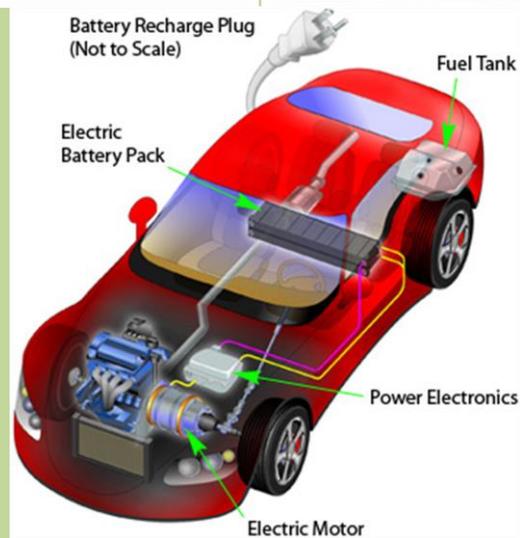


2024/25

Electrical Machines

EE 2408

Lecture Notes



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Dr. M. Abu-Khaizaran, BZU, 2024/25

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PART I

Introduction to Machinery Principles

Electric Machines: are a device that can convert either mechanical energy to electrical energy, or electrical energy to mechanical energy, through the action of magnetic fields; generators and motors.

Mechanical Energy $\xleftrightarrow{\text{Electrical Machine}}$ Electrical Energy

A Generator: a device that converts mechanical energy to electrical energy (generates power). The sources for the mechanical energy could be gas, diesel, steam, wind, coal, nuclear,...

A Motor: a device that converts electrical energy to mechanical energy, examples of its applications are: refrigerators, freezers, vacuum cleaners, air conditioners, fans,...

The electric energy is clean and can be easily transmitted from one place to another, over wires or in underground cables.

A Transformer: is a device that converts AC electric energy at one voltage level to AC electric energy at another voltage level. It reduces the energy loss between the point of generation and the point of consumption.

Systems of Units:

1. **System International (SI):** it is a metric system adopted as an international standard throughout most of the world; it uses centimeter, meter, second, kg, Volt, Ampere, Ohms, Watt,...
2. **English System:** uses inches, feet, slug, pounds, horse power, etc...

The conversion between the two systems can be achieved using tables.

Examples: 1slug = 14kg, 1kg = 2.205Pound mass, 1hp = 746W

Rotational Motion, Newton's Law and Power Relationships

Almost, the rotor in all electrical machines rotates about an axis called the **shaft**, which is normally mounted via **bearings**. The bearings are held on stands.



➔ The counterclockwise rotation is assumed to be positive.

Angular Position (θ): it is measured in radians or degrees, and it corresponds to the linear distance (r).



Angular Velocity (ω), or speed: it is the rate of change of the angular position with respect to time, it corresponds to velocity (v).

- For a linear motion, the velocity is, $v = \frac{dr}{dt}$, and is measured in $[m/s]$.
- For a rotational motion, the angular velocity is, $\omega = \frac{d\theta}{dt}$
- The angular velocity has the following symbols according to its units.
 - ω_m : angular velocity expressed in radians per second $[rad/s]$
 - f_m : angular velocity expressed in revolutions per second $[rps]$
 - n_m : angular velocity expressed in revolutions per minute $[rpm]$
- The subscript m stands for mechanical quantity.

where, $f_m = \frac{\omega_m}{2\pi}$

and

$$n_m = 60f_m$$

Angular Acceleration (α): is the rate of change in angular velocity with respect to time, and it corresponds to acceleration in the linear motion. It is measured in $[rad/s^2]$.

- For linear motion, the acceleration is $a = \frac{dv}{dt}$.
- For rotational motion, the angular acceleration is $= \frac{d\omega}{dt}$.

Torque (τ):

It is also called the **Twisting Force**. The torque on an object is defined as the product of the force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation.

$\tau = \text{Force applied} \times \text{Perpendicular distance}$

$$\tau = Fr \sin \theta$$

$$\tau = rF \sin \theta; \text{ Counter Clockwise}$$

Its units are Newton-meter or Pound-feet

Newton's Law:

For objects moving along a straight line,

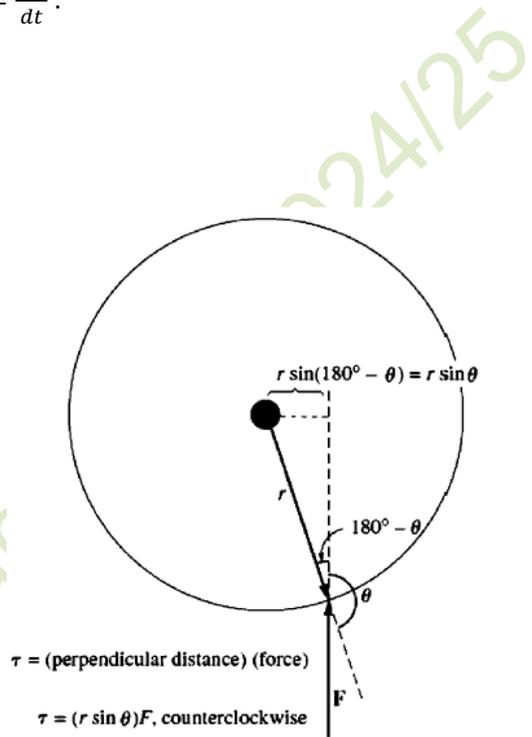
$$F = ma$$

where,

F: is the net force applied to the object measured in pound or Newton.

m: is the mass of the object measured in kg or Slug.

a: is the resulting acceleration measured in m/s^2 or $feet/s^2$.



Newton's Law for Rotational Motion:

$$\tau = J\alpha$$

where,

τ : is the torque measured in Newton-meter or pound-feet.

α : is the resulting angular acceleration measured in rad/s^2 .

J : is the moment of inertia measured in kg-meter² or slug-feet².

Work (W):

For a linear motion,

$$W = \int \mathbf{F} \cdot d\mathbf{r} \quad , \text{ the force is collinear with the direction of motion.}$$

It is measured in Joules or foot-pounds

For a rotational motion, the work is the application of torque through an angle;

$$W = \int \tau d\theta$$

If the torque, τ , is constant, then:

$$W = \tau\theta$$

Power (P):

It is the rate of doing work;

$$P = \frac{dW}{dt}$$

It is measured in joules per second (watts) or foot-pounds per second or in house-power

$$P = \frac{dW}{dt} = \frac{d(F.r)}{dt}$$

For a constant force, F , which is collinear with the direction of motion,

$$P = F \frac{dr}{dt}$$

$$P = Fv$$

Assuming a constant torque, for a rotational motion, the power is:

$$P = \frac{dW}{dt} = \frac{d(\tau\theta)}{dt} = \tau \frac{d\theta}{dt}$$

$$P = \tau\omega$$

Magnetic Fields

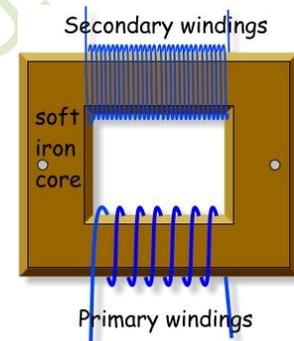
Magnetic fields are the fundamental mechanism by which energy is converted from one form to another in electric machines and transformers.

Basic Principles:

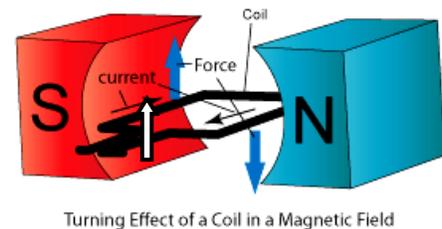
- 1) A current – carrying wire produces a magnetic field around it.



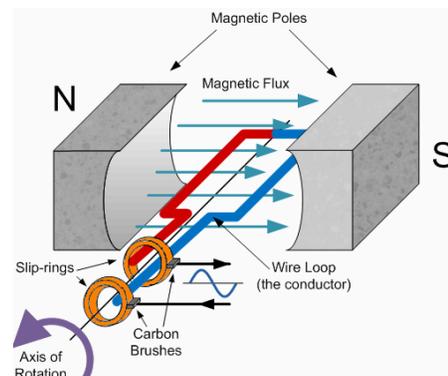
- 2) A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil (transformer action).



- 3) A current-carrying wire in the presence of a magnetic field has a force induced on it (motor action).



- 4) A moving wire in the presence of a magnetic field has a voltage induced in it (generator action).



Production of a Magnetic Field:

Ampere's Law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{net}$$

where,

\mathbf{H} : is the magnetic field intensity produced by the net current, and is measured in Ampere-turn/meter.

l : is the magnetic field path length, and is measured in meters.

I_{net} : is the net current producing the magnetic field, and is measured in Amperes.

Example:

Consider the simple magnetic core, shown in the Figure below, and made from a ferromagnetic material, to find the magnetic field intensity within the core.

Recall Ampere's Law,

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{net}$$

$$H \cdot l_c = Ni$$

$$H = \frac{Ni}{l_c}$$

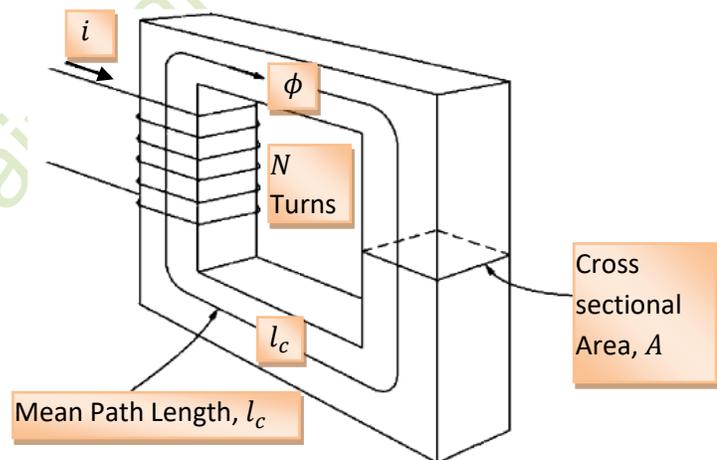
where,

N : is the number of turns in the coil.

i : is the current in the coil.

H : is a measure of the effort that the current is putting into the establishment of a magnetic field.

l_c : is the mean path length in the core.



Magnetic Flux Density (B): it is produced within a material, and is measured in Tesla (T) or Webbers per square meter. It is related to the magnetic field intensity by:

$$B = \mu H$$

μ : is the magnetic permeability of material, and it represents the relative ease of establishing a magnetic field in a given material (measured in Henrys per meter); [H/m].

Permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ H/m

The Relative Permeability:

$$\mu_r = \frac{\mu}{\mu_0}$$

Note: μ_r for steel ranges from 2000 to 6000 or more. Thus, most of the magnetic field essentially stays within the core!

For the previous Figure,

$$B = \mu H$$

$$B = \mu \frac{Ni}{l_c}$$

The Magnetic Flux:

The total flux is defined as:

$$\phi = \int \vec{B} \cdot d\vec{A}$$

where, A is the cross sectional area of the core, measured in meter square.

The flux is measured in **Webers or Tesla.m²**.

If $\vec{B} \perp d\vec{A}$ and B is constant through the area, then:

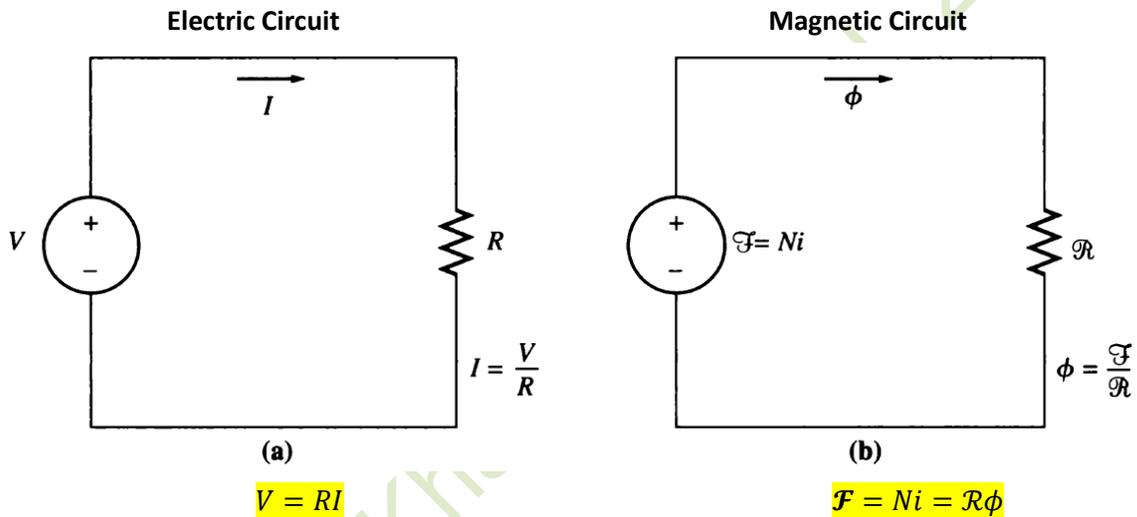
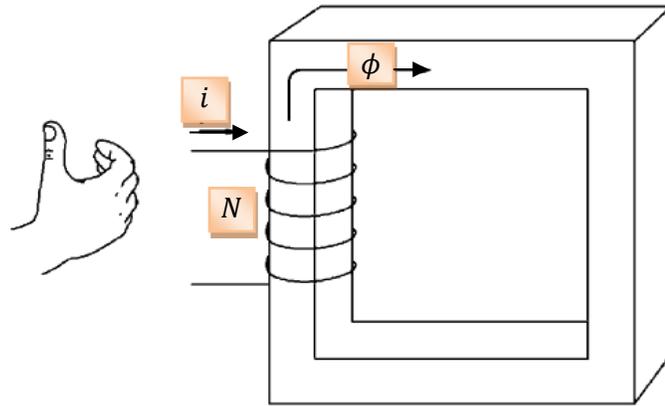
$$\phi = BA$$

For the previous Figure, the flux is:

$$\phi = \mu \frac{NiA}{l_c}$$

Magnetic Circuits:

A current in a coil of wire wrapped around a core produces a magnetic flux in the core. This is analogous to a voltage source in an electric circuit producing a current. This analogy simplifies the design of magnetic circuits.



The relationship between the magnetomotive force (*mmf*) and the flux is:

$$\mathcal{F} = Ni = \mathcal{R}\phi$$

where,

\mathcal{F} : is the magnetomotive force in the circuit; $\mathcal{F} = Ni$. It is measured in *Ampere.turns*.

ϕ : is flux of the circuit, and is measured in Webers (Wb).

\mathcal{R} : is the reluctance of the flux path in the circuit, and is measured in *Ampere.turn/Wb*.

The analogy between the magnetic and electric circuits is summarized in the table below,

Electric Circuit	Magnetic Circuit
Electromotive Force (V)	Magnetomotive Force (\mathcal{F})
Current (I)	Flux (ϕ)
Resistance (R)	Reluctance (\mathcal{R})
Conductance (G)	Permeance (\mathcal{P})

The permeance is the reciprocal of the reluctance; $\mathcal{P} = \frac{1}{\mathcal{R}}$.

Thus, $\phi = \mathcal{F} \mathcal{P}$

For the previous Magnetic circuit,

$$\phi = \mu \frac{NiA}{l_c}$$

Rearranging yields,

$$\phi = Ni \frac{\mu A}{l_c}$$

i.e.,

$$\phi = \mathcal{F} \frac{\mu A}{l_c}$$

$$\phi = \mathcal{F} \frac{1}{\mathcal{R}}$$

Therefore,

$$\mathcal{R} = \frac{l_c}{\mu A}$$

Reluctances in Series:

Reluctances are connected in series when they have the same flux. Reluctances in series are added together;

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 + \dots + \mathcal{R}_N$$

Reluctances in Parallel:

Reluctances are connected in parallel when they have the same magnetomotive force. Reluctances in parallel have the equivalent reluctance as:

$$\frac{1}{\mathcal{R}_{eq}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \dots + \frac{1}{\mathcal{R}_N}$$

Or

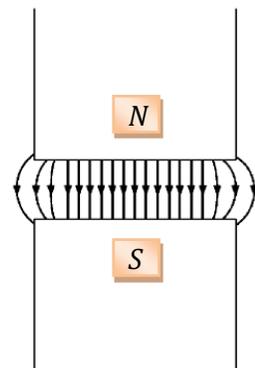
$$\mathcal{P}_{eq} = \mathcal{P}_1 + \mathcal{P}_2 + \dots + \mathcal{P}_N$$

Calculations of Flux:

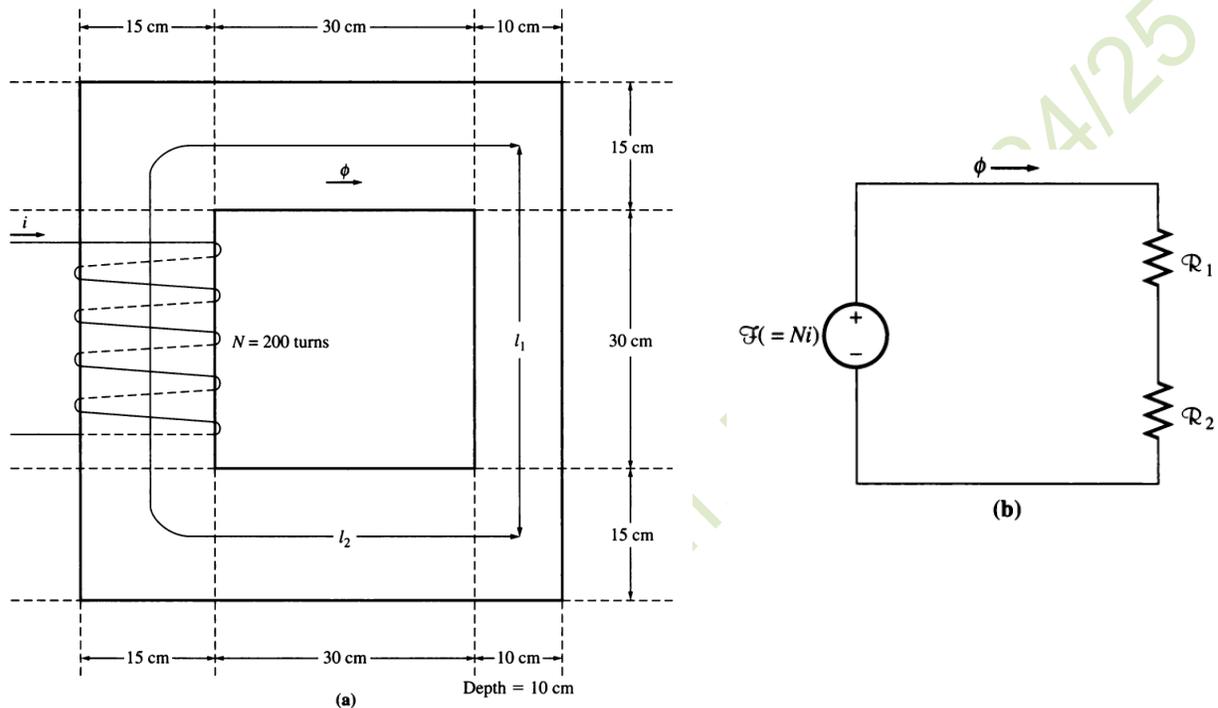
Calculations of flux in a core are approximations, and at the best they are accurate within about 5% for a number of reasons.

- 1) Assuming all the flux is confined within a magnetic core is not quite true; a small fraction of the flux escapes into the surrounding air, which is called a **leakage flux**.
- 2) Assuming a **mean path length and a cross sectional area** is not accurate especially at the corners.
- 3) In ferromagnetic materials, the **permeability varies with** the amount of **flux** already in the material.
- 4) If there are air gaps in the flux path, the **effective cross-sectional area of the airgap will be larger** than the cross-sectional area of the iron core on either side. The extra effective area is caused by the "**fringing effect**" of the magnetic field at the air gap.

Exact calculations may be found by Maxwell's equations.



Example 1–1. A ferromagnetic core is shown in Figure 1–7a. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10 cm, and the other dimensions are shown in the figure. There is a 200-turn coil wrapped around the left side of the core. Assuming relative permeability μ_r of 2500, how much flux will be produced by a 1-A input current?



Solution

We will solve this problem twice, once by hand and once by a MATLAB program, and show that both approaches yield the same answer.

Three sides of the core have the same cross-sectional areas, while the fourth side has a different area. Thus, the core can be divided into two regions: (1) the single thinner side and (2) the other three sides taken together. The magnetic circuit corresponding to this core is shown in Figure 1–7b.

The mean path length of region 1 is 45 cm, and the cross-sectional area is $10 \times 10 \text{ cm} = 100 \text{ cm}^2$. Therefore, the reluctance in the first region is

$$\begin{aligned} \mathcal{R}_1 &= \frac{l_1}{\mu A_1} = \frac{l_1}{\mu_r \mu_0 A_1} & (1-32) \\ &= \frac{0.45 \text{ m}}{(2500)(4\pi \times 10^{-7})(0.01 \text{ m}^2)} \\ &= 14,300 \text{ A} \cdot \text{turns/Wb} \end{aligned}$$

The mean path length of region 2 is 130 cm, and the cross-sectional area is $15 \times 10 \text{ cm} = 150 \text{ cm}^2$. Therefore, the reluctance in the second region is

$$\begin{aligned} \mathcal{R}_2 &= \frac{l_2}{\mu A_2} = \frac{l_2}{\mu_r \mu_0 A_2} & (1-32) \\ &= \frac{1.3 \text{ m}}{(2500)(4\pi \times 10^{-7})(0.015 \text{ m}^2)} \\ &= 27,600 \text{ A} \cdot \text{turns/Wb} \end{aligned}$$

Therefore, the total reluctance in the core is

$$\begin{aligned} \mathcal{R}_{\text{eq}} &= \mathcal{R}_1 + \mathcal{R}_2 \\ &= 14,300 \text{ A} \cdot \text{turns/Wb} + 27,600 \text{ A} \cdot \text{turns/Wb} \\ &= 41,900 \text{ A} \cdot \text{turns/Wb} \end{aligned}$$

The total magnetomotive force is

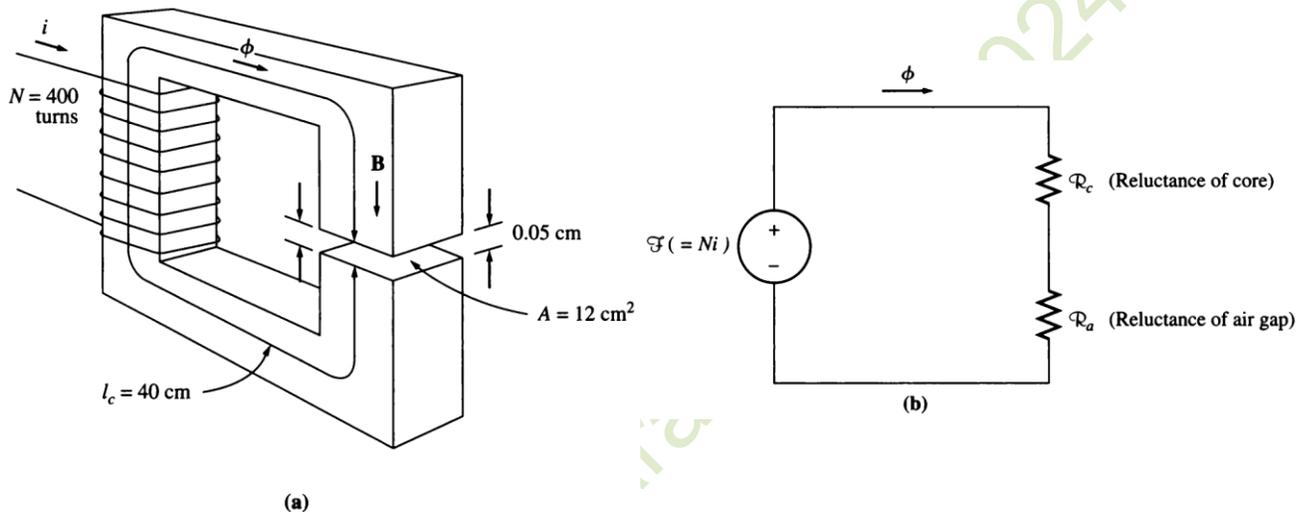
$$\mathcal{F} = Ni = (200 \text{ turns})(1.0 \text{ A}) = 200 \text{ A} \cdot \text{turns}$$

The total flux in the core is given by

$$\begin{aligned} \phi &= \frac{\mathcal{F}}{\mathcal{R}} = \frac{200 \text{ A} \cdot \text{turns}}{41,900 \text{ A} \cdot \text{turns/Wb}} \\ &= 0.0048 \text{ Wb} \end{aligned}$$

Dr. A.

Example 1–2. Figure 1–8a shows a ferromagnetic core whose mean path length is 40 cm. There is a small gap of 0.05 cm in the structure of the otherwise whole core. The cross-sectional area of the core is 12 cm², the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective cross-sectional area of the air gap by 5 percent. Given this information, find (a) the total reluctance of the flux path (iron plus air gap) and (b) the current required to produce a flux density of 0.5 T in the air gap.



Solution

The magnetic circuit corresponding to this core is shown in Figure 1–8b.

(a) The reluctance of the core is

$$\begin{aligned} \mathcal{R}_c &= \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} \\ &= \frac{0.4 \text{ m}}{(4000)(4\pi \times 10^{-7})(0.0012 \text{ m}^2)} \\ &= 66,300 \text{ A} \cdot \text{turns/Wb} \end{aligned}$$

The effective area of the air gap is $1.05 \times 12 \text{ cm}^2 = 12.6 \text{ cm}^2$, so the reluctance of the air gap is

$$\begin{aligned} \mathcal{R}_a &= \frac{l_a}{\mu_0 A_a} \quad (1-32) \\ &= \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7})(0.00126 \text{ m}^2)} \\ &= 316,000 \text{ A} \cdot \text{turns/Wb} \end{aligned}$$

Therefore, the total reluctance of the flux path is

$$\begin{aligned}\mathcal{R}_{eq} &= \mathcal{R}_c + \mathcal{R}_a \\ &= 66,300 \text{ A} \cdot \text{turns/Wb} + 316,000 \text{ A} \cdot \text{turns/Wb} \\ &= 382,300 \text{ A} \cdot \text{turns/Wb}\end{aligned}$$

Note that the air gap contributes most of the reluctance even though it is 800 times shorter than the core.

(b) Equation (1-28) states that

$$\mathcal{F} = \phi \mathcal{R} \quad (1-28)$$

Since the flux $\phi = BA$ and $\mathcal{F} = Ni$, this equation becomes

$$Ni = BA\mathcal{R}$$

so

$$\begin{aligned}i &= \frac{BA\mathcal{R}}{N} \\ &= \frac{(0.5 \text{ T})(0.00126 \text{ m}^2)(383,200 \text{ A} \cdot \text{turns/Wb})}{400 \text{ turns}} \\ &= 0.602 \text{ A}\end{aligned}$$

Notice that, since the *air-gap* flux was required, the effective air-gap area was used in the above equation.

Magnetic Behavior of Ferromagnetic Materials:

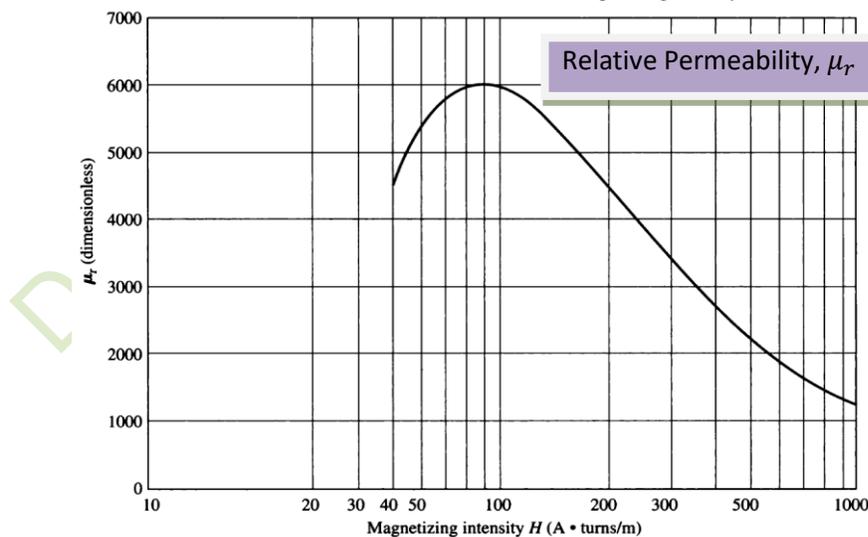
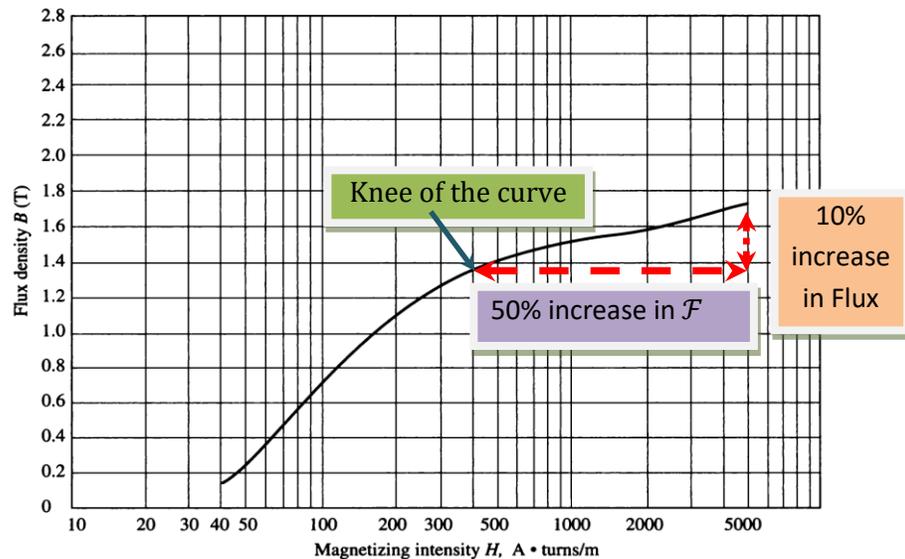
- ◆ The permeability (μ) was defined as:

$$\mu = \frac{B}{H}$$

- ◆ In other words, the permeability was assumed to be constant regardless of the magnetomotive force applied to the material. Although, this is true for free space, it is certainly untrue for iron and other ferromagnetic materials.

Saturation Curve or Magnetization Curve of a Ferromagnetic Core:

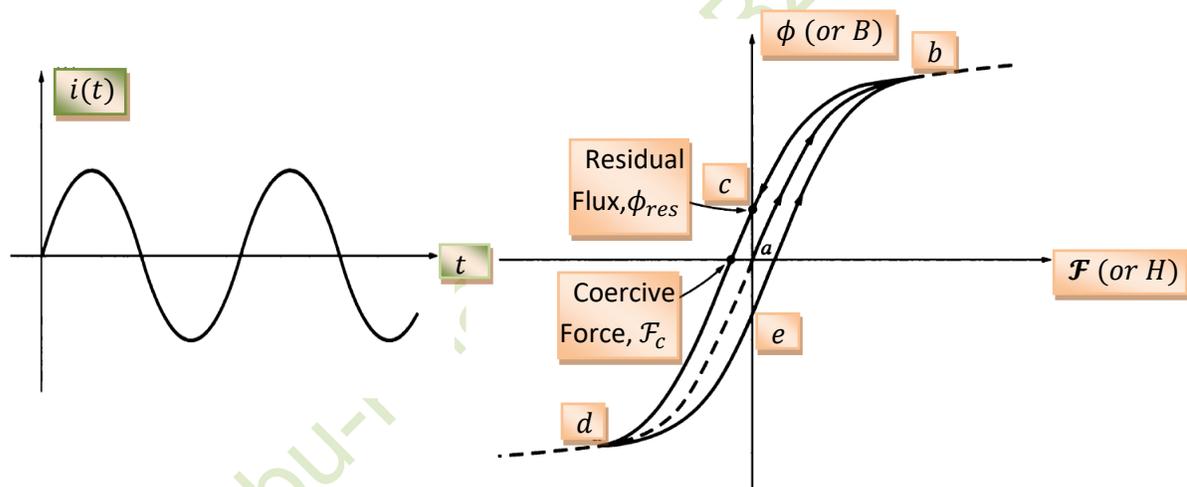
- ◆ The magnetization curve for a typical piece of steel is shown in the Figure below.



- ◆ If the resulting flux has to be proportional, or nearly so, to the applied magnetomotive force (i.e. current) then the core must be operated in the unsaturated region.
- ◆ Since real generators and motors depend on the magnetic flux to produce voltage and torque, they are designed to produce as much flux as possible, as a result most real machines operate near the knee of magnetization curve.

Energy Losses in a Ferromagnetic Core:

- Assuming that, an AC current is applied to a coil wrapped around a leg of a Ferromagnetic core, then the resulting flux versus the magnetomotive force is shown in the Figure below.



- The flux present in the core depends not only on the current applied to the winding, but also on the previous history of the flux.

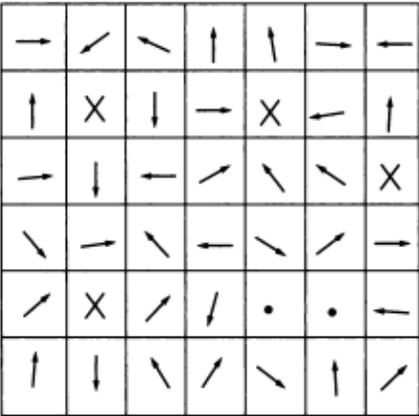
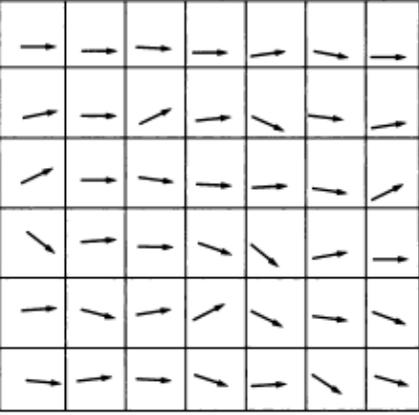
Hysteresis: is the dependence on the preceding flux history and the resulting failure to retrace the flux path.

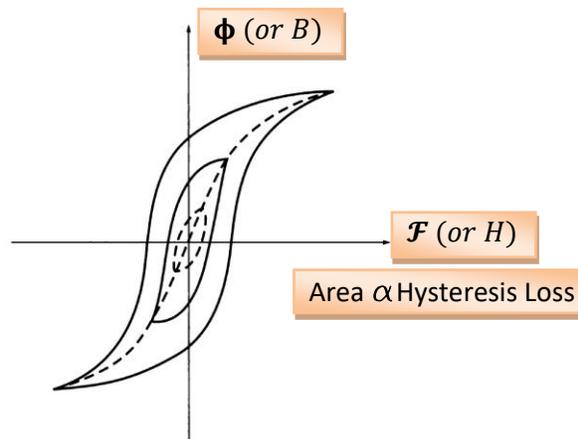
Hysteresis Loop: is the path 'bcdeb'.

Residual Flux (ϕ_{Res}): is the flux left in the core when the magnetomotive force is removed.

Coercive Magnetomotive Force (\mathcal{F}_c): is the amount of *mmf* that has to be applied in the opposite direction to force the flux to fall to zero.

Ferromagnetic Materials Structure:

- The atoms of iron and similar metals (Cobalt, Nickel and some of their alloys) tend to have their magnetic fields closely aligned with each other.
 - Within the metal, there are many small region called "domains".
 - In each domain, all the atoms are aligned with their magnetic fields pointing in the same direction, each domain acts a tiny permanent magnet.
 - These domains are oriented randomly → No Flux.
- 
- When a block of iron is placed in an external magnetic field, the domains, which point in the direction of the external magnetic field increase, increasing the magnetic flux in the iron (positive feedback) causing more and more domains oriented in the direction of the external magnetic field. That's why the iron has a higher permeability than air.
 - Once all the domains are aligned, any increase in the magnetic field can cause only the same flux increase that it would in the free space. At this point the iron is saturated with flux.
 - When the external magnetic field is removed, the domains do not completely randomize, because turning the atoms in them requires energy.
 - **Hysteresis Loss in a Core:** is the energy required to accomplish reorientation of domains during each cycle of alternating current applied to the core. It is proportional to the hysteresis loop area.
 - This loss causes heating of the core.
- 



Faraday's Law:

- It is the basis for a transformer operation.
- If a flux passes through a turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the rate of change in the flux with respect to time.

$$e_{ind} = -\frac{d\phi}{dt}$$

where,

e_{ind} : is the induced voltage in the turn of coil [V].

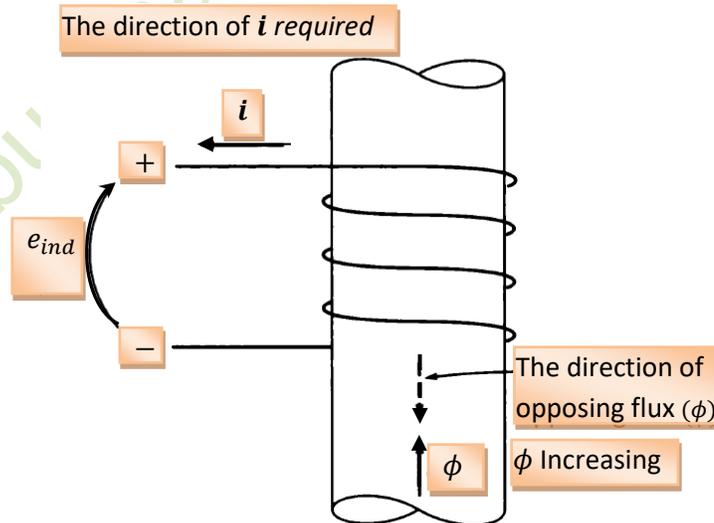
ϕ : is the flux passing through that turn [Webers].

For N turns,

$$e_{ind} = -N \frac{d\phi}{dt}$$

- The minus sign is an expression of **Lenz' law!**

Lenz' Law: the direction of the voltage build up in a coil is such that, if the coil ends were short-circuited, it would produce a current that would cause a flux opposing the original flux change.



However, the sign in Faraday's law is often left out as the polarity of the voltage can be determined from the physical considerations.

- If there was a flux leakage out of the core, then each turn might have a slightly different flux.
- Therefore, the voltage in the i^{th} turn is:

$$e_i = \frac{d\phi_i}{dt}$$

- **For N turns,**

$$e_{ind} = \sum_{i=1}^N e_i = \sum_{i=1}^N \frac{d\phi_i}{dt}$$

$$e_{ind} = \frac{d(\sum_{i=1}^N \phi_i)}{dt}$$

Define the **Flux Linkage** as the total flux linking all turns, and is measured in *Weber, turns*;

$$\lambda = \sum_{i=1}^N \phi_i$$

- Thus, Faraday's Law becomes,

$$e_{ind} = \frac{d\lambda}{dt}$$

- **Eddy Currents:**

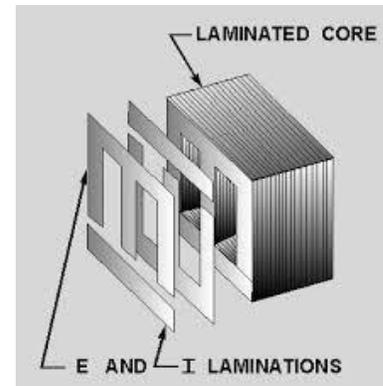
The time changing flux also induces voltages within the ferromagnetic core, in the same manner as it would in a wire wrapped around that core. These voltages cause eddy currents to flow within the core. These currents pass through the resistive core causing energy loss called **eddy current loss**.

Core Losses = Eddy Current Loss + Hysteresis Loss

- The **eddy current paths can be broken** by designing the core to have **laminations with insulating materials between them**.

- **Laminations:** are thin sheets of a magnetic material of a good permeability and low conductivity.

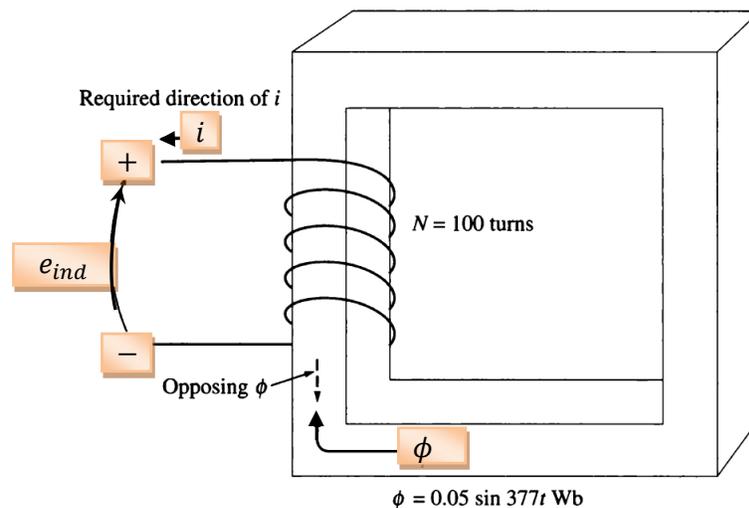
- The insulator between the sheets is **iron oxide, Silicon oxide, or special coating**. **Silicon-steel laminations** may be used with insulator of iron oxide, Silicon oxide, or special coating.



Example 1–6. Figure 1–15 shows a coil of wire wrapped around an iron core. The flux in the core is given by the equation

$$\phi = 0.05 \sin 377t \quad \text{Wb}$$

If there are 100 turns on the core, what voltage is produced at the terminals of the coil? Of what polarity is the voltage during the time when flux is *increasing* in the reference direction shown in the figure? Assume that all the magnetic flux stays within the core (i.e., assume that the flux leakage is zero).



Solution

By the same reasoning as in the discussion on pages 29–30, the direction of the voltage while the flux is increasing in the reference direction must be positive to negative, as shown in Figure 1–15. The *magnitude* of the voltage is given by

$$\begin{aligned} e_{ind} &= N \frac{d\phi}{dt} \\ &= (100 \text{ turns}) \frac{d}{dt} (0.05 \sin 377t) \\ &= 1885 \cos 377t \end{aligned}$$

or alternatively,

$$e_{ind} = 1885 \sin(377t + 90^\circ) \text{ V}$$

Production of Induced Force in a Wire:

➔ A major effect of the magnetic field is that it induces a force on a current-carrying wire (**motor action**).

➔ The Force (F) induced in a conductor is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

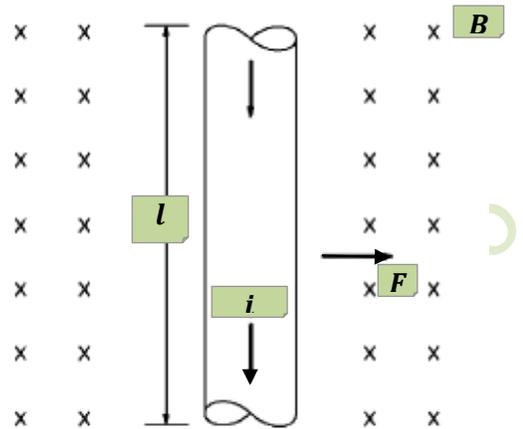
$$\vec{F} = ilB \sin \theta$$

where,

i : is the conductor current [A]

l : is the length of conductor [m]

B : is a uniform magnetic flux density [T]



Direction of Force:

The direction of the force is found by the right hand rule; the index finger is in the direction l (or i) the middle finger is in the direction of B , then the thumb points in the direction of the resultant force in the wire. Or **rotate from l (or i) to B and then the thumb points in the direction of the resultant force.**

The direction of F in the previous Figure is to the right!

Induced Voltage in a Conductor Moving in a Magnetic Field

❖ A moving conductor in the presence of a magnetic field has an induced voltage in it (**generator action**).

❖ The induced voltage (e_{ind}) in a conductor moving in a uniform magnetic field is:

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

where,

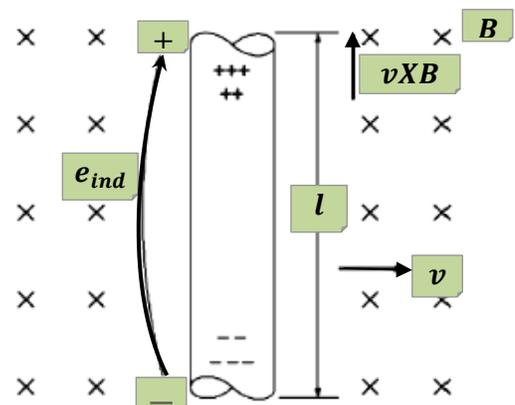
v : is the conductor velocity [m/s]

l : is the length of conductor [m]

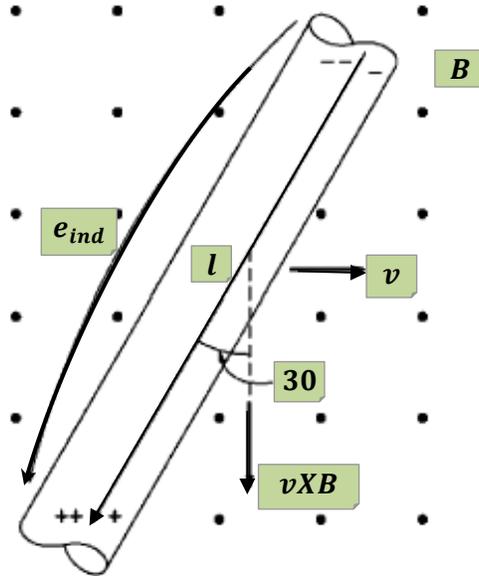
B : is a uniform magnetic flux density [T]

❖ Assume that \vec{l} makes the smallest angle with the **vector $\vec{v} \times \vec{B}$.**

❖ The voltage in the conductor will be built up so that the positive end is in the direction $\vec{v} \times \vec{B}$; in the previous Figure, $\vec{v} \times \vec{B}$ is upwards.



Example 1–9. Figure 1–18 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?



