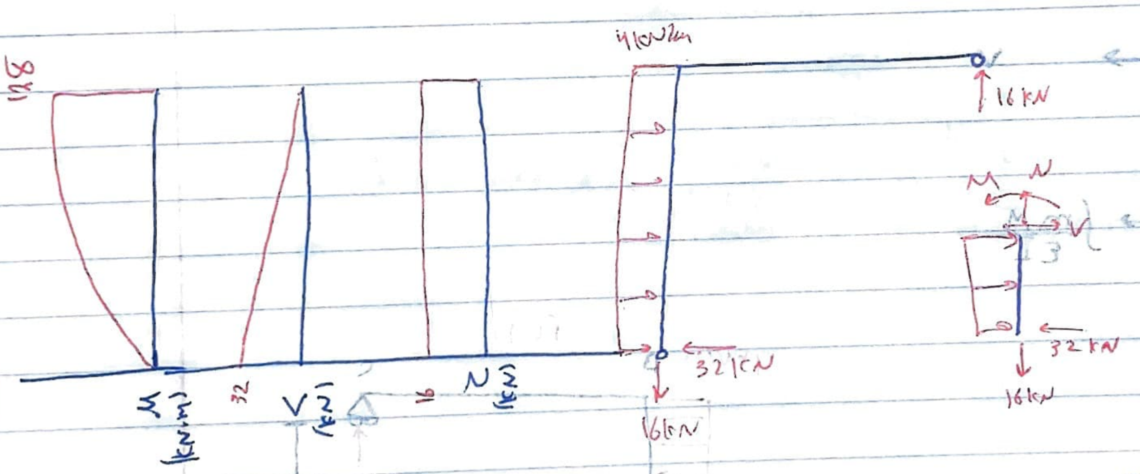
Horizontal displacement at point C.

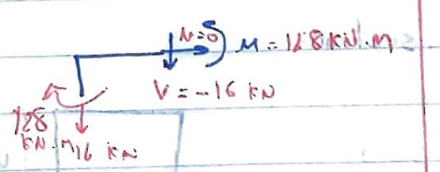
∴ Consider All int. displacements

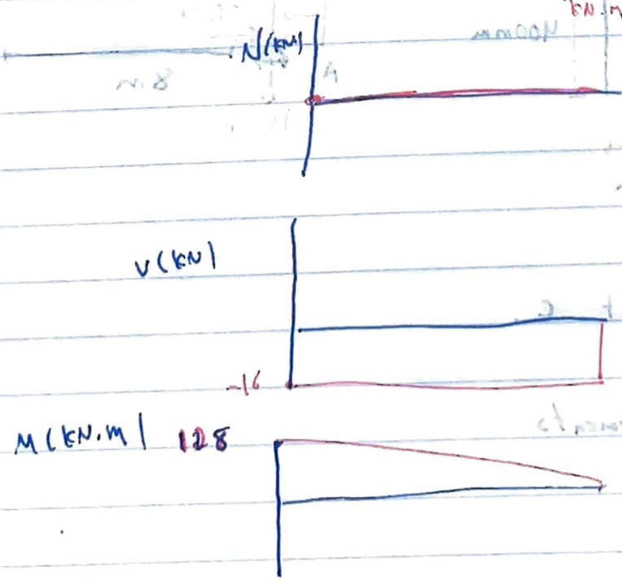
Virtual

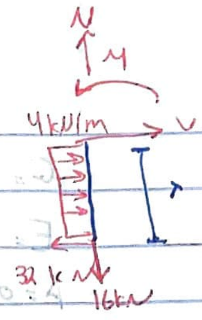


For the real:

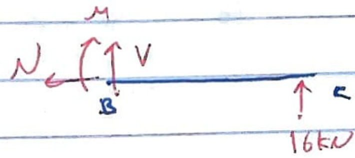




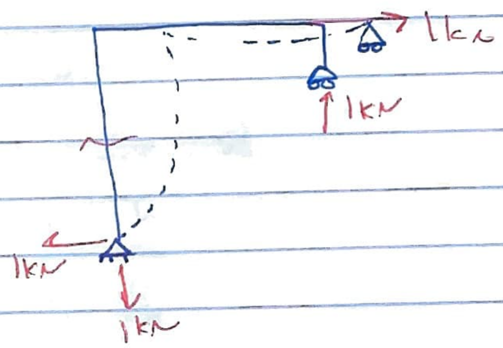




A→B $\begin{cases} N = 16 \\ V_c = 32-4x \\ M = -2x^2+32x \end{cases}$



B→C $\begin{cases} N=0 \\ V=-16kN \\ M=16x \end{cases}$



1 A→B virtual = $\begin{cases} \hat{N} = 1\ kN \\ \hat{V} = 1\ kN \\ \hat{M} = x \end{cases}$

B→C = $\begin{cases} \hat{N} = 1\ kN \\ \hat{V} = -1\ kN \\ \hat{M} = x \end{cases}$

Ext. virtual work = int. virtual work

$$1\times\Delta_c = \int_0^L \frac{nN}{AE}dx + \int_0^L \frac{vVk}{GA}dx + \int_0^L \frac{mM}{EI}dx$$

