Exercises : Continuous

Q8 : let X have the p.d.f , $f(x) = \begin{cases} \frac{(x+2)}{18} & , -2 < x < 4 \\ 0 & , \text{elsewhere} \end{cases}$. Find :

1. $E(X) = \int_{-2}^{4} x \frac{(x+2)}{18} \, dx = \frac{1}{18} \int_{-2}^{4} x^2 + 2x \, dx = \frac{1}{18} \left[\frac{x^3}{3} + x^2 \Big|_{-2}^{4} \right]$

$= \frac{1}{18} \left[\frac{64}{3} + 16 - \left(\frac{-8}{3} + 4 \right) \right]$

$= \frac{1}{18} \left[\frac{64}{3} + \frac{48}{3} + \frac{8}{3} - \frac{12}{3} \right]$

$= \frac{1}{18} \times \frac{108}{3} = \frac{36}{18} = 2$

2. $E[(x+2)^3] = \frac{1}{18}\int_{-2}^{4} (x+2)^3 (x+2)\, dx = \frac{1}{18}\int_{-2}^{4} (x+2)^4\, dx$

$= \frac{1}{18}\left[\frac{(x+2)^5}{5} \Big|_{-2}^{4} \right]$

$= \frac{1}{18}\left[\frac{7776}{5} - 0 \right] = \frac{7776}{90} = 86.4$

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3. $E[6x - 2(x+2)^3] = E(6x) - E(2(x+2)^3)$

$= 6E(x) - 2E((x+2)^3)$

$= 6 \times 2 - 2 \times 86.4$

$= -160.8$

Q81: X is a Random variable with p.d.f :

$f(x) = \begin{cases} \frac{1}{5}, & x = 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{cases}$ discrete

Find :

1. $E(x) = \sum_{x=1}^{5} x\left(\frac{1}{5}\right) = \frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5} + \frac{5}{5} = \frac{15}{5} = 3$

2. $E(x^2) = \sum_{x=1}^{5} x^2\left(\frac{1}{5}\right) = \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + \frac{16}{5} + \frac{25}{5} = \frac{55}{5} = 11$.

3. $E((x+2)^2) = E(x^2 + 4x + 4) = E(x^2) + E(4x) + E(4)$

$= 11 + 4(3) + 4$

$= 27$.

Q83 : let the p.d.f , $f(x) = \begin{cases} \text{positive} , & x = -1, 0, 1 \\ 0 , & \text{elsewhere} \end{cases}$ discreate .

a. If $f(0) = \frac{1}{4}$, Find $E(x^2)$:

$$E(x^2) = \sum_{x=-1}^{1} x^2 f(x) = \left[(-1)^2 f(-1) + 0 f(0) + 1 f(1) \right]$$

$$= f(-1) + f(1)$$

$$= 1 - f(0) \qquad \text{since } f(1) + f(-1) + f(0) = 1$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

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b. If $f(0) = \frac{1}{4}$ and if $E(x) = \frac{1}{4}$, determine $f(-1)$ and $f(1)$.

$\rightarrow$ $E(x) = \sum_{x=-1}^{1} x f(x) = -f(-1) + 0 f(0) + f(1)$

$$\frac{1}{4} = f(1) - f(-1) .$$

$\rightarrow$ $f(-1) + f(0) + f(1) = 1 \Rightarrow f(-1) + f(1) = 1 - f(0)$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

so we have two equ. $f(1) + f(-1) = \frac{3}{4}$ --- (1)

$f(1) - f(-1) = \frac{1}{4}$ --- (2) +

$2f(1) = 1$

so $\boxed{f(1) = \frac{1}{2}}$

$\Rightarrow f(-1) = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}$

Q84 : let X have the p.d.f. $f(x) = \begin{cases} 3x^2 , & 0 < x < 1 \\ 0 , & \text{elsewhere} \end{cases}$ continuous.

consider a random rectangle whose sides are X and $(1-X)$. determin the Expected value of the area of rectangle.

area $= X \cdot (1-X) = X - X^2$, so Find $E(X - X^2)$.

$$E(x-x^2) = \int_0^1 (x-x^2)(3x^2)\, dx = \int_0^1 (3x^3 - 3x^4)\, dx = \frac{3}{4}x^4 - \frac{3}{5}x^5 \Big|_0^1$$

$$= \frac{3}{4} - \frac{3}{5}$$

$$= \frac{15-12}{20} = \frac{3}{20}$$

Q87. let $f(x) = \begin{cases} 2x , & 0<x<1 \\ 0 , & \text{elsewhere} \end{cases}$ be the p.d.f of X. continuous.

a. compute $E(\frac{1}{X})$.

$$E(\tfrac{1}{x}) = \int_0^1 \frac{1}{x}(2x)\, dx = 2x \Big|_0^1 = 2$$

b. Find the distribution function and the p.d.f of $Y = 1\backslash X$.