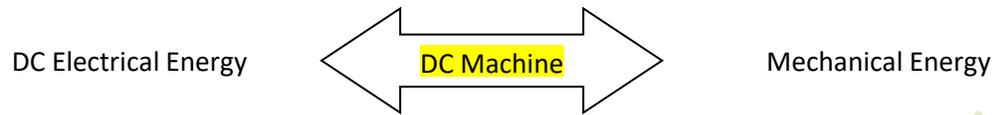
Solution

The direction of the quantity $\mathbf{v} \times \mathbf{B}$ is down. The wire is not oriented on an up-down line, so choose the direction of \mathbf{l} as shown to make the smallest possible angle with the direction of $\mathbf{v} \times \mathbf{B}$. The voltage is positive at the bottom of the wire with respect to the top of the wire. The magnitude of the voltage is

$$\begin{aligned}
 e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} && (1-45) \\
 &= (vB \sin 90^\circ) l \cos 30^\circ \\
 &= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30^\circ \\
 &= 4.33 \text{ V}
 \end{aligned}$$

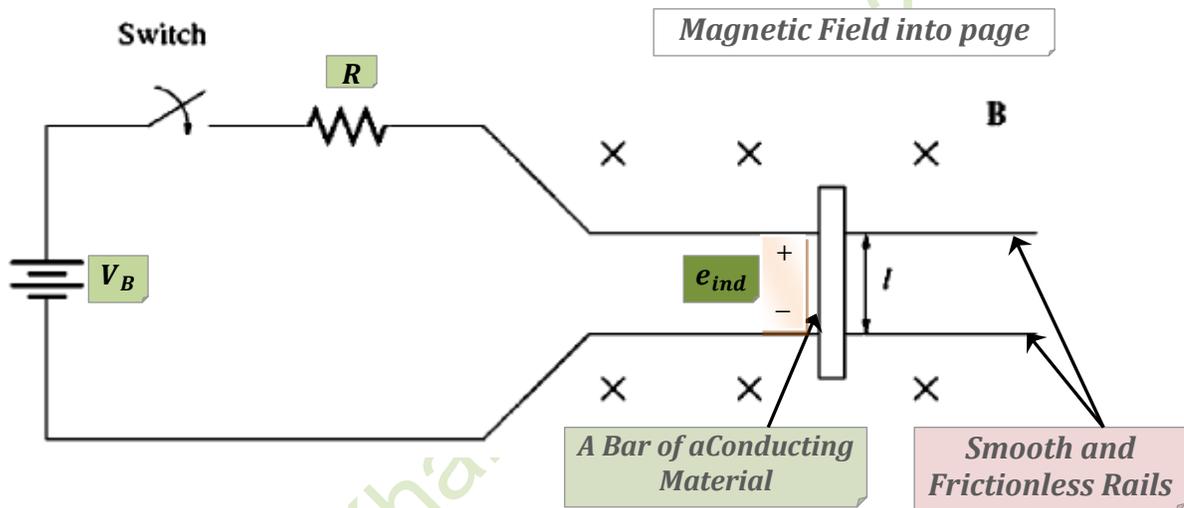
The induction of voltages in a wire moving in a magnetic field is fundamental to the operation of all types of generators. For this reason, it is called *generator action*.

Linear DC Machinery Fundamentals



- ➔ Most DC machines are like AC machines in that they have AC voltages and currents within them.
- ➔ However, DC machines have a DC output voltage because of the commutator and brushes mechanism.

A Linear DC Machine



The operation principle of the Linear DC machine can be determined from the application of four basic equations:

- 1- The force on a wire in the presence of a magnetic field is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

$$F = ilB \sin \theta$$

where,

i : is the conductor (bar) current [A]

l : is the length of conductor (bar) with direction of l as the direction of i [m]

B : is a uniform magnetic flux density [T]

2- The Voltage induced in a wire (bar) moving in a magnetic field is:

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

where,

v : is the conductor (bar) velocity [m/s]

l : is the length of conductor (bar) [m]

B : is the uniform magnetic flux density [T]

Assuming that \vec{l} makes the smallest angle with $\vec{v} \times \vec{B}$

3- Kirchhoff's Voltage Law:

$$V_B - iR - e_{ind} = 0$$

where,

i : is the machine's current [A]

V_B : is the battery voltage [V]

R : is the machine's equivalent resistance; including battery internal resistance, wiring, rails and bar resistances [Ω]

Thus, the current in the bar is:

$$i = \frac{V_B - e_{ind}}{R}$$

4- Newton's Law

The net force on the bar is:

$$F_{net} = ma$$

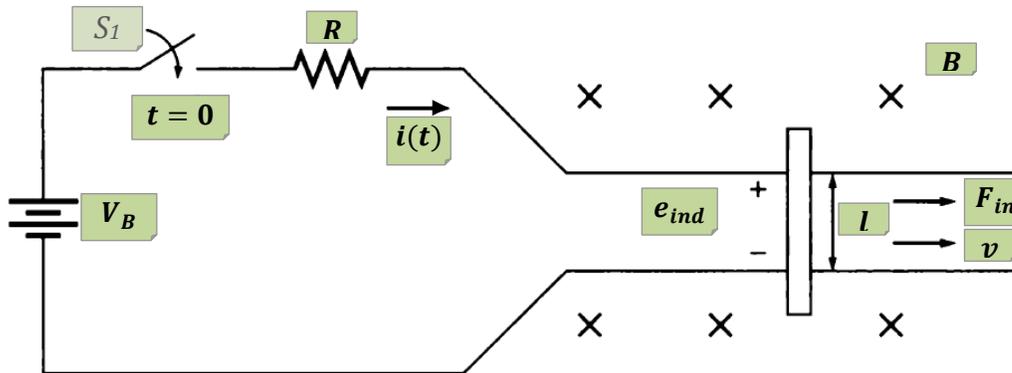
where,

m : is the mass of the bar [kg]

a : is the acceleration of the bar [m/s²]

Starting The Linear DC Machine

- Closing the switch starts the Linear DC machine.



- The current that flows in the bar is:

$$i = \frac{V_B - e_{ind}}{R}$$

- Initially, the machine (bar) is at rest, and $e_{ind} = 0$ ($= (\vec{v} \times \vec{B}) \cdot \vec{l}$). Thus, the starting current is:

$$i = \frac{V_B}{R}$$

- Therefore, the net force on the bar is:

$$\vec{F}_{ind} = i(\vec{l} \times \vec{B}); \quad \text{to the right}$$

$$\vec{F}_{ind} = ilB; \quad \text{to the right}$$

Consequently, the bar accelerates to the right!

- As the velocity of the bar increases, a voltage will be induced in the bar and is given by:

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

$$e_{ind} = vBl; \quad \text{positive upwards}$$

- The induced voltage reduces the current flowing in the bar because,

$$i \downarrow = \frac{V_B - e_{ind}}{R}$$

- The result of this action is that, the induced force decreases, and so does the acceleration. The bar speed continues increasing (since the acceleration is still positive) until reaching a constant steady state speed (no-load speed) at a zero acceleration, where the net force on the bar is zero.

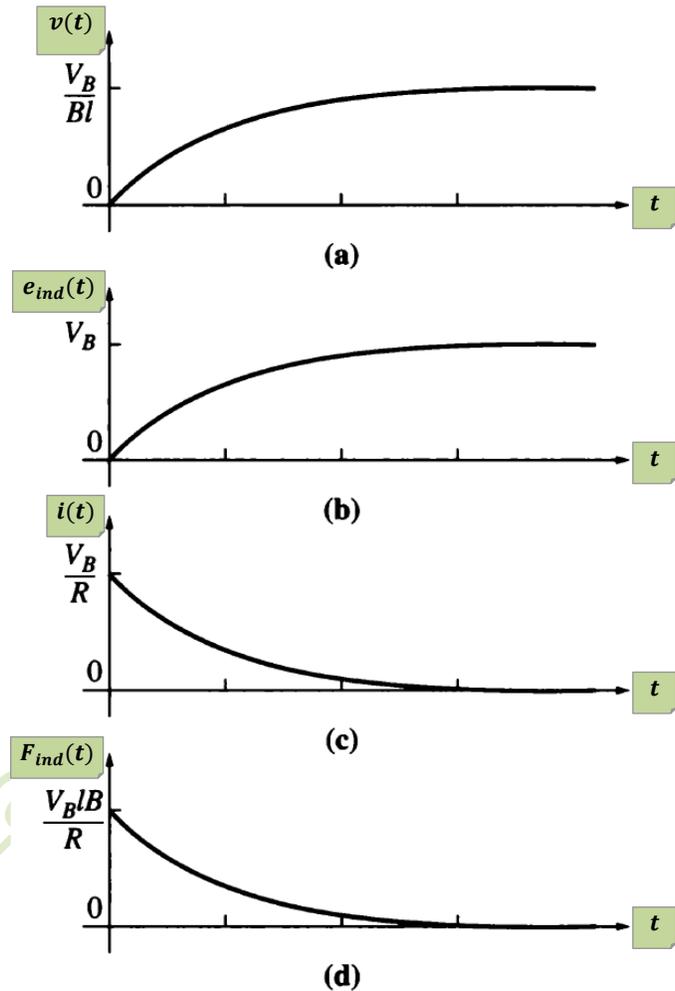
- In other words, the current becomes zero when:

$$V_B = e_{ind} = v_{ss}Bl$$

Therefore, the steady state speed is:

$$v_{ss} = \frac{V_B}{Bl}$$

- The waveforms of various parameters are shown in the Figure next.



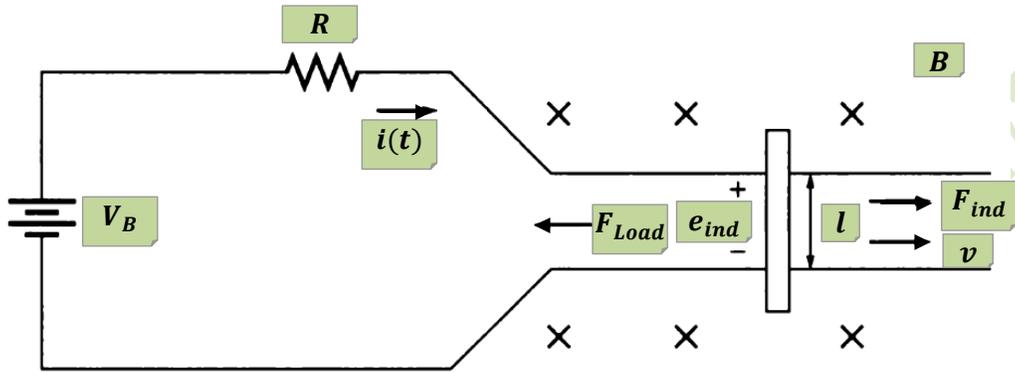
- ➔ To summarize, at starting, the linear DC machine behaves as follows:

- Closing the switch produces a current; $i = \frac{V_B}{R}$
- The current flow produces a force on the bar; $\bar{F}_{ind} = ilB$; to the right!
- The bar accelerates to the right producing an induced voltage, $e_{ind}(= vBl)$, upwards, as it speeds up.
- This induced voltage reduces the current flow.
- The induced force is thus decreased ($\bar{F}_{ind} = i \downarrow lB$), until eventually, $\bar{F}_{ind} = 0$. At this point, $V_B = e_{ind}$, $i = 0$, and the bar moves at a constant (steady state) no-load speed; $v_{ss} = \frac{V_B}{Bl}$.

This is precisely the **behavior of a Shunt DC motor** at starting!

The Linear DC Machine as a Motor

- ➔ Assuming that, the machine (bar) is running at a steady state (noload) speed to the right with a zero initial induced force, $\bar{F}_{ind} = 0$, before an external force \bar{F}_{load} is applied to it in the opposite direction of motion, as illustrated in the Figure below.



Therefore, the net force on the bar is:

$$\bar{F}_{net} = \bar{F}_{ind} - \bar{F}_{load};$$

But, $\bar{F}_{ind} = 0$;

Thus, $\bar{F}_{net} = -\bar{F}_{load}$; to the left

- ➔ This results in decelerating the bar;

$$a = \frac{-\bar{F}_{load}}{m}$$

- ➔ Therefore, the bar slows down and its speed decreases.

- ➔ Consequently, the induced voltage in the bar decreases as well, because,

$$e_{ind} = v \downarrow Bl; \quad \text{still positive upwards}$$

- ➔ As e_{ind} decreases, the current flow in the bar increases;

$$i \uparrow = \frac{V_B - e_{ind} \downarrow}{R}$$

- ➔ The increase in the current results in an increase in the induced force in the bar;

$$\bar{F}_{ind} = ilB$$

Until it equals the load force, where the bar reaches a steady state speed (the deceleration becomes zero);

$$\begin{aligned}\bar{F}_{ind} &= \bar{F}_{load} \\ \Rightarrow i' &= \frac{\bar{F}_{load}}{lB}\end{aligned}$$

- ➔ The induced voltage must equal:

$$\begin{aligned}e'_{ind} &= V_B - i'R \\ \Rightarrow v'_{ss}Bl &= V_B - \frac{\bar{F}_{load}}{lB}R\end{aligned}$$

- ➔ And the new steady state speed can be found as:

$$v'_{ss} = \frac{V_B - \frac{\bar{F}_{load}R}{lB}}{lB}$$

- ➔ This new steady state speed is certainly less than the previous steady state (noload) speed, which was

$$v_{ss} = \frac{V_B}{Bl}$$

- ➔ Now, there is an induced force in the direction of motion of the bar, and the electrical power converted to mechanical power, to keep the bar moving, is:

$$P_{conv} = e'_{ind}i' = F_{ind}v'_{ss}$$

- ➔ Hence, the machine is acting as a motor.

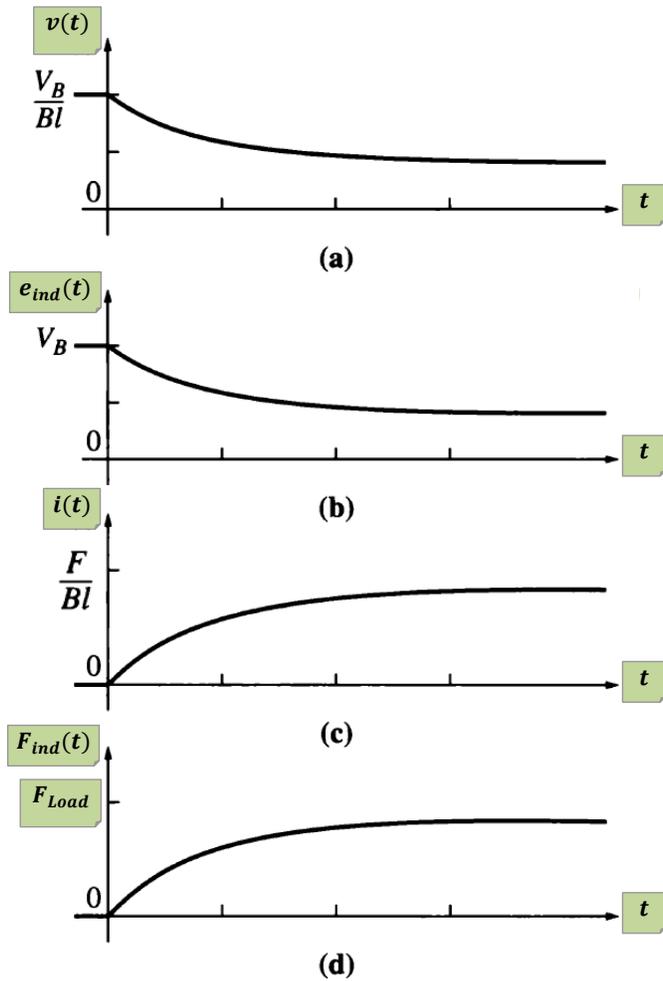
- ➔ The efficiency of the machine is:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{e'_{ind}i'}{V_B i'} \times 100\%$$

$$\eta = \frac{e'_{ind}}{V_B} \times 100\%$$

➔ The waveforms for various parameters for the motor case are shown in the Figure next.

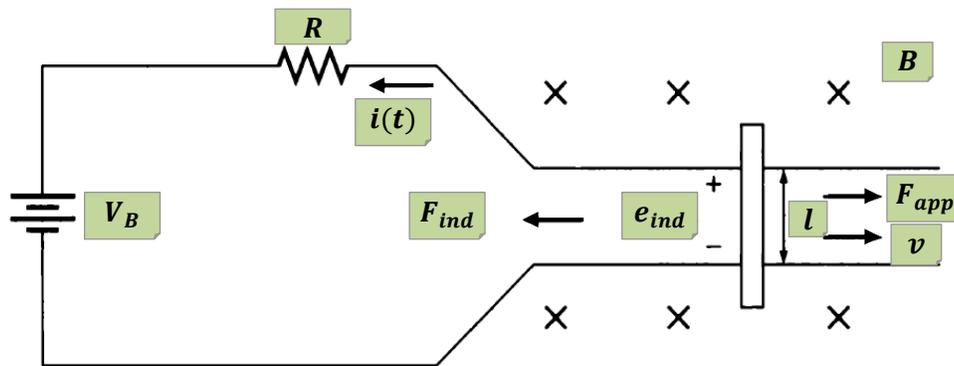


➔ To summarize the motor operation:

- 1- A force \bar{F}_{load} is applied opposite to the direction of motion of the bar, which results in a net force in the opposite direction of motion.
- 2- The resulting acceleration $a = \frac{\bar{F}_{net}}{m}$ is negative so the bar slows down.
- 3- The voltage induced falls; $e_{ind} \downarrow (= v \downarrow Bl)$, so the current increases; $i \uparrow = \frac{V_B - e_{ind}}{R}$
- 4- The induced force $\bar{F}_{ind} \uparrow (= i \uparrow lB)$ increases in the direction of motion, until $\bar{F}_{ind} = \bar{F}_{load}$ at a lower speed.
- 5- An amount of electric power ($e'_{ind}i'$) is being converted to mechanical power ($F_{ind}v'_{ss}$) and the machine is acting as a motor.

The Linear DC Machine as a Generator

- Assuming the machine is operating under no-load steady state speed, where $e_{ind} = V_B$ and $i = 0$, prior to applying a force \vec{F}_{app} in the direction of motion, as shown in the Figure below.



- The applied force causes the bar to accelerate in the direction of motion and the velocity (v) of the bar increases causing the induced voltage, $e''_{ind} = v \uparrow Bl$, to increase and to become larger than V_B (the battery is being charged).

- With $e''_{ind} > V_B$, the current reverses direction, and is given by:

$$i = \frac{e''_{ind} - V_B}{R}$$

- Since the current flows up in the bar, the induced force in the bar is:

$$\vec{F}_{ind} = i(\vec{l} \times \vec{B}); \quad \text{to the left}$$

$$\vec{F}_{ind} = ilB; \quad \text{to the left}$$

- This induced force opposes the applied force to the bar.

- The bar continues accelerating and its speed increases. Hence, the induced voltage and current continue increasing and so does the induced force, until the induced force equals the applied force. At this point, the bar's acceleration becomes zero, and the bar reaches a new steady state speed, which is higher than before;

$$\vec{F}_{ind} = \vec{F}_{app}$$

$$\Rightarrow i'' = \frac{F_{app}}{lB}$$

$$e''_{ind} = V_B + i''R$$

$$\Rightarrow e''_{ind} = V_B + \frac{\bar{F}_{app}}{lB} R$$

$$v''_{ss} = \frac{V_B + \frac{\bar{F}_{app}R}{lB}}{lB}$$

✚ Note that, $v''_{ss} > v_{ss}$!

✚ The battery is being charged and the machine is converting mechanical power ($\bar{F}_{app} v''_{ss}$) to electric power ($e''_{ind} i''$); hence it is a generator.

✚ The efficiency of the machine is:

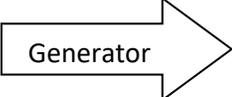
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{V_B i''}{e''_{ind} i''} \times 100\%$$

$$\eta = \frac{V_B}{e''_{ind}} \times 100\%$$

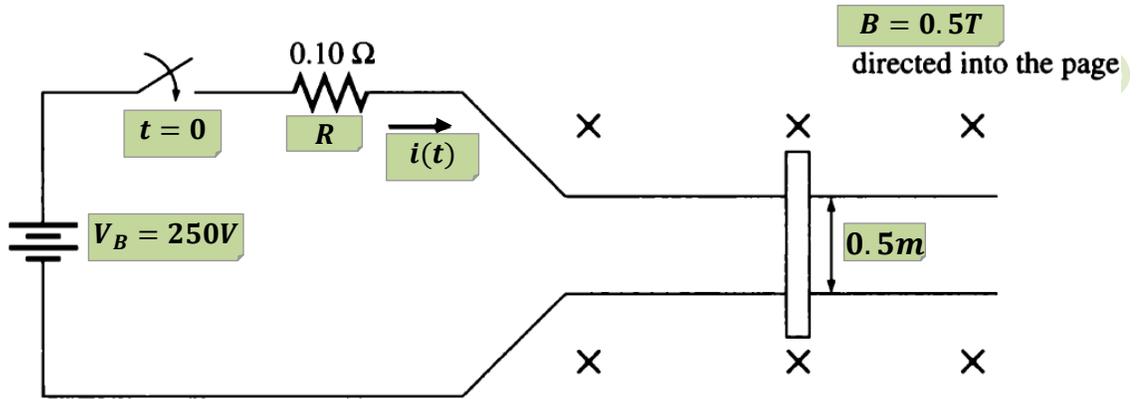
To Summarize:

- 1- A force \bar{F}_{app} is applied in the direction of motion. Consequently, F_{net} is in the same direction of motion.
- 2- The acceleration, $a = \frac{\bar{F}_{net}}{m}$, is positive so the bar speeds up ($v \uparrow$).
- 3- The voltage induced, $e''_{ind} (= v \uparrow l B)$, increases, and so does the current, $i = \frac{e''_{ind} \uparrow - V_B}{R}$, but in the reverse direction.
- 4- The induced force $\bar{F}_{ind} (= i \uparrow l B)$ increases, until $|\bar{F}_{ind}| = |\bar{F}_{app}|$, but at a higher speed.

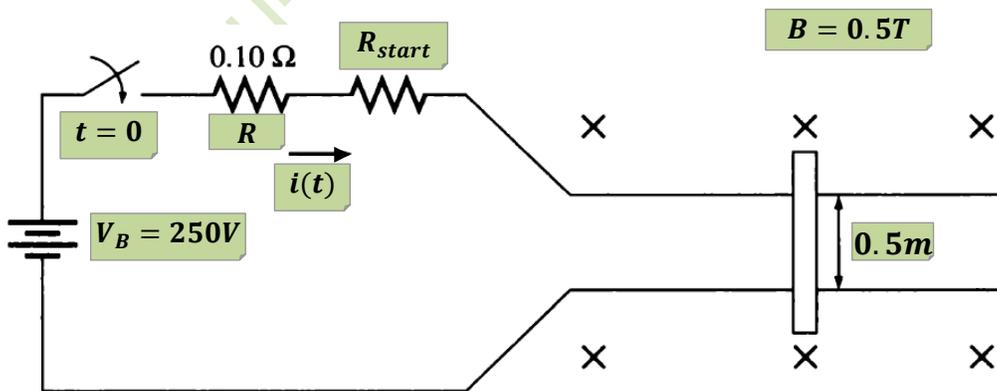
5- The power converted: $\bar{F}_{app} v''_{ss}$  $e''_{ind} i''$

Starting Problems with The Linear DC Machines

- ❖ ' R ' in the Figure below models the machine's internal resistance (0.1Ω is reasonable for medium size DC motor; e.g. 63kW).

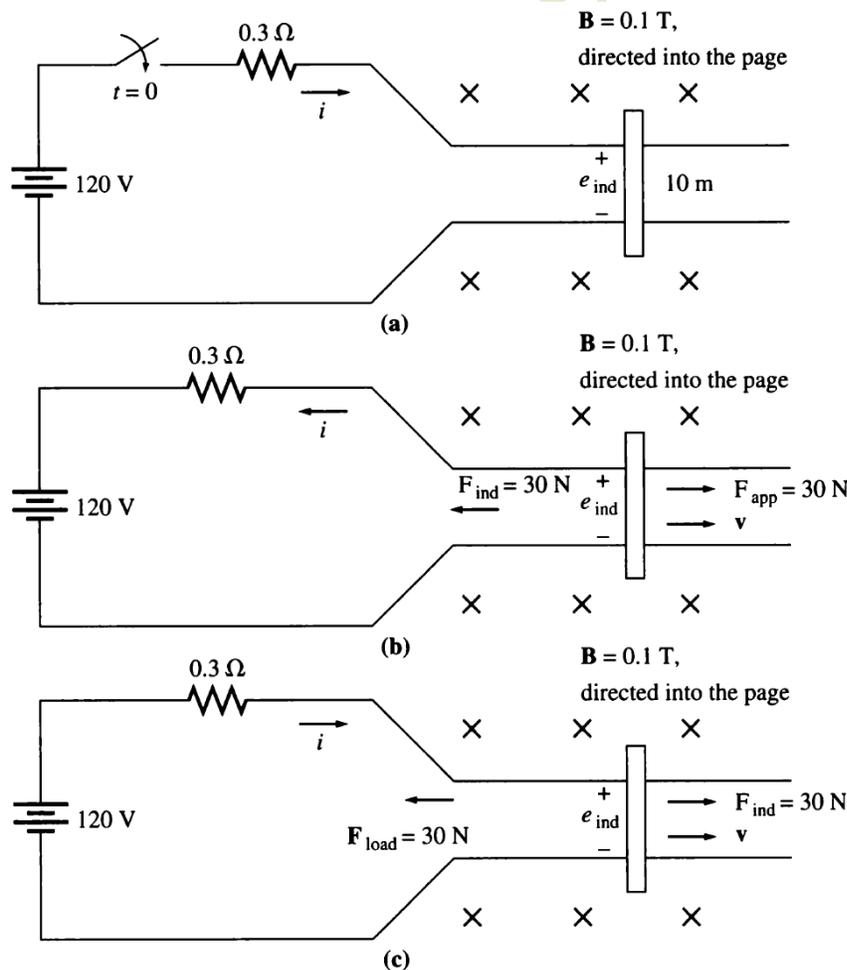


- ❖ At starting, when the switch is closed, $e_{ind} = 0$, and $i = \frac{V_B}{R} = \frac{250}{0.1} = 2500A!$
- ❖ This is a very high current and is almost 10 times the rated current. Such a current can cause damage to the motor.
- ❖ This problem can be overcome by inserting an extra resistor R_{start} into the circuit during starting, as shown in the Figure below, to limit the current flow until the induced voltage builds up enough to limit it.



Example 1–10. The linear dc machine shown in Figure 1–27a has a battery voltage of 120 V, an internal resistance of 0.3Ω , and a magnetic flux density of 0.1 T.

- What is this machine's maximum starting current? What is its steady-state velocity at no load?
- Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming? Explain the difference between these two figures. Is this machine acting as a motor or as a generator?
- Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?
- Assume that a force pointing to the left is applied to the bar. Calculate speed of the bar as a function of the force for values from 0 N to 50 N in 10-N steps. Plot the velocity of the bar versus the applied force.
- Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?



Solution

(a) At starting conditions, the velocity of the bar is 0, so $e_{\text{ind}} = 0$. Therefore,

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{120 \text{ V} - 0 \text{ V}}{0.3 \Omega} = 400 \text{ A}$$

When the machine reaches steady state, $F_{\text{ind}} = 0$ and $i = 0$. Therefore,

$$\begin{aligned} VB &= e_{\text{ind}} = v_{\text{ss}}Bl \\ v_{\text{ss}} &= \frac{V_B}{Bl} \\ &= \frac{120 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 120 \text{ m/s} \end{aligned}$$

(b) Refer to Figure 1–27b. If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force F_{ind} is equal and opposite to the applied force F_{app} , so that the net force on the bar is zero:

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

Therefore,

$$\begin{aligned} i &= \frac{F_{\text{ind}}}{lB} = \frac{30 \text{ N}}{(10 \text{ m})(0.1 \text{ T})} \\ &= 30 \text{ A} \quad \text{flowing up through the bar} \end{aligned}$$

The induced voltage e_{ind} on the bar must be

$$\begin{aligned} e_{\text{ind}} &= V_B + iR \\ &= 120 \text{ V} + (30 \text{ A})(0.3 \Omega) = 129 \text{ V} \end{aligned}$$

and the final steady-state speed must be

$$\begin{aligned} v_{\text{ss}} &= \frac{e_{\text{ind}}}{Bl} \\ &= \frac{129 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 129 \text{ m/s} \end{aligned}$$

The bar is *producing* $P = (129 \text{ V})(30 \text{ A}) = 3870 \text{ W}$ of power, and the battery is *consuming* $P = (120 \text{ V})(30 \text{ A}) = 3600 \text{ W}$. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator*.

(c) Refer to Figure 1–25c. This time, the force is applied to the left, and the induced force is to the right. At steady state,

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{ind}}}{lB} = \frac{30 \text{ N}}{(10 \text{ m})(0.1 \text{ T})}$$

$$= 30 \text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{\text{ind}} = V_B - iR$$

$$= 120 \text{ V} - (30 \text{ A})(0.3 \Omega) = 111 \text{ V}$$

and the final speed must be

$$v_{\text{ss}} = \frac{e_{\text{ind}}}{Bl}$$

$$= \frac{111 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 111 \text{ m/s}$$

This machine is now acting as a *motor*, converting electric energy from the battery into mechanical energy of motion on the bar.

(e) If the bar is initially unloaded, then $e_{\text{ind}} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B .

This fact can be used to determine the final speed of the bar. The *initial speed* was 120 m/s. The *final speed* is

$$VB = e_{\text{ind}} = v_{\text{ss}}Bl$$

$$v_{\text{ss}} = \frac{V_B}{Bl}$$

$$= \frac{120 \text{ V}}{(0.08 \text{ T})(10 \text{ m})} = 150 \text{ m/s}$$

Thus, when the flux in the linear motor weakens, the bar speeds up. The same behavior occurs in real dc motors: When the field flux of a dc motor weakens, it turns faster. Here, again, the linear machine behaves in much the same way as a real dc motor.

PART II

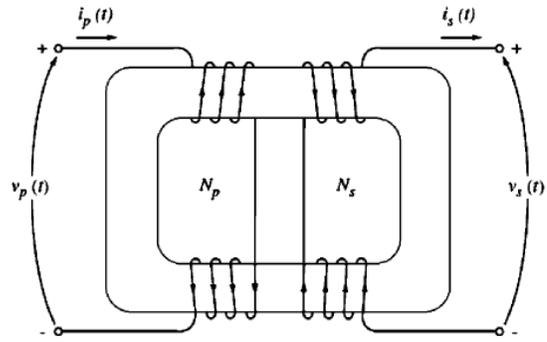
Electrical Transformers

- ➔ **A Transformers** is advice that converts AC electric energy at one voltage level to AC electric energy at another voltage level through the action of a magnetic field. It consists of two or more coils of wire wrapped around a common ferromagnetic core.
- ➔ Transformers are important to reduce losses in power systems by transmitting energy at high voltage levels (low current levels) over long distances.
- ➔ One of the transformer windings is connected to a source of AC electric power, and is called the **Primary or Input winding**.
- ➔ The second winding is at the load side or connected to the load, and is called the **Secondary or Output winding**.
- ➔ If there is a third winding, it is called the **Tertiary winding**.
- ➔ There are many **types** of Transformers: Power Transformers, Voltage or Potential Transformers (PT), Current Transformers (CT), or Impedance Transformers.

Types of Construction of Transformers:

1- Core Form

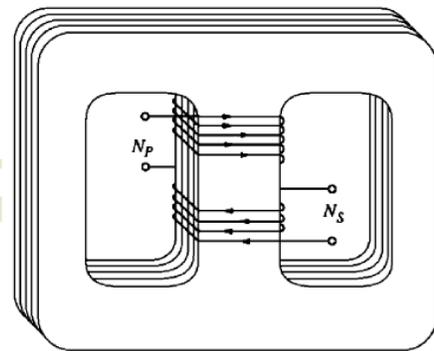
The core is constructed of thin laminations electrically isolated to minimize eddy currents. It has the shape shown in the Figure next.



2- Shell Form

It has the shape shown in the Figure next.

Its core is also laminated.



Note: In a physical transformer, the primary and secondary windings are wrapped one on top of the other with the low voltage winding innermost, such arrangement serves two purposes:

- 1- It simplifies the problem of insulating the high voltage windings from the core.
- 2- It results in much less leakage flux than would be the case if the two windings were separated by a distance on the core.



Power Transformers:

Power transformers are given a variety of different names:

- a) **Unit Transformers:** They are connected to the output of the generator (or power plant), at the sending end, to step up its voltage (to above 110kV). The generated voltage is in the range of 0.4kV to 25kV.
- b) **Substation Transformers:** They step down the voltage at the other end of the transmission line to distribution levels (2.3kV to 34.5kV).
- c) **Distribution Transformers:** they take the distribution voltage, at the receiving end, and steps it down to the final voltage at which the power is actually consumed (110V, 208V, 220V, or 380V).

Special (Instrument) Transformers:

These are other types of transformers used with electrical machines and power systems such as:

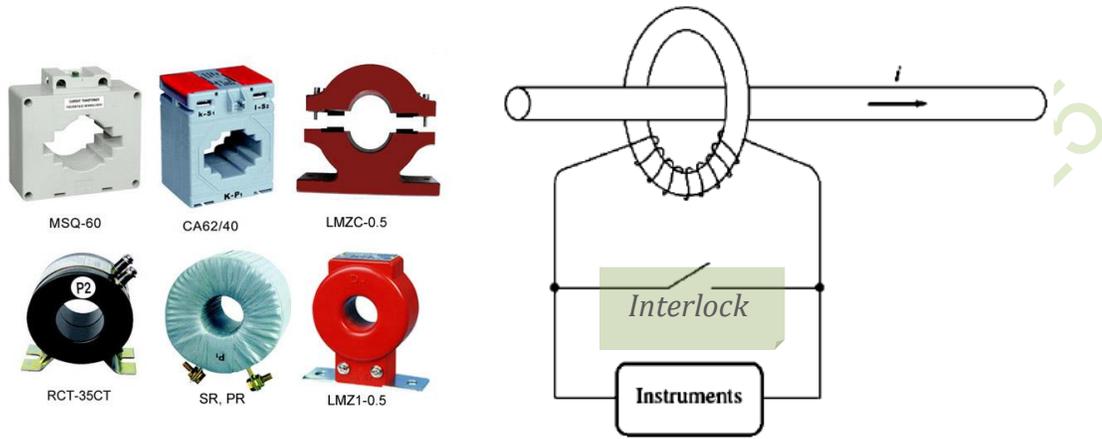
Potential Transformer (PT):

It has a high voltage primary and a low voltage secondary. It provides samples of the power system voltages to the instruments monitoring it, producing a low secondary voltage directly proportional to the power system voltages. It handles a very small current, and has a very low power rating, as its apparent power rating is in the range 50VA to 500VA. It must be accurate so as not to distort the true voltage badly.

Current Transformer (CT):

- It provides a secondary current, which is much smaller, but directly proportional to the primary (transmission line) current; it samples the line current and reduces it to a safe and a measurable level; 100: 5, 200: 5, 400: 5, 600: 5, 800: 5, 1000: 5,...
- **5A** is a standard value on the secondary of the CT.

- The secondary windings are loosely coupled, and thus the mutual flux is smaller than the leakage flux. Therefore, the voltage and current relationships do not apply!



- **CTs secondary should be loaded all the times via a very small burden resistor, whose value ranges from fractions to few Ohms, because high voltages may appear across its terminals when the secondary is open circuited.** Thus, a CT has a **shorting interlock**, which **must be shut** before the instrument's relay can be moved for inspection or adjustment.

Ideal Transformers

The voltage relationship:

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$$

The turns' ratio:

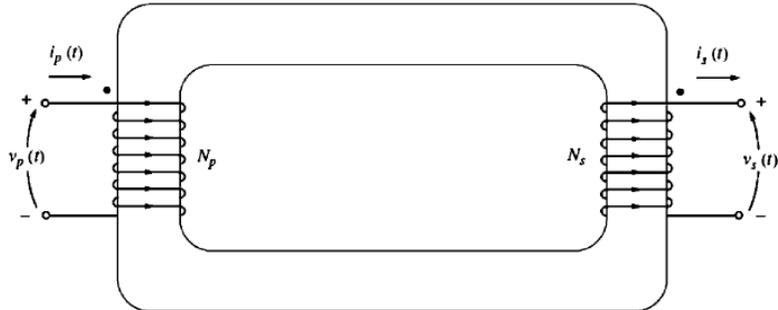
$$a = \frac{N_p}{N_s}$$

The current relationship:

$$\frac{i_p(t)}{i_s(t)} = \frac{N_s}{N_p} = \frac{1}{a}$$

In phasors:

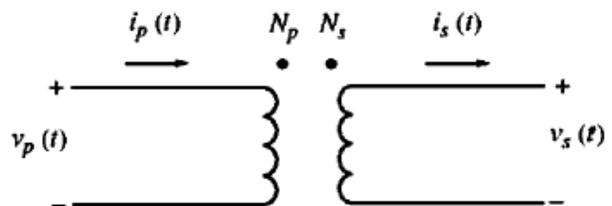
$$\frac{\bar{V}_p}{\bar{V}_s} = a \quad \text{and} \quad \frac{\bar{I}_p}{\bar{I}_s} = \frac{1}{a}$$



- In an ideal transformer, the primary voltage and current are in phase with the secondary's, so the transformer's turns' ratio affects the magnitude only, but not their angles.

The Dot Convention:

- If the primary voltage is positive at the dotted end, then the secondary voltage is positive at the dotted end.



- If the primary current is entering the dotted end of the primary winding, then the secondary current flows out of the dotted end of the secondary winding.

Power in an Ideal Transformer

The input real power:

$$P_{in} = V_p I_p \cos \theta_p$$

The output real power:

$$P_{out} = V_s I_s \cos \theta_s$$

But, since the ideal transformer does not affect the voltage and current angles, then $\theta_p = \theta_s = \theta$; i.e., the primary and secondary windings have the same power factor.

To find the relationship between the input and output powers,

$$P_{out} = V_s I_s \cos \theta$$

$$P_{out} = \frac{V_p}{a} a I_p \cos \theta$$

$$P_{out} = V_p I_p \cos \theta = P_{in}$$

The output power of an ideal transformer is equal to its input power.

The same relationship applies to the reactive power (Q) and the apparent power (S);

$$Q_{in} = V_p I_p \sin \theta = V_s I_s \sin \theta = Q_{out}$$

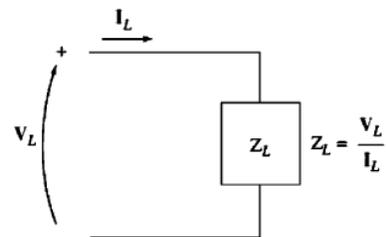
and

$$\overline{S}_{in} = \overline{V}_p \overline{I}_p^* = \overline{V}_s \overline{I}_s^* = \overline{S}_{out}$$

Impedance Transformation Through a Transformer

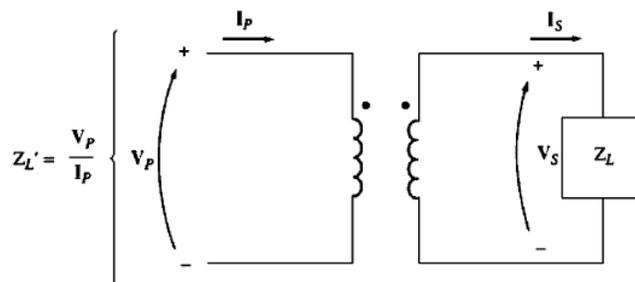
The impedance of a device is defined as:

$$Z_L = \frac{V_L}{I_L}$$



The load impedance is:

$$Z_L = \frac{V_s}{I_s}$$



- Because the transformer changes voltage and current levels, it changes the apparent impedance (Z_L') seen by the source;

$$Z_L' = \frac{\bar{V}_p}{\bar{I}_p}$$

But, $\bar{V}_p = a\bar{V}_s$ and $\bar{I}_p = \frac{\bar{I}_s}{a}$

Then,

$$Z_L' = \frac{a\bar{V}_s}{\frac{\bar{I}_s}{a}} = a^2 \frac{\bar{V}_s}{\bar{I}_s}$$

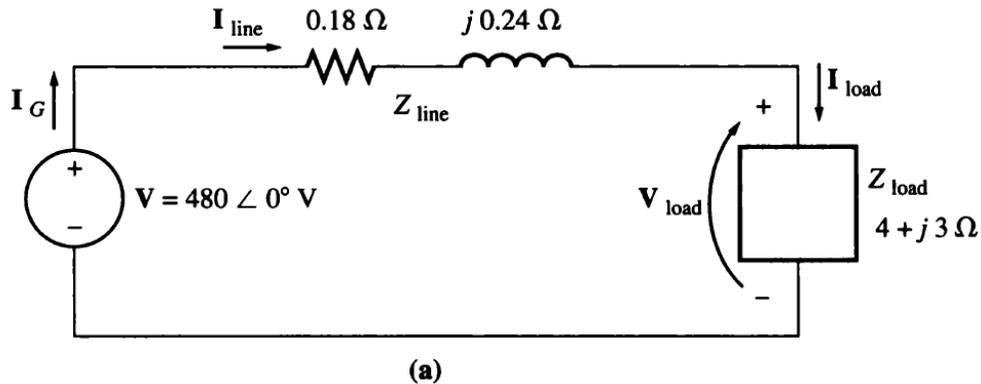
Thus,

$$Z_L' = a^2 Z_L$$

- It is possible to match the load impedance with the source impedance by picking up the proper turns' ratio; for example, maximum power transfer!
- To analyze a circuit containing an ideal transformer, the load impedance can be reflected to the primary side, and then the primary quantities will be calculated. The secondary quantities can be found from the turns' ratio relationships.

Example 2–1. A single-phase power system consists of a 480-V 60-Hz generator supplying a load $Z_{\text{load}} = 4 + j3 \Omega$ through a transmission line of impedance $Z_{\text{line}} = 0.18 + j0.24 \Omega$. Answer the following questions about this system.

- If the power system is exactly as described above (and shown in Figure 2–6a), what will the voltage at the load be? What will the transmission line losses be?
- Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (as shown in Figure 2–6b). What will the load voltage be now? What will the transmission line losses be now?



Solution

(a) Figure 2–6a shows the power system without transformers. Here $I_G = I_{\text{line}} = I_{\text{load}}$. The line current in this system is given by

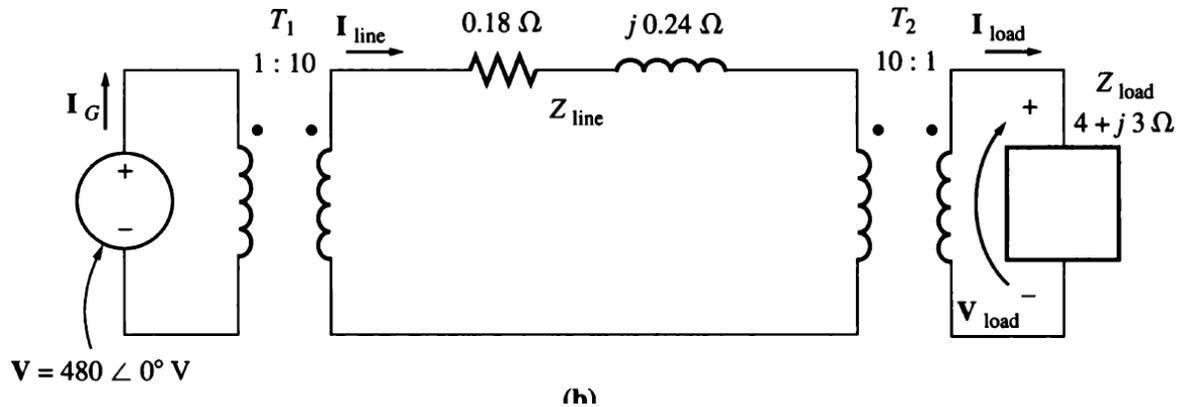
$$\begin{aligned}
 I_{\text{line}} &= \frac{V}{Z_{\text{line}} + Z_{\text{load}}} \\
 &= \frac{480 \angle 0^\circ \text{ V}}{(0.18 \Omega + j0.24 \Omega) + (4 \Omega + j3 \Omega)} \\
 &= \frac{480 \angle 0^\circ}{4.18 + j3.24} = \frac{480 \angle 0^\circ}{5.29 \angle 37.8^\circ} \\
 &= 90.8 \angle -37.8^\circ \text{ A}
 \end{aligned}$$

Therefore the load voltage is

$$\begin{aligned}
 V_{\text{load}} &= I_{\text{line}} Z_{\text{load}} \\
 &= (90.8 \angle -37.8^\circ \text{ A})(4 \Omega + j3 \Omega) \\
 &= (90.8 \angle -37.8^\circ \text{ A})(5 \angle 36.9^\circ \Omega) \\
 &= 454 \angle -0.9^\circ \text{ V}
 \end{aligned}$$

and the line losses are

$$\begin{aligned}
 P_{\text{loss}} &= (I_{\text{line}})^2 R_{\text{line}} \\
 &= (90.8 \text{ A})^2 (0.18 \Omega) = 1484 \text{ W}
 \end{aligned}$$



(b) Figure 2–6b shows the power system with the transformers. To analyze this system, it is necessary to convert it to a common voltage level. This is done in two steps:

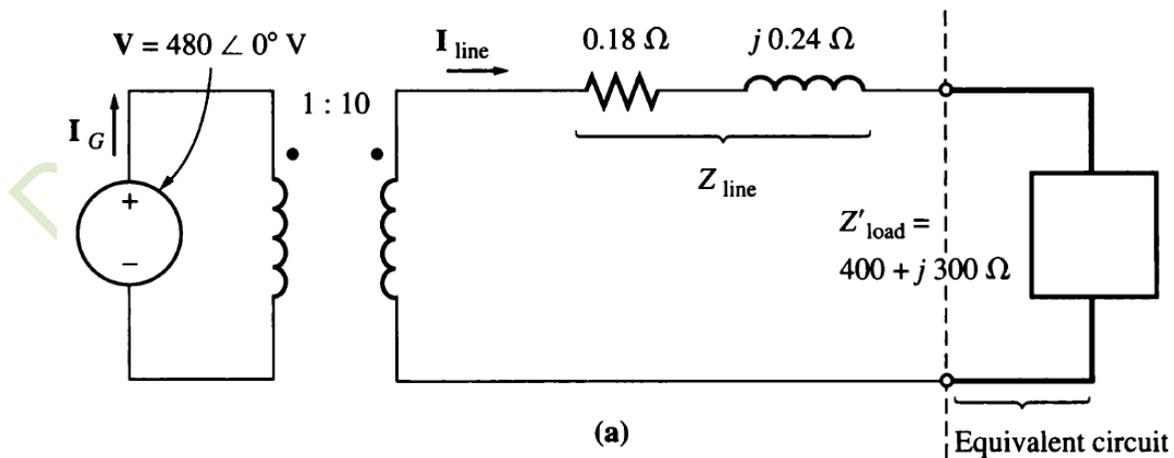
1. Eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2. Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.