AB, BC

$$= \frac{n_1N_1L}{AE} \quad \frac{n_2N_2L}{AE}$$

$k=1.2$

AB

$$= \frac{1\times16\times8}{AE} + \frac{1\times0\times L}{AE} + \int_0^8 \frac{1(32-4x)k\,dx}{GA} - \int_0^8 \frac{1(-16)k}{GA}dx$$

$$\int_0^8 \frac{(-2x^2+32x)\,x}{EI} \qquad \int_0^8 \frac{(16x)(x)}{EI}$$

$\frac{128}{AE} + \frac{3072}{GA} + \frac{6144}{EI}$

$= 4.27\times10^{-5} + 5.12\times10^{-3} + 0.1536$

$M \gg V$ and N

$\Delta C = 0.15876$

$G = 5\,GPa$

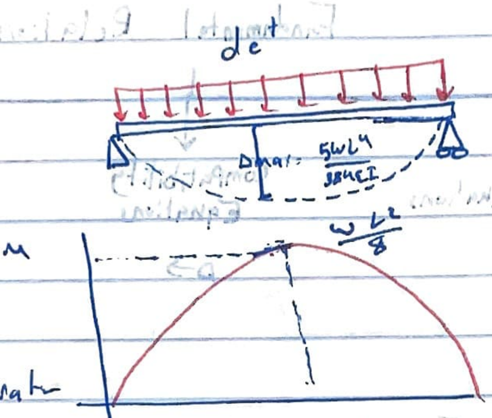
$E = 25\,GPa$

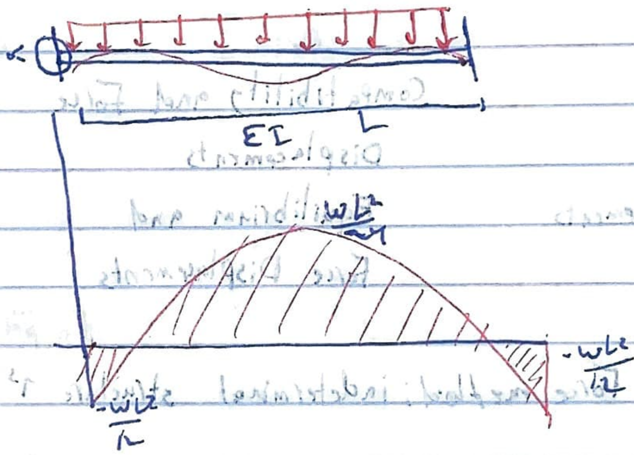
$A = 0.3\times0.4 = 0.012$

$I = \frac{0.3(0.4)^3}{12} = 1.6\times10^{-3}\,m^4$

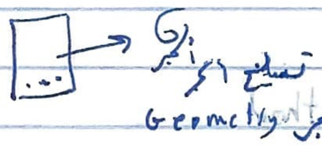
Analysis of indeterminate structure

إذا صار failure على عمد
determinate Beam في ال supports ال
في هاي الحالة رح يصير ال Beam
يعني ما بيكون ال Beam unstable
إذا صار failure في أحد indeterminate
ال supports رح يضل ال Beam يكون
stable





في حالة ال det. Beam رح يكون ال stresses
أعلى وبالتالي رح تتطلب تسليح أكثر
و geometry أكبر وبالتالي ال cost أعلى
يكون أعلى من في حالة ال ind.



Advantages:	disadvantages
Less stress deflections due to Loading	More stresses due to temperature and fabrication errors
Cost (member size)	Cost (support condition
Redundancy	

Analysis

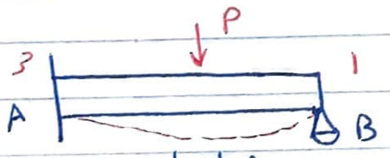
Fundamental Relations

- Equilibrium Equations: ΣF_x, ΣF_y, ΣM
- Compatibility Equations: $\Delta \rightarrow$
- Force-displacement Equations: $f \rightarrow \Delta$.

	Unknowns	Equations used for solution	Coefficients of the Unknowns
Force method	Forces	Compatibility and Force Displacements	Flexibility Coefficients
Displacement method	Displacements	Equilibrium and Force Displacements	Stiffness coefficents.

Structure 1 $\rightarrow$ Force method; indeterminat structure 1^{st} degree

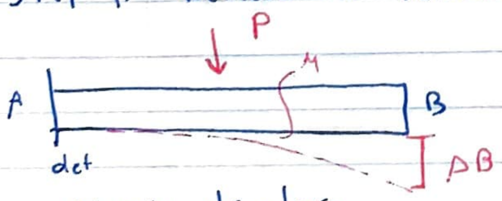
• Force Method



$r = 4 \rightarrow$ redundant reaction
eq $= 3$

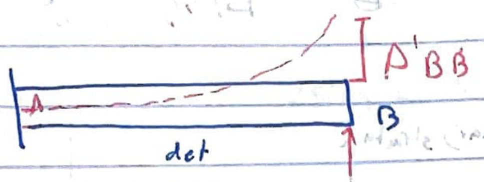
actual Beam

Step 1: Remove a redundant force

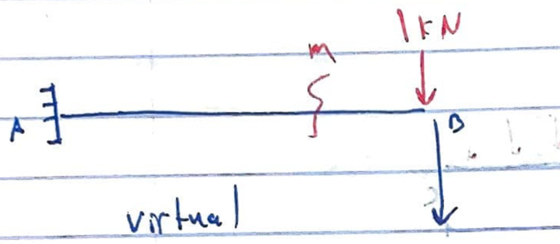


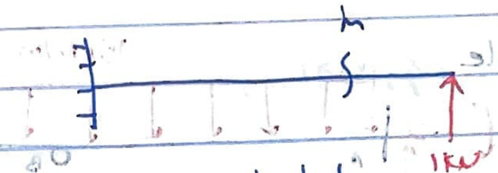
Primary structure

Step 2: Reapply redundant force



redundant By applied By





virtual

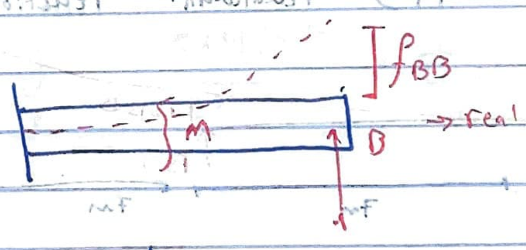
For calculating ΔB in First stem

For calculating Δ'BB is step 2.

