

Chapter 14 : Fluid dynamics of non-viscous fluids.

* an understanding of the physics of the fluid flow is vital to an understanding of biological system as diverse as the human circulatory system and the distribution of nutrients in plants

→ So, in this chapter we will introduce the physical foundations of fluid flow in the absence of viscosity.

14.2 : Definitions of some key terms.

* incompressible fluid : the fluid has a constant throughout

* Viscosity: the resistance of a fluid to flow

* Laminar flow: a situation in which layers of fluid slide smoothly past each other, (its characteristic of lower fluid velocities)

* turbulent flow: (non-laminar flow), The flow is irregular and complex, with mixing and eddies, This occurs at higher velocities or where there are objects in the flow producing large changes in velocity

stream lines a family of curved lines that are tangential to the velocity vector of the flow (i.e. always in the same direction as the flow). They provide a kind of snapshot of the flow throughout the fluid at an instant of time.

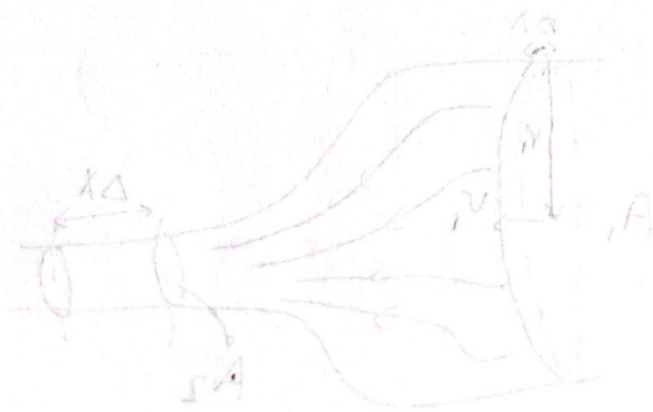
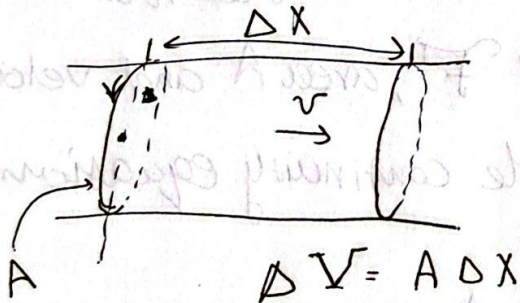
The equation of continuity:

- Volume flow rate tells us how much fluid is flowing across some surface (like pipe's cross section)

or = tell you how many cubic meters of fluid would be collected in 1 sec.

For an incompressible fluid, the volume flow rate is equal to the product of the cross-section area and the velocity.

$$Q = A v = A \frac{\Delta x}{\Delta t} = \frac{\Delta V}{\Delta t}$$



* Continuity of Flow:

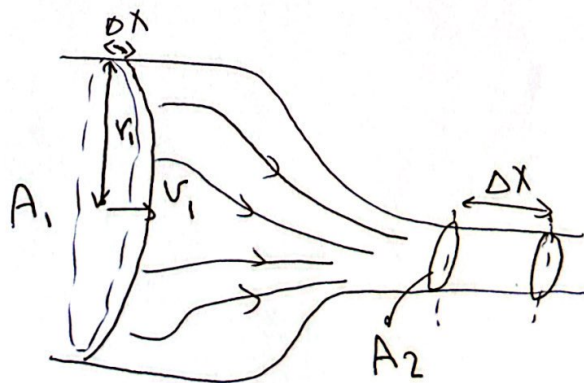
- The ~~let~~ volume flow rate is constant along a pipe or channel, this due to the conservation of mass.
- Conservation of mass: the amount of material entering one end of the pipe or channel must be the same as the amount coming out the other end.