The value of the load's impedance when reflected to the transmission system's voltage is

$$\begin{aligned}
 Z'_{\text{load}} &= a^2 Z_{\text{load}} \\
 &= \left(\frac{10}{1}\right)^2 (4 \Omega + j3 \Omega) \\
 &= 400 \Omega + j300 \Omega
 \end{aligned}$$

The total impedance at the transmission line level is now

$$\begin{aligned}
 Z_{\text{eq}} &= Z_{\text{line}} + Z'_{\text{load}} \\
 &= 400.18 + j300.24 \Omega = 500.3 \angle 36.88^\circ \Omega
 \end{aligned}$$



This equivalent circuit is shown in Figure 2–7a. The total impedance at the transmission line level ($Z_{\text{line}} + Z'_{\text{load}}$) is now reflected across T_1 to the source's voltage level:

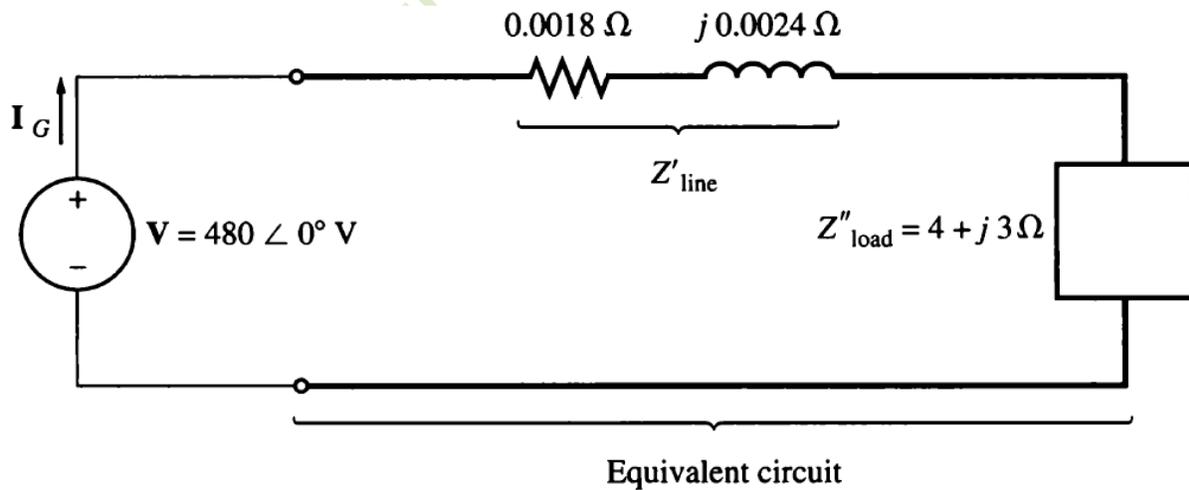
$$\begin{aligned} Z'_{\text{eq}} &= a^2 Z_{\text{eq}} \\ &= a^2(Z_{\text{line}} + Z'_{\text{load}}) \\ &= \left(\frac{1}{10}\right)^2 (0.18 \Omega + j0.24 \Omega + 400 \Omega + j300 \Omega) \\ &= (0.0018 \Omega + j0.0024 \Omega + 4 \Omega + j3 \Omega) \\ &= 5.003 \angle 36.88^\circ \Omega \end{aligned}$$

Notice that $Z''_{\text{load}} = 4 + j3 \Omega$ and $Z'_{\text{line}} = 0.0018 + j0.0024 \Omega$. The resulting equivalent circuit is shown in Figure 2–7b. The generator's current is

$$I_G = \frac{480 \angle 0^\circ \text{ V}}{5.003 \angle 36.88^\circ \Omega} = 95.94 \angle -36.88^\circ \text{ A}$$

Knowing the current I_G , we can now work back and find I_{line} and I_{load} . Working back through T_1 , we get

$$\begin{aligned} N_{P1} I_G &= N_{S1} I_{\text{line}} \\ I_{\text{line}} &= \frac{N_{P1}}{N_{S1}} I_G \\ &= \frac{1}{10} (95.94 \angle -36.88^\circ \text{ A}) = 9.594 \angle -36.88^\circ \text{ A} \end{aligned}$$



Working back through T_2 gives

$$\begin{aligned}N_{P2} \mathbf{I}_{\text{line}} &= N_{S2} \mathbf{I}_{\text{load}} \\ \mathbf{I}_{\text{load}} &= \frac{N_{P2}}{N_{S2}} \mathbf{I}_{\text{line}} \\ &= \frac{10}{1} (9.594 \angle -36.88^\circ \text{ A}) = 95.94 \angle -36.88^\circ \text{ A}\end{aligned}$$

It is now possible to answer the questions originally asked. The load voltage is given by

$$\begin{aligned}\mathbf{V}_{\text{load}} &= \mathbf{I}_{\text{load}} Z_{\text{load}} \\ &= (95.94 \angle -36.88^\circ \text{ A})(5 \angle 36.87^\circ \Omega) \\ &= 479.7 \angle -0.01^\circ \text{ V}\end{aligned}$$

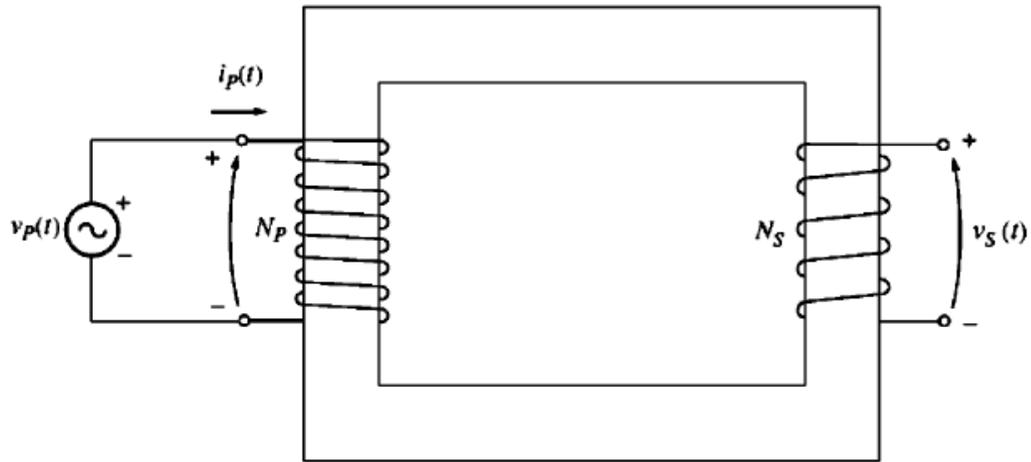
and the line losses are given by

$$\begin{aligned}P_{\text{loss}} &= (I_{\text{line}})^2 R_{\text{line}} \\ &= (9.594 \text{ A})^2 (0.18 \Omega) = 16.7 \text{ W}\end{aligned}$$

Notice that raising the transmission voltage of the power system reduced transmission losses by a factor of nearly 90! Also, the voltage at the load dropped much less in the system with transformers compared to the system without transformers. This simple example dramatically illustrates the advantages of using higher-voltage transmission lines as well as the extreme importance of transformers in modern power systems.

Dr. M. Abu-Kh...

Theory of Operation of Real Transformers



- ➔ The induced voltage is:

$$e_{ind} = \frac{d\lambda}{dt}$$

where, λ is the flux linkage, and it is the sum of flux passing through each turn in the coil added over all the turns of the coil, such that:

$$\lambda = \sum_{i=1}^N \phi_i$$

- ➔ Note that, λ does not equal to $N\phi$, because the flux passing through each turn is slightly different from the flux in the other turns depending on the position of the turn within the coil.
- ➔ However, the average flux is:

$$\bar{\phi} = \frac{\lambda}{N}$$

Thus,

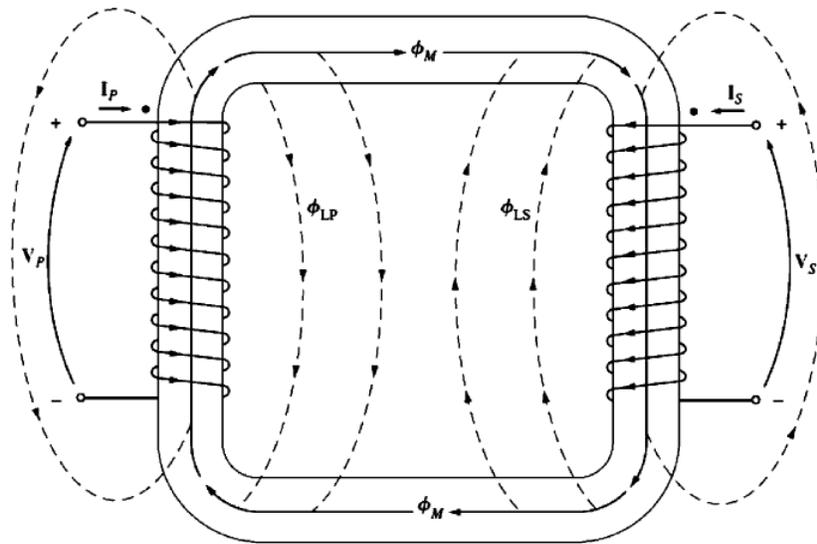
$$e_{ind} = N \frac{d\bar{\phi}}{dt}$$

- ➔ If $v_p(t)$ is applied across the primary coil, then the average flux is related to number of turns and applied voltage by:

$$v_p(t) = N_p \frac{d\bar{\phi}}{dt}$$

Solving the latter equation for the average flux yields:

$$\Rightarrow \bar{\phi} = \frac{1}{N_P} \int v_P(t) dt$$



- ➔ Not all the flux produced in the primary coil passes through the secondary coil; there is a **leakage flux**.

The **Leakage Flux** is the portion of the flux that goes through one of the transformer coils but not the other one.

- ➔ Thus, the primary flux has two components; a **leakage flux** and a **mutual flux**. The latter remains in the core and links both windings; i.e.,

$$\bar{\phi}_P = \phi_M + \phi_{LP}$$

where, $\bar{\phi}_P$ is the total average primary flux.

ϕ_M is the flux component linking both primary and secondary coils; **mutual flux**.

ϕ_{LP} is the primary leakage flux.

- ➔ Similarly, for the secondary coil,

$$\bar{\phi}_S = \phi_M + \phi_{LS}$$

where, $\bar{\phi}_S$ is the total average secondary flux.

ϕ_M is the flux component linking both primary and secondary coils; **mutual flux**.

ϕ_{LS} is the secondary leakage flux.

➔ Applying Faraday's Law, for the primary coil, yields:

$$v_P(t) = N_P \frac{d\bar{\phi}_P}{dt}$$

$$v_P(t) = N_P \frac{d\phi_M}{dt} + N_P \frac{d\phi_{LP}}{dt}$$

$$\Rightarrow v_P(t) = e_P(t) + e_{LP}(t)$$

➔ The primary voltage due to the mutual flux is:

$$e_P(t) = N_P \frac{d\phi_M}{dt}$$

$$\Rightarrow \frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} \dots\dots\dots(1)$$

➔ Similarly, the voltage on the secondary coil of the transformer can be expressed in terms of Faraday's Law as:

$$v_S(t) = N_S \frac{d\bar{\phi}_S}{dt}$$

$$v_S(t) = N_S \frac{d\phi_M}{dt} + N_S \frac{d\phi_{LS}}{dt}$$

$$\Rightarrow v_S(t) = e_S(t) + e_{LS}(t)$$

➔ The secondary voltage due to the mutual flux is:

$$e_S(t) = N_S \frac{d\phi_M}{dt}$$

$$\Rightarrow \frac{e_S(t)}{N_S} = \frac{d\phi_M}{dt} \dots\dots\dots(2)$$

➔ Equating eqs. (1) and (2) yields:

$$\frac{e_P(t)}{N_P} = \frac{d\phi_M}{dt} = \frac{e_S(t)}{N_S}$$

$$\frac{e_P(t)}{e_S(t)} = \frac{N_P}{N_S} = \mathbf{a}$$

➔ In well designed transformers, $\phi_M \gg \phi_{LP}$ and $\phi_M \gg \phi_{LS}$, therefore,

$$\frac{v_P(t)}{v_S(t)} \cong \frac{N_P}{N_S} \cong \mathbf{a}$$

➔ Note that, the smaller the leakage fluxes of the transformer are, the closer the total transformer voltages approximate that of an ideal transformer (the leakage voltages $e_{LP}(t)$ and $e_{LS}(t)$ are negligible).

The Magnetization Current in a Real Transformer

- When an AC power source is connected to a transformer primary, a current flows in the primary circuit even when the secondary terminals are open circuited. This current is required to produce the flux in a real ferromagnetic core and it consists of two components:

- The Magnetization Current** (i_m); It produces the flux in the core.
- The Core Loss Current** (i_{h+e}): The current required to make up for Hysteresis and Eddy current losses.

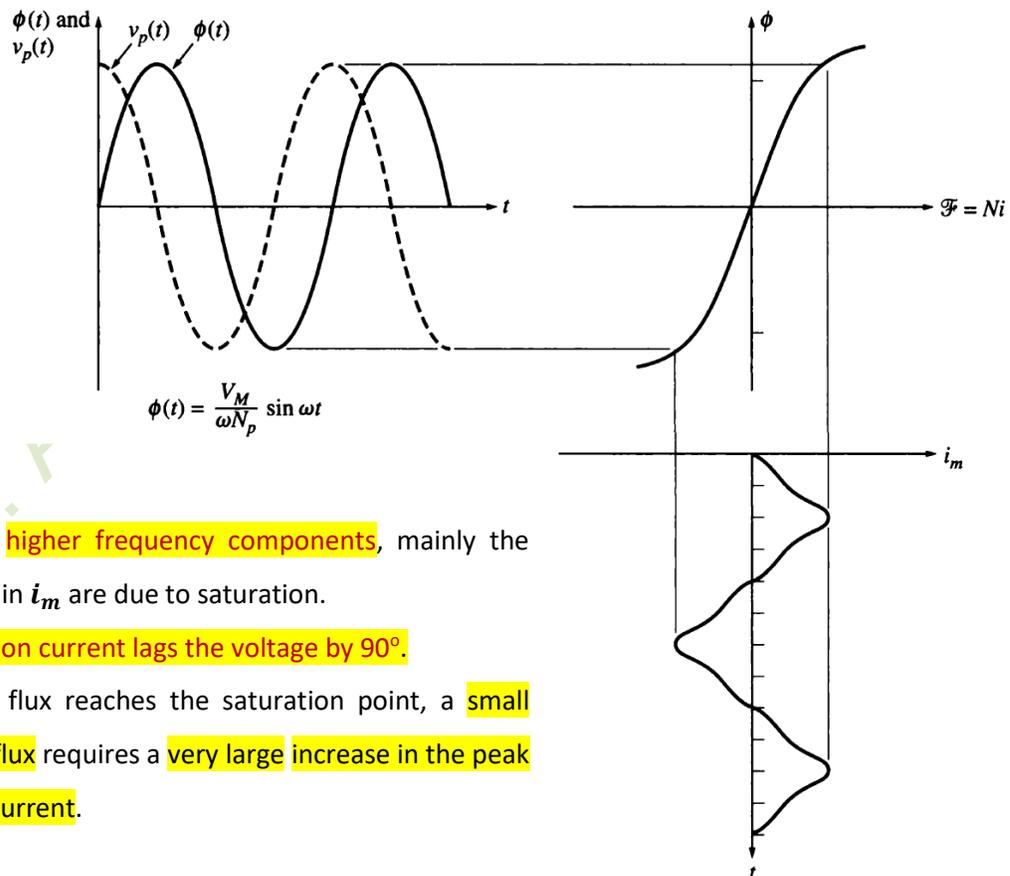
- Ignoring the effect of leakage flux, the average flux in the core is:

$$\bar{\phi}(t) = \frac{1}{N_p} \int v_p(t) dt$$

- If $v_p(t) = V_M \cos \omega t$, then:

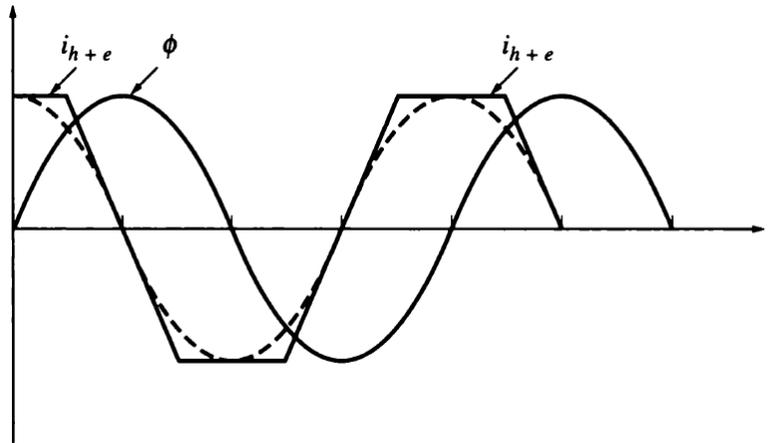
$$\bar{\phi}(t) = \frac{V_M}{N_p \omega} \sin \omega t$$

- The magnetization curve of a transformer is known. Therefore, comparing the values of current to the flux produced, a sketch of the magnetization current (i_m) can be obtained.



- Note that, the **higher frequency components**, mainly the third harmonic, in i_m are due to saturation.
- The magnetization current lags the voltage by 90°.**
- Once the peak flux reaches the saturation point, a **small increase in the flux** requires a **very large increase in the peak magnetization current**.

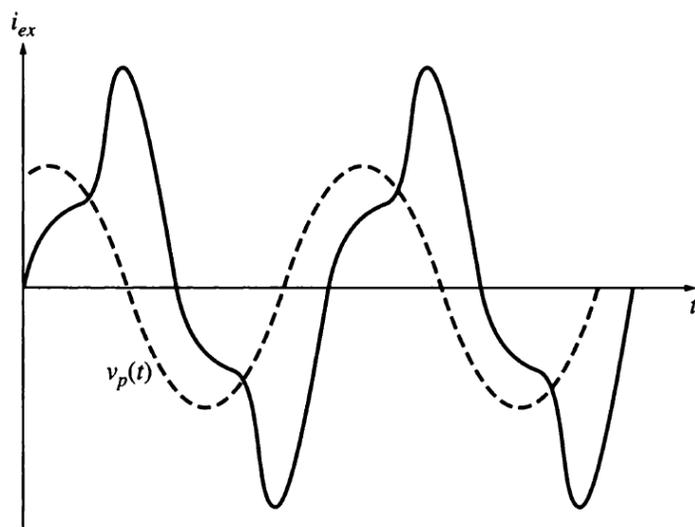
- The other component of the no-load current is the core loss current (i_{h+e}).
- Assuming a sinusoidal flux, the eddy current is proportional to $\frac{d\phi}{dt}$ and the eddy current has its maximum value when the flux in the core is passing through zero Wb (the peak of the cosine, which is the derivative of the sine, is at the zero crossing of a sine).
- The **Hysteresis loss is nonlinear**, and has its maximum when the flux passes through zero.
- Hence, the total current required to make up for core loss is shown in the Figure below.



- The **core loss current (i_{h+e}) is nonlinear** and its fundamental component is in phase with the applied voltage to the core.
- The total no-load current is called the **excitation current**;
Excitation Current = Magnetization Current + Core loss Current

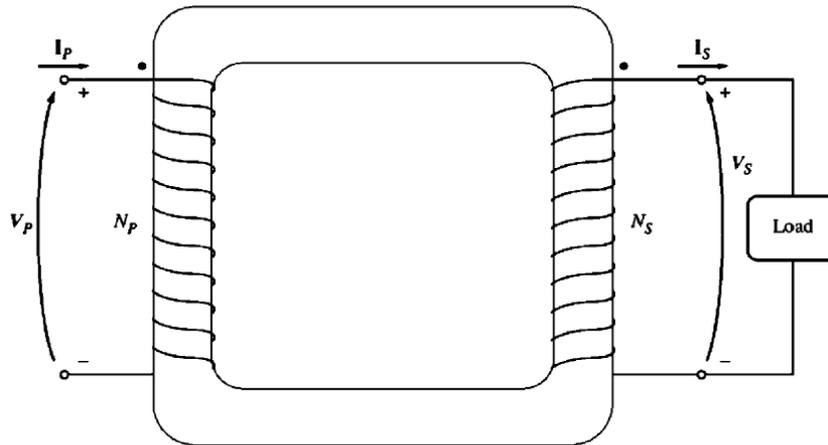
$$i_{ex} = i_m + i_{h+e}$$

i_{ex} is depicted in the Figure next.



The Current Ratio on a Transformer and Dot Convention

- Consider a loaded transformer as shown in the Figure below,



- As in an ideal transformer, the dots help in determining the polarity of voltages and currents in the windings without having physically to examine the windings.
- The physical significance of the dot convention is that, a **current flowing into the dotted end** of a winding **produces a positive magneto-motive force**, \mathcal{F} , whilst a current flowing into the undotted end of a winding produces a negative magneto-motive force.
- Therefore, two currents flowing into the dotted ends of their respective windings produce **magneto-motive forces that add**.
- If one current flows into a dotted end of a winding and one current flows out of a dotted end of the other, then the **magneto-motive forces will subtract** from each other.
- For the Figure above, the magneto-motive force for the primary is:

$$\mathcal{F}_P = N_P I_P$$

and the magneto-motive force for the secondary is:

$$\mathcal{F}_S = -N_S I_S$$

- The net magneto-motive force on the core must be:

$$\mathcal{F}_{net} = N_P I_P - N_S I_S = \mathcal{R} \phi$$

where ϕ is the net flux in the core, and \mathcal{R} is the Reluctance of the flux path!

- For a well designed transformer, the Reluctance (\mathcal{R}) is very small (nearly zero), until the core is saturated.

✦ Hence, $\mathcal{F}_{net} = N_P I_P - N_S I_S \cong 0$, as long as the core is unsaturated!

$$\Rightarrow N_P I_P \cong N_S I_S$$

$$\frac{I_P}{I_S} \cong \frac{N_S}{N_P} = \frac{1}{a}$$

✦ For the net magneto-motive force to be zero, the current must flow into one dotted end and out of the other dotted end.

✦ The **voltages must be built up in the same way with respect to the dots** on each winding in order to drive the currents in the direction required (the polarity can be determined from Lenz' Law).

Assumptions to Convert a Real Transformer into an Ideal Transformer:

1. The core must have no Hysteresis or Eddy current losses!
2. The magnetization curve must have the shape shown in the Figure next.

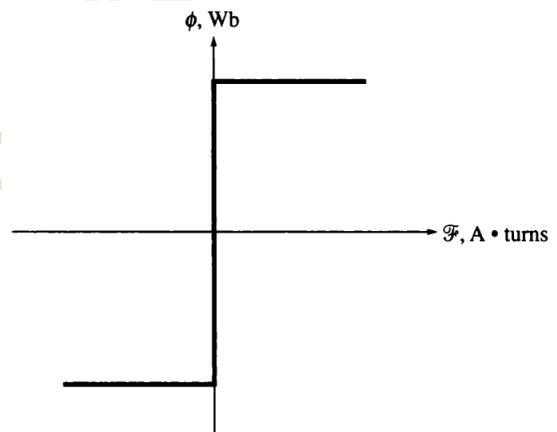
For unsaturated core, $\mathcal{F}_{net} = 0$, implying that,

$$N_P I_P = N_S I_S$$

Note that, for the curve, the slope = $\frac{1}{\mathcal{R}}$;

i.e., $\mathcal{R} = 0$, when the core is unsaturated

$\mathcal{R} = \infty$, when the core is saturated



3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
4. The resistance of the transformer windings must be zero.

The Equivalent Circuit of a Transformer

The losses in a real transformer have to be accounted for in any accurate model. These losses are:

1. **Copper Losses (I^2R):** They are the resistive heating losses in the windings.
2. **Eddy Current Losses:** They are the resistive heating losses in the core of a transformer. They are proportional to the square of the voltage applied to the transformer.
3. **Hysteresis Losses:** They are associated with the rearrangement of the magnetic domains during each half cycle. They are a complex nonlinear function of the voltage applied to the transformer.
4. **Leakage Flux:** The fluxes ϕ_{LP} & ϕ_{LS} which escape the core and pass through only one of the transformer windings are leakage fluxes. They produce a self inductance in the primary and secondary coils.

The Exact Equivalent Circuit of a Real Transformer:

- 1) Copper losses are represented by R_P resistor in the primary circuit, and a resistor R_S in the secondary circuit.
- 2) The Core loss current (i_{h+e}) is proportional to the applied voltage to the core and is in phase with the applied voltage.
Therefore, it can be modeled by a resistor R_C connected across the primary voltage source.
However, i_{h+e} is nonlinear, so R_C is an approximation.
- 3) The Magnetization current (i_m) is proportional to the voltage applied to the core and lagging the applied voltage by 90° (for the unsaturated region).
Therefore, it can be modeled by a reactance X_M , also connected across the primary voltage source.
Again, i_m is nonlinear, so X_M is also an approximation.
- 4) The leakage flux produces a voltage;
In the primary:
$$e_{LP} = N_P \frac{d\phi_{LP}}{dt}$$

In the secondary:
$$e_{LS} = N_S \frac{d\phi_{LS}}{dt}$$

But, the leakage flux passes in the air, which has a constant reluctance;

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = Ni\mathcal{F}$$

where, \mathcal{F} is the permeance of the flux path.

Consequently, the leakage fluxes of the primary and secondary windings, respectively, are:

$$\phi_{LP} = \mathcal{F}N_P i_P$$

and
$$\phi_{LS} = \mathcal{F}N_S i_S$$

where, i_P and i_S are primary and secondary currents, respectively!

The voltage e_{LP} and e_{LS} are:

$$e_{LP} = N_P \frac{d}{dt}(\mathcal{F}N_P i_P) = \mathcal{F}N_P^2 \frac{di_P}{dt}$$

$$e_{LP} = L_P \frac{di_P}{dt}$$

and
$$e_{LS} = N_S \frac{d}{dt}(\mathcal{F}N_S i_S) = \mathcal{F}N_S^2 \frac{di_S}{dt}$$

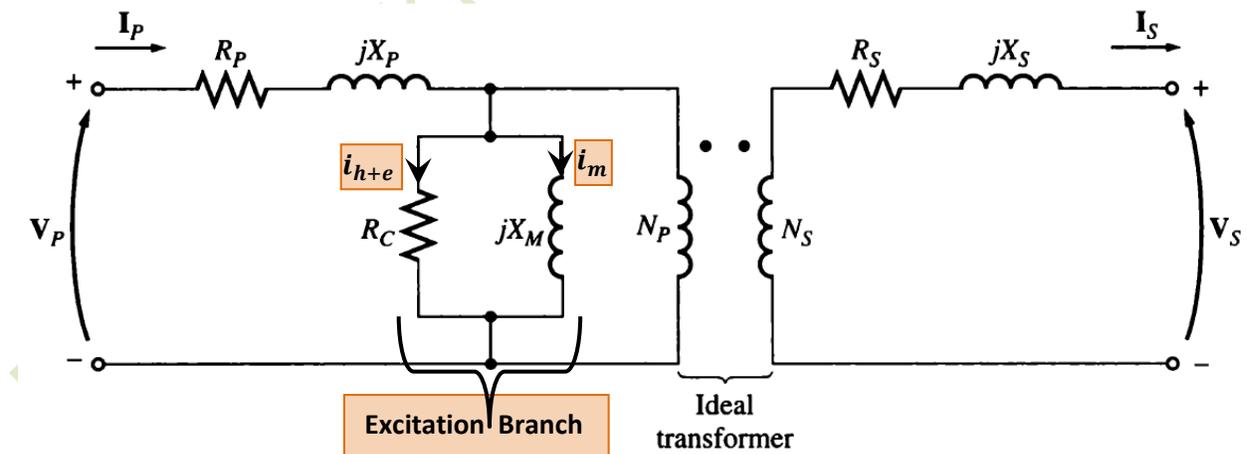
$$e_{LS} = L_S \frac{di_S}{dt}$$

where, $L_P = \mathcal{F}N_P^2$ is the self inductance of the primary coil,

and $L_S = \mathcal{F}N_S^2$ is the self inductance of the secondary coil.

Thus, the leakage fluxes are modeled by inductors.

Therefore, the **Equivalent Circuit Model of a Transformer** is:

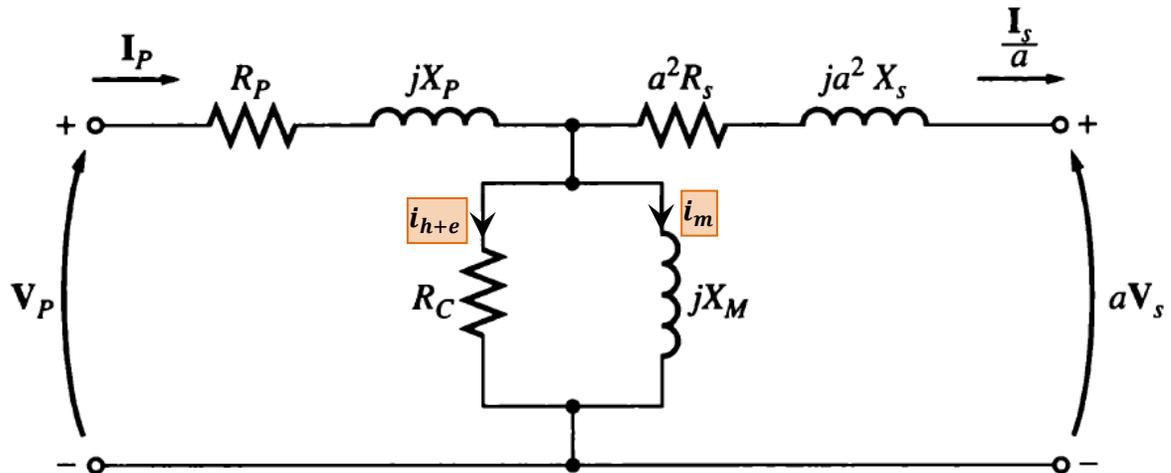


- The excitation branch is placed inside the primary resistance and inductance, because the actual voltage applied to the core is less than the input voltage by the internal voltage drop in the primary winding; a voltage drop in the primary winding's resistance and another voltage drop responsible for establishing

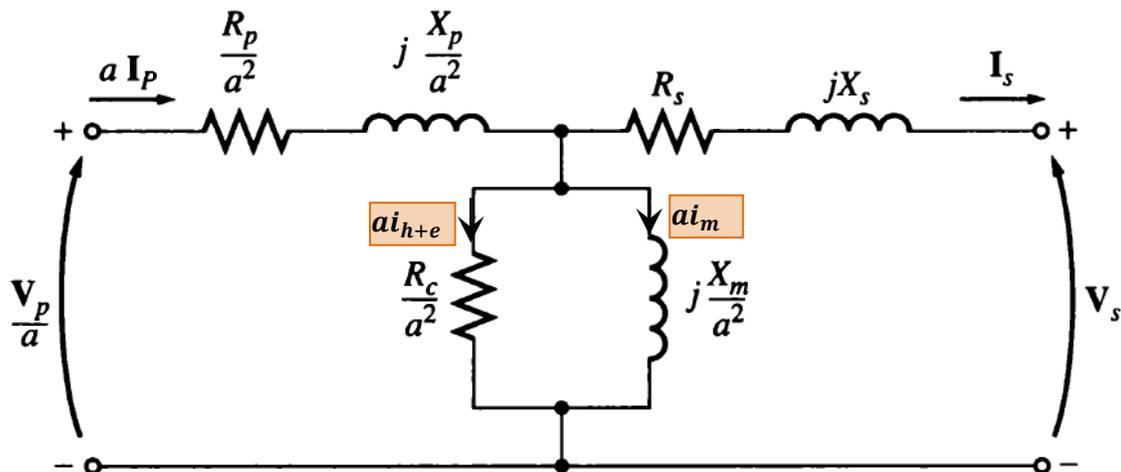
the primary leakage flux. The core loss and magnetization currents are related to mutual flux not the total flux; related to e_p , neither to v_p nor to e_{LP} .

- To be able to analyze practical circuits containing transformers, the transformer circuit should be converted to a single circuit, either referred to primary or secondary side.

➤ **The Exact Equivalent Circuit of a Transformer Referred to Primary Side**

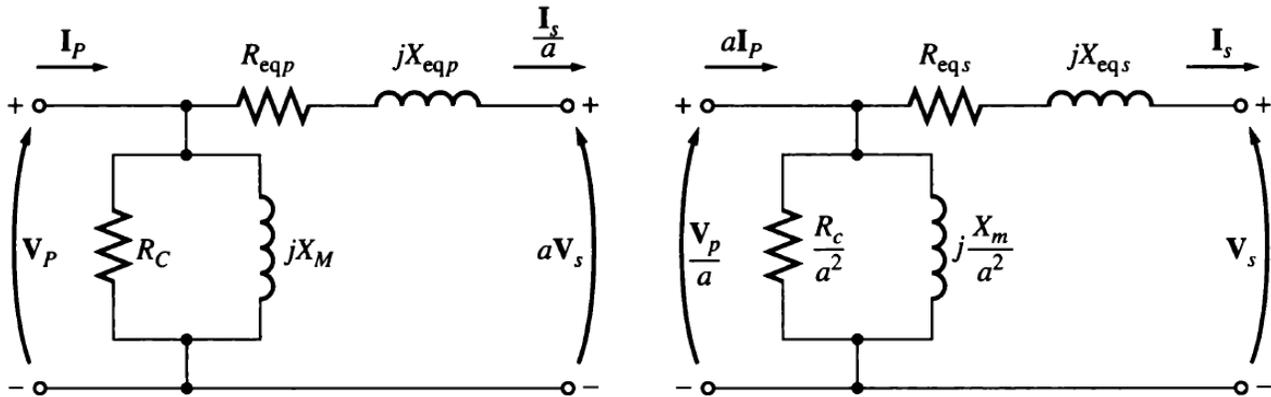


➤ **The Exact Equivalent Circuit of a Transformer Referred to Secondary Side**



Approximate Equivalent Circuits of a Transformer

- ➔ Usually, the excitation branch has a very small current compared to the load current of the transformer (R_C & X_M are very large compared to R_P & X_P).
- ➔ Therefore, the excitation branch can be moved to the front end of the transformer, and the primary and secondary impedances are left in series, as illustrated in the Figures below.



a) Referred to Primary side

$$R_{eqP} = R_P + a^2 R_S$$

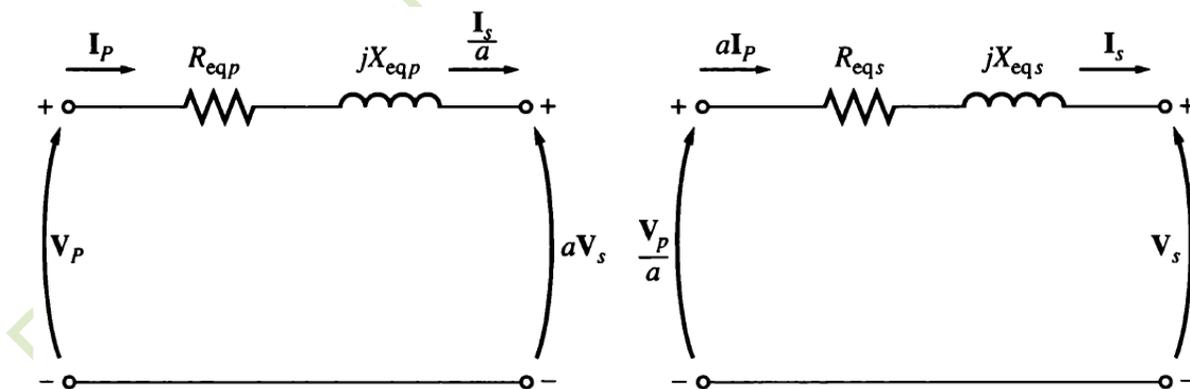
$$X_{eqP} = X_P + a^2 X_S$$

b) Referred to Secondary side

$$R_{eqS} = \frac{R_P}{a^2} + R_S$$

$$X_{eqS} = \frac{X_P}{a^2} + X_S$$

In some applications and very large transformers, the excitation branch can be neglected as shown in the Figures below.



c) Referred to Primary side

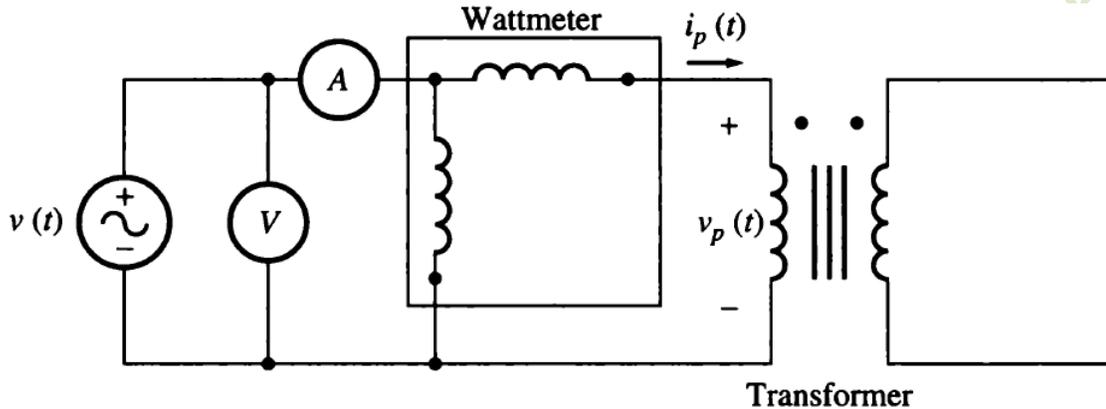
d) Referred to Secondary side

Determining the Values of Components in the Transformer Model

The following tests are conducted to find the values of the parameters of the model:

I. Open Circuit Test

- ✓ This test is conducted at full (rated) voltage.
- ✓ The transformer and the meters are connected as shown in the Figure below.



- ✓ During this test, the **input** current (I_{OC}), the input voltage (V_{OC}), and the input power (P_{OC}) are measured. The subscript 'OC' stands for the values of parameters during the Open Circuit test, but not in the Open circuit itself!
- ✓ Mind you that, the Voltmeter and Ammeter measure **the magnitude and rms values**.
- ✓ From this test, the magnitude of the excitation branch can be found (R_p and X_p are small compared to the excitation branch impedance; R_C and X_M).
- ✓ The conductance of the core-loss resistance is:

$$G_C = \frac{1}{R_C}$$

- ✓ The susceptance of the magnetizing inductance is:

$$B_M = \frac{1}{X_M}$$

- ✓ The admittance of the excitation branch is:

$$Y_{EX} = \frac{1}{R_C} - j \frac{1}{X_M} \dots\dots\dots(3)$$

- ✓ The magnitude of the excitation branch admittance referred to primary is:

$$|Y_{EX}| = \frac{I_{OC}}{V_{OC}}$$

- ✓ The angle of the admittance is found from the power factor as:

$$pf = \cos \theta = \frac{P_{OC}}{V_{OC}I_{OC}}$$

$$\Rightarrow \theta = \cos^{-1} \frac{P_{OC}}{V_{OC}I_{OC}}$$

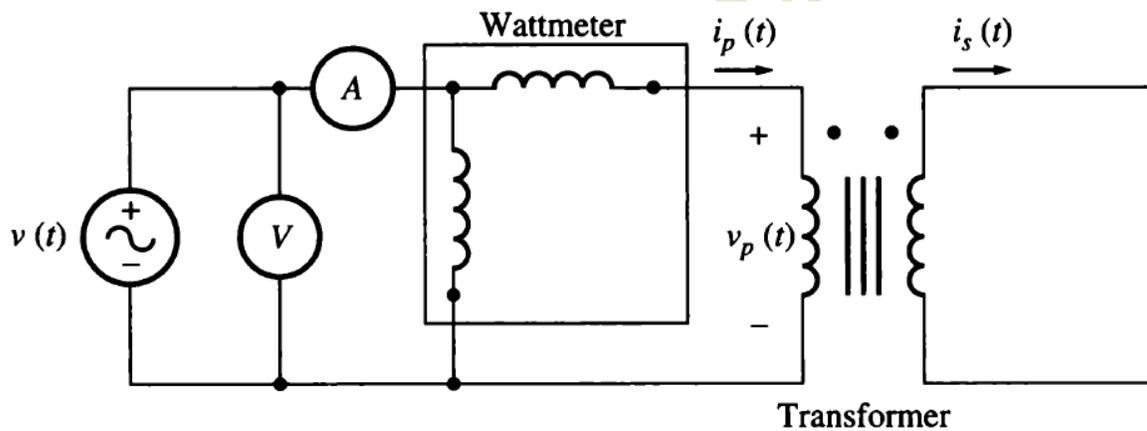
- ✓ Since the power factor in a transformer is always lagging,

$$Y_{EX} = \frac{I_{OC}}{V_{OC}} \angle -\theta \dots\dots\dots(4)$$

- ✓ Comparing equations (3) and (4) yields the values of R_C and X_M !

II. Short Circuit Test

- Before conducting this test, the **voltage source is set to zero**.
- **This test is conducted at a very low voltage and full (rated) Current**.
- The transformer and the meters are connected as shown in the Figure below.



- The voltage source is adjusted until the current in the short circuited winding is equal to its rated value.
- During this test, a negligible current flows in the excitation branch, and hence the branch can be ignored!
- During this test, the **input** current (I_{SC}), the input voltage (V_{SC}), and the input power (P_{SC}) are measured. The subscript 'SC' stands for the values of parameters during the Short Circuit test, but not in the short circuit itself!!
- The magnitude of the series impedances referred to the primary side are:

$$|Z_{SEP}| = \frac{V_{SC}}{I_{SC}}$$

- The power factor of the current is:

$$pf = \cos \theta = \frac{P_{SC}}{V_{SC} I_{SC}} \text{ (lagging)}$$

- Therefore,

$$\mathbf{Z}_{SEP} = \frac{V_{SC}}{I_{SC}} \angle \theta \text{(5)}$$

- But, the series impedance is equal to:

$$\begin{aligned} \mathbf{Z}_{SEP} &= R_{eqP} + \mathbf{j}X_{eqP} \text{(6)} \\ \Rightarrow \mathbf{Z}_{SEP} &= (R_p + a^2 R_s) + \mathbf{j}(X_p + a^2 X_s) \end{aligned}$$

- Comparing equations (5) and (6) yields the values of R_{eqP} and X_{eqP} !
- It is not easy to split the equivalent resistance and reactance into primary and secondary components!
- Sometimes, it is convenient to conduct these test (measurements) on the secondary side (because of the voltage levels), then the impedances obtained will be referred to the secondary!

Example 2–2. The equivalent circuit impedances of a 20-kVA, 8000/240 V, 60-Hz transformer are to be determined. The open-circuit test was performed on the secondary side of the transformer (to reduce the maximum voltage to be measured) and the short-circuit test were performed on the primary side of the transformer (to reduce the maximum current to be measured). The following data were taken:

Open-circuit test (on secondary)	Short-circuit test (on primary)
$V_{OC} = 240 \text{ V}$	$V_{SC} = 489 \text{ V}$
$I_{OC} = 7.133 \text{ A}$	$I_{SC} = 2.5 \text{ A}$
$P_{OC} = 400 \text{ W}$	$P_{SC} = 240 \text{ W}$

Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch that circuit.

Solution

The turns ratio of this transformer is $a = 8000/240 = 33.3333$. The power factor during the open-circuit test is

$$PF = \cos \theta = \frac{P_{OC}}{V_{OC} I_{OC}} \text{ (2-45)}$$

$$PF = \cos \theta = \frac{400 \text{ W}}{(240 \text{ V})(7.133 \text{ A})}$$

$$PF = 0.234 \text{ lagging}$$

The excitation admittance is given by

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -\cos^{-1} \text{PF} \quad (2-47)$$

$$Y_E = \frac{7.133 \text{ A}}{240 \text{ V}} \angle -\cos^{-1} 0.234$$

$$Y_E = 0.0297 \angle -76.5^\circ \text{ S}$$

$$Y_E = 0.00693 - j 0.02888 = \frac{1}{R_C} - j \frac{1}{X_M}$$

Therefore, the values of the excitation branch referred to the low-voltage (secondary) side

$$R_C = \frac{1}{0.00693} = 144 \Omega$$

$$X_M = \frac{1}{0.02888} = 34.63 \Omega$$

The power factor during the short-circuit test is

$$\text{PF} = \cos \theta = \frac{P_{SC}}{V_{SC} I_{SC}} \quad (2-49)$$

$$\text{PF} = \cos \theta = \frac{240 \text{ W}}{(489 \text{ V})(2.5 \text{ A})} = 0.196 \text{ lagging}$$

The series impedance is given by

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} \text{PF}$$

$$Z_{SE} = \frac{489 \text{ V}}{2.5 \text{ A}} \angle 78.7^\circ$$

$$Z_{SE} = 195.6 \angle 78.7^\circ = 38.4 + j192 \Omega$$

Therefore, the equivalent resistance and reactance referred to the high-voltage (primary) side are

$$R_{eq} = 38.4 \Omega \quad X_{eq} = 192 \Omega$$

The resulting simplified equivalent circuit referred to the high-voltage (primary) side can be found by converting the excitation branch values to the high-voltage side.

$$R_{C,P} = a^2 R_{C,S} = (33.333)^2 (144 \Omega) = 159 \text{ k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (33.333)^2 (34.63 \Omega) = 38.4 \text{ k}\Omega$$

The resulting equivalent circuit is shown in Figure 2-21.

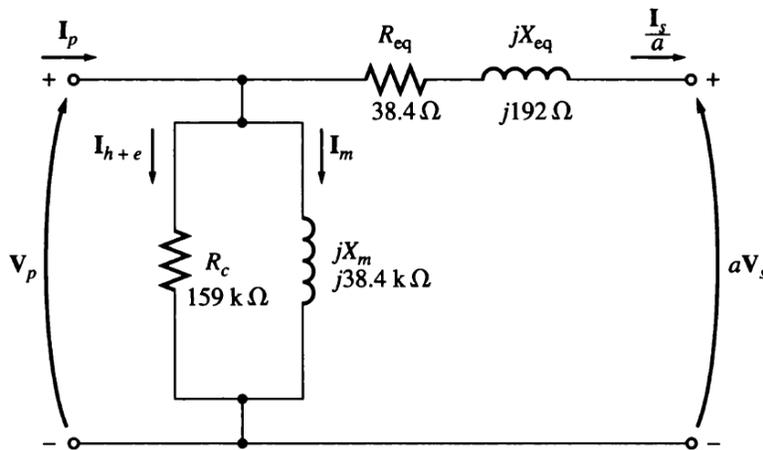


FIGURE 2-21

The Per-Unit System of Measurements

- ➔ In such a system, no need to worry about converting quantities from primary to secondary side and vice versa, as the per-unit system handles the conversions automatically.
- ➔ In a per-unit system, each electrical quantity (voltage, current, impedance, and power) is **not measured** in their usual 'SI' unit, but is measured as a decimal fraction of some base level.

$$\text{Quantity Per - Unit} = \frac{\text{Actual Quantity}}{\text{Base Value of Quantity}}$$

- ➔ Two base quantities are selected, usually apparent power ($S_{1\phi base}$) and voltage ($V_{1\phi base}$), and the other base quantities are related to them by the usual electrical laws.
- ➔ In a single phase system,

$$P_{1\phi base}, Q_{1\phi base}, \text{ or } S_{1\phi base} = V_{1\phi base} I_{1\phi base}$$

$$I_{1\phi base} = \frac{S_{1\phi base}}{V_{1\phi base}}$$

$$Z_{base} = \frac{V_{1\phi base}}{I_{1\phi base}}$$

$$Z_{base} = \frac{V_{1\phi base}^2}{S_{1\phi base}}$$

$$Y_{base} = \frac{I_{1\phi base}}{V_{1\phi base}}$$

- ➔ In a power system, a base apparent power ($S_{1\phi base}$) and voltage ($V_{1\phi base}$) are selected at a specific point of the system.
- ➔ The transformer has no effect on the base apparent power of the system, since the apparent power into a transformer equals the apparent power out of it.
- ➔ On the other hand, the voltage changes when it goes through a transformer, so the value of $V_{1\phi base}$ changes at every transformer in the system according to its turns' ratio.
- ➔ If more than one machine and one transformer are included in the system, but each is represented in per-unit based on its base quantities, then a unified base system should be adopted, such that the **$S_{1\phi base}$ is the greatest**, while the **base voltages are related to transformers turns' ratios**.
- ➔ Then the conversion of per-unit quantities from one base to another is as follows:

$$(P, Q, \text{ or } S)_{pu \text{ on base } 2} = (P, Q, \text{ or } S)_{pu \text{ on base } 1} X \frac{S_{1\phi base 1}}{S_{1\phi base 2}}$$

$$V_{pu \text{ on base } 2} = V_{pu \text{ on base } 1} X \frac{V_{1\phi base 1}}{V_{1\phi base 2}}$$

$$(R, X, \text{ or } Z)_{pu \text{ on base } 2} = (R, X, \text{ or } Z)_{pu \text{ on base } 1} X \frac{V_{1\phi base 1}^2}{V_{1\phi base 2}^2} X \frac{S_{1\phi base 2}}{S_{1\phi base 1}}$$

Because, $Z_{base} = \frac{V_{1\phi base}^2}{S_{1\phi base}}$

- ➔ Notes:

$$P_{pu} = V_{pu} I_{pu} \cos \theta$$

$$Q_{pu} = V_{pu} I_{pu} \sin \theta$$

$$\bar{S}_{pu} = \bar{V}_{pu} \bar{I}_{pu}^*$$

Example 2–3. A simple power system is shown in Figure 2–22. This system contains a 480-V generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer, and a load. The impedance of the transmission line is $20 + j60 \Omega$, and the impedance of the load is $10\angle 30^\circ \Omega$. The base values for this system are chosen to be 480 V and 10 kVA at the generator.