Step 3: Write Compatibility Equation

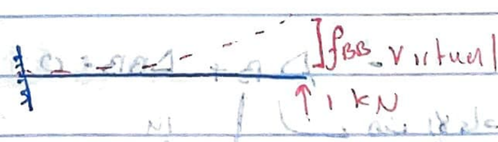
$$0 = -\Delta B + \Delta'_{BB}$$

Step 4. Find Displacements



$$0 = -\Delta B + B_y f_{BB}$$

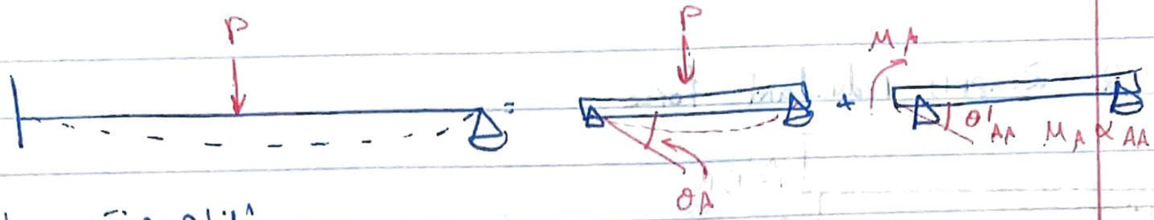
$$\Delta B = \int_0^L \frac{Mm}{EI} dx \qquad \Delta'_{BB} = B_y f_{BB}$$



$$f_{BB} = \int \frac{mm}{EI} dx$$

بنطبق بعدها بالمعادلة إلي فوق وبنطلع By

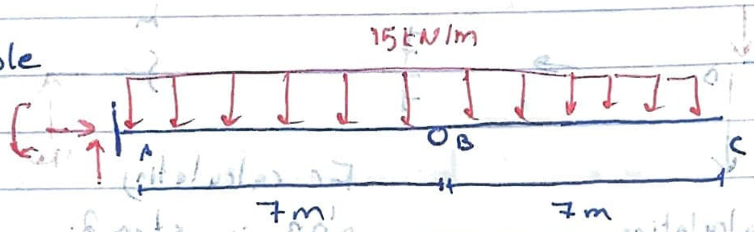
redundant force



Primary structure

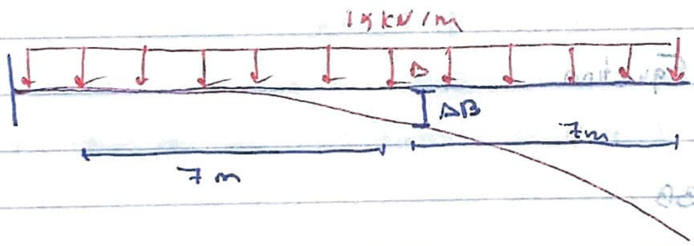
ثلثنا مومنت. وسهاي الحالة راح
يكون عنالك redundant هي عبارة
عن Rotation θ ، ونتعامل معها بنفس ال
concept تبع ال deflection ∇

Beam Example

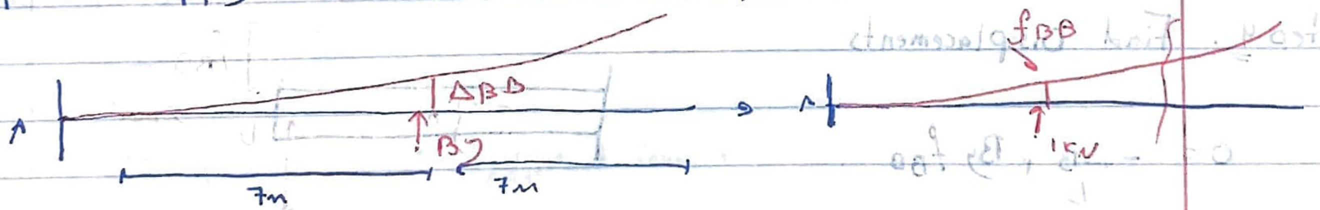


indet. 1st degree.

- step 1: removed redundant support reaction (By)



- step 2: Reapply redundant reaction (By)



- step 3: Compatibility equation of deflection (upward is positive)

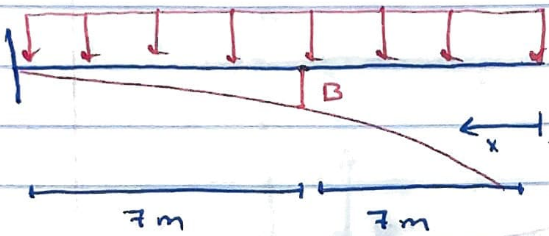
$$-\Delta_B + \Delta_{BB} = 0 \rightarrow -\Delta_B + f_{BB}\,B_y = 0$$

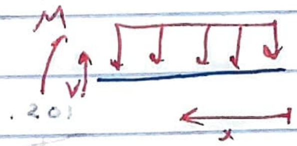
$$\int_0^{14} \frac{Mm}{EI} \rightarrow \int_0^{7} \frac{Mm}{EI} + \int_7^{14} \frac{Mm}{EI}$$

من الإتجاه إجا السالب
لأنو ال unitload لأعلى
وال deflection هي الأصلي
+

Step 4: find deflections (Δ_B, Δ_{BB})

- Find Δ_B, f_{BB} using virtual work.