→ and there must also be the same amount per unit time.

fixed mass → fixed volume → hence a constant:
Volume flow rate

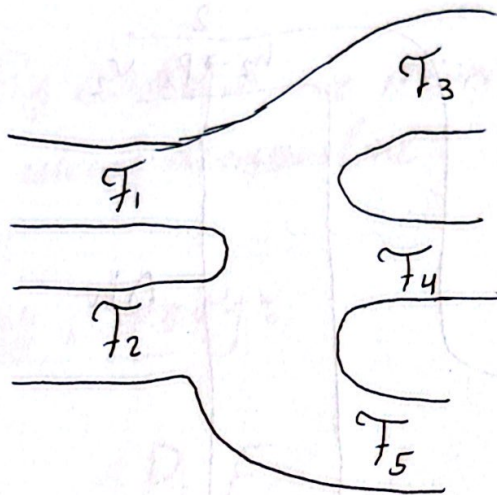
* we have established the relationship between ~~the~~ F , area A and velocity through the continuity equation

$$A_1 v_1 = A_2 v_2$$



→ This eq implies that when fluid enters a more constricted section, it will speed up.

* Even for multiple pipes joining, the volume of liquid flowing into a given location per sec is equal to the volume flowing out



* The sum of the volume flow rates into junction is equal to the sum out

see example 14.1

14.4 : Bernoulli's Equation

- Bernoulli's principle : when viscosity can be neglected, an increase in fluid velocity is accompanied by a decrease in pressure and decrease in gravitational potential energy.

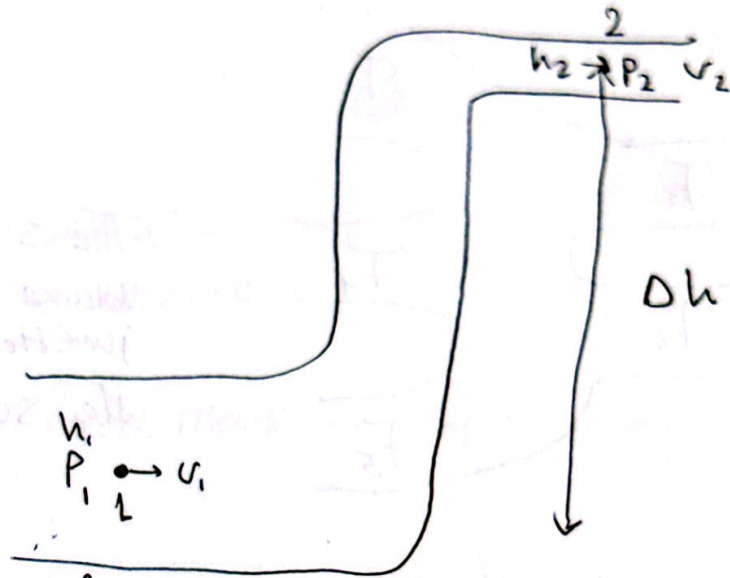


Figure 2

→ we can have eq relating together pressure, speed and elevation, and if the fluid flow laminar steady (independent of time), then we can ignore the effects of friction, then we have

Bernoulli's eq

$$P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$$

(For incompressible fluid) "No expansion or compression is happening"

P : pressure at a chosen point

v : the fluid velocity along a streamline at the point

h : is the height of the point above a selected reference level

ρ : density of the fluid

→ By constant, we mean that the sum is constant along a streamline.

* Energy Density:

$$P = \frac{F}{A}$$

$$W = Fd$$

$$F = \frac{W}{d}$$

$$\therefore P = \frac{W}{V} = 1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2} = \frac{\text{J m}^{-1}}{\text{m}^2}$$

$$= \frac{1 \text{ J}}{\text{m}^3}$$

→ So the pressure can also be thought of as energy per unit volume.

→ if we consider a gas under pressure in a cylinder with a movable piston. Suppose the gas exerts a force F on the piston, if the piston is allowed to move a very small distance Δx , then some work $W = F \Delta x$ has been done on the piston by the gas.

Thus the fluid under pressure can do work on some other system, and in order to increase the pressure of a fluid, work must be done on it.