- Find the base voltage, current, impedance, and apparent power at every point in the power system.
- Convert this system to its per-unit equivalent circuit.
- Find the power supplied to the load in this system.
- Find the power lost in the transmission line.

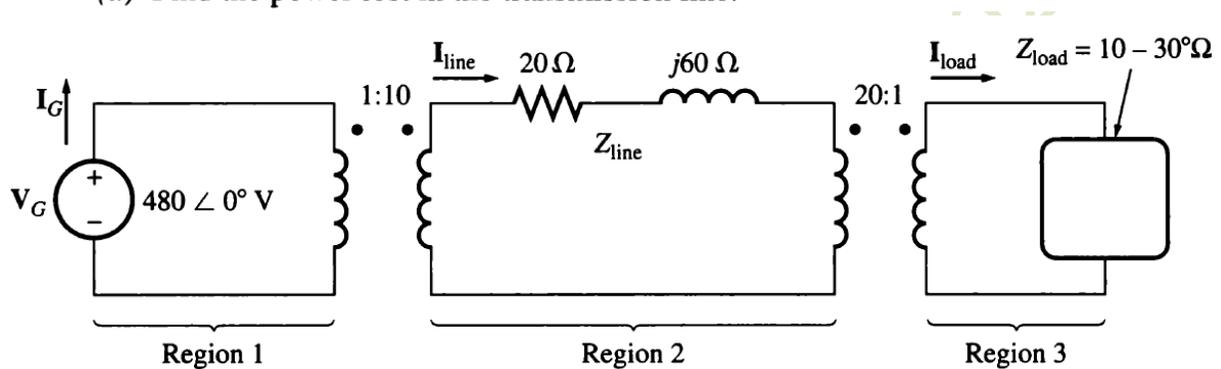


FIGURE 2–22

Solution

(a) In the generator region, $V_{\text{base}} = 480 \text{ V}$ and $S_{\text{base}} = 10 \text{ kVA}$, so

$$I_{\text{base } 1} = \frac{S_{\text{base}}}{V_{\text{base } 1}} = \frac{10,000 \text{ VA}}{480 \text{ V}} = 20.83 \text{ A}$$

$$Z_{\text{base } 1} = \frac{V_{\text{base } 1}}{I_{\text{base } 1}} = \frac{480 \text{ V}}{20.83 \text{ A}} = 23.04 \Omega$$

The turns ratio of transformer T_1 is $a = 1/10 = 0.1$, so the base voltage in the transmission line region is

$$V_{\text{base } 2} = \frac{V_{\text{base } 1}}{a} = \frac{480 \text{ V}}{0.1} = 4800 \text{ V}$$

The other base quantities are

$$S_{\text{base } 2} = 10 \text{ kVA}$$

$$I_{\text{base } 2} = \frac{10,000 \text{ VA}}{4800 \text{ V}} = 2.083 \text{ A}$$

$$Z_{\text{base } 2} = \frac{4800 \text{ V}}{2.083 \text{ A}} = 2304 \Omega$$

The turns ratio of transformer T_2 is $a = 20/1 = 20$, so the base voltage *in the load region* is

$$V_{\text{base } 3} = \frac{V_{\text{base } 2}}{a} = \frac{4800 \text{ V}}{20} = 240 \text{ V}$$

The other base quantities are

$$S_{\text{base } 3} = 10 \text{ kVA}$$

$$I_{\text{base } 3} = \frac{10,000 \text{ VA}}{240 \text{ V}} = 41.67 \text{ A}$$

$$Z_{\text{base } 3} = \frac{240 \text{ V}}{41.67 \text{ A}} = 5.76 \Omega$$

- (b) To convert a power system to a per-unit system, each component must be divided by its base value in its region of the system. The *generator's* per-unit voltage is its actual value divided by its base value:

$$V_{G,\text{pu}} = \frac{480 \angle 0^\circ \text{ V}}{480 \text{ V}} = 1.0 \angle 0^\circ \text{ pu}$$

The *transmission line's* per-unit impedance is its actual value divided by its base value:

$$Z_{\text{line,pu}} = \frac{20 + j60 \Omega}{2304 \Omega} = 0.0087 + j0.0260 \text{ pu}$$

The *load's* per-unit impedance is also given by actual value divided by base value:

$$Z_{\text{load,pu}} = \frac{10 \angle 30^\circ \Omega}{5.76 \Omega} = 1.736 \angle 30^\circ \text{ pu}$$

The per-unit equivalent circuit of the power system is shown in Figure 2–23.

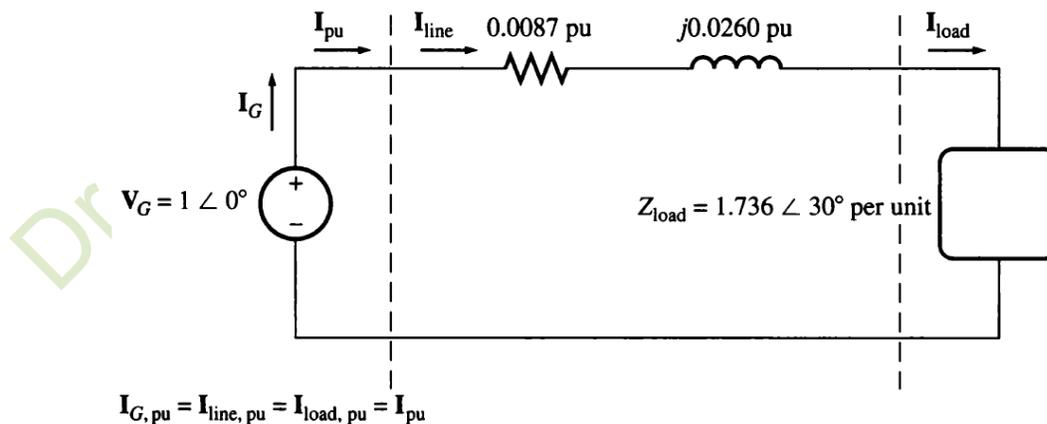


FIGURE 2–23

(c) The current flowing in this per-unit power system is

$$\begin{aligned} \mathbf{I}_{\text{pu}} &= \frac{\mathbf{V}_{\text{pu}}}{\mathbf{Z}_{\text{tot,pu}}} \\ &= \frac{1 \angle 0^\circ}{(0.0087 + j0.0260) + (1.736 \angle 30^\circ)} \\ &= \frac{1 \angle 0^\circ}{(0.0087 + j0.0260) + (1.503 + j0.868)} \\ &= \frac{1 \angle 0^\circ}{1.512 + j0.894} = \frac{1 \angle 0^\circ}{1.757 \angle 30.6^\circ} \\ &= 0.569 \angle -30.6^\circ \text{ pu} \end{aligned}$$

Therefore, the per-unit power of the load is

$$P_{\text{load,pu}} = I_{\text{pu}}^2 R_{\text{pu}} = (0.569)^2 (1.503) = 0.487$$

and the actual power supplied to the load is

$$\begin{aligned} P_{\text{load}} &= P_{\text{load,pu}} S_{\text{base}} = (0.487)(10,000 \text{ VA}) \\ &= 4870 \text{ W} \end{aligned}$$

(d) The per-unit power lost in the transmission line is

$$P_{\text{line,pu}} = I_{\text{pu}}^2 R_{\text{line,pu}} = (0.569)^2 (0.0087) = 0.00282$$

and the actual power lost in the transmission line is

$$\begin{aligned} P_{\text{line}} &= P_{\text{line,pu}} S_{\text{base}} = (0.00282)(10,000 \text{ VA}) \\ &= 28.2 \text{ W} \end{aligned}$$

Transformer Voltage Regulation

The Voltage Regulation is defined as:

$$VR = \frac{V_{Snl} - V_{Sfl}}{V_{Sfl}} \times 100\%$$

where V_{Snl} is the **magnitude** of the output voltage at noload.

V_{Sfl} is the **magnitude** of the output voltage at full load.

At noload, $V_{Snl} = \frac{V_P}{a}$, therefore,

$$VR = \frac{\frac{V_P}{a} - V_{Sfl}}{V_{Sfl}} \times 100\%$$

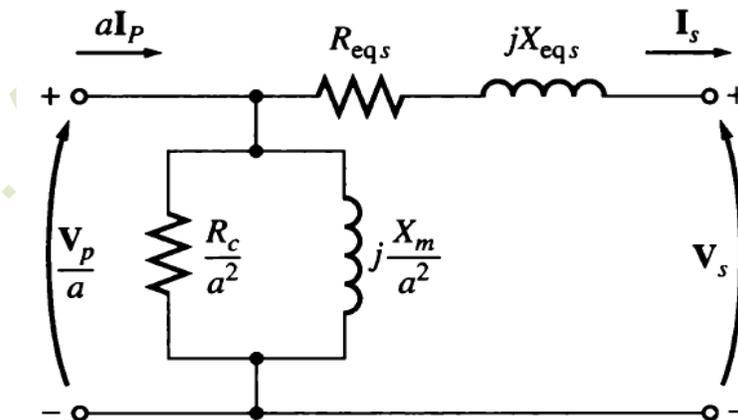
In per-unit,

$$VR = \frac{V_{Ppu} - V_{Sflpu}}{V_{Sflpu}} \times 100\%$$

Usually, it is a good practice to have a small voltage regulation, a low series impedance, (**ideal transformers have $VR = 0\%$**). However, high series impedance and high voltage regulation transformers are deliberately used to reduce the fault currents in power systems.

Transformer Phasor Diagram

- ✚ It can be utilized to calculate the voltage regulation of a transformer.
- ✚ The voltage regulation depends on the series impedance, the magnitude and the phase angle of the current flowing through a transformer (ignoring the effect of the excitation branch).



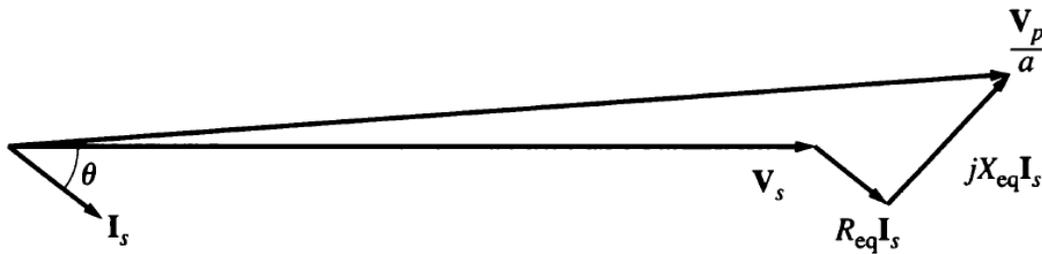
- Assuming that V_S is at an angle 0° and is known (rated voltage), the phasor diagrams for the three cases of load power factor are:

1- Lagging Power Factor

By applying KVL,

$$\frac{\bar{V}_P}{a} = \bar{V}_S + R_{eqS}\bar{I}_S + j X_{eqS}\bar{I}_S$$

The phasor diagram for this case is as shown below,



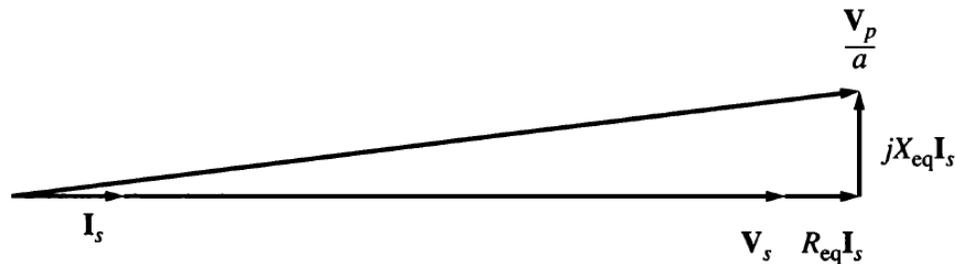
In this case (a lagging pf), $\frac{|\bar{V}_P|}{a} > |\bar{V}_S|$ and $VR > 0$

2- Unity Power Factor

By applying KVL,

$$\frac{\bar{V}_P}{a} = \bar{V}_S + R_{eqS}\bar{I}_S + j X_{eqS}\bar{I}_S$$

The phasor diagram for this case is as shown below,



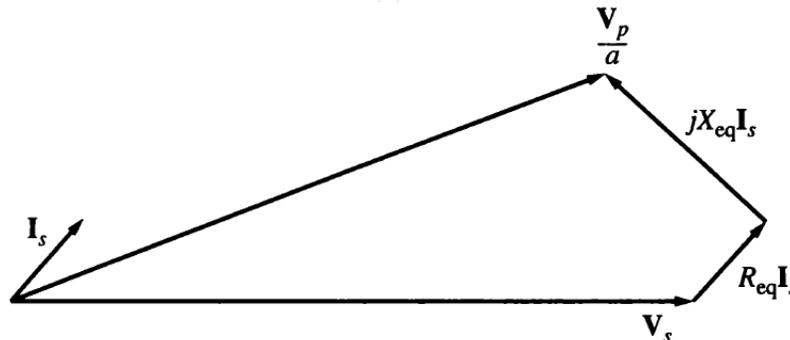
In this case (a unity pf), $\frac{|\bar{V}_P|}{a} > |\bar{V}_S|$ and $VR > 0$. However, the voltage regulation is smaller than that is for a lagging current.

3- Leading Power Factor

By applying KVL,

$$\frac{\bar{V}_P}{a} = \bar{V}_S + R_{eqS}\bar{I}_S + j X_{eqS}\bar{I}_S$$

The phasor diagram for this case is as shown below,



In this case (a leading pf), $\frac{|\bar{V}_P|}{a} < |\bar{V}_S|$ and $VR < 0$.

Transformer Efficiency

The efficiency is defined as:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

These equations apply to motors, generators as well as to transformers.

For a transformer, the output power is:

$$P_{out} = V_S I_S \cos \theta$$

The power losses are:

$$P_{loss} = P_{cu} + P_{core}$$

where,

$$P_{core} = P_{Hyst} + P_{Eddy}$$

such that:

P_{cu} : are the copper losses I^2R in the windings resistance

P_{Hyst} : is the Hysteresis power loss

P_{Eddy} : is the eddy current power loss

$$\eta = \frac{V_S I_S \cos \theta}{V_S I_S \cos \theta + P_{core} + P_{cu}} \times 100\%$$

Example 2-5. A 15-kVA, 2300/230-V transformer is to be tested to determine its excitation branch components, its series impedances, and its voltage regulation. The following test data have been taken from the transformer:

Open-circuit test (low voltage side)	Short-circuit test (high voltage side)
$V_{OC} = 230 \text{ V}$	$V_{SC} = 47 \text{ V}$
$I_{OC} = 2.1 \text{ A}$	$I_{SC} = 6.0 \text{ A}$
$P_{OC} = 50 \text{ W}$	$P_{SC} = 160 \text{ W}$

The data have been taken by using the connections shown in Figures 2-19 and 2-20.

- Find the equivalent circuit of this transformer referred to the high-voltage side.
- Find the equivalent circuit of this transformer referred to the low-voltage side.
- Calculate the full-load voltage regulation at 0.8 lagging power factor, 1.0 power factor, and at 0.8 leading power factor using the exact equation for V_p .
- Plot the voltage regulation as load is increased from no load to full load at power factors of 0.8 lagging, 1.0, and 0.8 leading.
- What is the efficiency of the transformer at full load with a power factor of 0.8 lagging?

Solution

- The turns ratio of this transformer is $a = 2300/230 = 10$. The excitation branch values of the transformer equivalent circuit referred to the secondary (low voltage) side can be calculated from the *open-circuit test* data, and the series elements referred to the primary (high voltage) side can be calculated from the *short-circuit test* data. From the open-circuit test data, the open-circuit impedance angle is

$$\theta_{OC} = \cos^{-1} \frac{P_{OC}}{V_{OC} I_{OC}}$$

$$\theta_{OC} = \cos^{-1} \frac{50 \text{ W}}{(230 \text{ V})(2.1 \text{ A})} = 84^\circ$$

The excitation admittance is thus

$$Y_E = \frac{I_{OC}}{V_{OC}} \angle -84^\circ$$

$$Y_E = \frac{2.1 \text{ A}}{230 \text{ V}} \angle -84^\circ \text{ S}$$

$$Y_E = 0.00913 \angle -84^\circ \text{ S} = 0.000954 - j0.00908 \text{ S}$$

The elements of the excitation branch referred to the secondary are

$$R_{C,S} = \frac{1}{0.000954} = 1050 \Omega$$

$$X_{M,S} = \frac{1}{0.00908} = 110 \Omega$$

From the short-circuit test data, the short-circuit impedance angle is

$$\theta_{SC} = \cos^{-1} \frac{P_{SC}}{V_{SC} I_{SC}}$$

$$\theta_{SC} = \cos^{-1} \frac{160 \text{ W}}{(47 \text{ V})(6 \text{ A})} = 55.4^\circ$$

The equivalent series impedance is thus

$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \theta_{SC}$$

$$Z_{SE} = \frac{47 \text{ V}}{6 \text{ A}} \angle 55.4^\circ \Omega$$

$$Z_{SE} = 7.833 \angle 55.4^\circ = 4.45 + j6.45 \Omega$$

The series elements referred to the primary side are

$$R_{eq,P} = 4.45 \Omega \quad X_{eq,P} = 6.45 \Omega$$

The resulting simplified equivalent circuit referred to the primary side can be found by converting the excitation branch values to the primary side.

$$R_{C,P} = a^2 R_{C,S} = (10)^2 (1050 \Omega) = 105 \text{ k}\Omega$$

$$X_{M,P} = a^2 X_{M,S} = (10)^2 (110 \Omega) = 11 \text{ k}\Omega$$

This equivalent circuit is shown in Figure 2–28a.

(b) To find the equivalent circuit referred to the low-voltage side, it is simply necessary to divide the impedance by a^2 . Since $a = N_p/N_s = 10$, the resulting values are

$$R_C = 1050 \Omega \quad R_{eq} = 0.0445 \Omega$$

$$X_M = 110 \Omega \quad X_{eq} = 0.0645 \Omega$$

The resulting equivalent circuit is shown in Figure 2–28b.

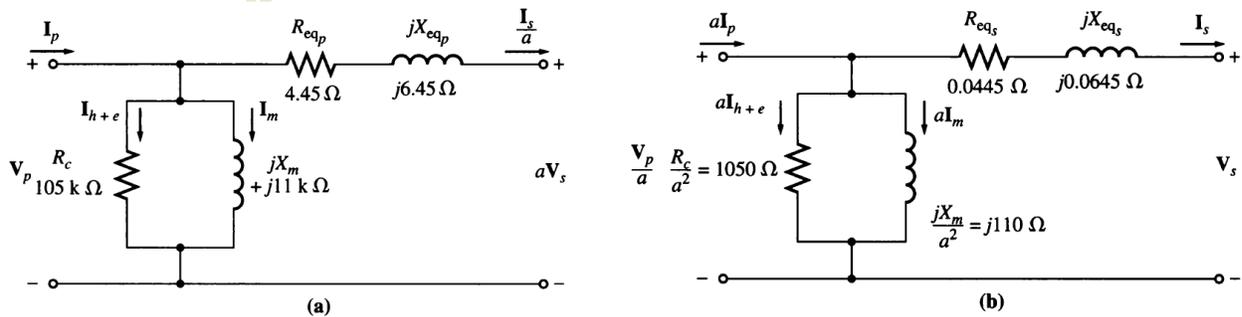


FIGURE 2–28

(c) The full-load current on the secondary side of this transformer is

$$I_{S,\text{rated}} = \frac{S_{\text{rated}}}{V_{S,\text{rated}}} = \frac{15,000 \text{ VA}}{230 \text{ V}} = 65.2 \text{ A}$$

To calculate V_p/a , use Equation (2-64):

$$\frac{V_p}{a} = V_s + R_{\text{eq}} I_s + jX_{\text{eq}} I_s \quad (2-64)$$

At PF = 0.8 lagging, current $I_s = 65.2 \angle -36.9^\circ \text{ A}$. Therefore,

$$\begin{aligned} \frac{V_p}{a} &= 230 \angle 0^\circ \text{ V} + (0.0445 \Omega)(65.2 \angle -36.9^\circ \text{ A}) + j(0.0645 \Omega)(65.2 \angle -36.9^\circ \text{ A}) \\ &= 230 \angle 0^\circ \text{ V} + 2.90 \angle -36.9^\circ \text{ V} + 4.21 \angle 53.1^\circ \text{ V} \\ &= 230 + 2.32 - j1.74 + 2.52 + j3.36 \\ &= 234.84 + j1.62 = 234.85 \angle 0.40^\circ \text{ V} \end{aligned}$$

The resulting voltage regulation is

$$\begin{aligned} \text{VR} &= \frac{V_p/a - V_{S,\text{fl}}}{V_{S,\text{fl}}} \times 100\% \\ &= \frac{234.85 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = 2.1\% \end{aligned} \quad (2-62)$$

At PF = 1.0, current $I_s = 65.2 \angle 0^\circ \text{ A}$. Therefore,

$$\begin{aligned} \frac{V_p}{a} &= 230 \angle 0^\circ \text{ V} + (0.0445 \Omega)(65.2 \angle 0^\circ \text{ A}) + j(0.0645 \Omega)(65.2 \angle 0^\circ \text{ A}) \\ &= 230 \angle 0^\circ \text{ V} + 2.90 \angle 0^\circ \text{ V} + 4.21 \angle 90^\circ \text{ V} \\ &= 230 + 2.90 + j4.21 \\ &= 232.9 + j4.21 = 232.94 \angle 1.04^\circ \text{ V} \end{aligned}$$

The resulting voltage regulation is

$$\text{VR} = \frac{232.94 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = 1.28\%$$

At PF = 0.8 leading, current $I_s = 65.2 \angle 36.9^\circ \text{ A}$. Therefore,

$$\begin{aligned} \frac{V_p}{a} &= 230 \angle 0^\circ \text{ V} + (0.0445 \Omega)(65.2 \angle 36.9^\circ \text{ A}) + j(0.0645 \Omega)(65.2 \angle 36.9^\circ \text{ A}) \\ &= 230 \angle 0^\circ \text{ V} + 2.90 \angle 36.9^\circ \text{ V} + 4.21 \angle 126.9^\circ \text{ V} \\ &= 230 + 2.32 + j1.74 - 2.52 + j3.36 \\ &= 229.80 + j5.10 = 229.85 \angle 1.27^\circ \text{ V} \end{aligned}$$

The resulting voltage regulation is

$$VR = \frac{229.85 \text{ V} - 230 \text{ V}}{230 \text{ V}} \times 100\% = -0.062\%$$

Each of these three phasor diagrams is shown in Figure 2–29.

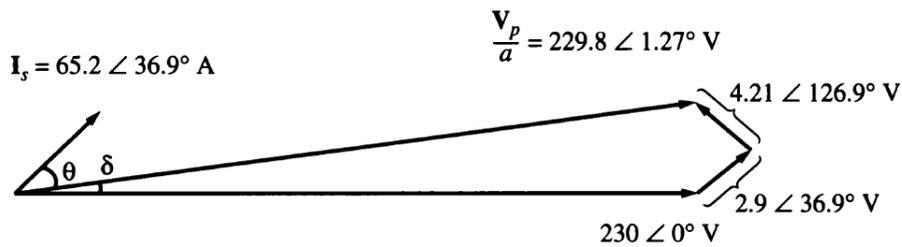
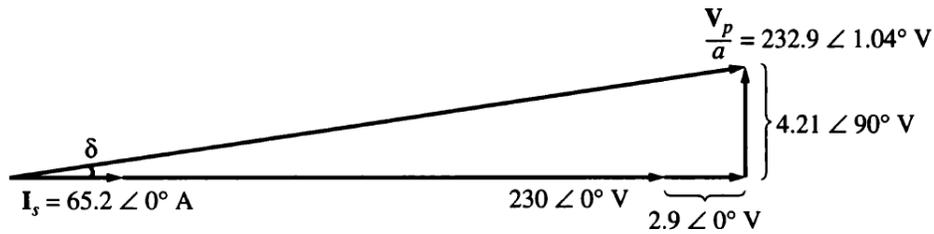
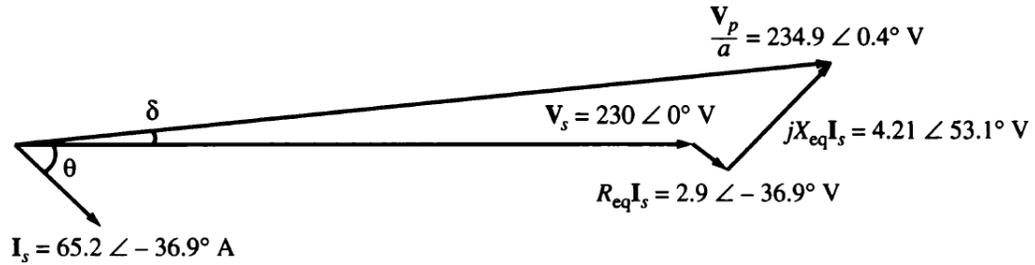


FIGURE 2–29

(e) To find the efficiency of the transformer, first calculate its losses. The copper losses are

$$P_{\text{Cu}} = (I_S)^2 R_{\text{eq}} = (65.2 \text{ A})^2 (0.0445 \Omega) = 189 \text{ W}$$

The core losses are given by

$$P_{\text{core}} = \frac{(V_P/a)^2}{\frac{R_C}{a^2}} = \frac{(234.85 \text{ V})^2}{1050 \Omega} = 52.5 \text{ W}$$

The output power of the transformer at this power factor is

$$\begin{aligned} P_{\text{out}} &= V_S I_S \cos \theta \\ &= (230 \text{ V})(65.2 \text{ A}) \cos 36.9^\circ = 12,000 \text{ W} \end{aligned}$$

Therefore, the efficiency of the transformer at this condition is

$$\begin{aligned} \eta &= \frac{V_S I_S \cos \theta}{P_{\text{Cu}} + P_{\text{core}} + V_S I_S \cos \theta} \times 100\% && (2-68) \\ &= \frac{12,000 \text{ W}}{189 \text{ W} + 52.5 \text{ W} + 12,000 \text{ W}} \times 100\% \\ &= 98.03\% \end{aligned}$$

Transformer Taps and Voltage Regulators

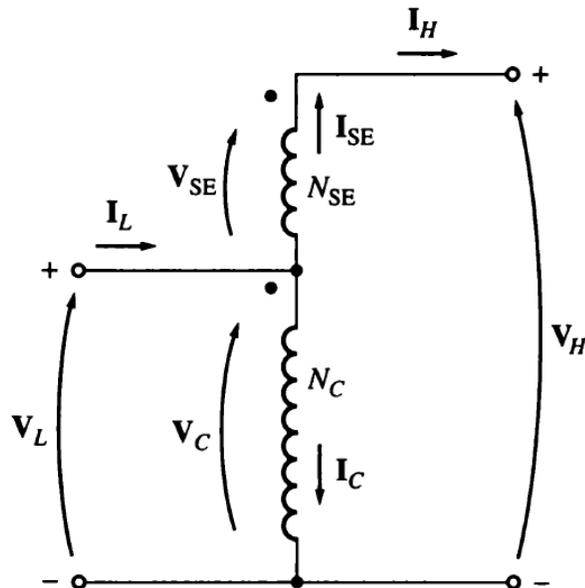
- The voltage variation at the output of a transformer is due to the high impedance between the generators and loads, and due to variation of loads.
- Distribution transformers have a series of taps in the windings to permit small changes in the turns' ratio of the transformer.
- Typically, transformers have four taps in addition to the nominal setting with spacing of 2.5% of full load voltage between them.
- Such arrangement provides adjustments up to 5% above or below the nominal voltage rating of the transformer.
- However, these taps normally cannot be adjusted while power is being applied to the transformer.
- One solution to this problem is a special transformer called Tap Changing Under Load (TCUL) transformer or voltage regulator.
- The taps are connected at the high voltage (low current) side of the transformer.
- A TCUL transformer has a built-in voltage sensing circuitry that changes taps automatically to keep the voltage almost constant.
- TCUL transformers operate while power is connected to the transformer, and they are very common in modern power systems.

Autotransformers

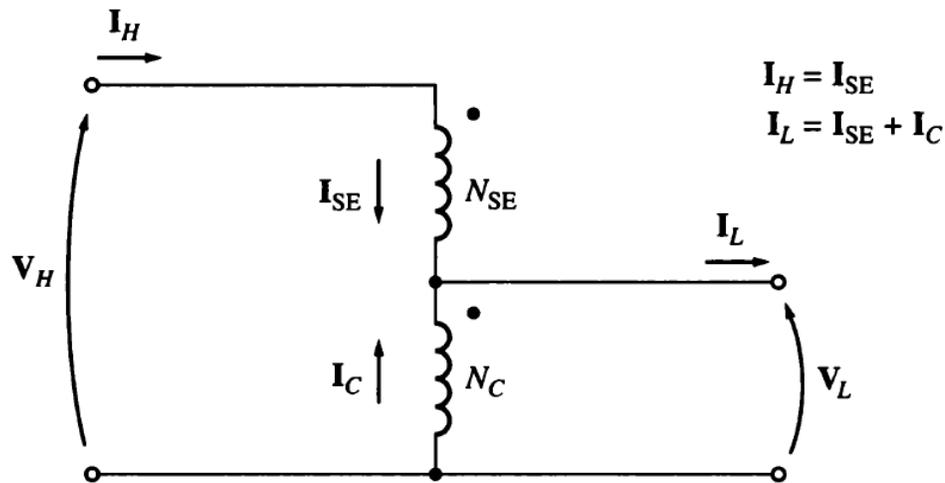
- ✦ They are used to change the voltage levels by a small amount.
- ✦ An example of autotransformer is the variable autotransformer for AC (**VARIAC**), which is used to provide an adjustable AC voltage.



- ✦ The autotransformer consists of two windings; a **Common winding**, which is common to input and output ports and has N_C turns. The other winding is called the **Series winding**, which is connected in series with the Common winding and is smaller than the Common winding, and has N_{SE} turns.
- ✦ The autotransformer can be configured as a step-up transformer as depicted in the Figure below.



- The other configuration is the step-down autotransformer, which is depicted in the Figure below.



- The voltages in the Common and Series windings are related according to:

$$\frac{|\bar{V}_C|}{|\bar{V}_{SE}|} = \frac{N_C}{N_{SE}}$$

- The currents are related according to:

$$\frac{|\bar{I}_C|}{|\bar{I}_{SE}|} = \frac{N_{SE}}{N_C}$$

- The terminal voltages are:

$$\bar{V}_L = \bar{V}_C$$

$$\bar{V}_H = \bar{V}_C + \bar{V}_{SE}$$

- And the terminal currents are:

$$\bar{I}_L = \bar{I}_C + \bar{I}_{SE}$$

$$\bar{I}_L = \bar{I}_{SE}$$

- Rearranging the above equations yields the terminal voltages relationship as:

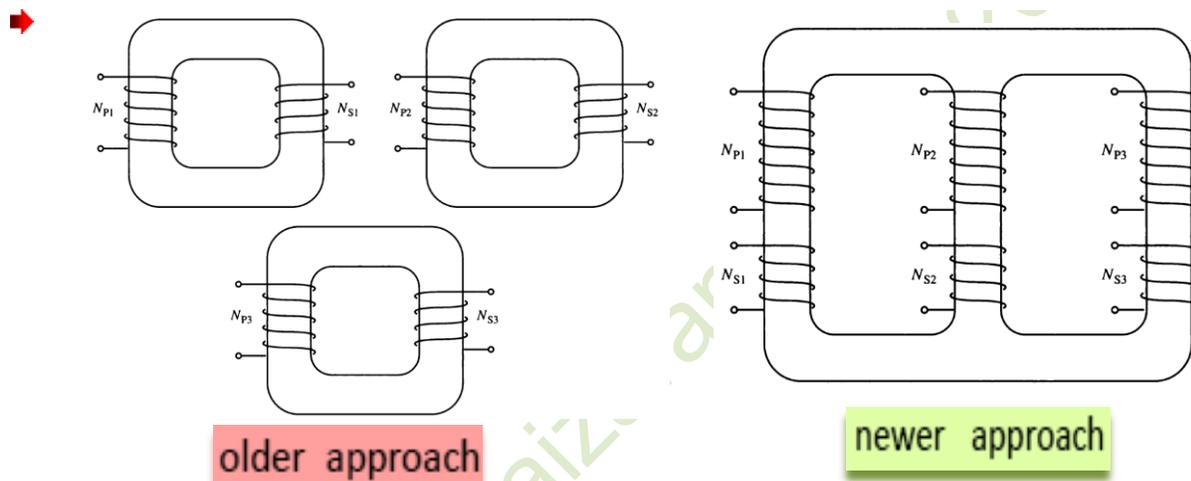
$$\frac{|\bar{V}_L|}{|\bar{V}_H|} = \frac{N_C}{N_C + N_{SE}}$$

and the terminal currents relationship as:

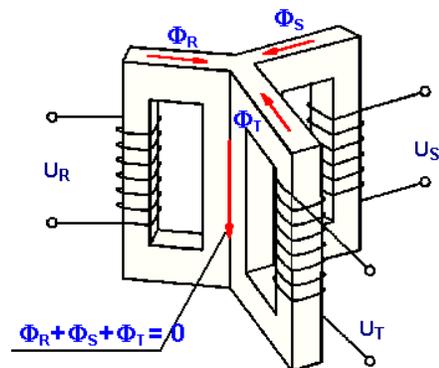
$$\frac{|\bar{I}_L|}{|\bar{I}_H|} = \frac{N_C + N_{SE}}{N_C}$$

Three-Phase Transformers

- ➔ Almost all major power generation and distribution systems in the world today are three-phase AC power systems.
- ➔ Three-phase transformers can be constructed either by connecting three single phase transformers in a three-phase bank, or by wrapping three sets of windings on a common core and connecting them in a three-phase bank.



- ➔ The older approach is to connect three independent transformers to compose a three-phase transformer as shown in the Figure above. This approach has the advantage that each unit could be replaced individually in the event of trouble. However, the combined three-phase transformer is preferred.
- ➔ The newer approach is to construct a three-phase-transformer on a single core as it is lighter, smaller in size, cheaper, and slightly more efficient. These advantages outweigh the advantage of the older approach.
- ➔ Note that, the middle leg of the core can be removed as it has zero flux.



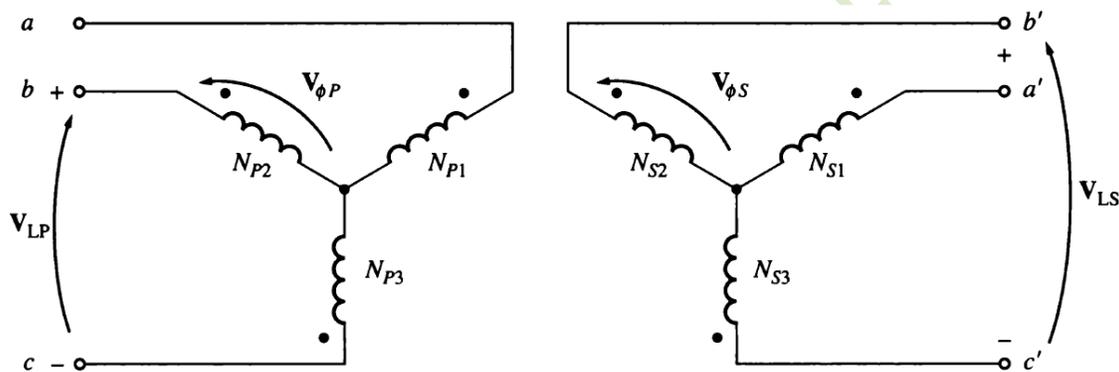
Three-Phase Transformers' Connections

The primaries and secondaries can be independently connected in a WYE (Y) or Delta (Δ). Hence, four combinations are possible:

- 1) WYE - WYE ($Y - Y$)
- 2) WYE - Delta ($Y - \Delta$)
- 3) Delta - WYE ($\Delta - Y$)
- 4) Delta-Delta ($\Delta - \Delta$)

1) WYE - WYE ($Y - Y$) Connection

- The windings are connected as shown in the Figure below.



- The primary voltage on each phase is related to the primary line voltage by:

$$V_{LP} = \sqrt{3}V_{\phi P}$$

- The primary phase voltage is related to the secondary phase voltage by the turns' ratio (a):

$$a = \frac{V_{\phi P}}{V_{\phi S}} = \frac{N_P}{N_S}$$

- The secondary voltage on each phase is related to the secondary line voltage by:

$$V_{LS} = \sqrt{3}V_{\phi S}$$

- The overall voltage ratio on the transformer is:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

$$\frac{V_{LP}}{V_{LS}} = a$$

- Note that, the magnetization current of a transformer is non-sinusoidal and contains a very large third harmonic current, which is necessary to overcome saturation in order to produce a sinusoidal flux.
- WYE – WYE connection has **two serious problems**, and therefore **it is rarely used**:
 - a) An **unbalanced load** can cause the phase voltages to be severely unbalanced.
 - b) A **Third Harmonic problem** due to non linearity of the core:
 - ✚ The magnetization current for each phase has a fundamental component and a third harmonic component. In other words, for phase 'a':

$$i_a(t) = I_{a1} \sin \omega_1 t + I_{a3} \sin 3(\omega_1 t) + \dots$$

$$\Rightarrow i_a(t) = I_1 \sin \omega_1 t + I_3 \sin (\omega_3 t) + \dots$$

where, $\omega_3 = 3\omega_1$ is the third harmonic frequency.

For Phase 'b':

$$i_b(t) = I_{b1} \sin(\omega_1 t - 120) + I_{b3} \sin 3(\omega_1 t - 120) + \dots$$

$$\Rightarrow i_b(t) = I_1 \sin(\omega_1 t - 120) + I_3 \sin (\omega_3 t - 360) + \dots$$

For Phase 'c':

$$i_c(t) = I_{c1} \sin(\omega_1 t + 120) + I_{c3} \sin 3(\omega_1 t + 120) + \dots$$

$$i_c(t) = I_1 \sin(\omega_1 t + 120) + I_3 \sin (\omega_3 t + 360) + \dots$$

- ✚ The fundamental components of the three-phase currents are 120° apart. However, **the 3rd harmonic currents of the three phases are 360° out of phase; i.e. in phase.**
- ✚ Because all the third harmonic currents are in phase with each other in the three windings, and they must add up to zero at the Neutral (when the Neutral is ungrounded), they are all zero;

$$\bar{I}_{a3} + \bar{I}_{b3} + \bar{I}_{c3} = I_N = 0$$

$$I_3 \angle 0 + I_3 \angle -360 + I_3 \angle 360 = 0$$

$$I_3 \angle 0 + I_3 \angle 0 + I_3 \angle 0 = 0$$

$$3I_3 \angle 0 = 0$$

Thus, the third harmonic component itself, of any of the three phases, is zero;

$$\Rightarrow I_3 = 0$$

In other words, **no triplen harmonic currents can flow in a Y-Y connection if there is no connection from the transformer windings' Neutral back to the sources' Neutral.**

- ✦ As a result, the magnetization currents will be sinusoidal, without the third harmonic component flowing, resulting in a flat-topped flux, which does not vary sinusoidally.
- ✦ Since, the induced voltage is proportional to the flux amplitude and its frequency, then the induced voltages will be also flat-topped, which have a large third harmonic component (as large as the fundamental component); **the induced voltages in the secondaries are not sinusoidal.**

Solutions to Y – Y Problems:

I) Solidly Grounding the Neutrals of the Transformers, especially the Primary Transformers' Neutral

This permits the additive **third harmonic components** to cause a current **flow in the Neutral** instead of building up large third harmonic voltages. The neutral also provides a return path for any current imbalances in the load.

II) Adding a Third (or Tertiary) Winding Connected in Delta

Thus, the third harmonic voltages in the Delta will add up, **causing a circulating third harmonic current to flow in the Tertiary** to supply the needed component of the magnetizing current, hence restoring the flux to a near sinusoidal waveform. This suppresses the third harmonic components of voltages in a similar manner to grounding the Neutral.

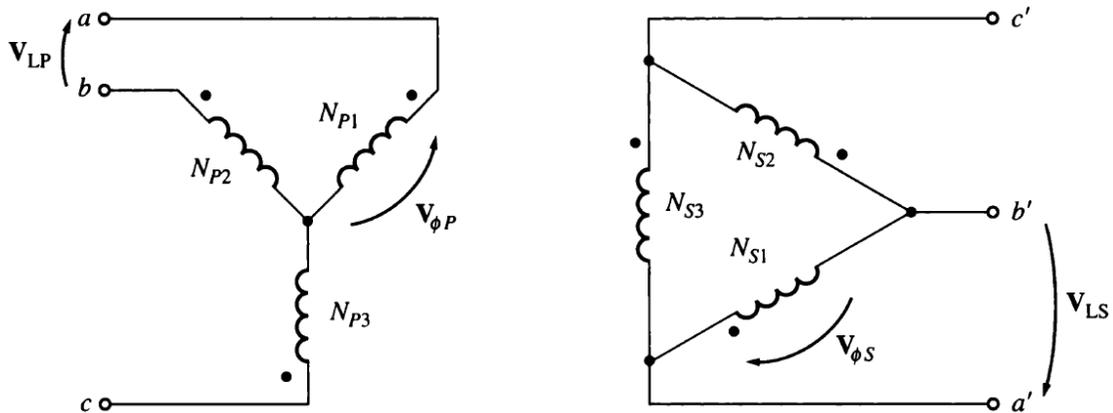
The Tertiary winding is often used to **supply lights, motors and other applications** in the substation.

The rated power of the Tertiary winding is about $\frac{1}{3rd}$ that of the main windings, to be able to handle the large circulating (third harmonic) currents.

The Delta connected Tertiaries also couple the three phases together. Any fundamental voltage imbalance between the phases, due to secondary line imbalance, **causes a circulating fundamental current in the Tertiaries.** This tends to equalize the phase voltages and distribute the secondary load unbalance more evenly among the primary phases.

2) WYE-Delta (Y – Δ) Connection

The windings are connected as shown in the Figure below.



The primary voltage on each phase is related to the primary line voltage by:

$$V_{LP} = \sqrt{3}V_{\phi P}$$

The primary phase voltage is related to the secondary phase voltage by the turns' ratio (a):

$$a = \frac{V_{\phi P}}{V_{\phi S}} = \frac{N_P}{N_S}$$

The secondary voltage on each phase is related to the secondary line voltage by:

$$V_{LS} = V_{\phi S}$$

The overall voltage ratio on the transformer is:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}} = \sqrt{3} a$$

$$\frac{V_{LP}}{V_{LS}} = \sqrt{3} a$$

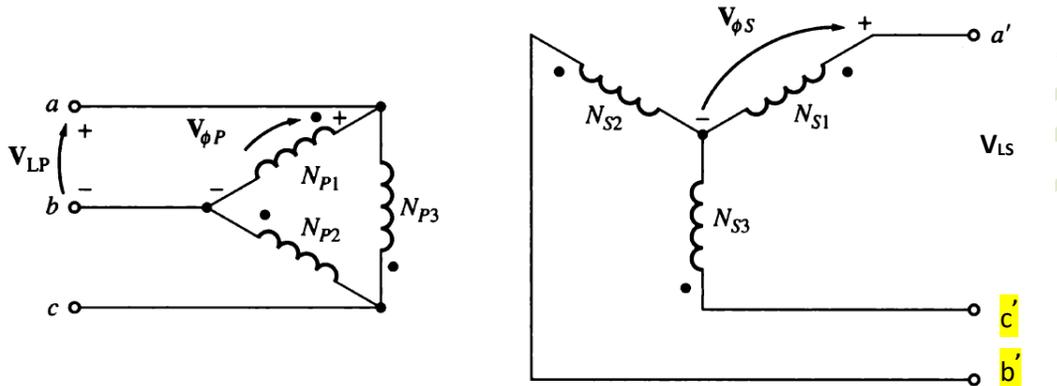
This connection has no problem with the third harmonic components in its voltages, since they are consumed in a circulating current in the Delta side.

This connection is more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs.

A feature might be considered as a disadvantage for this connection is that, the secondary voltage is shifted 30° with respect to primary voltage for abc sequence; the secondary line voltage lags the primary line voltage by 30°.

3) Delta – WYE ($\Delta - Y$) Connection

- ✓ The windings are connected as shown in the Figure below.



- ✓ The primary voltage on each phase is equal to the primary line voltage;

$$V_{LP} = V_{\phi P}$$

- ✓ The primary phase voltage is related to the secondary phase voltage by the turns' ratio (a):

$$a = \frac{V_{\phi P}}{V_{\phi S}} = \frac{N_P}{N_S}$$

- ✓ The secondary voltage on each phase is related to the secondary line voltage by:

$$V_{LS} = \sqrt{3}V_{\phi S}$$

- ✓ The overall voltage ratio on the transformer is:

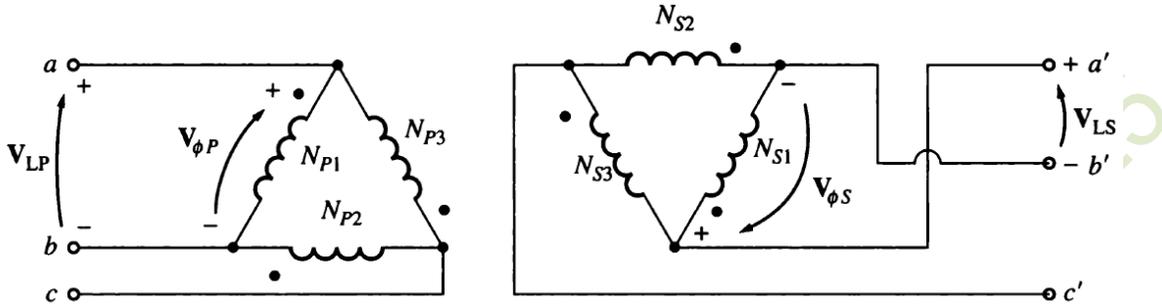
$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}} = \frac{a}{\sqrt{3}}$$

$$\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}}$$

- ✓ It has the same advantages as the $Y - \Delta$ connection, but it causes the secondary line voltage to lead the primary line voltage by 30° for 'abc' sequence.