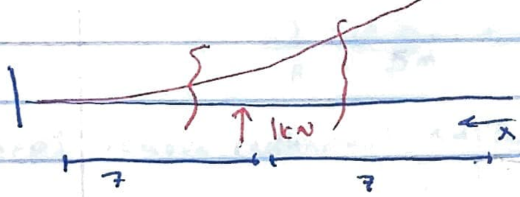


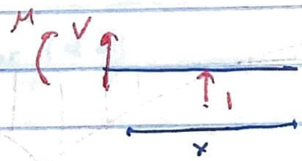


$$-M - 15(x)\left(\frac{x}{2}\right) = 0$$

Real structure

$$M = -\frac{15}{2}x^2 \qquad 0 \to 14$$





$m = 0, \qquad 0 < x < 7$

$m = x - 7 \qquad 7 < x < 14$

$$f_{BB} = \int_0^7 \frac{mM}{EI} + \int_7^{14} \frac{mm}{EI} = 0 + \int_7^{14} \frac{(x-7)^2}{EI}dx = \frac{1}{EI}\int x^2 - 14x + 49\,dx \quad \frac{114.3}{EI}$$

$$\Delta_B = \int_0^7 \frac{Mm}{EI}dx + \int_7^{14} \frac{Mm}{EI} = 0 + \int_7^{14} \frac{(-15x^2)(x-7)}{EI} = \frac{1}{EI}\int_7 \left(-\frac{15}{2}x^3 + 52.5x^2\right)dx$$

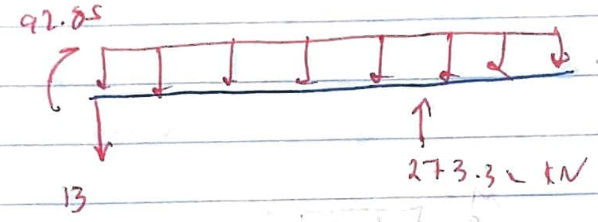
$$= -\frac{25510}{EI}$$

- Step 5: Apply compatibility and solve for By

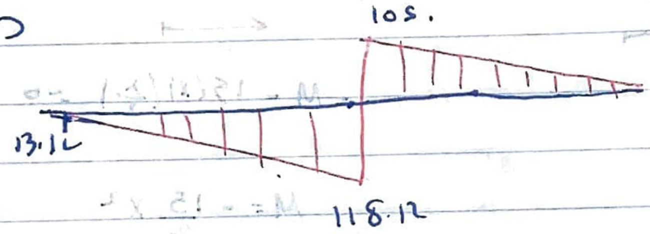
Compatibility $= -\frac{25510}{EI} + \frac{114.3}{EI}B_y = 0$

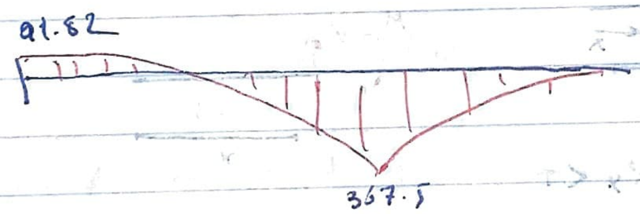
$B_y = 223.2\,kN$

- step 6 Find reactions At A and draw shear and moment diagrams



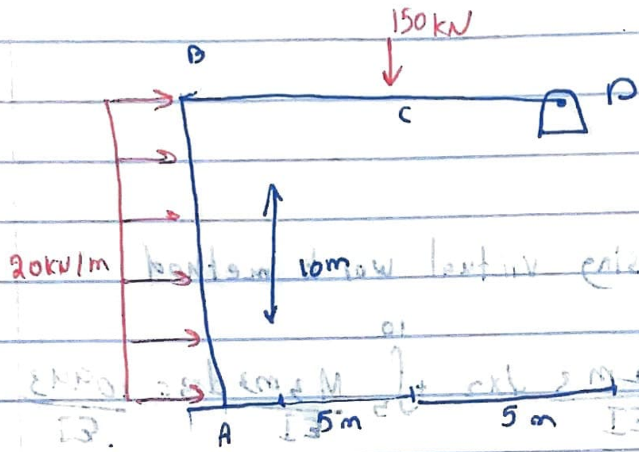
S.B.D



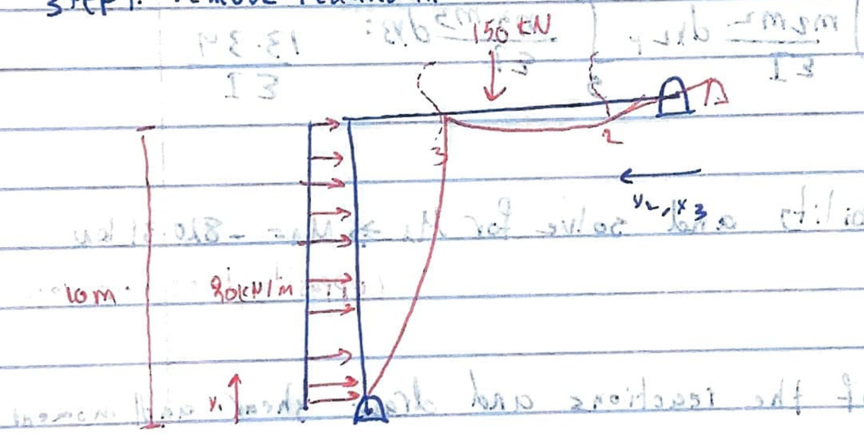


Frame Example

Draw axial shear and moment diagrams for



step1: remove redundant support reaction (M a)