HW : Show that each term of Bernoulli's eq has units of $\frac{J}{m^3}$ (energy per unit volume)

pressure and Velocity:

- another way to write (Bernoulli's eq) relates the parameter values at 2 points on a streamline labelled 1 and 2

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

- consider a case where there is no change in height, so the gravitational potential energy is not changing

$$\therefore \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 = P_1 - P_2$$

- So the change in pressure gives us the change in K.E per unit volume, as in figure 2
- the v_2 is higher then P_2 is lower?!
 - From the continuity eq that the fluid speeds up as it moves from point 1 to 2, so it gains K.E
 - as the pipe is horizontal, there is no change in the fluid gravitational energy.
 - positive work must have done on the mass of the fluid in order to speed it up.
 - The only force available to do this work is the pressure difference between point 1 and 2

- To do positive work on a mass of fluid

from point 1 to 2 the pressure must be high.

at point 1 than at 2, so the pressure is indeed

lower in the more constricted region

where the velocity is higher.

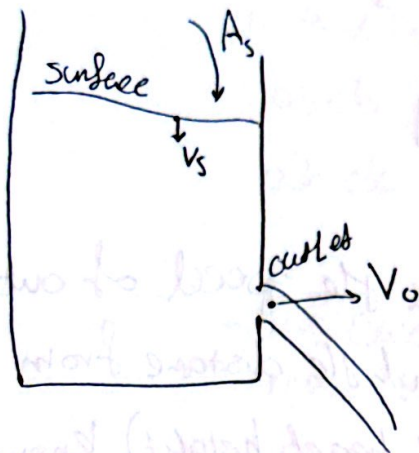
See Example 14.2

$$P_1 - P_2 = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2$$

- The change in pressure gives us the
change in K.E. per unit volume, as in figure 14.2
→ the v_2 is higher than v_1 in figure 14.2!
- from the continuity of flow the fluid speeds up
as it moves from point 1 to 2, so it gains K.E.
- as the pipe is horizontal, there is no change
in the fluid gravitational energy.
- positive work must have done on the mass of
fluid in order to speed it up.
- the only force available to do this work is
the pressure difference between point 1 and 2

Applications of Bernoulli's Eq

1- Fluid Flow out of a Tank



By applying Bernoulli's eq

s → tank

o → outlet

$$P_s + \frac{1}{2} \rho v_s^2 + \rho g h_s = P_o + \frac{1}{2} \rho v_o^2 + \rho g h_o$$

- Now assume that the ~~the~~ surface of the tank and the tank outlet are at atmospheric pressure

$$\therefore \frac{1}{2} \rho v_s^2 + \rho g h_s = \frac{1}{2} \rho v_o^2 + \rho g h_o$$

if the surface area of the tank is much larger than the cross-sectional area of the outlet
 $\therefore$ then the continuity eq suggests

$$v_s \ll v_o$$

- in other words, the speed at which the surface of water in the tank drops is (much) ~~more~~ less than the speed at which water leaves the tank outlet pipe

$$\therefore \rho g h_s \approx \frac{1}{2} \rho v_0^2 + \rho g h_0$$

we can neglect v_s

$$v_0 = \sqrt{2g(h_s - h_0)}$$

- This relationship between the speed of outflow (the speed of efflux) and the distance from the liquid surface (called head height) known as "Torricelli's Theorem"

Chap 2. Plaque Deposits and Aneurysms:

- when the height is unchanged we looked at the ΔP and the liquid speeds up or slows down, This has consequences for blood vessels that narrowed by plaque, or widened at the site of an aneurysm.
- This case called (stenosis), the blood velocity must be increased, which decreases the pressure, and may result in further narrowing, leading the artery to close entirely
- when the artery is narrowed, the flow will also become more turbulent, possibly damaging the arterial wall
- To make matters worse, the decrease in elasticity changes the wall's vibrational character which can lead to resonant vibrations which can dislodge the deposits.
- An aneurysm is ~~located~~ localised, balloon-like bulge in an artery, As the radius increases and velocity decreases, the pressure increases. as the wall is already likely to be weakened this further increases the chances of rupture

Example: 14.3