4) Delta-Delta ($\Delta - \Delta$) Connection

- ✓ The windings are connected as shown in the Figure below.



- ✓ The primary voltage on each phase is equal to the primary line voltage;

$$V_{LP} = V_{\phi P}$$

- ✓ The primary phase voltage is related to the secondary phase voltage by the turns' ratio (a):

$$a = \frac{V_{\phi P}}{V_{\phi S}} = \frac{N_P}{N_S}$$

- ✓ The secondary voltage on each phase is related to the secondary line voltage by:

$$V_{LS} = V_{\phi S}$$

- ✓ The overall voltage ratio on the transformer is:

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a$$

$$\frac{V_{LP}}{V_{LS}} = a$$

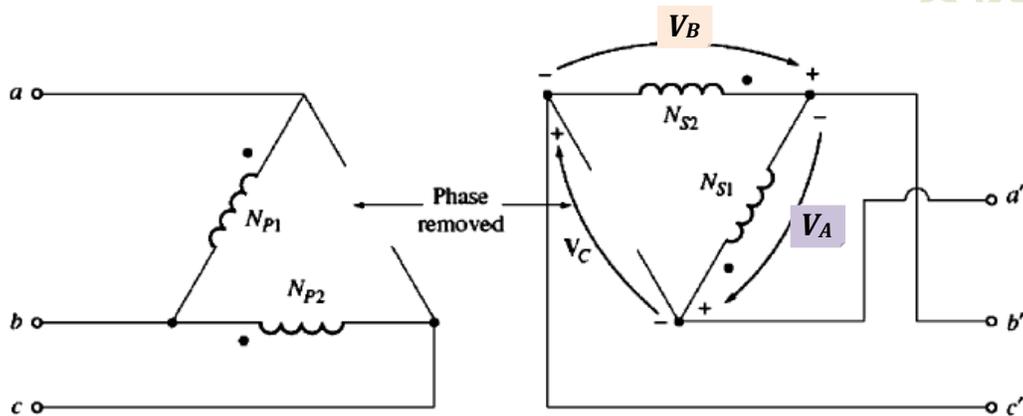
- ✓ This transformer **has no phase shift associated with it, no problem with harmonics, or unbalanced loads.**

Three-Phase Transformation Using Two Transformers

- 1- Open- Δ ($V - V$) Connection
- 2- Open- Y Open- Δ Connection
- 3- Scott-T Connection

1. Open- Δ ($V - V$) Connection

- ➔ Assuming that the winding of one phase of the $\Delta - \Delta$ connected transformer is damaged, and has been removed for repair, as shown in the Figure below,



- ➔ If the secondary voltages are:

$$\bar{V}_A = V \angle 0^\circ \text{ V}$$

and

$$\bar{V}_B = V \angle -120^\circ \text{ V}$$

Then, the voltage across the gap, where the third winding is removed, is:

$$\bar{V}_C = -\bar{V}_A - \bar{V}_B$$

$$\bar{V}_C = -V \angle 0^\circ - V \angle -120^\circ$$

$$\bar{V}_C = -V - (-0.5V - j0.866V)$$

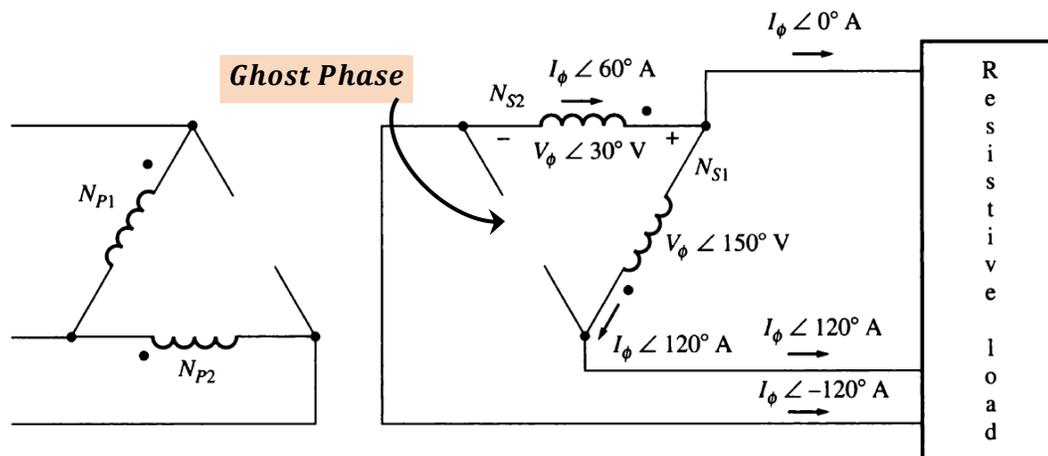
$$\bar{V}_C = -0.5V + j0.866V$$

$$\bar{V}_C = V \angle 120^\circ \text{ V}$$

- ➔ This (\bar{V}_C) is the exact voltage that would be if the third winding still exists; \bar{V}_C is called **a Ghost phase!**
- ➔ Thus, **balanced three-phase voltages can be obtained at the output of a transformer with only two transformer windings connected as open- Δ !**

Real Power of Two-Winding Transformer

- Consider a two-winding transformer supplying a three-phase Δ -connected resistive load, as shown in the Figure below,



- The load phase voltage is the same as the load line voltage and it equals the transformer secondary voltage; for phase A, $V_\phi \angle 150^\circ$.
- Because the load is resistive, then the load phase current is at the same angle as the load phase voltage; $I_{R\phi} \angle 150^\circ$.
- The line current lags the load phase current by 30° ; $I_\phi \angle 120^\circ$.
- Since one winding is missing, the line currents are the same as the phase current in the secondary windings.
- Because the current and voltage angles differ in each winding, the real power of each is examined individually.
- For winding 1 of the transformer secondaries, its phase current lags its voltage by 30° .**

The real power of winding 1 is:

$$P_1 = V_\phi I_\phi \cos(150 - 120)$$

$$P_1 = V_\phi I_\phi \cos(30)$$

$$P_1 = \frac{\sqrt{3}}{2} V_\phi I_\phi$$

- For winding 2 of the transformer secondaries, the phase voltage is at angle of 30° and the current must be leaving the dotted end of winding 2 to be able to deliver a real power; i.e., $-(I_\phi \angle -120^\circ)$, which can be written as: $I_\phi \angle 60^\circ$.

Thus, the current angle is 60° . The real power of winding 2 is:

$$P_2 = V_\phi I_\phi \cos(30 - 60)$$

$$P_2 = V_\phi I_\phi \cos(-30)$$

$$P_2 = \frac{\sqrt{3}}{2} V_\phi I_\phi$$

- The total maximum real power of the *open*- Δ transformer is:

$$P_T = \sqrt{3} V_\phi I_\phi$$

- The ratio between the real power of the *open*- Δ and the available real power of normal three-phase bank is:

$$\frac{P_{open-\Delta}}{P_{3phase}} = \frac{\sqrt{3} V_\phi I_\phi}{3 V_\phi I_\phi}$$

$$\frac{P_{open-\Delta}}{P_{3phase}} = \frac{1}{\sqrt{3}} = 0.577$$

- The available real power out of an *open*- Δ bank is only 57.7% of the original bank's rating.
- Note, the real power of the *open*- Δ bank is not $\frac{2}{3}$ or 66.7% of original bank's rating, as could be anticipated!
- Note also, the real power output of the *open*- Δ bank is only 86.6% of the two remaining windings' real power; $0.866 \times \frac{2}{3} P_{3phase} = 0.577 P_{3phase}$!

- The reactive power of winding 1 is:

$$Q_1 = V_\phi I_\phi \sin(150 - 120)$$

$$Q_1 = V_\phi I_\phi \sin(30)$$

$$Q_1 = \frac{1}{2} V_\phi I_\phi$$

- The reactive power of winding 2 is:

$$Q_2 = V_\phi I_\phi \sin(30 - 60)$$

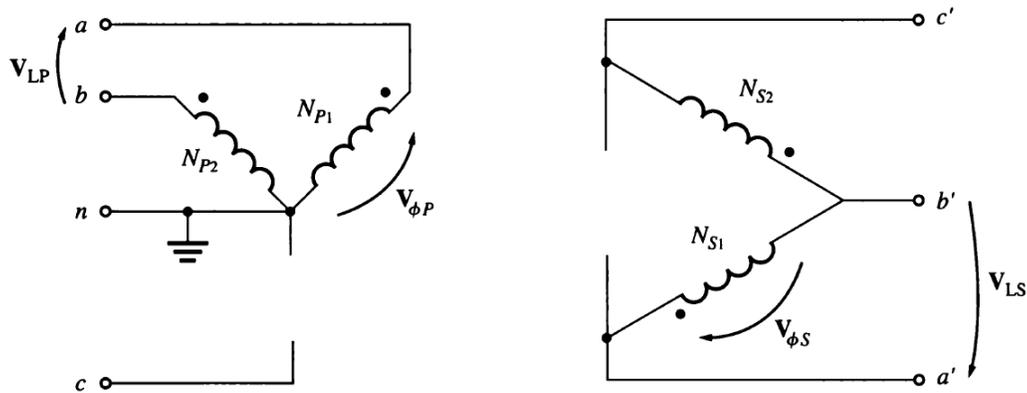
$$Q_2 = V_\phi I_\phi \sin(-30)$$

$$Q_2 = -\frac{1}{2} V_\phi I_\phi$$

- Thus, **one transformer is producing reactive power which the other one is consuming** (though the load is purely resistive). That's why the real power with two windings is limited to 57.7% of the original bank's rating.

2. Open-Y Open-Δ Connection

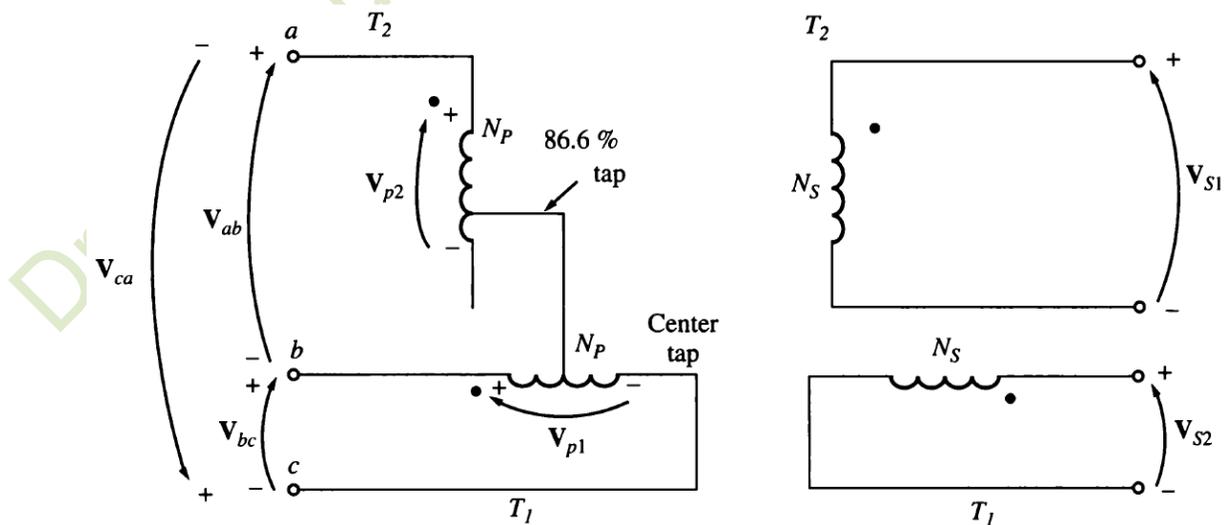
It is very similar to the Open-Δ connection except that the primary voltages are derived from two phases and a Neutral, as shown below.



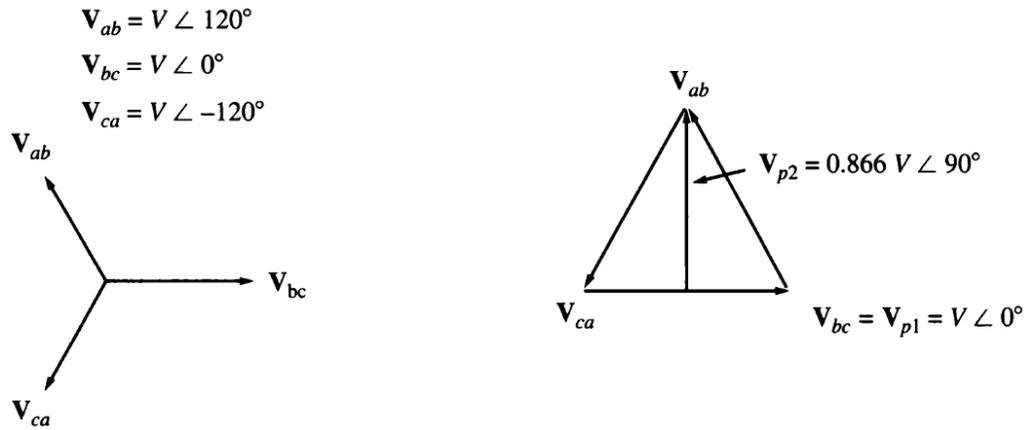
By this connection, a three-phase service can be obtained from two lines only. However, a very large current flows in the Neutral of the primary circuit.

3. Scott-T Connection

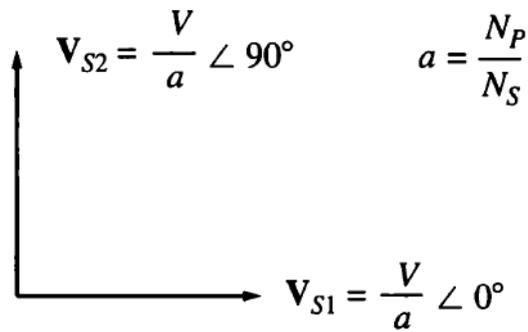
- ✚ It is used to derive two phases 90° apart from three-phase power supply.
- ✚ Scott-T consists of two single phase transformers with identical ratings.
- ✚ One transformer has taps on its primary and secondary windings at 86.6 percent of the full load voltage.
- ✚ The other transformer has a center tap on its primary winding.
- ✚ Scott-T connection is shown in the Figure below.



✚ The equivalent phasor diagrams and voltages are:



✚ The resulting secondary voltages are:



The Per-Unit System for Three-phase Transformers

- The single phase base equations apply to three-phase systems on a per phase basis.
- The base Volt-Ampere value of a single (one) transformer is related to three-phase base by:

$$S_{1\phi base} = \frac{S_{base}}{3}$$

where, S_{base} is the three-phase Volt-Ampere.

Also,
$$I_{1\phi base} = \frac{S_{1\phi base}}{V_{1\phi base}}$$

Thus,
$$I_{1\phi base} = \frac{S_{base}}{3V_{1\phi base}}$$

$$Z_{base} = \frac{V_{1\phi base}^2}{S_{1\phi base}}$$

$$Z_{base} = \frac{3V_{1\phi base}^2}{S_{base}}$$

- The relationship between the base line voltage and the base phase voltage of the transformer depends on the connection of the windings.

For Δ –Connection,

$$V_{Lbase} = V_{1\phi base}$$

For Y –Connection,

$$V_{Lbase} = \sqrt{3}V_{1\phi base}$$

- The base line current in a three-phase transformer bank is:

$$I_{Lbase} = \frac{S_{base}}{\sqrt{3}V_{Lbase}}$$

Transformer Name Plate:

Information on a name plate includes rated voltage, rated kilo Volt-Ampere, rated frequency, and transformer per-unit series impedance. Also, the voltage rating of each tap on the transformer and wiring schematic of the transformer are shown.

Transformer Ratings and Related Problems

Voltage and Frequency Ratings

The voltage rating of a transformer serves two functions:

- 1- **Protecting the winding insulation** from breakdown due to an excessive voltage applied to it!
- 2- **Setting the maximum acceptable magnetizing current (i_m)** in the core.

Assuming that the steady state voltage applied to the primary winding is:

$$v(t) = V_M \sin \omega t$$

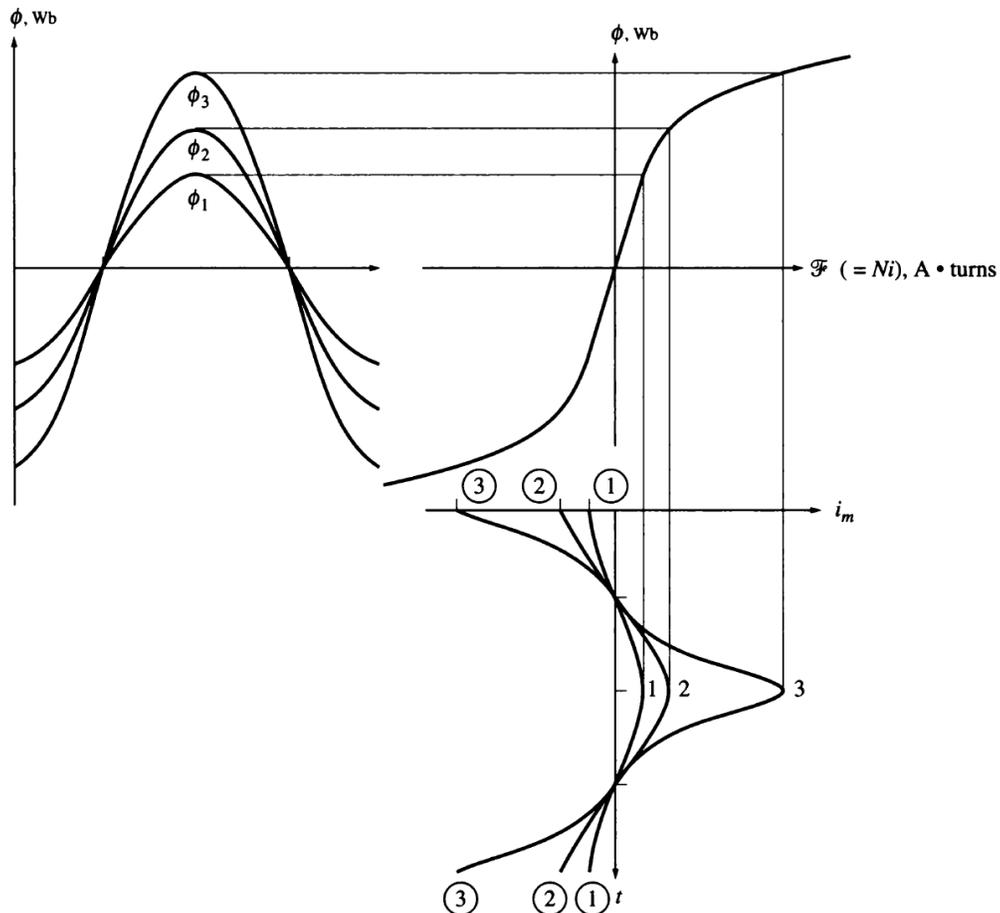
Then, the flux in the core of the transformer is:

$$\phi(t) = \frac{1}{N_p} \int v(t) dt$$

$$\phi(t) = \frac{1}{N_p} \int V_M \sin \omega t dt$$

$$\phi(t) = -\frac{V_M}{N_p \omega} \cos \omega t$$

Considering the magnetization curve of a transformer core for 3 different values of flux,



In the unsaturated region of the magnetizing curve, an increase in $v(t)$ by 10% results in a 10% increase in the flux. However, **in the saturated region, an increase in $v(t)$ by 10% results in a 10% increase in the flux, but results in a large increase in the magnetizing current (i_m)**, as in case 3, which was illustrated in the previous Figure.

Therefore, **the maximum applied voltage (therefore, the rated voltage) is set by the maximum acceptable magnetization current** in the core.

Also, since $\phi_M = \frac{V_M}{N_P \omega}$, the ratio $\frac{V}{f}$ must be kept constant to avoid excessive magnetizing currents.

In other words, **if the frequency is reduced, then the rated voltage must be reduced accordingly** (this is called **Derating of the transformer**), and vice versa; if the frequency is increased, the rated voltage of the transformer can be increased if this does not cause insulation problems.

Apparent Power Rating of a Transformer

- The apparent power rating and voltage rating set the maximum current flowing in the windings.
- The rated current is:

$$I_{rated} = \frac{S_{rated}}{V_{rated}}$$

- The rated current sets the copper losses (I^2R), which results in heating the windings. **Overheating of windings results in shortening the life time of insulation.**
- **Forced cooling** of transformers may allow operation at a higher Volt-Ampere rating.
- If the **transformer voltage is reduced**, then **the apparent power rating must be reduced** by an equal amount.

The Problem of Current Inrush

- The inrush current **at starting** is dependent on the voltage level of the transformer.
- Assuming that, a voltage whose phase shift θ ,

$$v(t) = V_M \sin(\omega t + \theta)$$

is applied to the transformer at the moment of connection to a power line.

- Then, the flux in the core is:

$$\phi(t) = \frac{1}{N_P} \int v(t) dt$$

$$\phi(t) = \frac{1}{N_P} \int V_M \sin(\omega t + \theta) dt$$

$$\phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + \theta) + \phi_C \quad \dots\dots\dots(7)$$

where, ϕ_C is the constant of integration and is a **transient flux** that **quickly decays as a result of excess eddy current and primary copper losses (I^2R)**.

- After few cycles, the flux becomes:

$$\phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + \theta)$$

Such that, the maximum flux in steady state is:

$$\phi_{max} = \frac{V_M}{N_P \omega}$$

- If **the residual flux in the core is ϕ_R** , then ϕ_C can be found by substituting $t = 0$ in the general flux equation (eq.(7));

$$\Rightarrow \phi(t = 0^+) = -\frac{V_M}{N_P \omega} \cos(\theta) + \phi_C = \phi_R$$

Therefore,

$$\phi_C = \frac{V_M}{N_P \omega} \cos(\theta) + \phi_R$$

- If the residual flux is zero ($\phi_R = 0$), then:

$$\phi_C = \frac{V_M}{N_P \omega} \cos(\theta)$$

- Thus, the core flux (in eq. (7)) becomes:

$$\phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + \theta) + \frac{V_M}{N_P \omega} \cos(\theta)$$

If $\theta = 90^\circ$, then:

$$\phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + 90) + \frac{V_M}{N_P \omega} \cos(90)$$

$$\Rightarrow \phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + 90)$$

$$\Rightarrow \phi(t) = \frac{V_M}{N_P \omega} \sin(\omega t)$$

This does not cause any problem, as the flux does not exceed its maximum value!

- However, if $\theta = 0^\circ$, then the flux becomes:

$$\phi(t) = -\frac{V_M}{N_P \omega} \cos(\omega t + 0) + \frac{V_M}{N_P \omega} \cos(0)$$

$$\phi(t) = -\phi_{max} \cos(\omega t) + \phi_{max}$$

At $\omega t = 0$, $\phi = 0$, no problem in that!

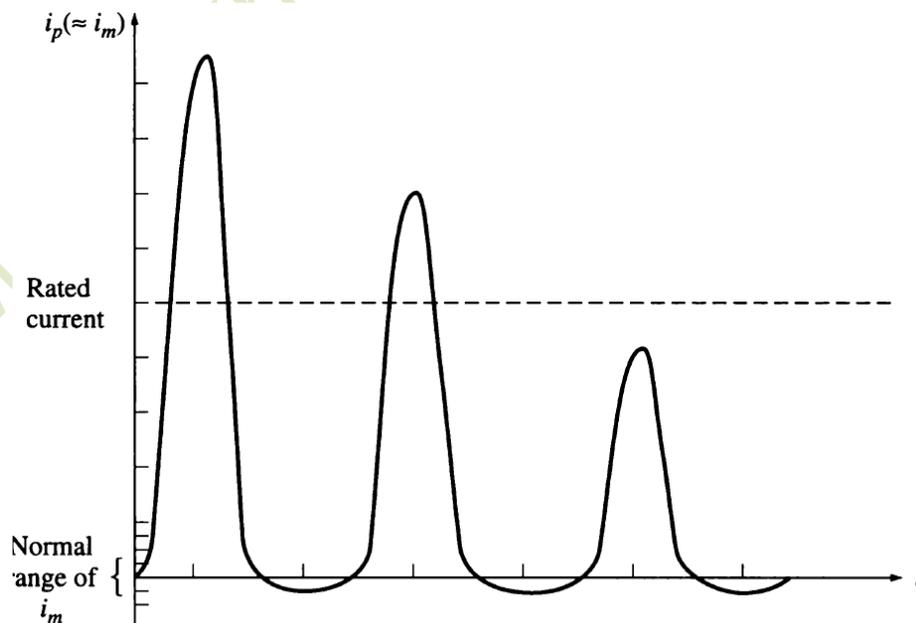
But, at $\omega t = \pi$, after the first half cycle, the flux is:

$$\phi = -\phi_{max} \cos(\pi) + \phi_{max}$$

$$\phi = 2\phi_{max}$$

Which is the worst case (at $\theta = 0^\circ$, and $\omega t = \pi$), as it results in a very large magnetization current!

- Note that, the maximum flux is twice as high as the normal steady state flux.
- Doubling the maximum flux results in an enormous increase in the magnetizing current, as the transformer is designed to operate at the knee of the magnetization curve.
- For part of a cycle, the transformer **looks like a short circuit** and a very large starting current flows; **Inrush current!**
- An example of the inrush current waveform is shown in the Figure below. **It has a second harmonic component!**



- The maximum flux height reached depends on the phase angle of the voltage at the moment it is applied.
- Zero phase shift is the worst moment to connect the transformer to the power line!
- Normally, the angle (phase shift) is not controlled, and the transformer and the power system must be able to withstand these huge starting currents.

Dr. M. Abu-Khaizaran, BZU, 2024/25

PART III

AC Machinery Fundamentals

- ➔ AC Machines are either generators that convert mechanical energy to AC electrical energy, or motors that convert AC electrical energy to mechanical energy.

Mechanical Energy $\xleftrightarrow{\text{AC Electrical Machine}}$ AC Electric Energy

➔ **Types of AC Machines:**

- 1- **Synchronous Machines:** They are motors or generators, whose magnetic field current is supplied by a **separate DC power source**.
- 2- **Induction (Asynchronous) Machines:** They are motors or generators, whose field current is supplied by **magnetic induction (transformer action)** into their field winding.

➔ **Construction of AC Machines:**

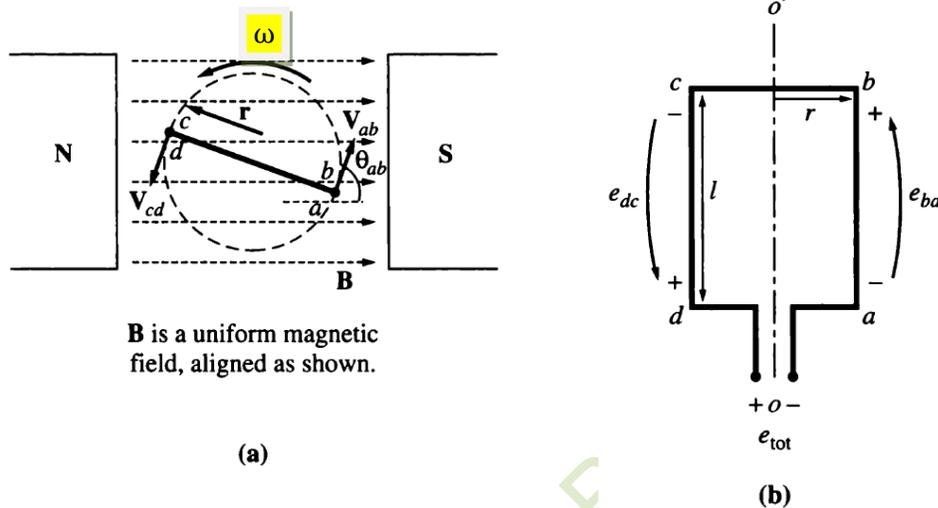
The AC machine has **two main parts**:

- a) **Rotor:** the rotating part of the machine,
- b) **Stator:** the stationary part of the machine, and handles the high current!

- ➔ The **field circuits of most AC machines are located on their rotor**.

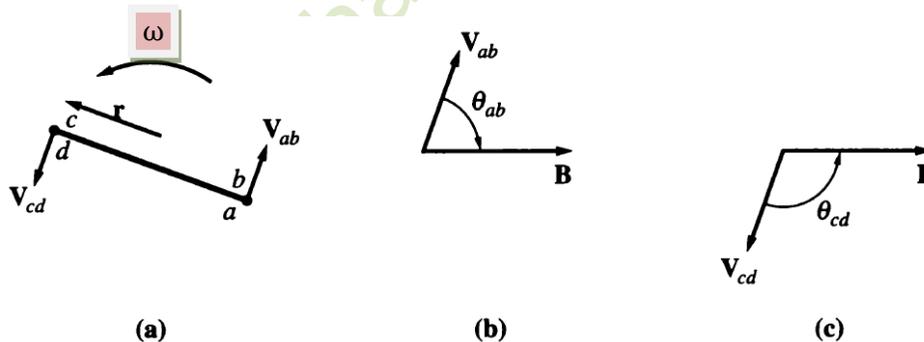
A Simple Rotating Loop between Flat Pole Faces

- Consider a simple loop placed between flat pole faces of a uniform magnetic field, as shown in Figure (a) below. The top view of the loop is shown in Figure (b) below.



- The loop is rotated at an angular velocity (ω) in the counter clockwise rotation.
- The magnetic flux density (B) is uniform between the pole faces.

Voltage Induced in the Simple Loop



- When the loop is turned at an angular velocity (ω) around the axis $o - o'$, a voltage will be induced at its terminals, which is the sum of individual voltages induced in each segment of the loop.

- The tangential velocity is:

$$v = \omega r$$

where ' r ' is the radius of the loop; from the axis of rotation to the edge of the loop.

- Each segment of the loop will have an induced voltage in it;

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

- The total induced voltage in the loop (e_{tot}) is the summation of the induced voltages in individual segments. The individual segments are:

1. Segment 'ba'

$$e_{ba} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

$$e_{ba} = vBl \sin \theta_{ab};$$

It is **positive into the page** as shown in the Figure next.

2. Segment 'cb'

$(\vec{v} \times \vec{B})$ is either into the page or out of the page, whilst \vec{l} is in the plane of the page; i.e., $(\vec{v} \times \vec{B}) \perp \vec{l}$. Therefore,

$$(\vec{v} \times \vec{B}) \cdot \vec{l} = 0$$

Thus,

$$e_{cb} = 0$$

3. Segment 'dc'

$$e_{dc} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

$$e_{dc} = vBl \sin \theta_{cd};$$

It is **positive out of the page**, as shown in the Figure next.

4. Segment 'ad'

$(\vec{v} \times \vec{B})$ is either into the page or out of the page, whilst \vec{l} is in the plane of the page. Therefore,

$$(\vec{v} \times \vec{B}) \cdot \vec{l} = 0$$

$$e_{ad} = 0$$

The total induced voltage, therefore, is:

$$e_{tot} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$e_{tot} = vBl \sin \theta_{ab} + 0 + vBl \sin \theta_{cd} + 0$$

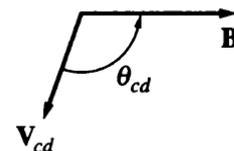
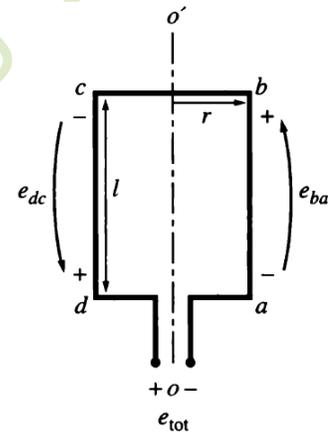
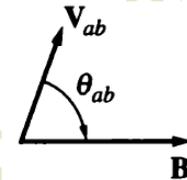
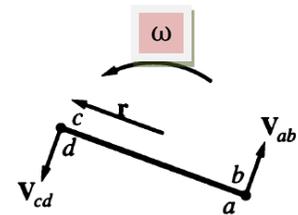
$$e_{tot} = vBl \sin \theta_{ab} + vBl \sin \theta_{cd}$$

But,

$$\theta_{ab} = 180 - \theta_{cd}$$

Recall the triangular identity,

$$\sin \theta_{ab} = \sin(180 - \theta_{cd}) = \sin \theta$$

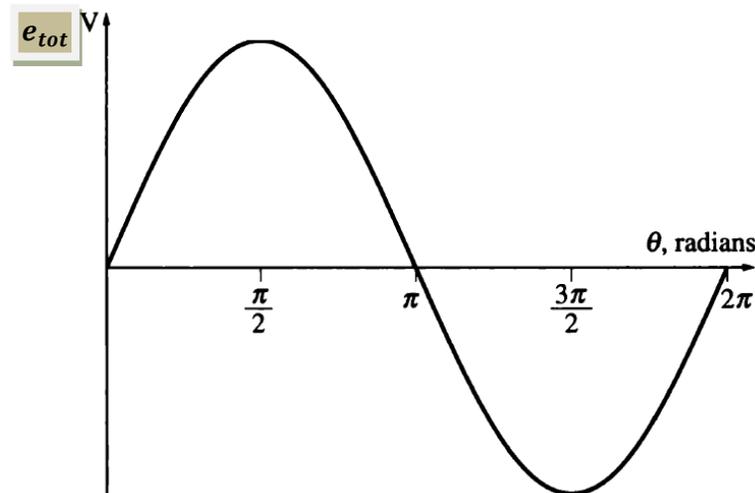


Consequently,

$$e_{tot} = vBl \sin(180 - \theta_{cd}) + vBl \sin \theta_{cd}$$

$$e_{tot} = 2vBl \sin \theta$$

- The total voltage is maxima when $\theta = 90^\circ$; i.e. the loop in the horizontal position.



- If the loop is rotated at an angular velocity (ω), then the angle (θ) will increase linearly with time;

$$\theta = \omega t$$

and the tangential velocity is:

$$v = \omega r$$

where, ' r ' is the radius of the loop; from the axis of rotation to the edge of the loop.

- Then, e_{tot} can be expressed in terms of the angular velocity as:

$$e_{tot} = 2\omega rBl \sin \omega t$$

But, the area of the loop is:

$$A = 2rl$$

Then,

$$e_{tot} = AB\omega \sin \omega t$$

The maximum flux through the loop (ϕ) occurs when the area is perpendicular to the magnetic flux density (B), and is:

$$\phi = AB$$

Thus,

$$\Rightarrow e_{tot} = \phi \omega \sin \omega t$$

For number of loop (turns), the maximum voltage is:

$$\Rightarrow e_{tot} = e_{max} \sin \omega t$$

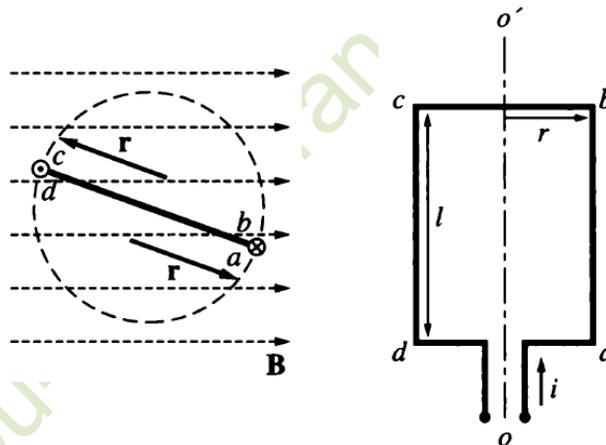
where,

$$e_{max} = k\phi\omega$$

Therefore, the induced voltage depends on the flux in the machine, speed of rotation, and a constant (k) representing the construction of the machine such as shape, and number of turns,...

Induced Torque in a Current-Carrying Loop

Assume that, the rotor loop is at some arbitrary angle (θ) with respect to the magnetic flux density (B), and that a DC current is flowing in the loop as shown in the Figure below. The magnetic flux density (B) is uniform, and is pointing from left to right.



Note:

- ⊗ means that the current is flowing into the page
- ⊙ means that the current is flowing out of the page

The force on each segment of the loop is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

The torque induced on each segment of the loop is:

$$\tau = \text{The Perpendicular Distance} \times \text{force applied}$$

$$\tau = \vec{r} \times \vec{F}$$

$$\tau = rF \sin \theta$$

where, θ is the angle between the line of action of \vec{F} and \vec{r} .

The torque induced on the loop is the net torque of individual torques on each loop segment;

1- **Segment 'ab'**

The force on the segment is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

$$\vec{F} = ilB;$$

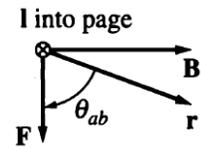
Downwards!

The resulting torque is:

$$\tau_{ab} = rF \sin \theta_{ab}$$

$$\tau_{ab} = rilB \sin \theta_{ab};$$

Clockwise!



(a)

2- **Segment 'bc'**

The force on the segment is:

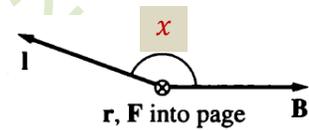
$$\vec{F} = i(\vec{l} \times \vec{B})$$

$$\vec{F} = ilB \sin x;$$

Into the page!

Since the force is parallel to the axis of rotation $\mathbf{o} - \mathbf{o}'$, the torque

τ_{bc} is zero!



$$\tau_{bc} = 0$$

(b)

3- **Segment 'cd'**

The force on the segment is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

$$\vec{F} = ilB;$$

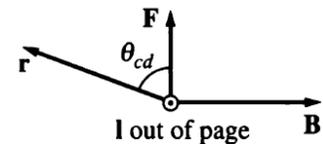
Upwards!

The resulting torque is:

$$\tau_{cd} = rF \sin \theta_{cd}$$

$$\tau_{cd} = rilB \sin \theta_{cd};$$

Clockwise!



(c)

4- **Segment 'da'**

The force on the segment is:

$$\vec{F} = i(\vec{l} \times \vec{B})$$

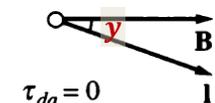
$$\vec{F} = ilB \sin y;$$

Out of the page!

Again, since the force is parallel to the axis of rotation $\mathbf{o} - \mathbf{o}'$, the torque

τ_{da} is zero!

\mathbf{r}, \mathbf{F} out of page



$$\tau_{da} = 0$$

(d)

The **net torque induced** on the loop (τ_{ind}) is the sum of torques on individual loop segments; i.e.,

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da}$$

$$\tau_{ind} = rilB \sin \theta_{ab} + 0 + rilB \sin \theta_{cd} + 0$$

But, $\theta_{ab} = \theta_{cd} = \theta$,

$$\therefore \tau_{ind} = 2rlBi \sin \theta; \quad \text{Clockwise}$$

Note that, the area of the loop is:

$$A = 2rl$$

$$\tau_{ind} = ABi \sin \theta; \quad \text{Clockwise} \quad \dots\dots\dots(8)$$

And the maximum flux in the loop is:

$$\phi = AB$$

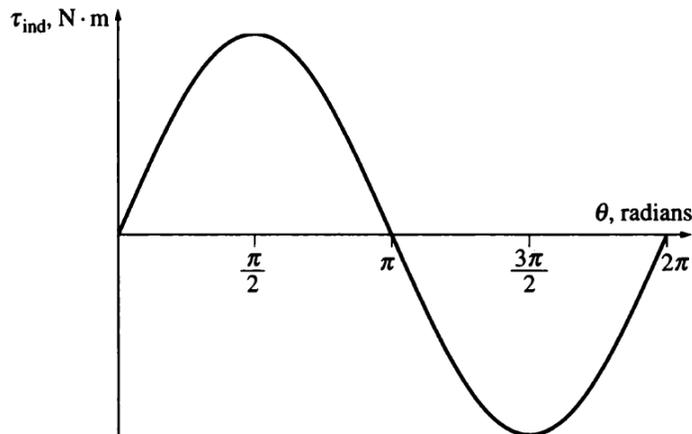
Therefore, the net induced torque is:

$$\tau_{ind} = \phi i \sin \theta; \quad \text{Clockwise}$$

For number of turns in the loop, then the induced torque is:

$$\tau_{ind} = k\phi i \sin \theta; \quad \text{Clockwise}$$

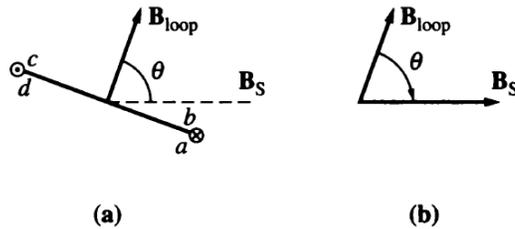
- ❖ Note that, the torque is maximum when the loop is in the horizontal position ($\theta = 90^\circ$), and the torque is zero when the loop is in the vertical position ($\theta = 0^\circ$ or 180°), as shown in the Figure below.



- ❖ The torque depends on the flux in the machine, the current in the machine loop, and on the construction of the machine; shape and number of turns...

Alternative Way to Express The Torque

- ➔ Assume that, the loop is subject to a stator magnetic flux density (\vec{B}_S), which is pointing to the right.
- ➔ The current in the loop generates a magnetic flux density (\vec{B}_{loop}), whose direction is perpendicular to the loop surface, as shown in the Figure below.



- ➔ The loop magnet flux density is proportional to its current (i) and can be found from **Biot-Savart Law**;

$$B_{loop} = \frac{\mu i}{G}$$

where G is a factor depending on the loop geometry; for example it is $2r$ for a circle!

- ➔ Thus, the loop current, i ,

$$i = \frac{GB_{loop}}{\mu}$$

- ➔ Substituting ' i ' in the torque equation (eq. (8)) yields:

$$\tau_{ind} = A \frac{GB_{loop}}{\mu} B_S \sin \theta$$

- ➔ Let $K = \frac{AG}{\mu}$, then:

$$\tau_{ind} = KB_{loop}B_S \sin \theta; \quad \text{Clockwise}$$

- ➔ θ is also the angle between \vec{B}_{loop} and \vec{B}_S !

- ➔ Therefore, the induced torque can be expressed as:

$$\tau_{ind} = k\vec{B}_{loop} \times \vec{B}_S$$

k is a constant, which depends on the construction of the machine and its geometry.

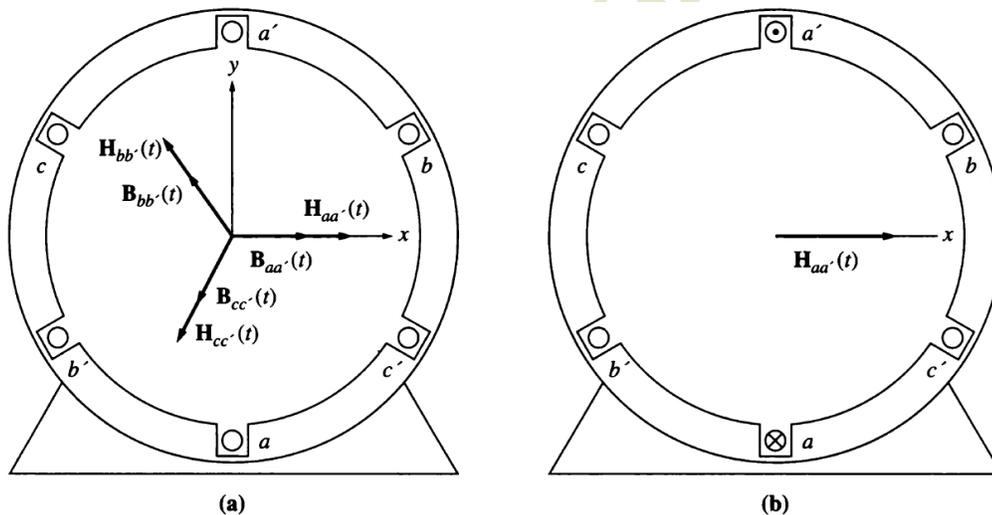
- ➔ The torque produced is into the page implying that the torque is clockwise (rotate from \vec{B}_{loop} to \vec{B}_S to find the torque direction).
- ➔ The magnetic fields tend to align (lineup) themselves with each other.

➔ **The Torque Induced in any Real Machine depends on:**

- The strength of the rotor magnetic field,
- The strength of the external (stator) magnetic field,
- The sine of the angle between the two magnetic fields,
- A constant 'k' which depends on the construction of the machine (geometry).

Rotating Magnetic Field

- It was shown that, the torque tends to turn the rotor (loop) and align itself with the stator magnetic field.
- If the stator magnetic field was rotating, then the rotor will chase it and rotate.
- Consider a three-phase windings spaced 120° electrical degrees (also 120° mechanical degrees for two-pole machine) apart around the surface of the stator of the machine.



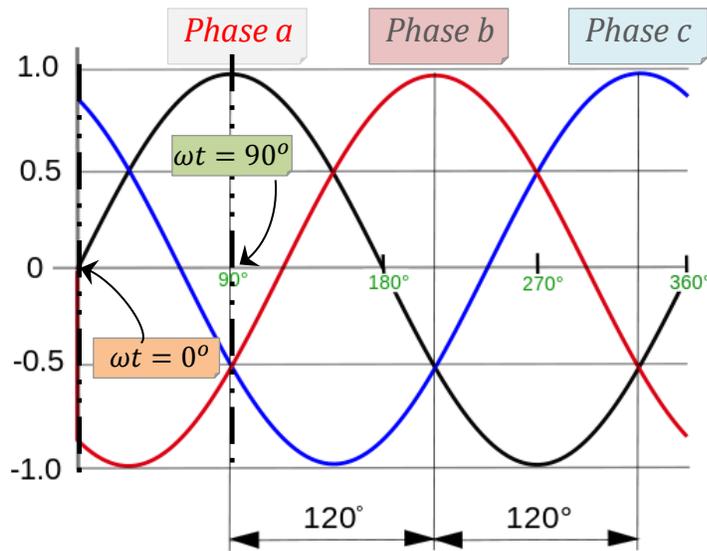
- The current is assumed to be positive when entering the unprimed end, and vice versa.
- Assuming that, the three-phase currents applied to the coils are:

$$i_{aa'}(t) = I_m \sin \omega t \text{ A}$$

$$i_{bb'}(t) = I_m \sin(\omega t - 120) \text{ A}$$

$$i_{cc'}(t) = I_m \sin(\omega t + 120) \text{ A}$$

- The current in coil (aa') flows into "a – end" of the coil, and out of the " a' – end" of the coil, and so forth for the other coils...