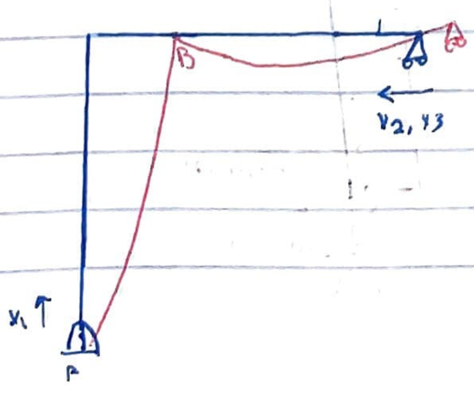


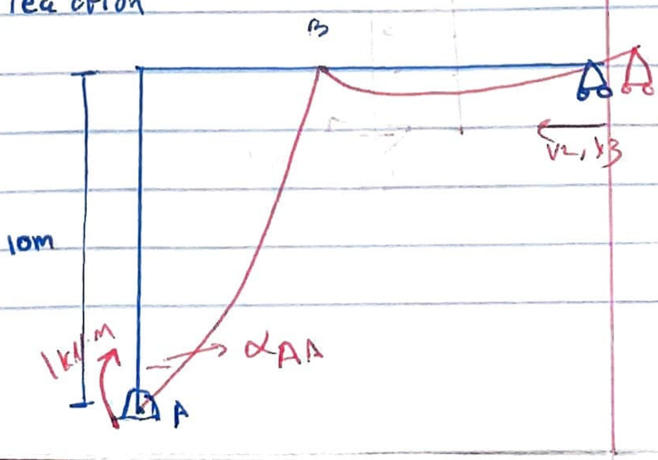
$$M_1 = 200x - 10x^2 \rightarrow 0 < x_1 < 10$$

$$M_2 = 175x \rightarrow 0 < x_2 < 5$$

$$M_3 = 25x + 750 \rightarrow 5 < x_2 < 10$$

- step 2: Reapply redundant reaction





- step 3: Write compatibility equation (ccw. rotation positive)

$$\theta_A + \theta_{AA} = 0 \rightarrow \theta_A + \alpha_{AA} M_A = 0$$

- step 4: find deflections

  - Find θ_A and α_{AA} using virtual work method

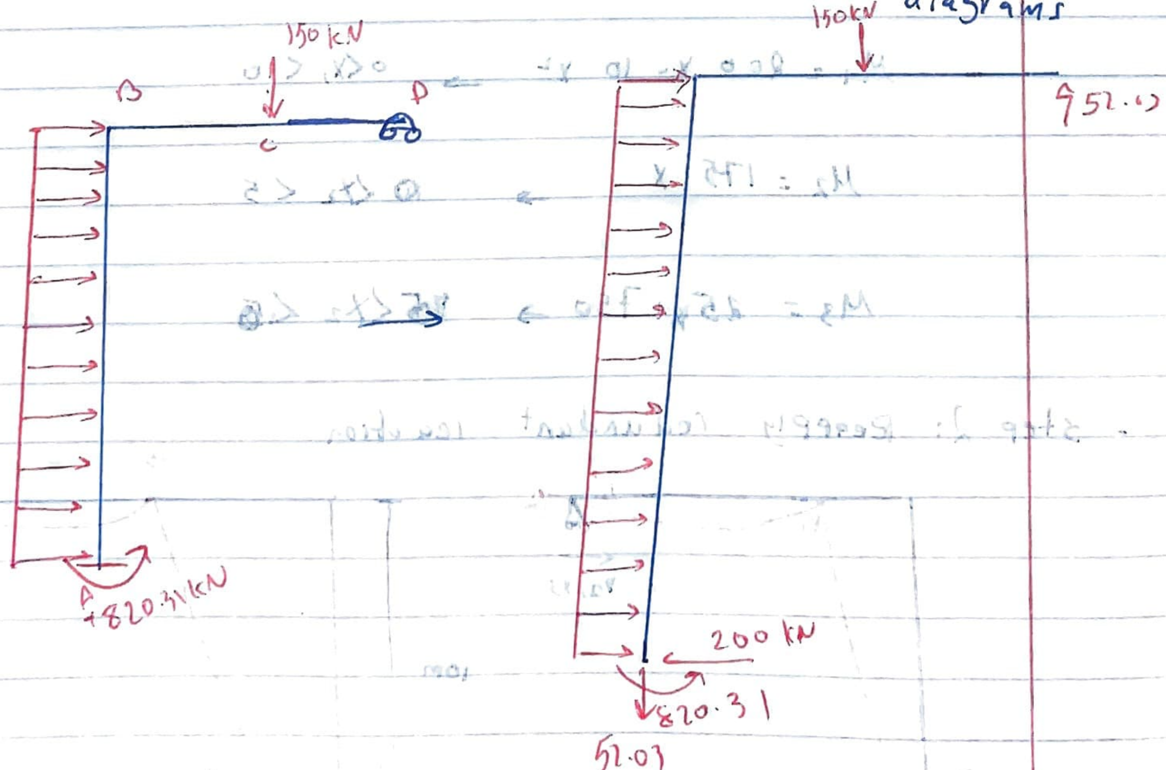
$$\theta_{AA} = \int_0^{10} \frac{M_1 m_1}{EI} dx_1 + \int_0^{5} \frac{M_2 m_2}{EI} dx_2 + \int_5^{10} \frac{M_3 m_3}{EI} dx_3 = \frac{10943}{EI}$$

$$\alpha_{AA} = \int_0^{10} \frac{m_1 m_1}{EI} dx_1 + \int_0^{5} \frac{m_2 m_2}{EI} dx_2 + \int_5^{10} \frac{m_3 m_3}{EI} dx_3 = \frac{13.34}{EI}$$