- ◆ The currents produce magnetic field intensities:

$$\bar{H}_{aa'}(t) = H_m \sin \omega t \angle 0^\circ \text{ A.turns/m}$$

$$\bar{H}_{bb'}(t) = H_m \sin(\omega t - 120^\circ) \angle 120^\circ \text{ A.turns/m}$$

$$\bar{H}_{cc'}(t) = H_m \sin(\omega t + 120^\circ) \angle -120^\circ \text{ A.turns/m}$$

- ◆ The directions of magnetic field intensities can be found by the right hand rule and are shown in the Figure next.

- ◆ But,

$$\bar{B} = \mu \bar{H}$$

Therefore, the magnetic flux densities are:

$$\bar{B}_{aa'}(t) = B_m \sin \omega t \angle 0^\circ \text{ T}$$

$$\bar{B}_{bb'}(t) = B_m \sin(\omega t - 120^\circ) \angle 120^\circ \text{ T}$$

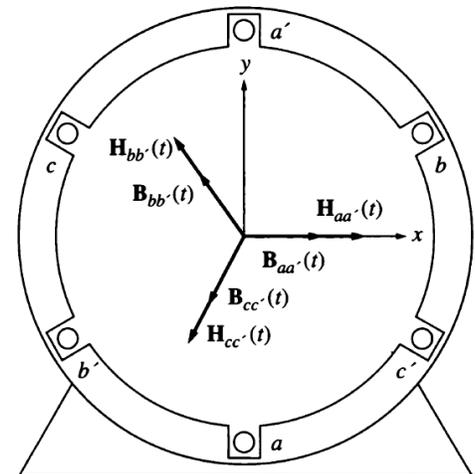
$$\bar{B}_{cc'}(t) = B_m \sin(\omega t + 120^\circ) \angle -120^\circ \text{ T}$$

where, $B_m = \mu H_m$!

- ◆ The net magnetic flux density is:

$$\bar{B}_{net} = \bar{B}_{aa'} + \bar{B}_{bb'} + \bar{B}_{cc'}$$

- ◆ Within a cycle, the direction of \bar{B}_{net} depends on the instantaneous values of individual magnetic flux densities, which in turn depend on the instantaneous currents.



- ◆ For example, at $\omega t = 0^\circ$,

$$\bar{B}_{aa'}(t) = 0 \angle 0^\circ \text{ T}$$

$$\bar{B}_{bb'}(t) = B_m \sin(-120) \angle 120^\circ \text{ T}$$

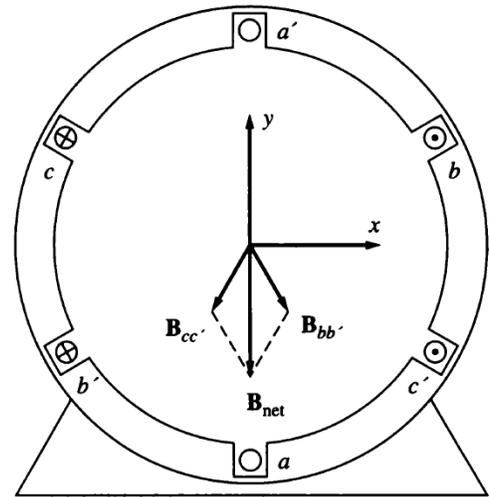
$$\bar{B}_{cc'}(t) = B_m \sin(+120) \angle -120^\circ \text{ T}$$

Thus,

$$\bar{B}_{net} = 0 + B_m \left(\frac{-\sqrt{3}}{2}\right) \angle 120^\circ + B_m \left(\frac{\sqrt{3}}{2}\right) \angle -120^\circ$$

...

$$\bar{B}_{net} = 1.5 B_m \angle -90^\circ$$



$\omega t = 0^\circ$

- ◆ But, at $\omega t = 90^\circ$,

$$\bar{B}_{aa'}(t) = B_m \angle 0^\circ \text{ T}$$

$$\bar{B}_{bb'}(t) = B_m \sin(90 - 120) \angle 120^\circ \text{ T}$$

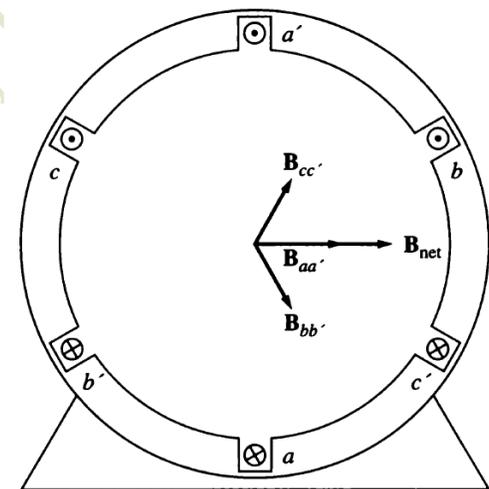
$$\bar{B}_{cc'}(t) = B_m \sin(90 + 120) \angle -120^\circ \text{ T}$$

Thus,

$$\bar{B}_{net} = B_m + B_m(-0.5) \angle 120^\circ + B_m(-0.5) \angle -120^\circ$$

...

$$\bar{B}_{net} = 1.5 B_m \angle 0^\circ$$



$\omega t = 90^\circ$

- ◆ A similar procedure can be followed to find out the magnitude and direction of the magnetic flux density at other angles.
- ◆ The **resulting magnetic flux density (\bar{B}_{net}) has a constant magnitude ($1.5 B_m$), but rotating counter clockwise at an angular velocity (ω).**
- ◆ **Balanced three-phase currents (equal magnitudes, 120° apart in phase angle, and have the same frequency) produce a rotating magnetic field of a constant magnitude if they were allowed to flow in three-phase windings spaced 120° electrical degrees apart around the surface of the machine.**
- ◆ The **direction of rotation of the magnetic field may be reversed by swapping the phase currents in two coils!**

Relationship between Electrical Frequency and Speed of Magnetic Field Rotation

✦ The order of stator windings sequence, $a - c' - b - a' - c - b'$, produces one north and one south poles, which complete one mechanical rotation around the stator surface for each electrical cycle of the applied current.

✦ The Figure next shows the location of poles at $\omega t = 90^\circ$!

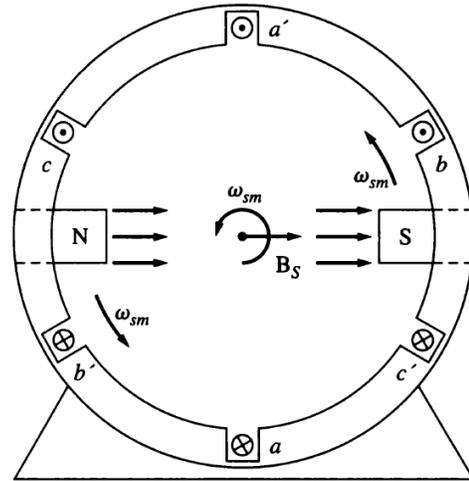
✦ Therefore, the **mechanical speed of rotation of the magnetic field in revolutions per second is equal to the electric frequency in Hertz.**

✦ Therefore, for a two pole machine,

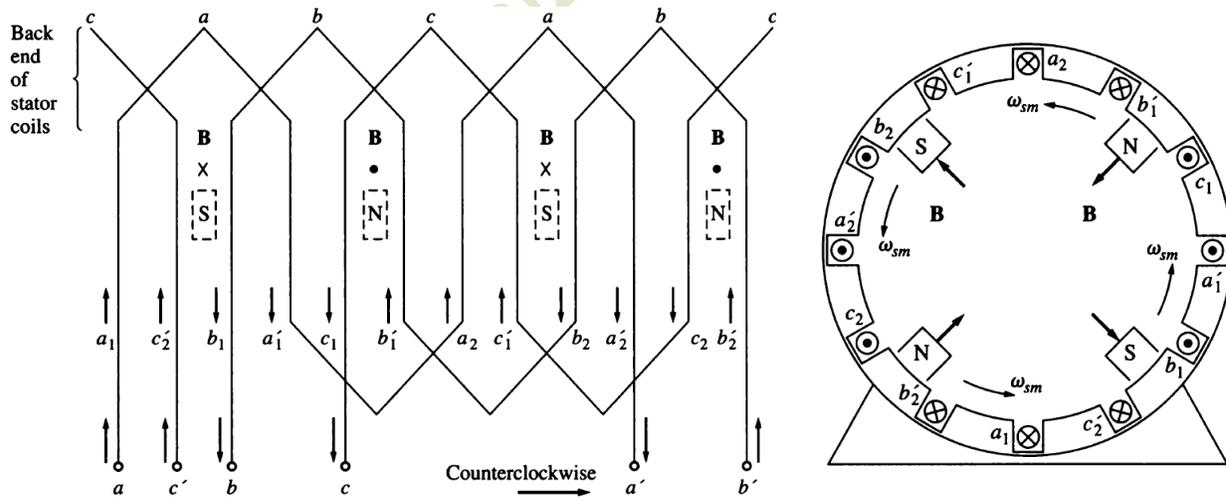
$$f_e = f_m$$

$$\omega_e = \omega_m$$

where, f_e is measured in *Hz*, f_m is measured in *revolution/s*, ω_e and ω_m are both measured in *rad/s*.



✦ If the pattern of stator windings sequence was repeated twice; $a - c' - b - a' - c - b' - a - c' - b - a' - c - b'$, then a four-pole machine results as shown in the Figure below.



✦ Three-phase currents applied to the latter stator windings **produce two north and two south poles.**

- In this winding sequence, four-pole windings, the pole moves only half way around the stator surface in one electrical cycle; i.e., 360° electrical degrees produce 180° mechanical degrees, then:

$$\theta_e = 2\theta_m$$

$$f_e = 2f_m$$

$$\omega_e = 2\omega_m$$

- In general, if the number of poles in an AC machine stator is P , then there are $\frac{P}{2}$ repetitions of the winding sequence $a - c' - b - a' - c - b'$ around its inner surface, and

$$\theta_e = \frac{P}{2}\theta_m$$

$$\omega_e = \frac{P}{2}\omega_m$$

$$f_e = \frac{P}{2}f_m$$

Since, $f_m = \frac{n_m}{60}$, then:

$$f_e = \frac{P n_m}{2 \cdot 60}$$

$$f_e = \frac{P n_m}{120}$$

$$n_m = \frac{120 f_e}{P};$$

in Revolution Per Minute [rpm]

Reversing the Direction of Magnetic Field Rotation

If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed; i.e., **the direction of rotation of an AC motor can be reversed by switching (swapping) the connections of any of two of the three coils.**

Induced Voltage in a Three-Phase Set of Coils

- Assuming that, the rotor is rotated at an angular velocity (ω) and has a uniform and a constant magnetic flux density (B), then **the flux density in the airgap varies sinusoidally** with the mechanical angle, as shown in the Figure next; i.e.,

$$\bar{B} = B_M \sin \alpha$$

- If the reference of α was selected as in the second Figure below, then:

$$\bar{B} = B_M \cos \alpha$$

$$\bar{B} = B_M \cos \omega t$$

- If three coils each of N_c turns are placed around the rotor magnetic field, as before, then the voltage induced in each of them will be the same in magnitude, but will differ in phase by 120° !

- The resulting voltages in the three coils are:

$$e_{aa'}(t) = N_c \phi \omega \sin \omega t \text{ V}$$

$$e_{bb'}(t) = N_c \phi \omega \sin(\omega t - 120) \text{ V}$$

$$e_{cc'}(t) = N_c \phi \omega \sin(\omega t + 120) \text{ V}$$

- Therefore, **a three-phase set of currents can generate a uniform magnetic field in a machine stator, and a uniform rotating magnetic field can generate a three-phase set of voltages in such a stator.**

- The maximum voltage in each phase is:

$$E_{max} = N_c \phi \omega$$

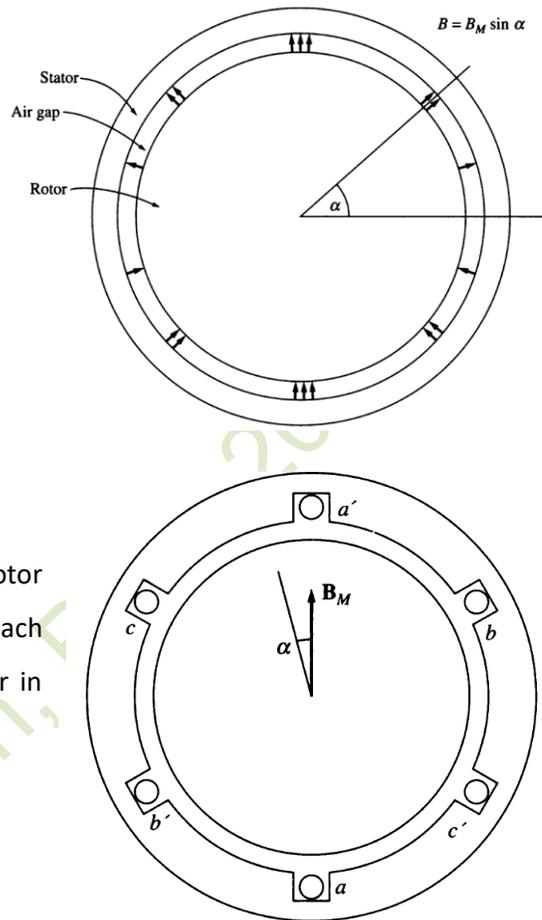
$$E_{max} = N_c \phi (2\pi f)$$

- The rms value of any phase voltage is:

$$E_A = \sqrt{2} \pi f N_c \phi \quad \text{or} \quad E_A = k \phi \omega$$

- The terminal voltage depends on the stator windings' connection;

- For Y: $V_T = \sqrt{3} E_A$
- For Δ : $V_T = E_A$



Induced Torque in an AC Machine

- The interaction between the stator magnetic field and the rotor magnetic field produces a torque in the machine (both magnetic fields tend to line up with each other).
- Assuming that, the stator flux has a sinusoidal distribution and peaks upwards as shown in the Figure next.

$$\bar{B}_s = B_s \sin \alpha$$

- Assuming also that, a single coil of wire is mounted on the rotor.
- The force induced on the right coil segment is:

$$\bar{F}_{ind1} = i(\bar{l} \times \bar{B})$$

$$\bar{F}_{ind1} = ilB_s \sin \alpha; \quad \text{Upwards!}$$

The resulting torque is:

$$\tau_{ind1} = rilB_s \sin \alpha; \quad \text{CCW}$$

Similarly, the torque induced on the other segment is:

$$\tau_{ind2} = rilB_s \sin \alpha; \quad \text{CCW}$$

- The net torque induced is:

$$\tau_{ind} = 2rilB_s \sin \alpha; \quad \text{CCW}$$

- The torque can be derived also by finding the rotor magnetic field, which is proportional to the rotor current (i); i.e.,

$$\bar{H}_R = k_x i$$

where, k_x is a constant.

- Note that, the angles are related by:

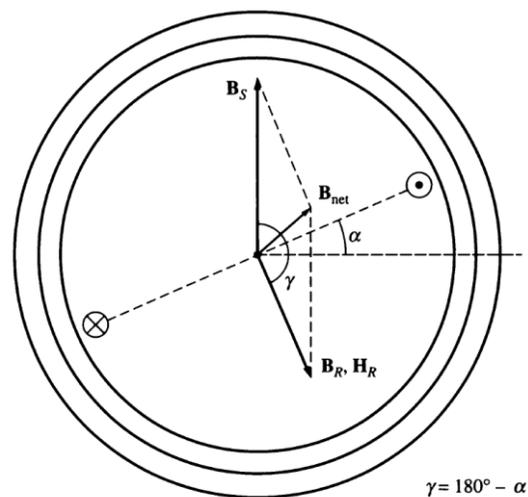
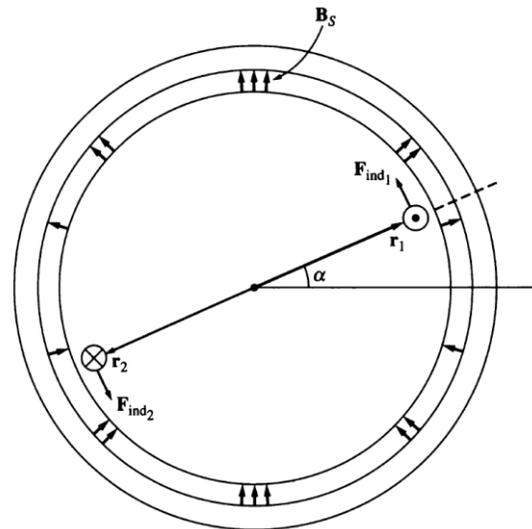
$$\gamma = 180 - \alpha$$

- Hence, $\sin \gamma = \sin(180 - \alpha) = \sin \alpha$

- Therefore,

$$\tau_{ind} = K_y H_R B_s \sin \gamma$$

$$\tau_{ind} = K_y \bar{H}_R \times \bar{B}_s$$

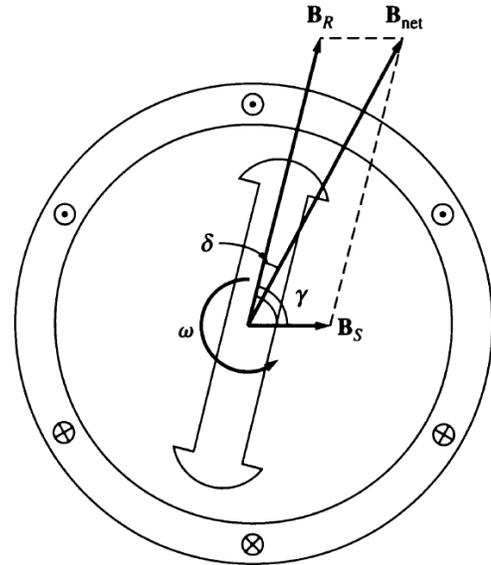


- Since, $\bar{B}_R = \mu\bar{H}_R$, then the torque can be re-expressed as:

$$\tau_{ind} = k\bar{B}_R \times \bar{B}_S$$

- But, the net magnetic field is,

$$\begin{aligned}\bar{B}_{net} &= \bar{B}_R + \bar{B}_S \\ \Rightarrow \bar{B}_S &= \bar{B}_{net} - \bar{B}_R \\ \therefore \tau_{ind} &= k\bar{B}_R \times (\bar{B}_{net} - \bar{B}_R) \\ \tau_{ind} &= k\bar{B}_R \times (\bar{B}_{net}) - k\bar{B}_R \times (\bar{B}_R)\end{aligned}$$



Note that, the cross product of a vector by itself is zero!

$$\tau_{ind} = k\bar{B}_R \times \bar{B}_{net}$$

$$\tau_{ind} = kB_R B_{net} \sin \delta$$

where, δ is called the torque angle, and it is the angle between \bar{B}_R and \bar{B}_{net} !

- Note:**

When the torque induced is opposite to the direction of rotation of the rotor, the machine operates as a generator. Whilst, when both the torque induced and speed are in the same direction then the machine is operating as a motor.

Winding Insulation in an AC Machine

- Winding insulation breakdown due to **overheating can be partially prevented** by providing a cooling air circulation over them.
- A rule of thumb, the **life expectancy** of a motor with a given type of insulation **is halved** for each **10% rise** in temperature above the rated temperature of windings.
- **Insulation System Classes** according to the **National Electrical Manufacturers Association (NEMA)** in USA, for a continuous operation, are:

Insulation System Class	Temperature Limit in Degree Celsius
<i>Class A</i>	60
<i>Class B</i>	80
<i>Class F</i>	105
<i>Class H</i>	125

- There are other standards defined by **International Electromechanical Commission (IEC)**.

AC Machine Efficiency and Losses

The **efficiency** is defined as:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} \times 100\%$$

Or

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

Types of Losses in AC Machines:

1. Electrical or Copper Losses ($I^2 R$):

They are losses occurring in the resistance of the windings of the machine;

For a three-phase machine, the stator (armature) copper losses are:

$$P_{SCL} = 3I_A^2 R_A; \quad R_A: \text{The armature (stator) winding resistance}$$

The rotor (field) copper losses:

$$P_{RCL} = I_F^2 R_F; \quad R_F: \text{The field (rotor) winding resistance}$$

2. **Core Losses:** They are Hysteresis and Eddy Current losses occurring in the metal of the machine. They are dependent of B^2 and $n^{1.5}$; n is the speed of rotation of magnetic fields.

3. **Mechanical Losses:** they are friction losses caused by bearings and windage losses caused by the friction between moving parts and the air. They vary as a cube of rotational speed (n^3).

$$\text{No Load Rotational Losses} = \text{Core Losses} + \text{Mechanical Losses}$$

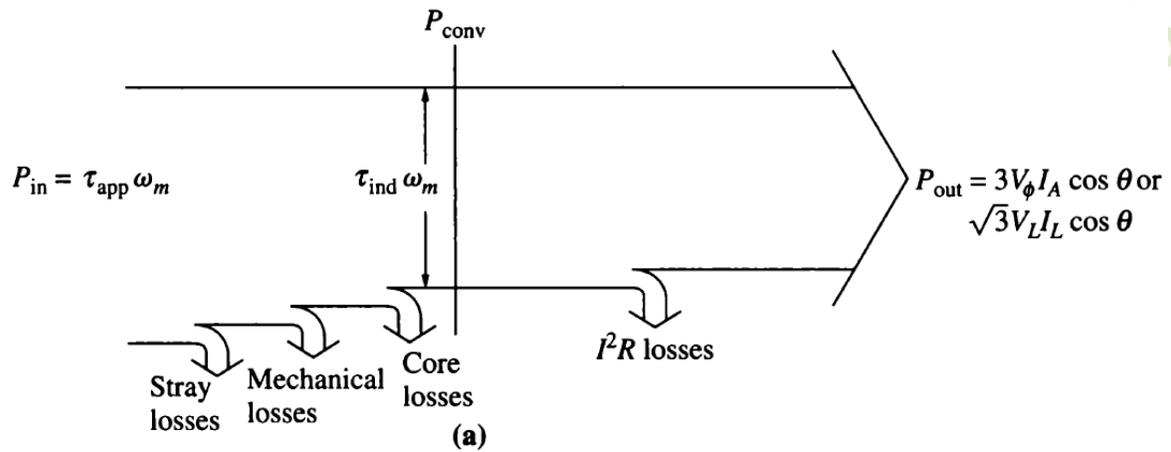
4. **Stray (Miscellaneous) Losses:** They are losses which are not accounted for. They represent 1% of full load power.

Power Flow in an AC Machine

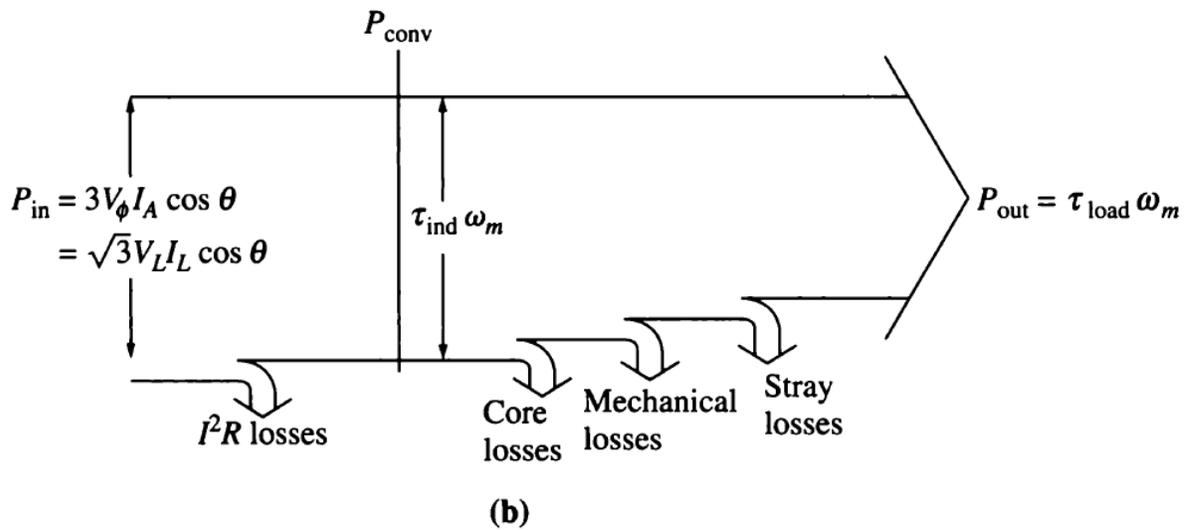
The power that is converted from mechanical power to electrical power, or vice versa, is:

$$P_{conv} = \tau_{ind} \omega_m$$

For a generator, the power flow diagram is shown in the Figure below.



For a motor, the power flow diagram is shown in the Figure below.



Voltage Regulation (for generators)

- The Voltage Regulation is a measure of the ability of a generator to keep a constant voltage at its terminals as the load varies. It is a measure of the output voltage–current characteristic.
- The Voltage Regulation is defined as:

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

- Ideally, $VR = 0\%$!
- If $VR > 0$, then the voltage drops as the load increases. On the other hand, if $VR < 0$, then the terminal voltage increases as the load increases.

Speed Regulation (for motors)

- The Speed Regulation is a measure of the ability of a motor to keep a constant speed as the load torque varies. It is a measure of the torque–speed characteristic.
- The Speed Regulation is defined as:

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\%$$

- Ideally, $SR = 0\%$; e.g. Synchronous motors!
- If $SR > 0$, then the motor speed drops as the load torque increases (as is the case in most motors). On the other hand, if $SR < 0$, then the motor speed increases as the load torque increases, which is the case in unstable motors (e.g. a Differentially Compounded DC motor).

Part IV

Synchronous Generators

Introduction:

Synchronous Generators or Alternators are Synchronous Machines used to convert mechanical power to AC electric power.

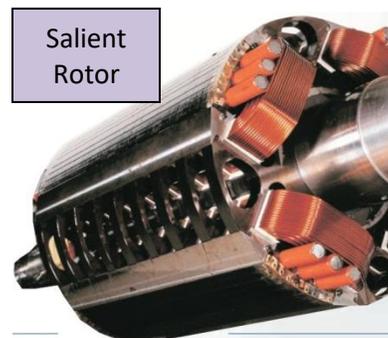
Synchronous Generator Construction:

It consists of a stationary part called a **stator** and a rotating part called a **rotor**.

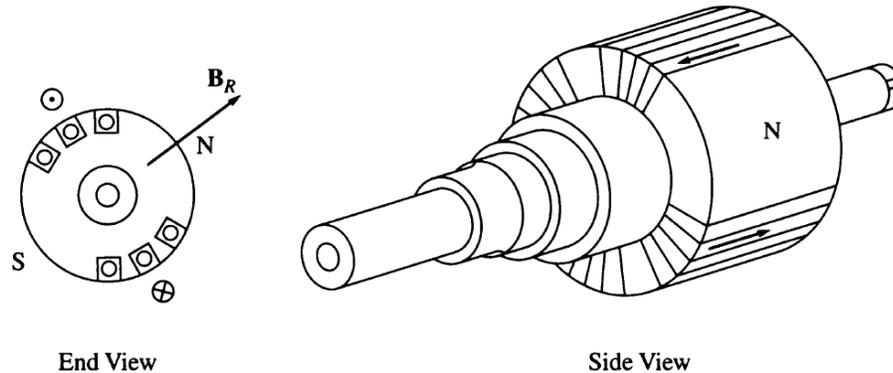
- ➔ DC current is applied to the rotor winding to produce a rotor magnetic field,
- ➔ The rotor is turned by a Prime Mover producing a rotating magnetic field within the machine.
- ➔ The rotating magnetic field induces three phase voltages in the stator windings.
- ➔ **Windings:**
 1. **Field Winding (Rotor Winding):** produces the main magnetic field
 2. **Armature Winding (Stator Winding):** where the main voltages are induced

Types of Rotors:

1. **Salient Pole Rotor:** The Magnetic poles are striking out from the surface of the rotor (machines with a number of poles greater than 4).



2. Cylindrical (Nonsalient) Pole rotor: the magnetic poles are constructed flush with the surface of the rotor (machines with number of poles 4 or 2).



Notes:

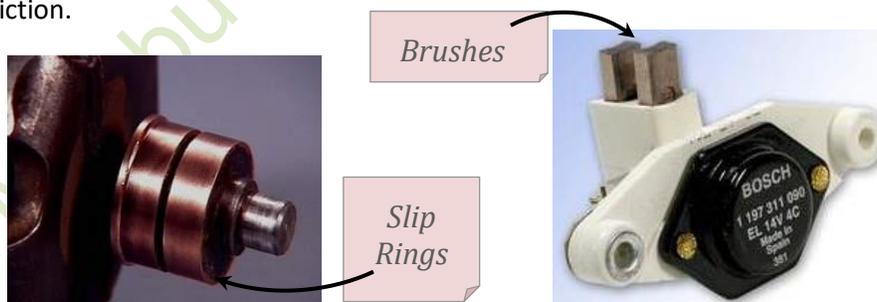
- ◆ The rotor is subject to changing magnetic fields, thus it is constructed from laminations to reduce eddy currents!
- ◆ The number of rotor poles equals the number of stator poles to interlock with each other.

Methods for Supplying a DC Current to the Rotor Windings:

1. External DC Power Supply by means of Slip Rings and Brushes

Slip Rings: are metal rings completely encircling the shaft of the machine but insulated from it.

Brushes: Blocks of Graphite-like Carbon Compound that conduct electricity freely, but have very low friction.

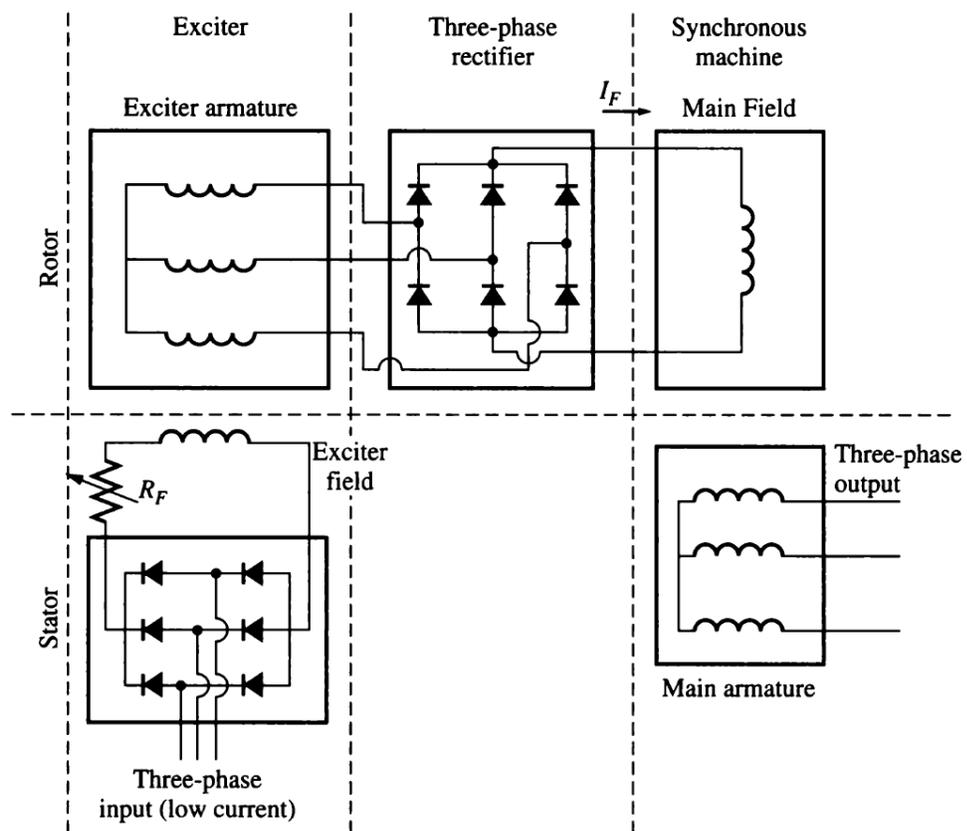


However, this arrangement has two main problems:

- a. Brushes need regular maintenance and must be checked for wear.
- b. Brushes' voltage drop causes losses especially with very large currents.

2. Brushless Exciter

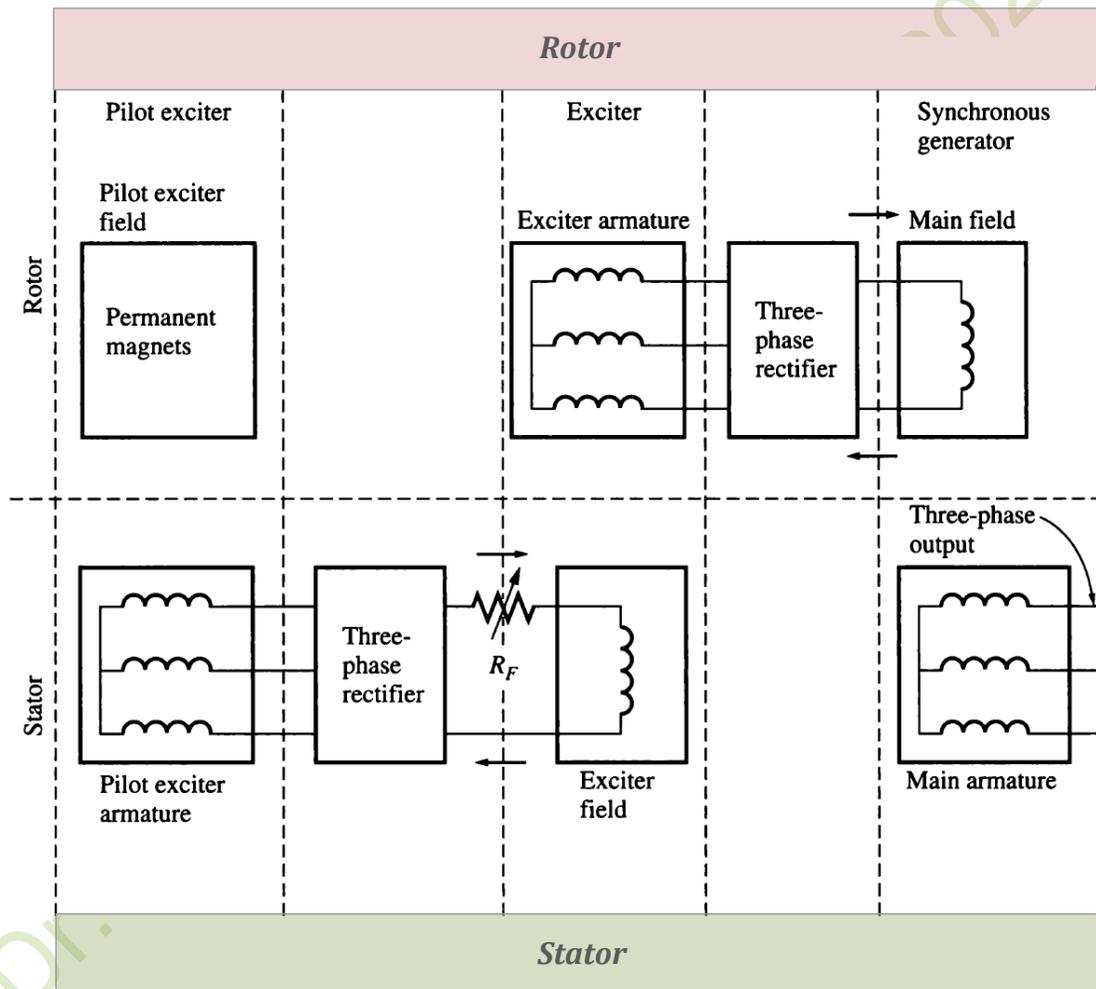
- It is used in Large AC Machines
- It is a small AC generator with its field mounted on the stator and the armature mounted on the rotor.
- The layout of a Brushless Exciter is shown in the Figure below.
- The three phase out of the exciter generator is rectified by a three-phase rectifier, also mounted on the shaft of the generator, and then is fed in the main DC field circuit.
- The exciter field itself is fed from an external three phase supply of a low power.



3. Brushless Exciter with a Pilot Exciter

A small Pilot Exciter is added to the Brushless exciter to make the excitation of the generator completely independent of any external circuit, as shown in the Figure below.

The **Pilot Exciter** is a small AC generator with Permanent Magnets mounted on the rotor and three phase windings on the stator. It produces the power for the field circuit of the exciter, which in turn controls the field circuit of the main machine, as illustrated in the Figure.



Speed of Rotation of a Synchronous Generator

The electrical frequency produced is locked or synchronized to the mechanical speed of rotation of a Synchronous generator and related by:

$$f_e = \frac{n_m P}{120} \quad \text{or}$$

$$n_m = \frac{120f_e}{P}$$

where f_e = electrical frequency in Hz

P = number of poles

n_m = mechanical speed of the rotor in r/min

Examples: For 2 poles and 50Hz, the speed of rotation is 3000rpm

For 4 poles and 50Hz, the speed of rotation is 1500rpm

For 2 poles and 60Hz, the speed of rotation is 3600rpm

For 4 poles and 60Hz, the speed of rotation is 1800rpm

Internally Generated Voltage of a Synchronous Generator

The magnitude of the voltage induced in a given stator phase is:

$$E_A = k\phi\omega$$

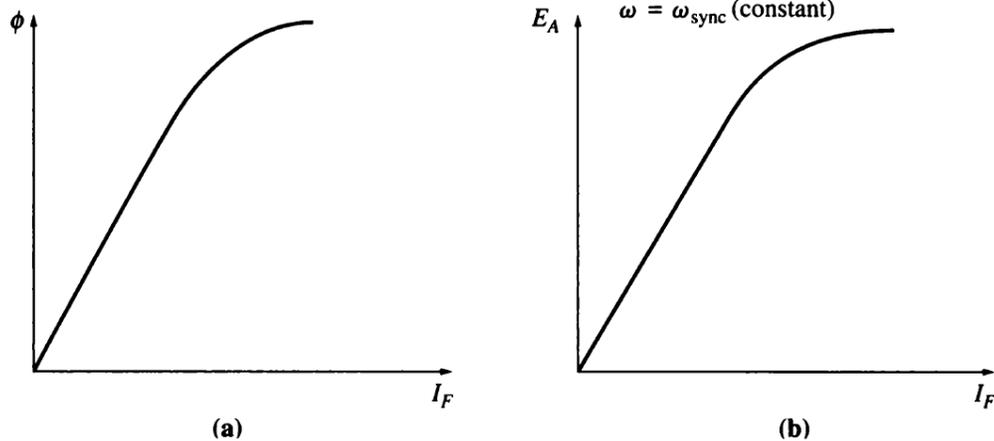
where,

k is a constant representing the machine construction; geometry, number of turns,...

ω is the angular speed in rad/s

ϕ is the magnetic flux in Tesla

E_A is directly proportional to the flux, and the flux is related to the field current (I_f); i.e. E_A is directly proportional to the field current. Hence, ϕ versus I_f , magnetization characteristics can be scaled (by $k\omega$) to produce a plot of E_A versus I_f , as shown in the Figure below.



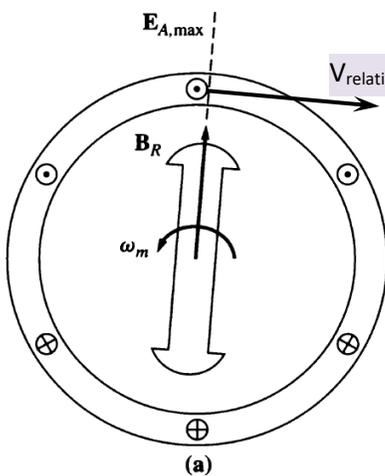
Equivalent Circuit of a Synchronous Generator

The terminal phase voltage (V_ϕ) is different from the internally generated voltage (E_A), except when the generator is unloaded. This is due to four factors:

1. **Armature Reaction Effect:** the distortion of the airgap magnetic field by the current flowing in the stator
2. The **Self Inductance** of the Armature (stator) coils
3. The **Resistance** of the Armature (stator) coils
4. The effect of salient pole rotor shape; **saliency effect** will be ignored in the analysis. Saliency produces X_d and X_q .

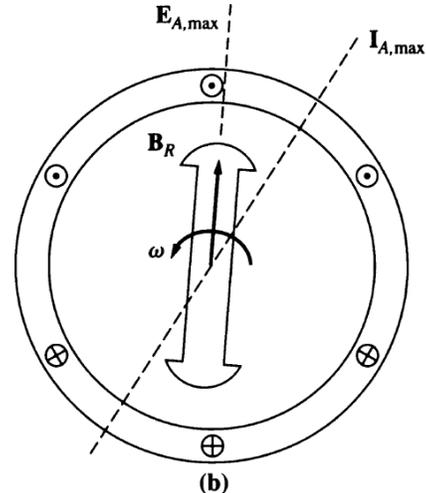
1. Armature Reaction Effect:

The load current flowing in the stator windings produces a magnetic field, which distorts the original rotor magnetic field, and this is called **Armature Reaction Effect**. Hence, the distortion will change the resulting voltage.



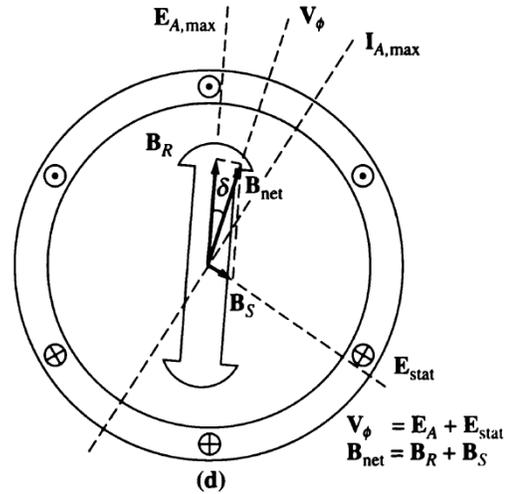
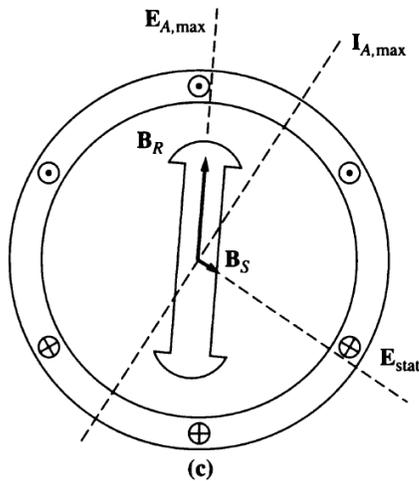
➤ The peak of the voltage ($E_{A,max}$) coincides with the direction of B_R .

➤ Note: $e_{ind} = (\overline{V_{relative}} \times \overline{B_R}) \cdot \vec{L}$



➤ The voltage E_A produces a lagging current when connected to a load with a lagging power factor.

➤ The peak current ($I_{A,max}$) will occur at an angle behind the peak voltage ($E_{A,max}$).



- The stator current, I_A , produces its own magnetic field, B_S , which produces a voltage (E_{stat}) in the stator windings.
- E_{stat} is the armature reaction voltage.
- The peak of the voltage (E_{stat}) coincides with the direction of B_S .

- ◆ The field B_S adds to B_R distorting it into B_{net}
- ◆ The voltage E_{stat} adds to the voltage E_A producing V_ϕ .
- ◆ E_{stat} lags $I_{A,max}$ by 90°

The total terminal phase voltage (V_ϕ) is the sum of voltages:

$$\overline{V_\phi} = \overline{E_A} + \overline{E_{stat}}$$

And the net magnetic field is:

$$\overline{B_{net}} = \overline{B_R} + \overline{B_S}$$

Modeling the Armature Reaction Effect is based on the following facts:

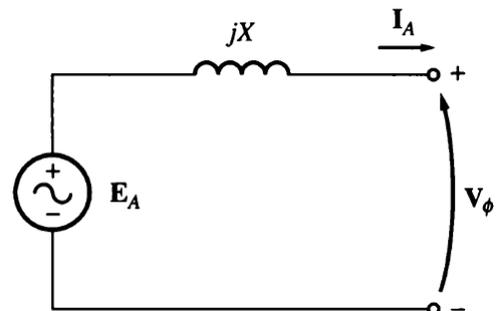
E_{stat} lags $I_{A,max}$ by 90° , and E_{stat} is directly proportional to I_A . Therefore, E_{stat} can be expressed as:

$$\overline{E_{stat}} = -jX\overline{I_A}$$

The total terminal phase voltage (V_ϕ) becomes:

$$\Rightarrow \overline{V_\phi} = \overline{E_A} - jX\overline{I_A}$$

The Armature Reaction Effect is modeled by an inductor connected in series with the internally generated voltage E_A , as shown in the Figure next.



2. & 3. The Self Inductance and Resistance of Stator Windings

- ➔ The stator coil has a self inductance L_A and a resistance R_A .
- ➔ The total difference in voltage between E_A and V_ϕ is then:

$$\overline{V_\phi} = \overline{E_A} - jX\overline{I_A} - jX_A\overline{I_A} - R_A\overline{I_A}$$

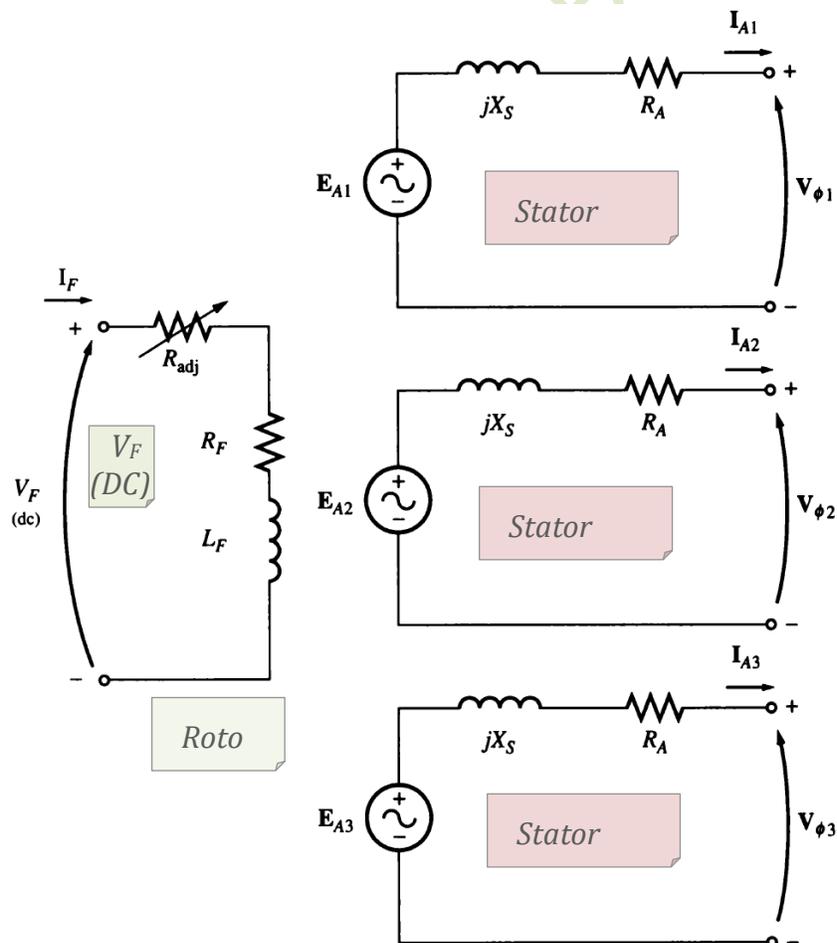
- ➔ The **Armature Reaction Effect and the self inductance are combined together in the synchronous reactance of the machine (X_s)**;

$$X_s = X + X_A$$

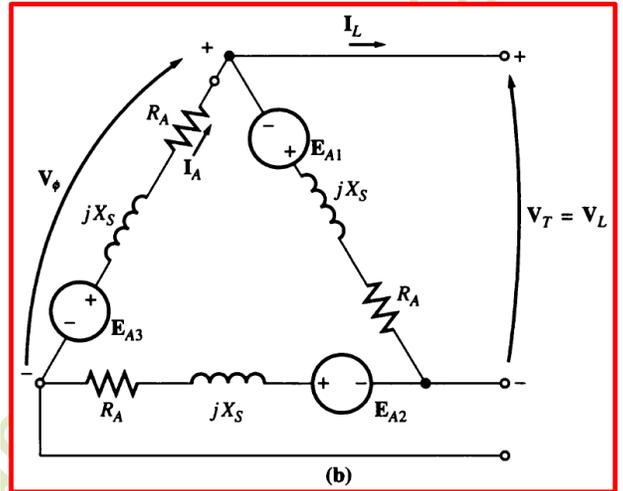
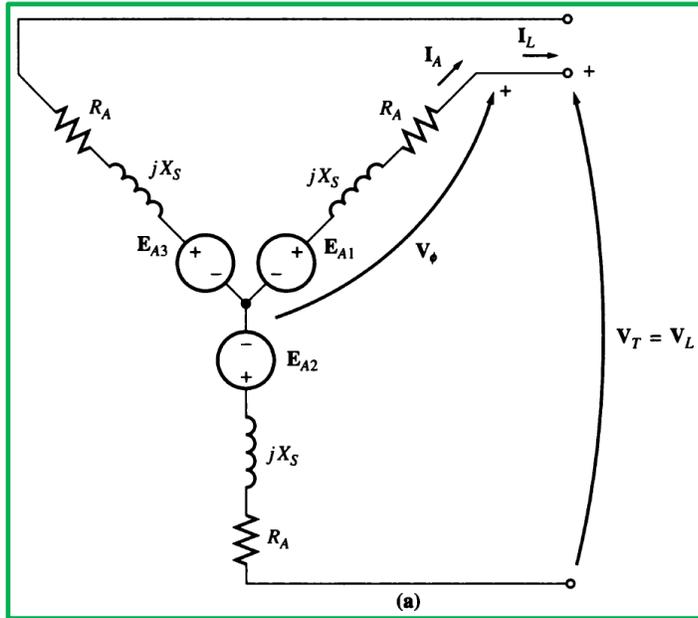
Therefore, the terminal phase voltage becomes:

$$\overline{V_\phi} = \overline{E_A} - jX_s\overline{I_A} - R_A\overline{I_A}$$

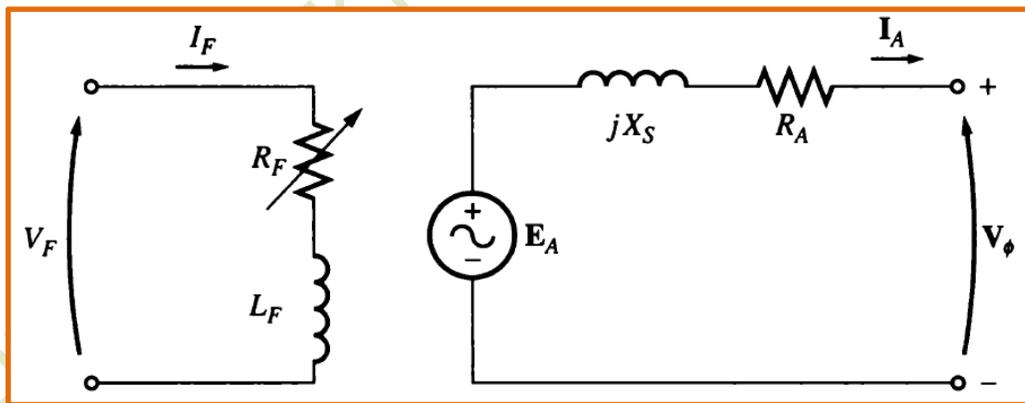
- ➔ The complete equivalent circuit of the Synchronous generator is:



- The stator windings may be connected as Y or Δ .
- The terminal line voltage depends on the connection of stator windings;
- For Y-connection: $\overline{V_T} = \sqrt{3} \overline{V_\phi} \angle 30^\circ$
- For Δ -connection: $\overline{V_T} = \overline{V_\phi}$

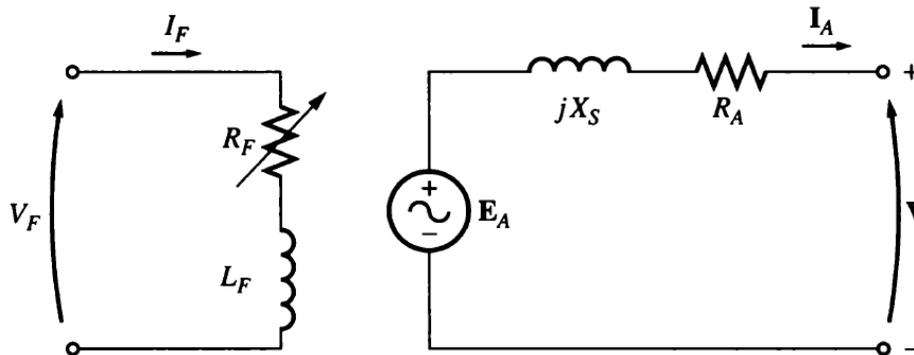


- For a balanced load, analyzing the per-phase equivalent circuit is sufficient.



The Phasor Diagram of a Synchronous Generator

Recall, the per-phase equivalent circuit:



Applying KVL for stator in the per-phase equivalent circuit yields:

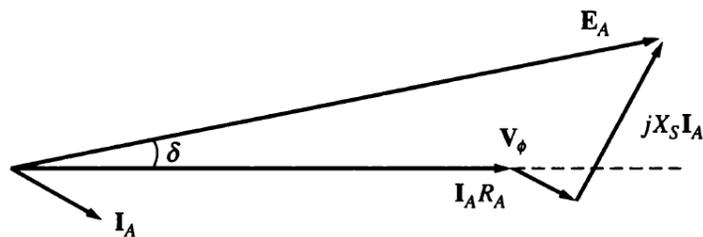
$$\overline{V}_\phi = \overline{E}_A - jX_S \overline{I}_A - R_A \overline{I}_A$$

Assuming \overline{V}_ϕ is the reference and is kept at zero angle, the phasor diagram representing the above equation for various power factors is as follows:

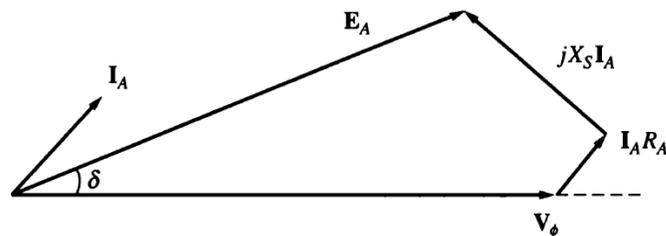
1. Unity Power Factor



2. Lagging Power Factor



3. Leading Power Factor



Comments:

- ◆ For a given V_ϕ and I_A , a larger E_A is needed for a lagging power factor; a larger field current is needed because $E_A = k\phi\omega = k'I_f\omega$.
- ◆ In other words, for a given field current (i.e. E_A) and magnitude of Armature current (I_A), the terminal voltage (V_ϕ) is lower for a lagging power factor and is higher for a leading power factor.
- ◆ Usually R_A is neglected in studying voltage variation because $R_A \ll X_S$.

Power and Torque in a Synchronous Generator

The input power to the generator from the Prime Mover (Diesel Engine, Steam Turbine, Gas Turbine, Hydraulic Turbine, ...) is:

$$P_{in} = \tau_{app}\omega_m$$

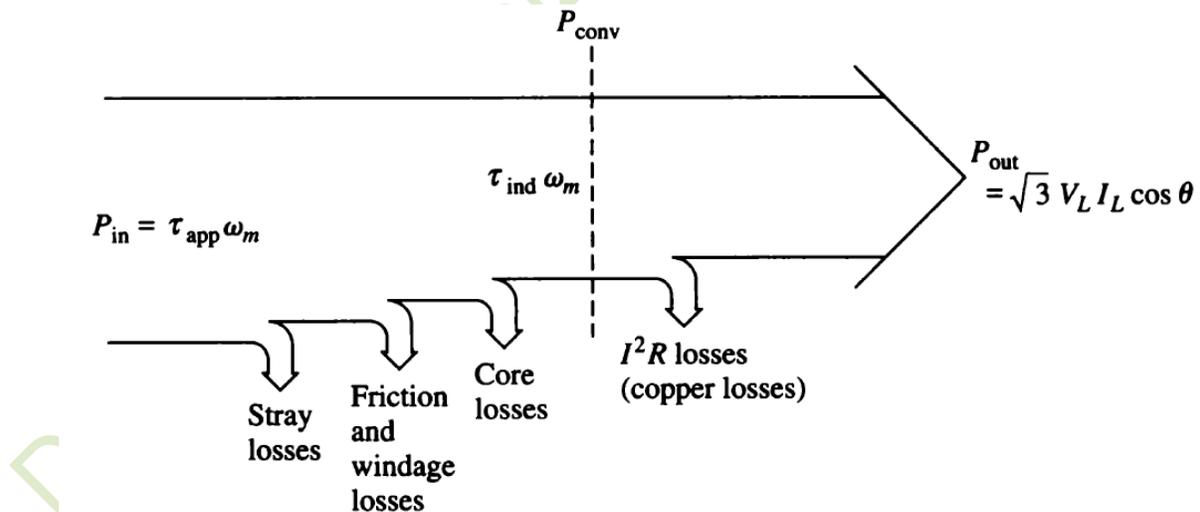
The power converted from mechanical to electrical form is:

$$P_{conv} = \tau_{ind}\omega_m$$

$$P_{conv} = 3 E_A I_A \cos \gamma$$

where γ is the angle between E_A and I_A

The Power Flow Diagram in a Synchronous Generator is:



The **Real Output Power** is:

$$P_{out} = \sqrt{3}V_T I_L \cos \theta$$

$$\text{or } P_{out} = 3V_\phi I_A \cos \theta$$

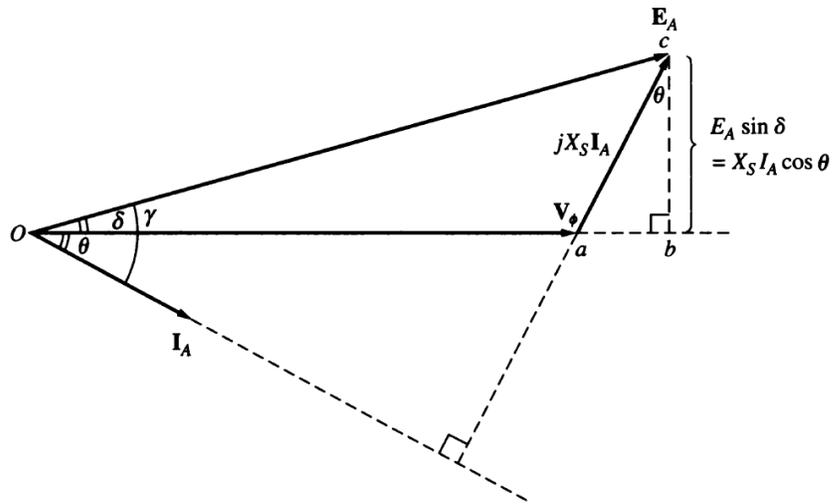
The **Reactive Output Power** is:

$$Q_{out} = \sqrt{3}V_T I_L \sin \theta$$

$$\text{or } Q_{out} = 3V_\phi I_A \sin \theta$$

A Simplified Power Formula:

If $R_A \ll X_S$ and can be ignored, then the phasor diagram becomes as shown next.



From the Figure,

$$X_S I_A \cos \theta = E_A \sin \delta$$

or

$$I_A \cos \theta = \frac{E_A \sin \delta}{X_S}$$

Substituting for $I_A \cos \theta$ in the Real output power equation yields;

$$P_{out} = 3V_\phi \frac{E_A \sin \delta}{X_S}$$

$$\Rightarrow P_{out} = \frac{3V_\phi E_A}{X_S} \sin \delta$$

where δ is the torque angle and is typically between 15° and 20° .

When the torque angle $\delta = 90^\circ$, the **maximum Real power a generator can supply occurs**,

$$P_{max} = \frac{3V_\phi E_A}{X_S}$$

which is called the **Static Stability Limit** of the generator.

A Simplified Torque Formula:

It was shown earlier that,

$$\tau_{ind} = k \overline{B_R} X \overline{B_s}$$

or

$$\tau_{ind} = k \overline{B_R} X \overline{B_{net}}$$

The magnitude of the torque is:

$$\tau_{ind} = k B_R B_{net} \sin \delta$$

where δ is the angle between the rotor magnetic field and the net magnetic field.

Since B_R produces E_A , and B_{net} produces V_ϕ , the angle δ between E_A and V_ϕ is the same as the angle δ between B_R and B_{net} .

Because R_A is assumed zero; no copper losses, then:

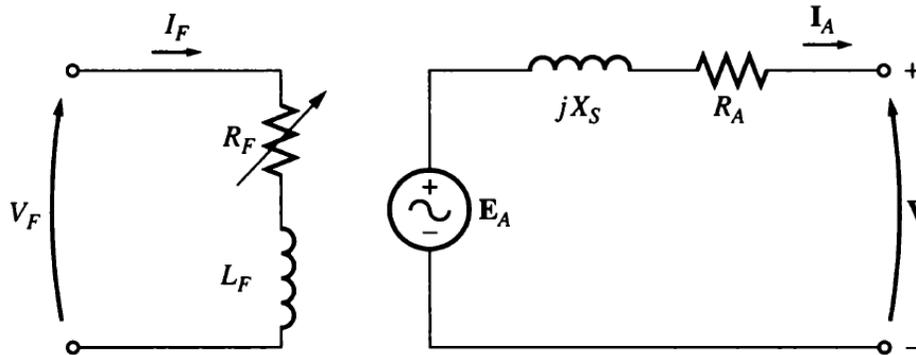
$$P_{conv} = P_{out}$$
$$\tau_{ind} \omega_m = \frac{3V_\phi E_A}{X_s} \sin \delta$$

$$\tau_{ind} = \frac{3V_\phi E_A}{\omega_m X_s} \sin \delta$$

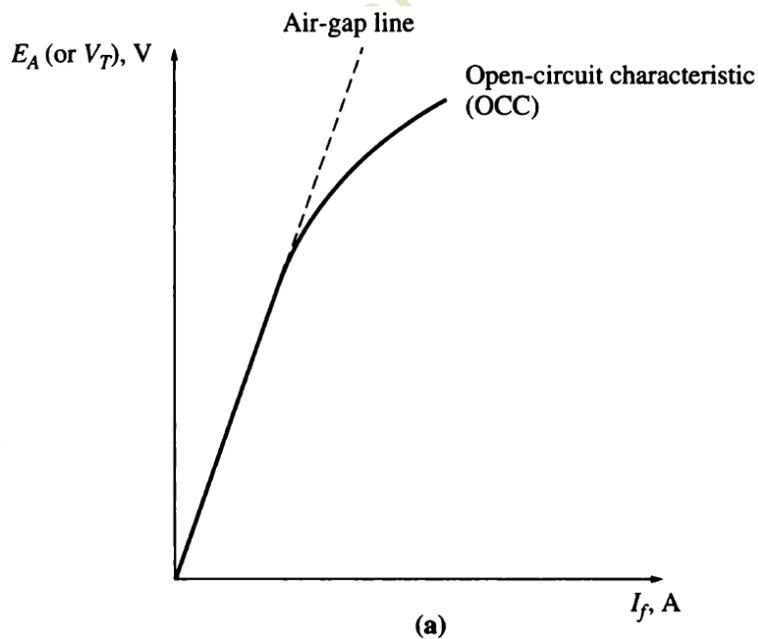
Measuring the Synchronous Generator Model Parameters

1. Open Circuit Test

- It is conducted at rated speed and at no-load.

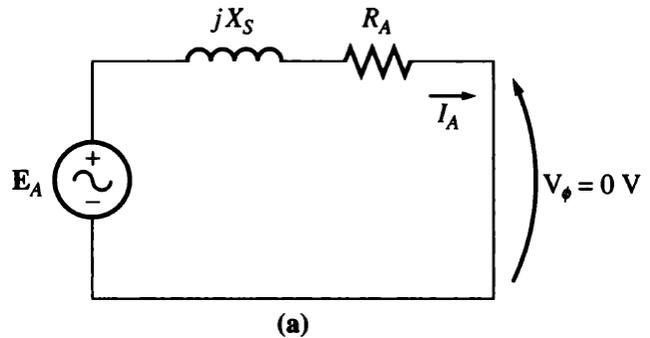


- Initially, set the field current (I_f) to zero, then increase it in steps and measure the terminal voltage (V_ϕ), which equals E_A at no-load
- Plot E_A (or V_ϕ or V_T) versus I_f ; the Open-Circuit Characteristic of the generator (OCC)
- The Open-Circuit Characteristic (OCC) of the generator is shown in the Figure below:



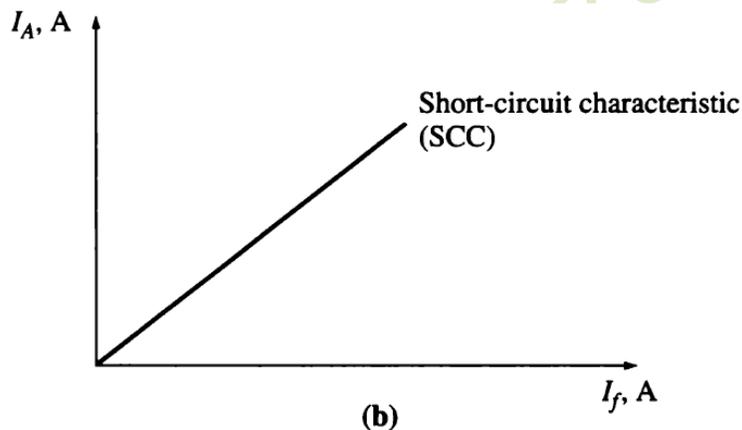
2. Short Circuit Test

- Initially, adjust the field current (I_f) to zero
- Short circuit the generator terminals through Ammeters
- When the terminals are shorted, the Armature current is:

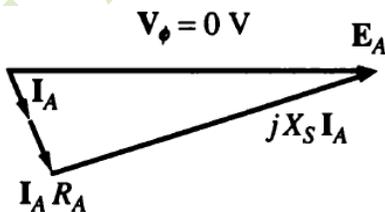


$$I_A = \frac{E_A}{R_A + jX_S}$$

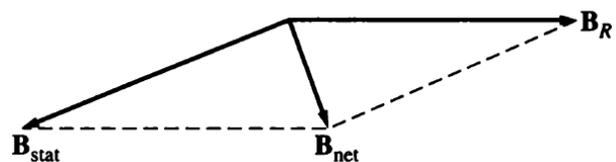
- Increase the field current (I_f) and measure the Armature current (I_A)!
- Plot I_A versus I_f ; the Short Circuit Characteristic of the generator (SCC) is as shown below,



- Note that, the SCC is linear and the Machine does not saturate due to small B_{net} .



Phasor Diagram during Short Circuit



Magnetic Fields during Short Circuit

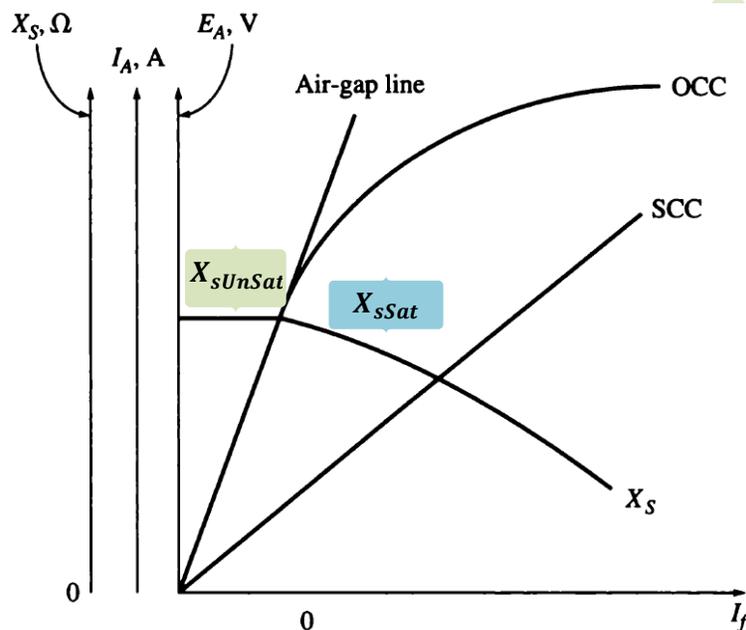
- Note also that, the internal impedance of the machine is:

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A}$$

- Since $X_S \gg R_A$ then,

$$X_S \approx \frac{E_A}{I_A} = \frac{V_{\phi,oc}}{I_A}$$

- For any particular value of I_f , E_A and I_A are known, therefore X_S can be found.
- A Plot of X_S versus I_f is shown below.

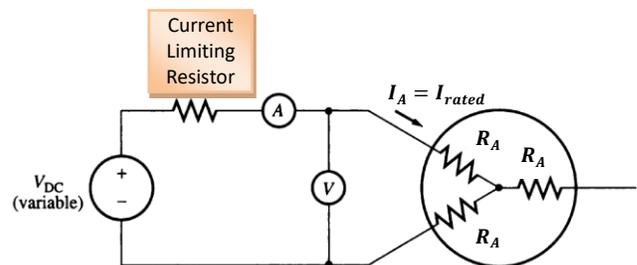


- The **unsaturated X_S (X_{sUnSat}) is greater than the saturated X_S (X_{sSat}).**

3. DC Test

R_A can be found by applying a DC voltage to the armature circuit, while the machine is stationary, **until rated current flows**, to heat the windings to operating Temperature, and measuring the DC current. Neglecting the skin effect,

$$R_A = \frac{1 V_{dc}}{2 I_{dc}}$$



The Synchronous Generator Operating Alone

The effect of load changes on a Synchronous Generator operating alone, which has a constant field current (constant flux, ϕ) and a constant Prime Mover speed (ω), and hence constant frequency; i.e. constant $E_A (= k\phi\omega)$, will be explored.

If the load is increased at the same power factor, then $|I_A|$ increases. Consequently, the voltage $jX_S I_A$ is larger than before, but at the same angle.

Neglecting R_A , KVL equation describing the per-phase equivalent circuit of the machine stator becomes:

$$\overline{V_\phi} = \overline{E_A} - jX_S \overline{I_A}$$

The effect of load changes is explored for the 3 cases of load power factor:

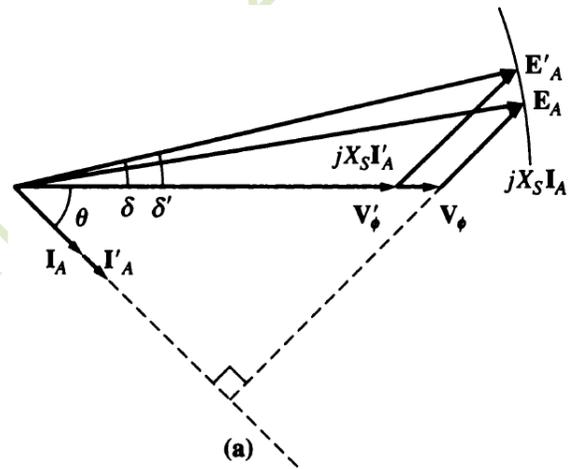
1. Lagging Power Factor

- As the load increases, the voltage V_ϕ decreases sharply.

- The voltage regulation:

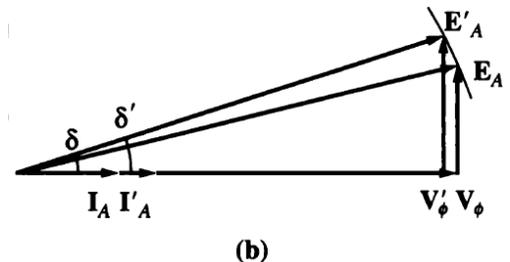
$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

is rather large and positive.



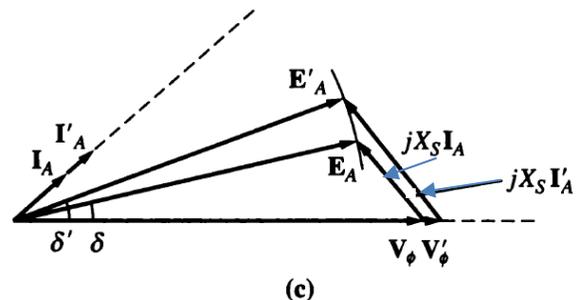
2. Unity Power Factor

- As the load increases, the voltage V_ϕ decreases slightly.
- VR is small and positive.



3. Leading Power Factor

- As the load increases, the voltage V_ϕ increases.
- VR is negative.



Notes:

It is desirable to keep the load voltage constant even with varying the load. The obvious approach is to vary the magnitude of the internally generated voltage E_A to compensate for load changes.

Since the Prime Mover speed is constant ($\omega = \text{constant}$), and because $E_A = k\phi\omega$, ϕ should be changed accordingly.

$$\text{as: } R_f \downarrow \Rightarrow I_f \uparrow \Rightarrow \phi \uparrow \Rightarrow E_A \uparrow \Rightarrow V_\phi \uparrow$$

or

$$\text{as: } R_f \uparrow \Rightarrow I_f \downarrow \Rightarrow \phi \downarrow \Rightarrow E_A \downarrow \Rightarrow V_\phi \downarrow$$

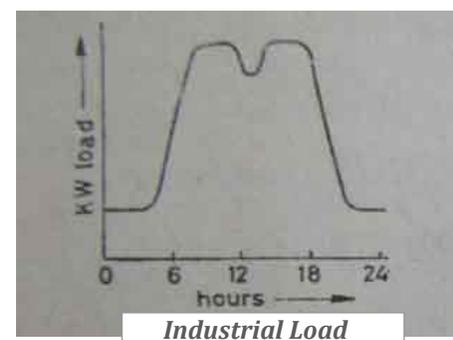
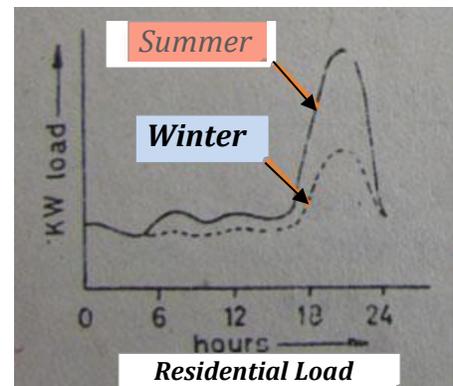
Thus, by adjusting the field current (I_f), the terminal voltage can be regulated. This can be achieved by the **Automatic Voltage Regulator (AVR)** in real Synchronous Generators.

Study Examples in Text Book!

Parallel Operation of AC Generators

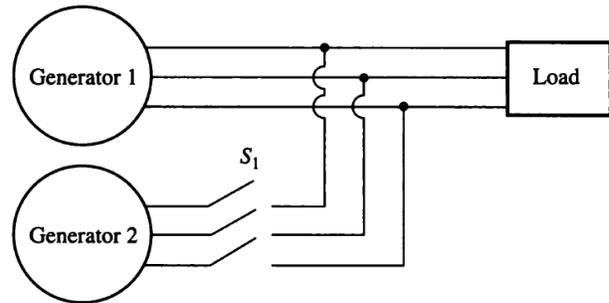
In power plants and large power systems, more than one generator is connected in parallel. Parallel operation has many advantages, some of these are:

- I. Several generators can **supply a bigger load** than one machine can do by itself.
- II. Several smaller generators allow **operation near full load, and thus more efficiently**. When one generator is not operating near full load, it will be relatively inefficient. Some units can be shut down when the load is low causing the remaining ones to operate near full load. Examples of daily load variation are shown in the Figures next.
- III. Having many generators connected in parallel **increases the reliability** of the power system; the failure of any one of them does not cause a total power loss to the load.
- IV. Parallel operation allows one or more of the generators to be removed for **shut down and maintenance**.



Conditions Required for Paralleling Generators

The closing time of a switch (S_1) connecting two generators together cannot be selected arbitrarily, otherwise the generators may be damaged due to huge currents and power transients!

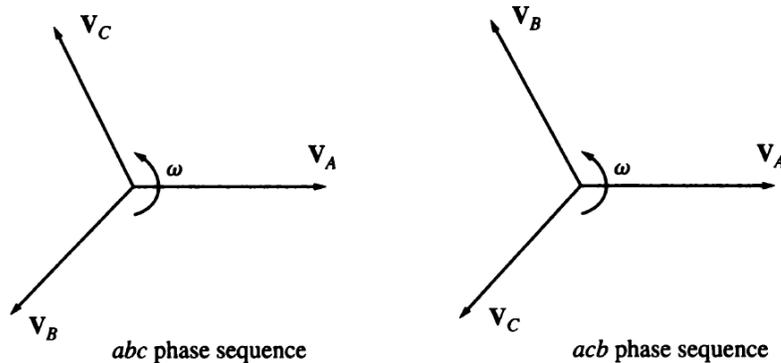


To avoid such a risk, each of the three phases must have exactly the same voltage magnitude and phase angle as the conductor to which it will be connected. To achieve this, the following conditions must be satisfied:

- I. The **rms line voltages** of the two generators must be equal
- II. The **phase angles** of the two "a"-phases must be equal
- III. The two generators must have the **same phase sequence** ('abc' or 'acb' sequence!)
- IV. The **frequency** of the new generator, Generator 2, (called the **Oncoming Generator**) must be **slightly higher** than the frequency of the running system (or generator)

Comments:

- ➔ Conditions 'I' and 'II' assures that the voltages are identical (in magnitude and angle) in both systems.
- ➔ If the phase sequence is different, there would be no problem in one phase, but the other two phases would have a huge current flow. **Swap any two phases!**



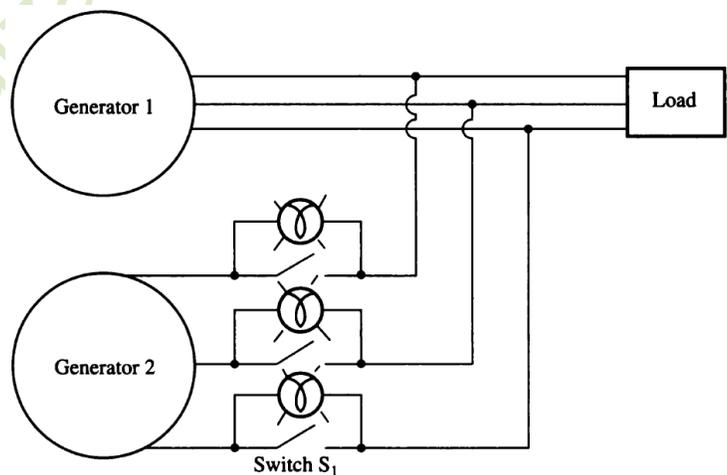
- ➡ If the frequencies of the two generators are not very nearly equal when they are connected together, large power transients will occur until the two generators stabilize at a common frequency.
- ➡ The frequencies of the two generators must differ by a small amount, so that the phase angles of the oncoming generator will change slowly with respect to the phase angles of the running system. When the systems are in phase the switch can be closed.

General Procedure for Paralleling Generators

1. Adjusting the field current of the oncoming generator to equate its terminal voltage with the system voltage. A **Voltmeter** can be used to measure the voltages.
2. Checking the phase sequence either by:
 - a) **A Small Induction Motor** connected to each system; the motor must have the same direction of rotation in each case.
 - b) **A Three-Light Bulb Method**

As the phase changes between the two systems, the light bulbs first get bright (a large phase difference) and then get dim (a small phase difference).

If all the light bulbs get bright and dark together, then the systems have the same phase sequence.



If the bulbs brighten in succession, then the systems have opposite phase sequences.

3. Observing the Frequency by a Frequency Meter

Adjust the frequency of the oncoming generator to be slightly higher than the system frequency.

Higher frequency of the oncoming generator's so that the oncoming generator comes on line supplying power as a generator, instead of consuming it as a motor.

Slightly higher to make the change in phase slow, and to be able to react in appropriate time.

4. Observing the Phase Changes in the Two Systems

The small difference in frequency, between the two systems, will cause the voltages in the two systems to change phase with respect to each other very slowly.

When the phases' angles are equal, the switch connecting the two systems together can be closed.

The two systems are in phase can be determined by:

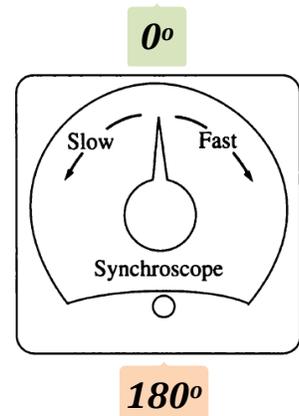
I. Observing Three-Light Bulbs:

When the three-light bulbs all go out (are off), the voltage difference across them is zero and the systems are in phase.

II. Employing a Synchroscope:

A Synchroscope is a meter that measures the difference in phase angle between the 'a' phases of the two systems. The Figure next shows the front view of a Synchroscope.

- ◆ When the dial (needle) is pointing at the top, the phase difference is zero, and when it is pointing at the bottom it is 180° .
- ◆ Since the frequencies of the two systems are slightly different, the phase angle on the meter changes slowly.
- ◆ If the oncoming generator is faster (has higher frequency) than the running system, the phase angle advances and Synchroscope needle rotates clockwise.
- ◆ If the oncoming generator is slower, the needle rotates counterclockwise.
- ◆ When the Synchroscope needle is pointing upwards in the vertical position, the voltages are in phase and the switch can be closed to connect both systems together.
- ◆ In large generators and power systems, the whole process is automated or computerized.



Frequency-Real Power Characteristic of a Synchronous Generator

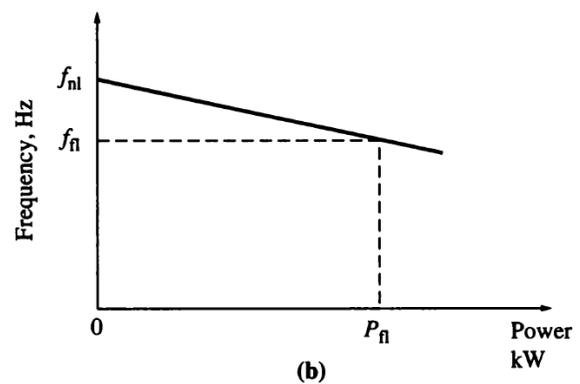
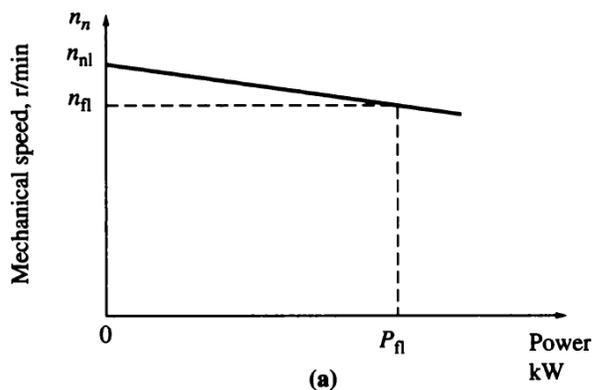
- ✚ The Prime Mover (Diesel Engine, Steam Turbine, Gas Turbine, Hydraulic Turbine, ...) is the source of input mechanical power to the generator.
- ✚ As the power drawn from the Prime Mover increases, the speed at which it turns decreases nonlinearly. But, the **Governor mechanism**, which is included in the Prime Mover, **tends to make the decrease in speed linear with an increase in load Real Power demand** from the generator.
- ✚ The Governor is adjusted to provide a slight drooping characteristic with increasing load power.
- ✚ The Speed Droop (SD) is expressed as:

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

where n_{nl} is the Prime Mover no-load speed

n_{fl} is the Prime Mover full load speed

- ✚ Typically, Speed Droop is in the range: **2% to 4%**
- ✚ Governors have some type of set-point adjustment to allow the no-load speed of the turbine (Prime Mover) to be varied.
- ✚ The frequency versus Real Power of a generator is a scaled plot of the speed versus Real Power of the Prime Mover, as illustrated in the Figures below.



- ✚ The Relationship between frequency and Real Power can be described quantitatively by:

$$P = S_P(f_{nl} - f_{fl})$$

where, P is the Real Power output of the generator.

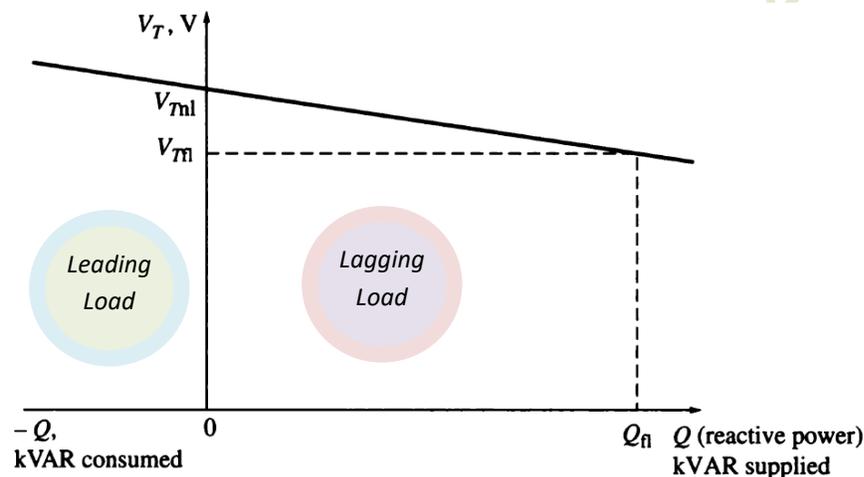
S_P is 1/slope of the curve measured in kW/Hz or MW/Hz.

f_{nl} is the no-load frequency of the generator.

f_{fl} is the full load frequency of the generator, and it equals the frequency of the system, f_{sys} .

Reactive Power and Terminal Voltage Characteristic of a Synchronous Generator

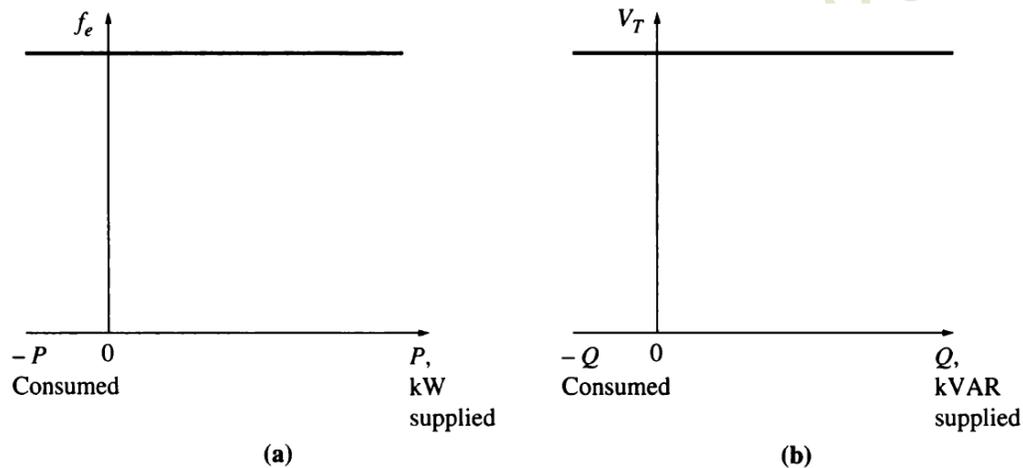
- When a lagging load is added to a Synchronous generator, its terminal voltage drops. Likewise, when a leading load is added to the Synchronous generator, its terminal voltage increases.
- This voltage variation is not intrinsically linear. But, generators have **Voltage Regulators**, which make the characteristic linear.
- It is possible to plot the terminal voltage versus the Reactive Power. The plot has a drooping characteristic, as shown in the Figure below.



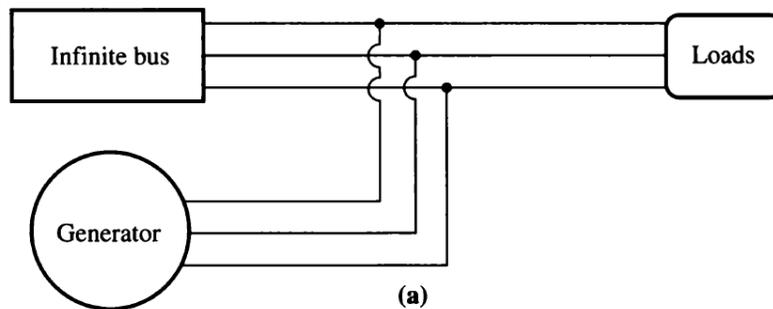
- The characteristic curve can be moved up or down by changing the no-load terminal voltage's set-point on the Voltage Regulator.
- For a single generator operating alone, the Real and Reactive Powers (P and Q) supplied by the generator will be the amount demanded by the load attached to the generator.
- For a given Real Power (P), the Governor's set-point controls the generator's operating frequency, and for a given Reactive Power (Q), the field current controls the generator's terminal voltage (the Voltage Regulator's set-point controls the terminal voltage).

Operation of Generators in Parallel with a Large Power System

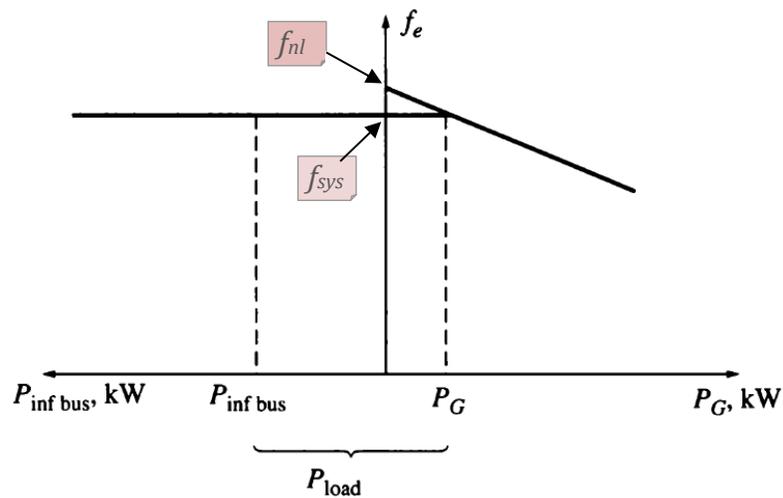
- ◆ When a generator is connected to a large power system, which has an infinite bus, the generator has no effect on the system's voltage or frequency.
- ◆ **Infinite Bus:** is a power system so large that **its voltage and frequency do not vary** regardless of how much Real Power and/or Reactive Power are drawn from or supplied to it.
- ◆ The Frequency-Real Power characteristic and the Terminal Voltage- Reactive Power characteristic of an Infinite Bus are shown below in Figures (a) and (b), respectively.



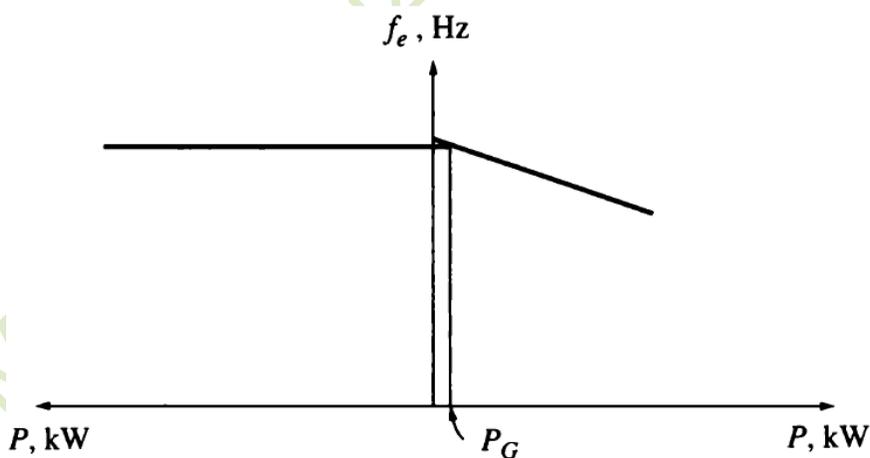
- ◆ When a generator is connected in parallel with another generator, or a large system, **the frequency and the terminal voltage of all machines must be the same**, since their output conductors are tied together.



- ◆ Frequency-Real Power characteristic of an infinite Bus and a generator are drawn back to back in a sketch called **House Diagram**, shown in the Figure below.

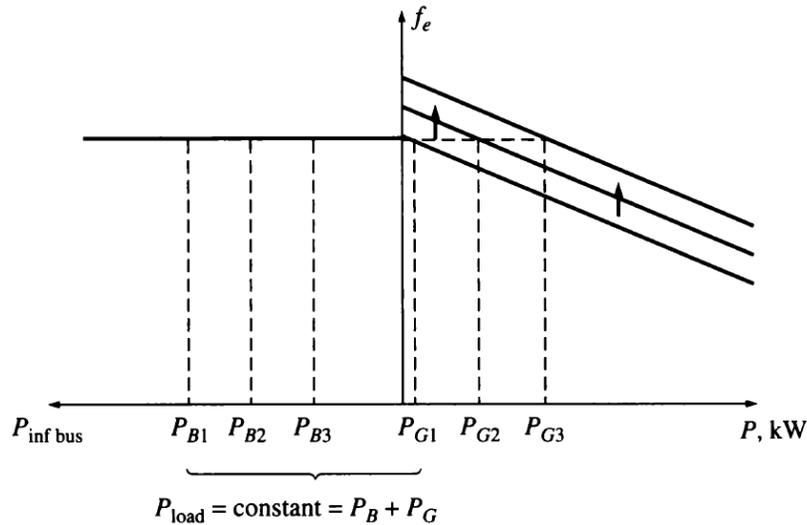


- ◆ If the generator, had been paralleled, is at a slightly higher frequency ($f_{nl} > f_{sys}$), then the generator Real Power (P_G) is positive; the generator supplies power, as illustrated in the Figure above.
- ◆ **If the oncoming generator's frequency was slightly less than the system frequency, the generator Real Power (P_G) is negative, and it actually consumes Real Power, from the Infinite Bus, and runs as a motor.** The Figure below illustrates this situation.