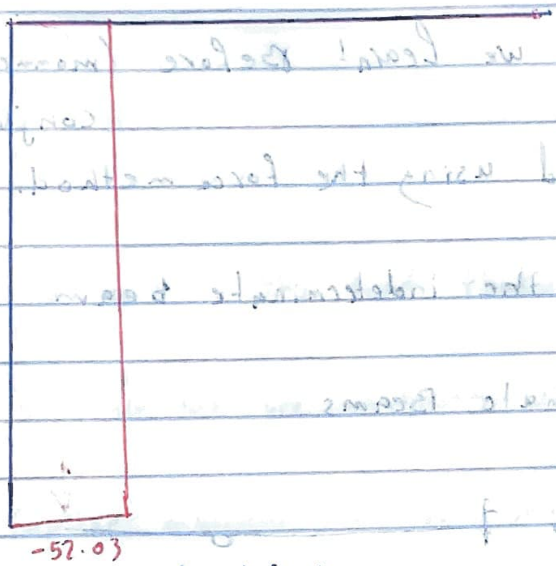
- step 5: Apply compatibility and solve for $M_A \Rightarrow M_A = -820.31$ kN

(opposite to assumption)

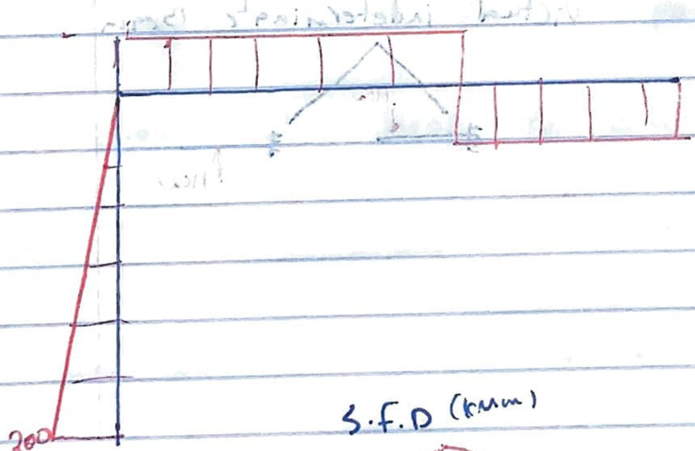
- Step 6: Find the rest of the reactions and draw shear and moment diagrams



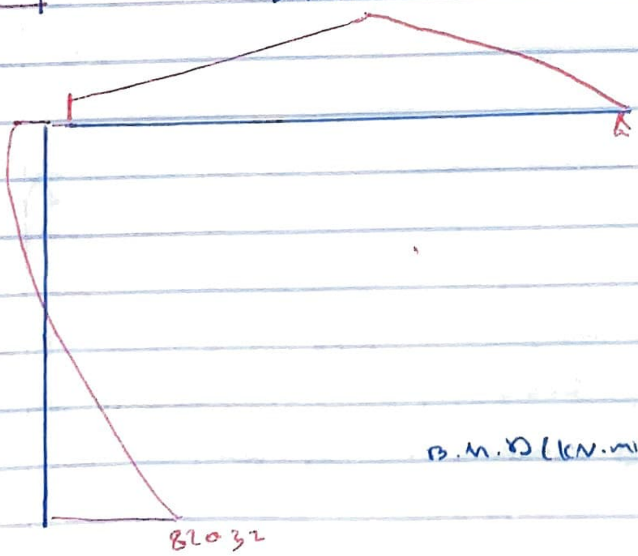
Diagrams.



Axial (kN)



S.F.D (kN)



B.M.D (kN.m)

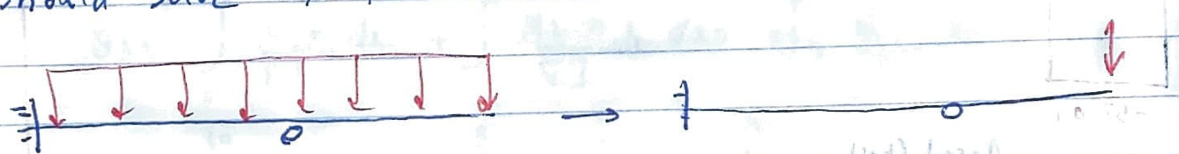
for the first Beam, if the question asked for the deflection

also, we can apply the methods we learn! Before (moment area, conjugate) ..

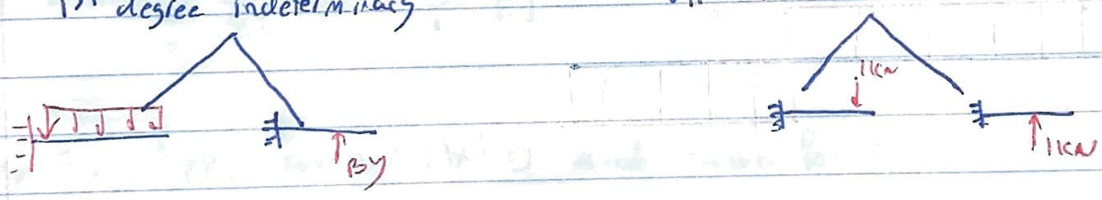
And use the diagrams we determined using the force method.

if we apply the virtual work on the indeterminate beam

we should solve ~~the~~ two indeterminate Beams.



1st degree indeterminacy   virtual indeterminate Beam



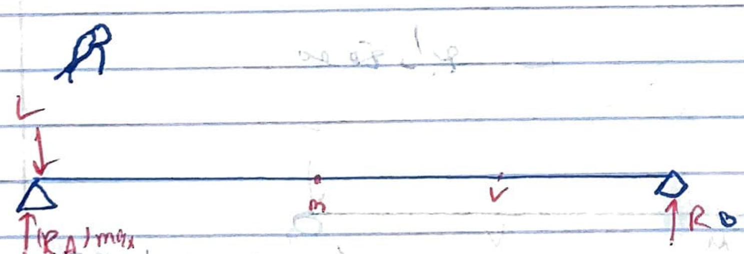
Influence lines

- Types of loads ?

بالسلايد.

The live load is the load that changes (place, value), As engineers in any structure we are analysing, we should consider worst case, We search for the maximum moment, shear..., since live loads changes we are seek to look for the point where this load case maximum deflection and moment, for example.

for the cantilever Beam, the maximum deflection and M is at the طرف



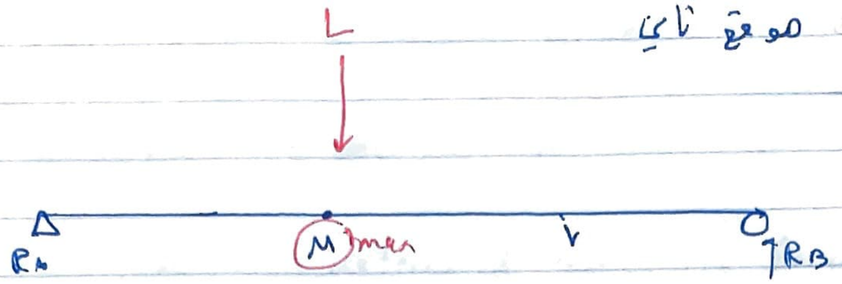
الموقع الأول: لما يكون الجسم فوق ال support

كل ال Load الجسم راح يصير فوق ال support

وراح يكون ال reaction بهاي الحالة عند A

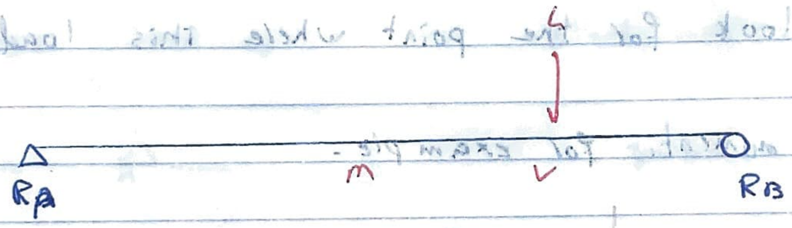
maximum، وباقي ال Beam صفر V, m, R

موقع ثاني



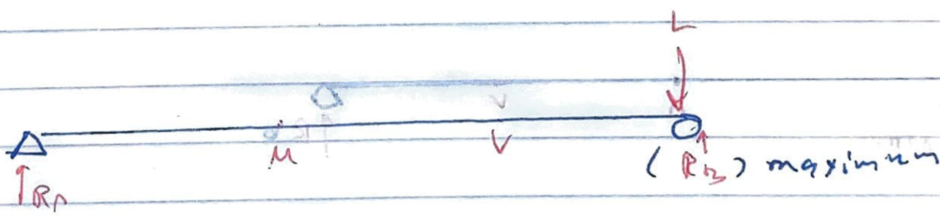
راح يتوزع ال Load ع Ra, Rb ، واذا بدنا نحسب المومنت عند m
وكأنه ال Load مباشرة عليها ، بكونه ال moment في هاي الحالة maximum

موقع ثالث.



بتوزع ال Load ع Ra, Rb مع ملاحظة انه Rb
قاعدة بتزيد كل ما قربنا باتجاهها.

موقع رابع



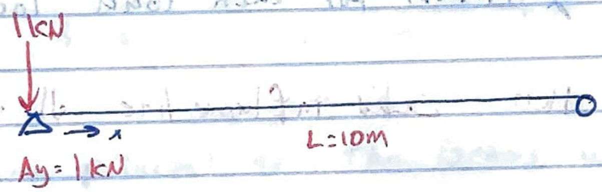
بتشبه ال First case ، كل ال Load مكونة ع ال support B
وبالتالي ال reaction عند B أعلى اشي ، والباقي صفر

STUDENTS-HUB.com

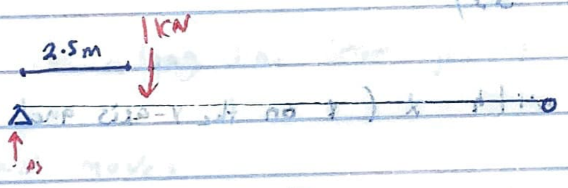
Example.

- An influence line shows the variation of either the reaction, axial, shear or moment at a specific point in the structure due to the movement of a concentrated force along the structure

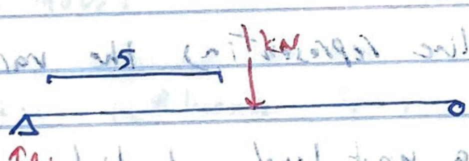
- Draw influence line of the reaction at A



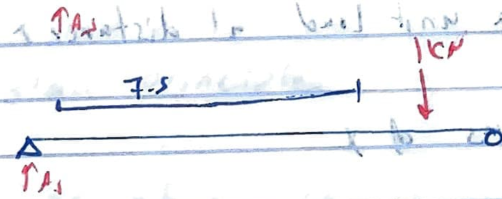
$x=0$, $A_y = 1$ kN



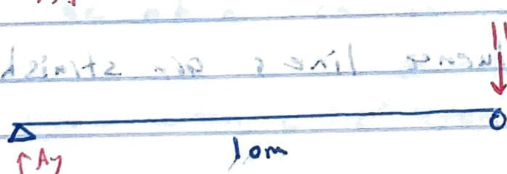
$x=2.5$, $A_y = 0.75$



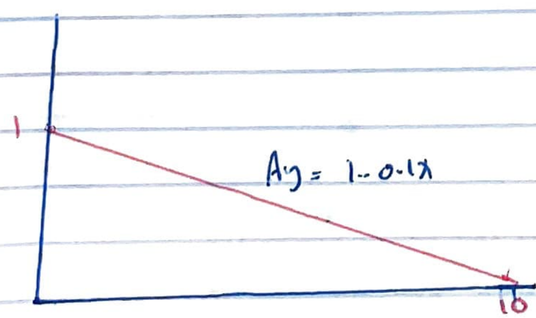
$x=5$, $A_y = 0.5$ kN



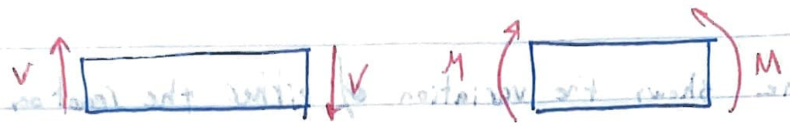
$x=7.5$, $A_y = 0.25$ kN



$x=10$, $A_y = 0$



Sign convention



Steps:

- place a unit load at various locations on the structure and calculate the quantity of interest for each load location

لرسم ال influence line بنطبق ال (unit load) و أركز شو معي

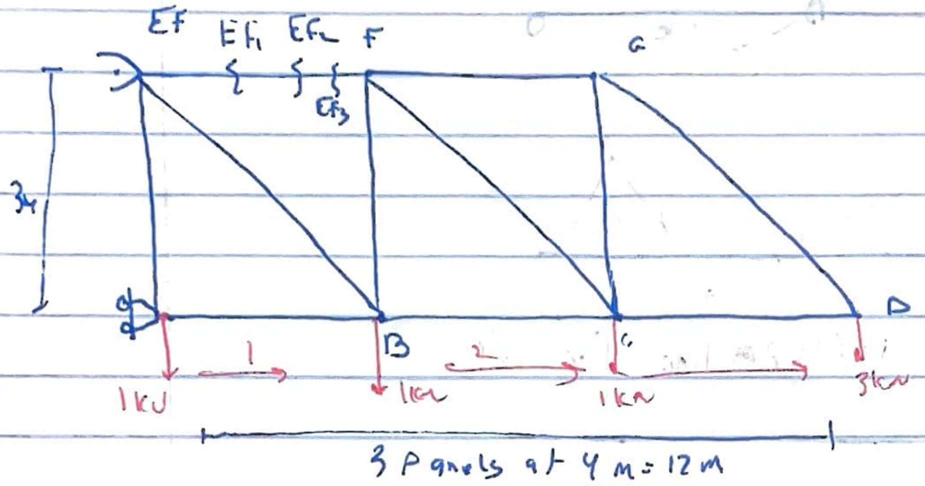
- Tabulate your results (x vs. SF)

- Draw the variation of SF with x (x on the x-axis and SF on the y-axis)

- you can get equation for the line representing the variation of SF with load location by placing a unit load at distance x from the reference and get SF in terms of x

- For determinate structures, Influence lines are straight lines (1st order)

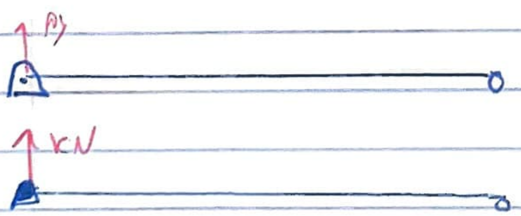
Influence line for truses

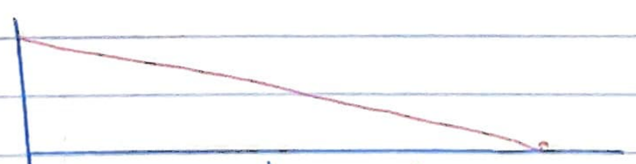


we use same principal as the Beams, we use a unit load of 1kN and move it along the Beam joints, And determine the Value of FEF when 1 kN moves

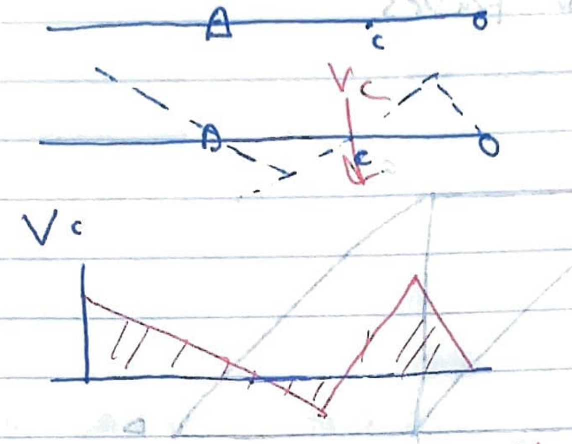
- Qualitative influence lines
- Muller - Breslau principle

The influence line of a Beam is to the same scale as the deflected shape of the Beam when acted upon by the function

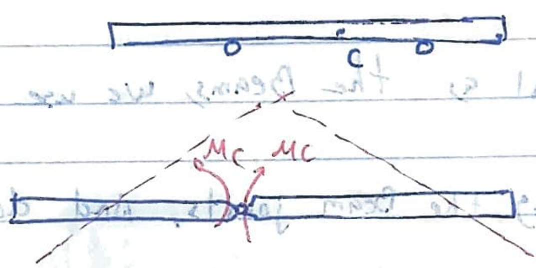


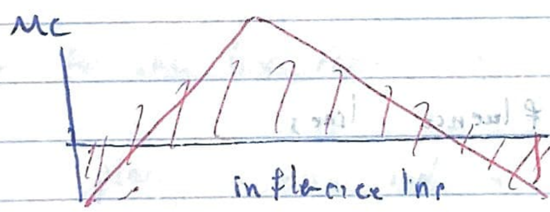


influence line for Ay



inflence line for Vc





positive

لو كان عنا uniform load وبدنا ال maximum عند نقطة C مثلا، بنطبق

ال live load على المناطق الموجبة المعينة