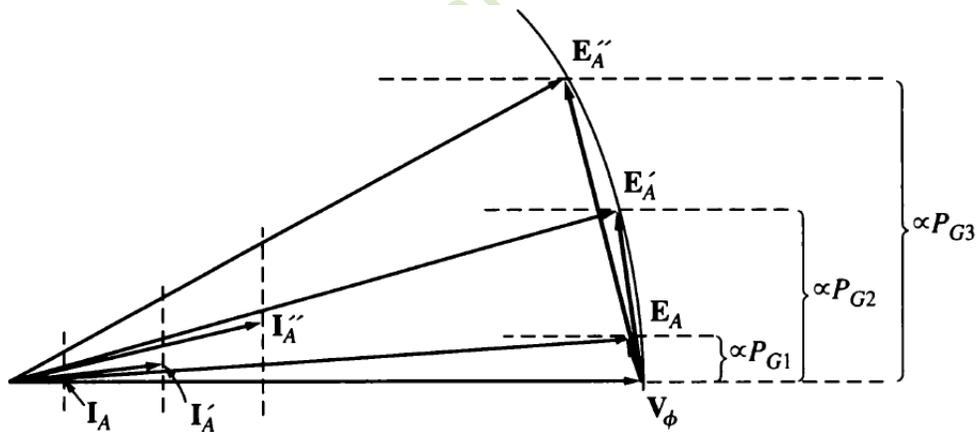


- ◆ Note that many generators have a **Reverse-Power Trip** connected to them, so that it is imperative that they be paralleled with their frequency higher than that of the running system, i.e. if such a generator ever starts to consume Real Power, it will be automatically disconnected from the line.

- ◆ Once the generator is connected with the Infinite Bus, if the Governor's set-point was increased, the no-load frequency is **shifted upwards and the Real Power supplied by the generator increases**, as illustrated in the Figure below.



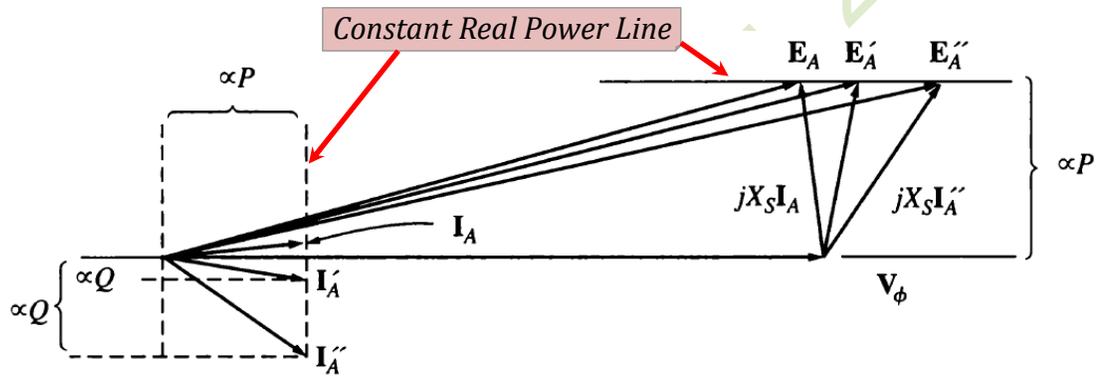
- ◆ The effect of increasing the Governor's set-point on the phasor diagram is shown in the following Figure.



- ◆ E_A ($E_A = k\phi\omega$) is constant because it is connected to an Infinite Bus (frequency is constant), and ϕ is constant (no change in the field current), V_ϕ is also constant, but $E_A \sin \delta$ has increased.
- ◆ In the aforementioned Figure, the generator is operating at a leading power factor, supplying a negative Reactive Power; consuming Reactive Power from the Infinite Bus.
- ◆ If the generator's Real Power is increased to exceed that of the load, the extra Real Power generated flows back to the Infinite Bus.

Adjustment of the Reactive Power of the Oncoming Generator

- ➔ Once the generator Real Power has been adjusted to the desired value via the Governor's set-point then $E_A \sin \delta$ and $I_A \cos \theta$ must remain constant.
- ➔ Since the generator is tied to the Infinite Bus, then ω is also constant.
- ➔ If the field current (I_f) is increased, the flux ϕ increases, therefore, $E_A (= k\phi\omega)$ increases and slides along the line of constant Real Power. Since V_ϕ is constant (because connected to Infinite Bus), the angle of $jX_S I_A$ changes. Thus, the angle and magnitude of I_A change.



- ➔ For a lagging power factor, the distance proportional to generator's Reactive Power (Q), $I_A \sin \theta$, increases, with increasing I_f .
- ➔ In other words, increasing the field current of a Synchronous generator, operating in parallel with an Infinite Bus, increases the Reactive Power output of the generator when the power factor is lagging, and reduces the consumed Reactive Power when the power factor is leading.

Operation of Generators in Parallel with Other Generators of the Same Size

- ❖ When a single generator operated alone, **P and Q** supplied by the generator were constrained to be **equal to those demanded by the load**. The frequency and the voltage were varied by the Governor's and Voltage Regulator's set-points, respectively.
- ❖ When a generator is operated in parallel with an Infinite Bus, the **frequency and the terminal voltage** were constrained **to be constant** by the Infinite Bus, and P and Q were varied by the Governor's set-point and field current, respectively.

❖ Parallel Operation of Generators of the Same Size

- ✓ The basic constraint is that the sum of Real and Reactive Powers supplied by the two generators must equal P and Q demanded by the load; i.e.,

$$P_{tot} = P_L = P_{G1} + P_{G2}$$

and

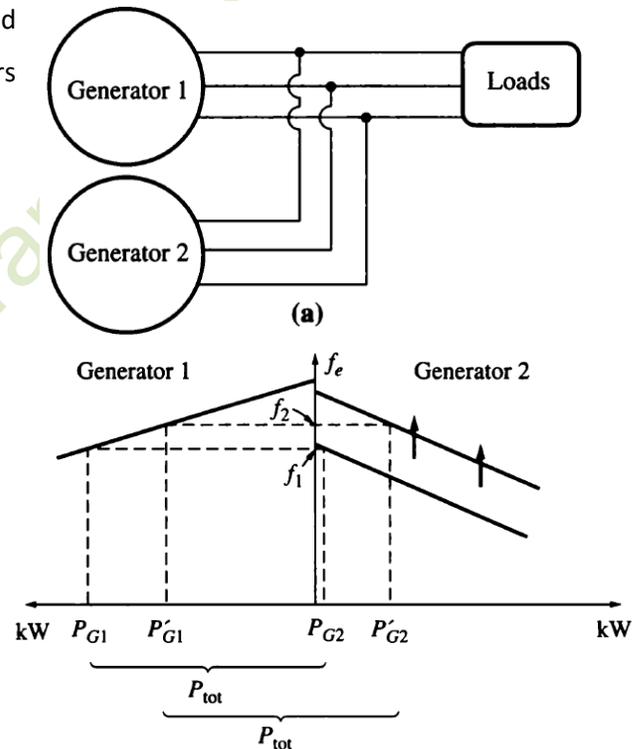
$$Q_{tot} = Q_L = Q_{G1} + Q_{G2}$$

- ✓ The system frequency is not constant and neither is the Real Power of a given generator constrained to be constant.

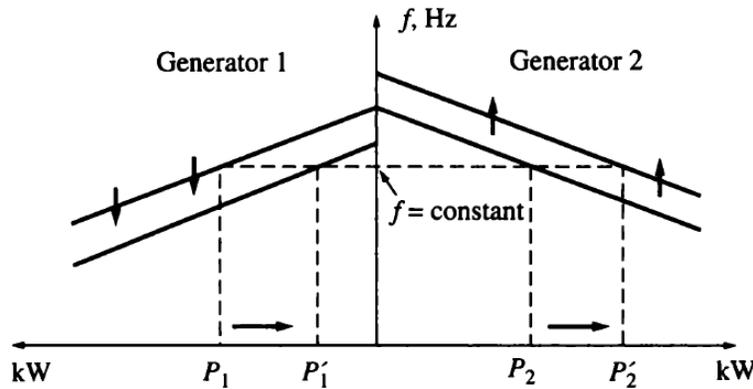
- ✓ Increasing the Governor's set-point of one generator

1. **Increases the system frequency**
2. **Increases the Real Power supplied by that generator** while reducing the Real Power supplied by the other generator.

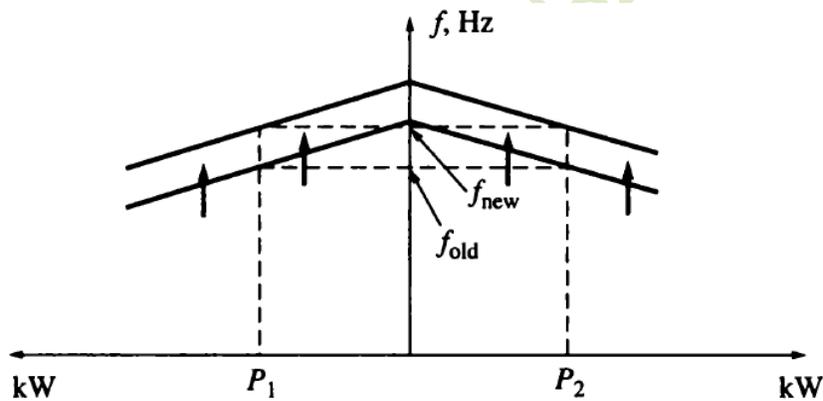
- ✓ A change in the Governor's set-point of one of the generators, **changes both the system frequency and the Real Power sharing**.



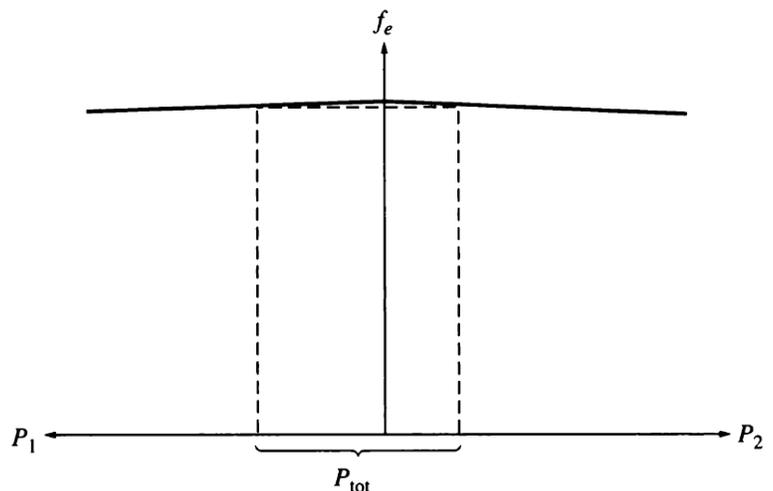
- ✓ To adjust the Real Power sharing without changing the system frequency, the Governor's set-point of one of them is increased and simultaneously the Governor's set-point of the other is decreased.



- ✓ Similarly, to adjust the system frequency without changing the Real Power sharing, the Governors' set-points of the two generators are increased or decreased, simultaneously.



- ✓ If the generators have flat drooping frequency-Real Power characteristic, then the Real Power sharing can vary widely with small changes in the no-load speed. Therefore, to ensure a good control of power sharing between generators, they should have speed droops in the range of 2% to 4%.

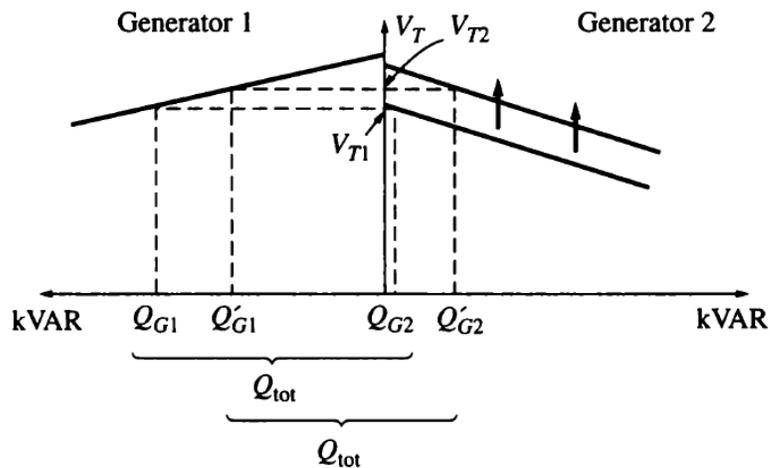


Effect of Field Current:

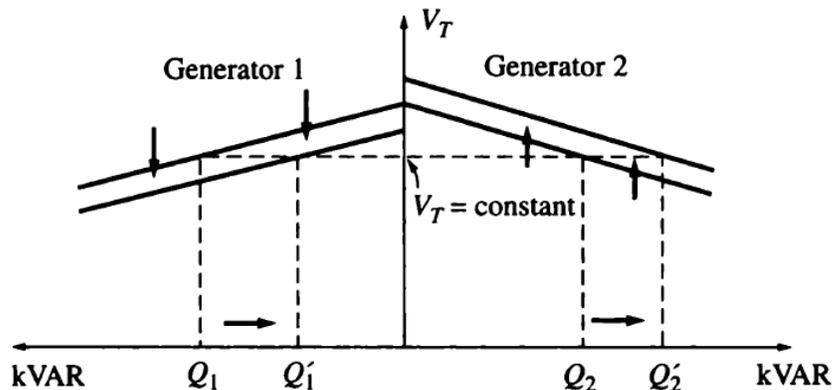
- ✓ The sum of Reactive Power supplied by the two generators must equal to Q demanded by the load;

$$Q_{tot} = Q_L = Q_{G1} + Q_{G2}$$

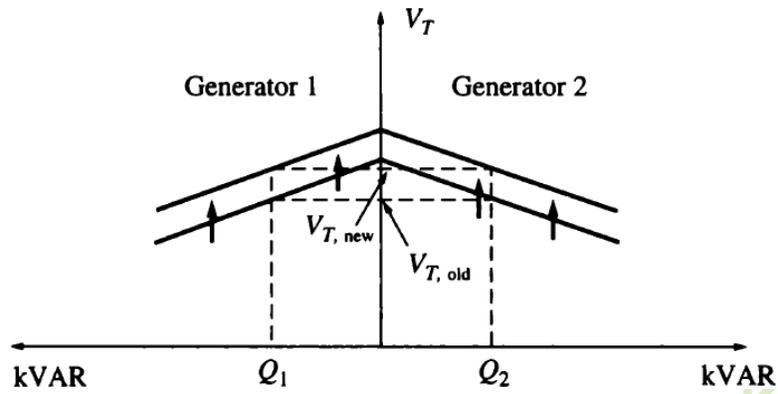
- ✓ Increasing the field current's set-point of one generator,
 - Increases the system terminal voltage
 - Increases the Reactive Power supplied by that generator, while reducing the Reactive Power supplied by the other generator.



- ✓ To shift the Reactive Power sharing without changing the terminal voltage (V_T), the field current of one generator is increased, and simultaneously the field current of the other generator is decreased.



- ✓ To change the system's terminal voltage without changing the Reactive Power sharing, the field currents' set-points of the two generators are increased or decreased, simultaneously.



Example 4–3. A 480-V, 50-Hz, Y-connected, six-pole synchronous generator has a per-phase synchronous reactance of 1.0Ω . Its full-load armature current is 60 A at 0.8 PF lagging. This generator has friction and windage losses of 1.5 kW and core losses of 1.0 kW at 50 Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480 V at no load.

- (a) What is the speed of rotation of this generator?
 (b) What is the terminal voltage of this generator if the following are true?
1. It is loaded with the rated current at 0.8 PF lagging.
 2. It is loaded with the rated current at 1.0 PF.
 3. It is loaded with the rated current at 0.8 PF leading.
- (c) What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?
 (d) How much shaft torque must be applied by the prime mover at full load? How large is the induced countertorque?
 (e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

Solution

This generator is Y-connected, so its phase voltage is given by $V_\phi = V_T / \sqrt{3}$. That means that when V_T is adjusted to 480 V, $V_\phi = 277$ V. The field current has been adjusted so that $V_{T,nl} = 480$ V, so $V_\phi = 277$ V. At *no load*, the armature current is zero, so the armature reaction voltage and the $I_A R_A$ drops are zero. Since $I_A = 0$, the internal generated voltage $E_A = V_\phi = 277$ V. The internal generated voltage $E_A (= K\phi\omega)$ varies only when the field current changes. Since the problem states that the field current is adjusted initially and then left alone, the magnitude of the internal generated voltage is $E_A = 277$ V and will not change in this example.

- (a) The speed of rotation of a synchronous generator in revolutions per minute is given by Equation (3–34):

$$f_{se} = \frac{n_m P}{120} \tag{3-34}$$

Therefore,

$$\begin{aligned} n_m &= \frac{120 f_{se}}{P} \\ &= \frac{120(50 \text{ Hz})}{6 \text{ poles}} = 1000 \text{ r/min} \end{aligned}$$

Alternatively, the speed expressed in radians per second is

$$\begin{aligned} \omega_m &= (1000 \text{ r/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) \\ &= 104.7 \text{ rad/s} \end{aligned}$$

- (b) 1. If the generator is loaded down with rated current at 0.8 PF lagging, the resulting phasor diagram looks like the one shown in Figure 4–24a. In this phasor diagram, we know that V_ϕ is at an angle of 0° , that the magnitude of E_A is 277 V, and that the quantity $jX_S I_A$ is

$$jX_S I_A = j(1.0 \Omega)(60 \angle -36.87^\circ \text{ A}) = 60 \angle 53.13^\circ \text{ V}$$

The two quantities not known on the voltage diagram are the magnitude of V_ϕ and the angle δ of E_A . To find these values, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 4–24a, the right triangle gives

$$E_A^2 = (V_\phi + X_S I_A \sin \theta)^2 + (X_S I_A \cos \theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF lagging is

$$(277 \text{ V})^2 = [V_\phi + (1.0 \Omega)(60 \text{ A}) \sin 36.87^\circ]^2 + [(1.0 \Omega)(60 \text{ A}) \cos 36.87^\circ]^2$$

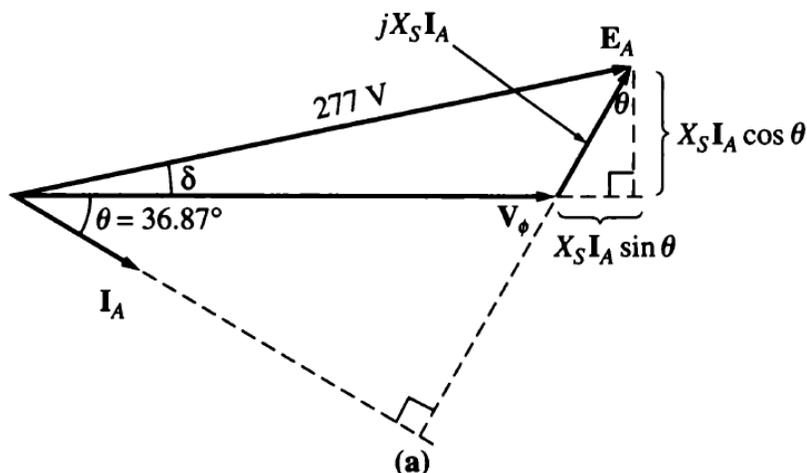
$$76,729 = (V_\phi + 36)^2 + 2304$$

$$74,425 = (V_\phi + 36)^2$$

$$272.8 = V_\phi + 36$$

$$V_\phi = 236.8 \text{ V}$$

Since the generator is Y-connected, $V_T = \sqrt{3} V_\phi = 410 \text{ V}$.



2. If the generator is loaded with the rated current at unity power factor, then the phasor diagram will look like Figure 4–24b. To find V_ϕ here the right triangle is

$$\begin{aligned}
 E_A^2 &= V_\phi^2 + (X_S I_A)^2 \\
 (277 \text{ V})^2 &= V_\phi^2 + [(1.0 \Omega)(60 \text{ A})]^2 \\
 76,729 &= V_\phi^2 + 3600 \\
 V_\phi^2 &= 73,129 \\
 V_\phi &= 270.4 \text{ V}
 \end{aligned}$$

Therefore, $V_T = \sqrt{3}V_\phi = 468.4 \text{ V}$.

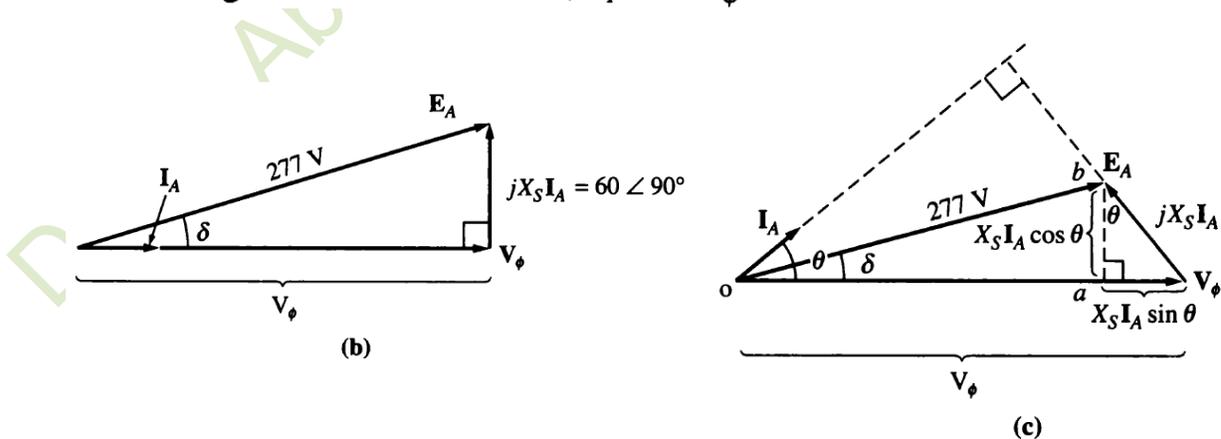
3. When the generator is loaded with the rated current at 0.8 PF leading, the resulting phasor diagram is the one shown in Figure 4–24c. To find V_ϕ in this situation, we construct the triangle OAB shown in the figure. The resulting equation is

$$E_A^2 = (V_\phi - X_S I_A \sin \theta)^2 + (X_S I_A \cos \theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF leading is

$$\begin{aligned}
 (277 \text{ V})^2 &= [V_\phi - (1.0 \Omega)(60 \text{ A}) \sin 36.87^\circ]^2 + [(1.0 \Omega)(60 \text{ A}) \cos 36.87^\circ]^2 \\
 76,729 &= (V_\phi - 36)^2 + 2304 \\
 74,425 &= (V_\phi - 36)^2 \\
 272.8 &= V_\phi - 36 \\
 V_\phi &= 308.8 \text{ V}
 \end{aligned}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 535 \text{ V}$.



(c) The output power of this generator at 60 A and 0.8 PF lagging is

$$P_{\text{out}} = 3V_{\phi} I_A \cos \theta \\ = 3(236.8 \text{ V})(60 \text{ A})(0.8) = 34.1 \text{ kW}$$

The mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}} \\ = 34.1 \text{ kW} + 0 + 1.0 \text{ kW} + 1.5 \text{ kW} = 36.6 \text{ kW}$$

The efficiency of the generator is thus

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{34.1 \text{ kW}}{36.6 \text{ kW}} \times 100\% = 93.2\%$$

(d) The input torque to this generator is given by the equation

$$P_{\text{in}} = \tau_{\text{app}} \omega_m$$

so

$$\tau_{\text{app}} = \frac{P_{\text{in}}}{\omega_m} = \frac{36600}{104.7} = 349.57 \text{ N.m.}$$

The induced countertorque is given by

$$P_{\text{conv}} = \tau_{\text{ind}} \omega_m$$

so

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{P_{\text{out}}}{\omega_m} = \frac{34100}{104.7} = 325.69 \text{ N.m.}$$

(e) The voltage regulation of a generator is defined as

$$\text{VR} = \frac{V_{\text{nl}} - V_{\text{fl}}}{V_{\text{fl}}} \times 100\% \quad (3-67)$$

By this definition, the voltage regulation for the lagging, unity, and leading power-factor cases are

$$1. \text{ Lagging case: } \text{VR} = \frac{480 \text{ V} - 410 \text{ V}}{410 \text{ V}} \times 100\% = 17.1\%$$

$$2. \text{ Unity case: } \text{VR} = \frac{480 \text{ V} - 468 \text{ V}}{468 \text{ V}} \times 100\% = 2.6\%$$

$$3. \text{ Leading case: } \text{VR} = \frac{480 \text{ V} - 535 \text{ V}}{535 \text{ V}} \times 100\% = -10.3\%$$

In Example 4–3, lagging loads resulted in a drop in terminal voltage, unity-power-factor loads caused little effect on V_T , and leading loads resulted in an increase in terminal voltage.

Example 4-6. Figure 4-38a shows two generators supplying a load. Generator 1 has a no-load frequency of 61.5 Hz and a slope s_{P1} of 1 MW/Hz. Generator 2 has a no-load frequency of 61.0 Hz and a slope s_{P2} of 1 MW/Hz. The two generators are supplying a real load totaling 2.5 MW at 0.8 PF lagging. The resulting system power-frequency or house diagram is shown in Figure 4-39.

- At what frequency is this system operating, and how much power is supplied by each of the two generators?
- Suppose an additional 1-MW load were attached to this power system. What would the new system frequency be, and how much power would G_1 and G_2 supply now?
- With the system in the configuration described in part *b*, what will the system frequency and generator powers be if the governor set points on G_2 are increased by 0.5 Hz?

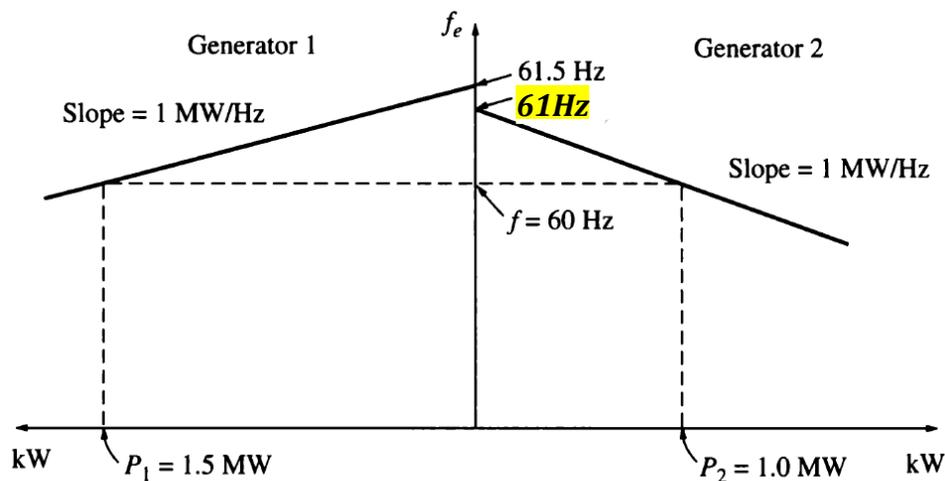
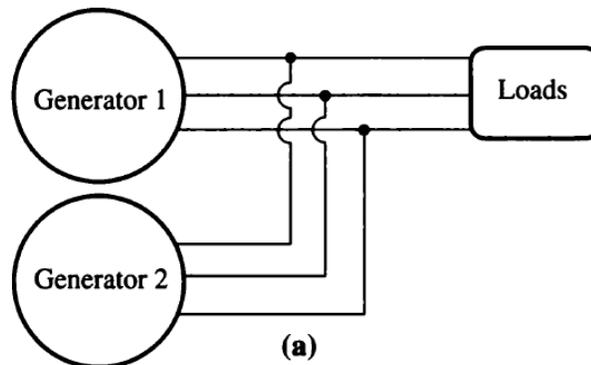


FIGURE 4-39

Solution

The power produced by a synchronous generator with a given slope and no-load frequency is given by Equation (4–28):

$$P_1 = s_{P1}(f_{nl,1} - f_{sys})$$

$$P_2 = s_{P2}(f_{nl,2} - f_{sys})$$

Since the total power supplied by the generators must equal the power consumed by the loads,

$$P_{load} = P_1 + P_2$$

These equations can be used to answer all the questions asked.

- (a) In the first case, both generators have a slope of 1 MW/Hz, and G_1 has a no-load frequency of 61.5 Hz, while G_2 has a no-load frequency of 61.0 Hz. The total load is 2.5 MW. Therefore, the system frequency can be found as follows:

$$\begin{aligned} P_{load} &= P_1 + P_2 \\ &= s_{P1}(f_{nl,1} - f_{sys}) + s_{P2}(f_{nl,2} - f_{sys}) \\ 2.5 \text{ MW} &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - f_{sys}) + (1 \text{ MW/Hz})(61 \text{ Hz} - f_{sys}) \\ &= 61.5 \text{ MW} - (1 \text{ MW/Hz})f_{sys} + 61 \text{ MW} - (1 \text{ MW/Hz})f_{sys} \\ &= 122.5 \text{ MW} - (2 \text{ MW/Hz})f_{sys} \end{aligned}$$

therefore $f_{sys} = \frac{122.5 \text{ MW} - 2.5 \text{ MW}}{(2 \text{ MW/Hz})} = 60.0 \text{ Hz}$

The resulting powers supplied by the two generators are

$$\begin{aligned} P_1 &= s_{P1}(f_{nl,1} - f_{sys}) \\ &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - 60.0 \text{ Hz}) = 1.5 \text{ MW} \end{aligned}$$

$$\begin{aligned} P_2 &= s_{P2}(f_{nl,2} - f_{sys}) \\ &= (1 \text{ MW/Hz})(61.0 \text{ Hz} - 60.0 \text{ Hz}) = 1 \text{ MW} \end{aligned}$$

(b) When the load is increased by 1 MW, the total load becomes 3.5 MW. The new system frequency is now given by

$$\begin{aligned}
 P_{\text{load}} &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) + s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
 3.5 \text{ MW} &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - f_{\text{sys}}) + (1 \text{ MW/Hz})(61 \text{ Hz} - f_{\text{sys}}) \\
 &= 61.5 \text{ MW} - (1 \text{ MW/Hz})f_{\text{sys}} + 61 \text{ MW} - (1 \text{ MW/Hz})f_{\text{sys}} \\
 &= 122.5 \text{ MW} - (2 \text{ MW/Hz})f_{\text{sys}}
 \end{aligned}$$

therefore $f_{\text{sys}} = \frac{122.5 \text{ MW} - 3.5 \text{ MW}}{(2 \text{ MW/Hz})} = 59.5 \text{ Hz}$

The resulting powers are

$$\begin{aligned}
 P_1 &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) \\
 &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - 59.5 \text{ Hz}) = 2.0 \text{ MW} \\
 P_2 &= s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
 &= (1 \text{ MW/Hz})(61.0 \text{ Hz} - 59.5 \text{ Hz}) = 1.5 \text{ MW}
 \end{aligned}$$

(c) If the no-load governor set points of G_2 are increased by 0.5 Hz, the new system frequency becomes

$$\begin{aligned}
 P_{\text{load}} &= s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) + s_{P2}(f_{\text{nl},2} - f_{\text{sys}}) \\
 3.5 \text{ MW} &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - f_{\text{sys}}) + (1 \text{ MW/Hz})(61.5 \text{ Hz} - f_{\text{sys}}) \\
 &= 123 \text{ MW} - (2 \text{ MW/Hz})f_{\text{sys}} \\
 f_{\text{sys}} &= \frac{123 \text{ MW} - 3.5 \text{ MW}}{(2 \text{ MW/Hz})} = 59.75 \text{ Hz}
 \end{aligned}$$

The resulting powers are

$$\begin{aligned}
 P_1 &= P_2 = s_{P1}(f_{\text{nl},1} - f_{\text{sys}}) \\
 &= (1 \text{ MW/Hz})(61.5 \text{ Hz} - 59.75 \text{ Hz}) = 1.75 \text{ MW}
 \end{aligned}$$

Notice that the system frequency rose, the power supplied by G_2 rose, and the power supplied by G_1 fell.

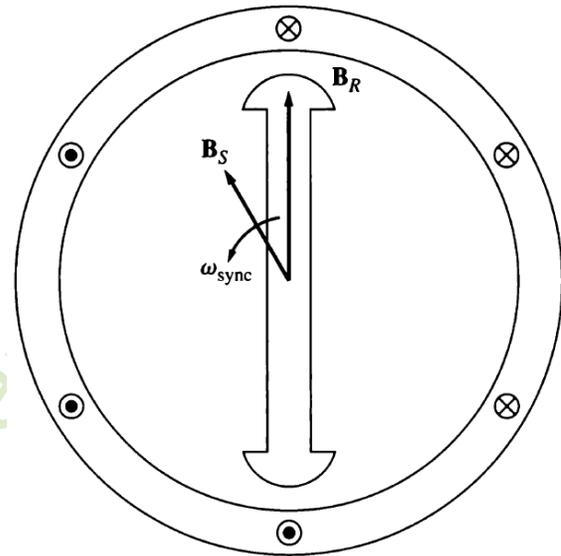
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PART V

Synchronous Motors

Basic Principle of Operation

- ➔ The field current produces the rotor magnetic field, B_R .
- ➔ The 3-phase currents applied to stator (Armature) windings produce a uniform rotating magnetic field; B_S .
- ➔ The rotor has the same number of poles (P) as the stator and carries the field circuit.
- ➔ The magnetic fields in a two-pole machine are shown in the Figure next.
- ➔ The rotor magnetic field (and hence the rotor attached to it) chases the stator magnetic field around a circle and never quite catches up with it.
- ➔ The induced torque is:



$$\tau_{ind} = k \overline{B_R} \times \overline{B_S}; \text{ counter clockwise}$$

It was also shown in previous parts that:

$$\tau_{ind} = k \overline{B_R} \times \overline{B_{net}}$$

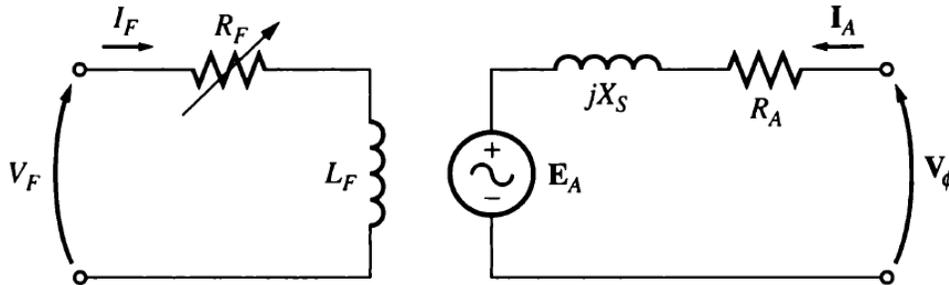
The magnitude of the torque is:

$$\tau_{ind} = k B_R B_{net} \sin \delta; \text{ counter clockwise}$$

where δ is the angle between the rotor magnetic field and the net magnetic field.

The Equivalent Circuit of a Synchronous Motor

The per-phase equivalent circuit of a Synchronous motor is shown below:



Applying KVL to the stator circuit yields:

$$\overline{V_\phi} = \overline{E_A} + R_A \overline{I_A} + jX_s \overline{I_A}$$

$$\overline{E_A} = \overline{V_\phi} - R_A \overline{I_A} - jX_s \overline{I_A}$$

The direction of power flow, and therefore the Armature current, is reversed in a Synchronous motor compared to that in a Synchronous generator.

Synchronous Motor from a Magnetic Field Perspective

- The phasor diagram of a Synchronous generator operating with a large field current and in parallel with an Infinite bus is shown in Figure (a) next.
- It was shown in previous chapters that:

$$\overline{B_R} \text{ corresponds to } \overline{E_A}$$

$$\overline{B_{net}} \text{ corresponds to } \overline{V_\phi}$$

and $\overline{B_S}$ corresponds to $\overline{E_{stat}} (-jX_s \overline{I_A})$; the Armature Reaction voltage

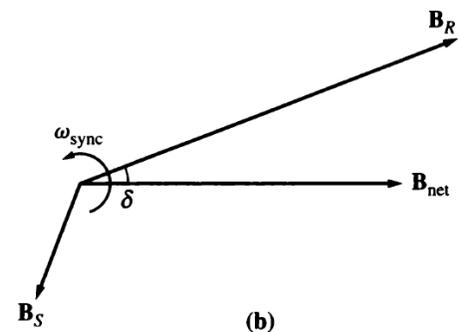
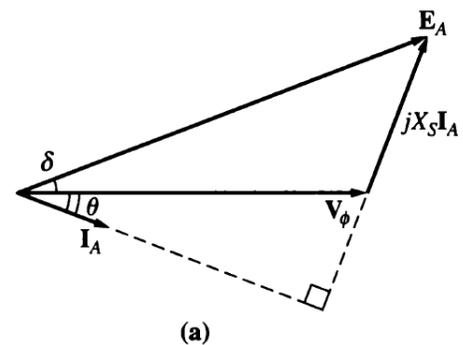
- Hence, the corresponding magnetic fields diagram is shown in Figure (b) next.
- The torque induced is:

$$\tau_{ind} = k \overline{B_R} \times \overline{B_{net}}$$

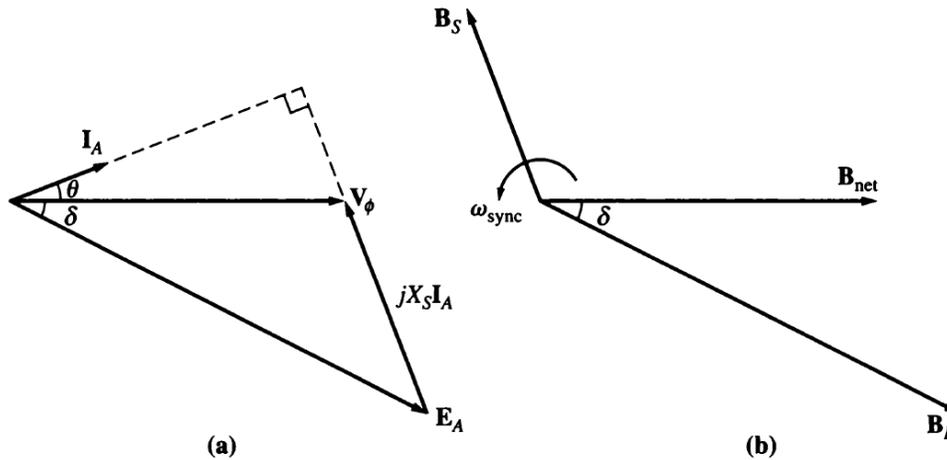
The magnitude of the torque is:

$$\tau_{ind} = k B_R B_{net} \sin \delta ;$$

- The direction of the induced torque is clockwise, which is opposite to the speed and the applied torque of the Prime Mover in the counter clockwise direction. Hence, the machine is a generator.



If the Prime Mover suddenly loses power, the rotor slows down and the rotor itself (and its magnetic field) falls behind the net magnet field. Consequently, the induced torque ($\tau_{ind} = k\overline{B_R} \times \overline{B_{net}}$) will be in the counter clockwise direction, and the machine acts as a motor. The corresponding voltage and magnetic field diagrams describing the new situation are shown in the Figures (a) and (b) below, respectively.



The increasing torque angle (δ) results in a larger and larger induced torque in the direction of rotation, until eventually the motor's induced torque equals the load torque on the shaft. At this point, the machine reaches steady state and synchronous speed operating as a Synchronous motor.

Note that, for a given field current (E_A) and a lagging power factor operation in a generator, results in a leading power factor in the motor operation for the same E_A (field current). But, in both cases above, the Reactive power is supplied by the machine.

In a generator: $\overline{E_A}$ lies ahead of $\overline{V_\phi}$ or $\overline{B_R}$ lies ahead of $\overline{B_{net}}$

In a motor: $\overline{E_A}$ lies behind $\overline{V_\phi}$ or $\overline{B_R}$ lies behind $\overline{B_{net}}$

Steady State Operation of a Synchronous Motor

Synchronous Motor Torque-Speed Characteristic Curves

- Synchronous motors are usually connected to large power systems; the terminal voltage and the frequency are constant regardless of the amount of power drawn by the motor.
- The speed of rotation is constant regardless of the motor load torque.
- The Speed Regulation is:

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = 0\%$$

where n_{nl} is the no-load speed in rpm ($= n_s$)

n_{fl} is the full load speed in rpm ($= n_s$)

- The torque induced is the resultant of the cross product between rotor and the net magnetic fields;

$$\tau_{ind} = k \overline{B_R} \times \overline{B_{net}}$$

Hence, the magnitude of the torque is:

$$\tau_{ind} = k B_R B_{net} \sin \delta ;$$

or

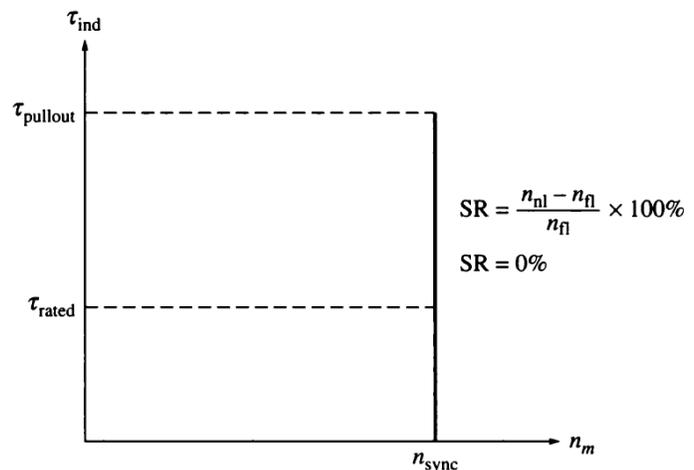
$$\tau_{ind} = \frac{3V_\phi E_A}{\omega_m X_s} \sin \delta$$

- Pull-out Torque: is the maximum torque that a motor can supply. It is about 2.5 to 3 times the full load (rated) torque.
- The Pull-out (maximum) torque occurs when $\delta = 90^\circ$.

Hence, $\tau_{max} = \frac{3V_\phi E_A}{\omega_m X_s}$

- The torque-speed characteristic of a Synchronous Motor is shown in the Figure next.

- Clearly **increasing the field current** (hence E_A) **increases the Pull-out torque**. Therefore, there is a **stability advantage** in operating the motor with large field currents (a large E_A).



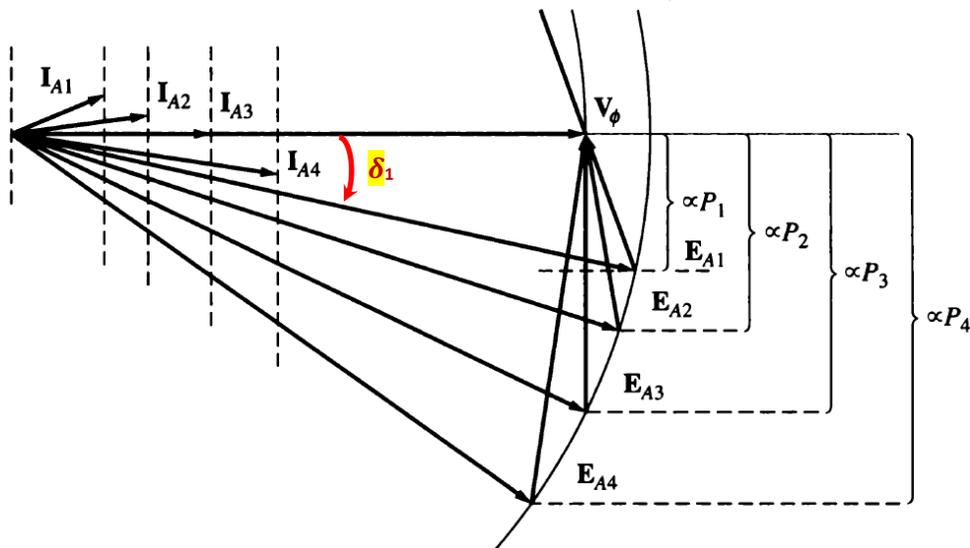
- **When the torque** on the shaft **exceeds the Pull-out torque**, the rotor (and its magnetic field) is no longer locked to stator or net magnetic field. It slips behind them. As the rotor slows down, the stator magnetic field **laps it repeatedly**, and the direction of **induced torque** on the rotor **reverses with each pass**.
- This **causes huge torque surges and severe vibrations**.
- **Slipping Poles Phenomenon** is the loss of synchronization after the Pull-out torque is exceeded.
- Clearly the rotor rotates at synchronous speed as long as the load torque is less than the Pull-out torque, beyond which the motor loses synchronism and encounters the **Slipping Poles Phenomenon**.

The Effect of Load Changes on a Synchronous Motor

- ◆ If the load on the shaft increases, the rotor will initially slow down. As it does so, the torque angle (δ) becomes larger, and the torque induced increases.
- ◆ The Increase in the induced torque eventually speeds up the rotor backup, and the rotor again runs at synchronous speed with a larger torque angle (δ).
- ◆ In summary:

as: $\tau_{Load} \uparrow \Rightarrow \omega_r \downarrow \Rightarrow \delta \uparrow \Rightarrow \tau_{ind} \uparrow \Rightarrow \omega_r$ (until reaching synchronous speed but at larger δ)

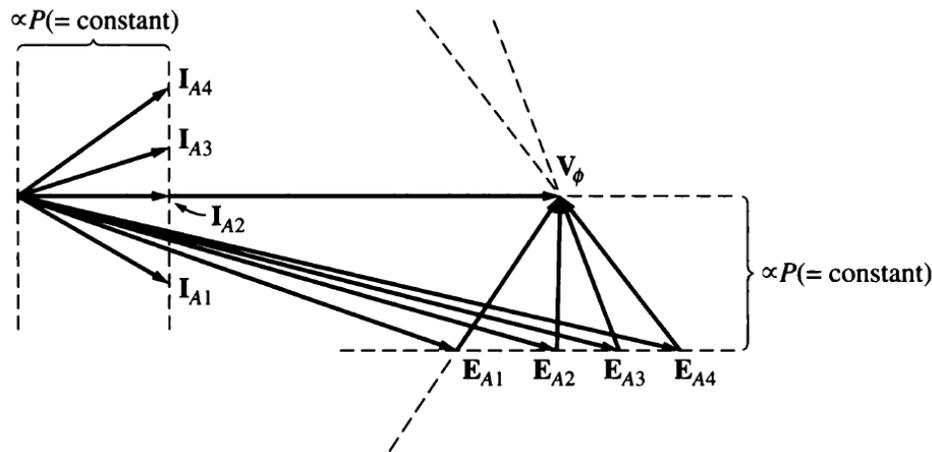
- ◆ Assuming that ω (frequency is constant) and I_f are constants during load changes, then $|E_A|$ is also constant.
- ◆ As the load increases, $E_A \sin \delta$ and $I_A \cos \theta$ increase!
- ◆ The phasor diagram representing load increases for an initially leading power factor operation, case "1", is shown in the Figure below.



- ◆ As the load increases, $\overline{E_A}$ swings down, while $|jX_s \overline{I_A}|$ increases; i.e. the Armature current (I_A) increases, also the power factor angle decreases, becomes zero (at unity power factor) and then becomes lagging, and more and more lagging.
- ◆ Note that, the motor was supplying Reactive Power to the Infinite Bus during case "1" and "2", has zero Reactive Power in case "3", and consumes Reactive Power during case "4".

The Effect of Field Current Changes on a Synchronous Motor

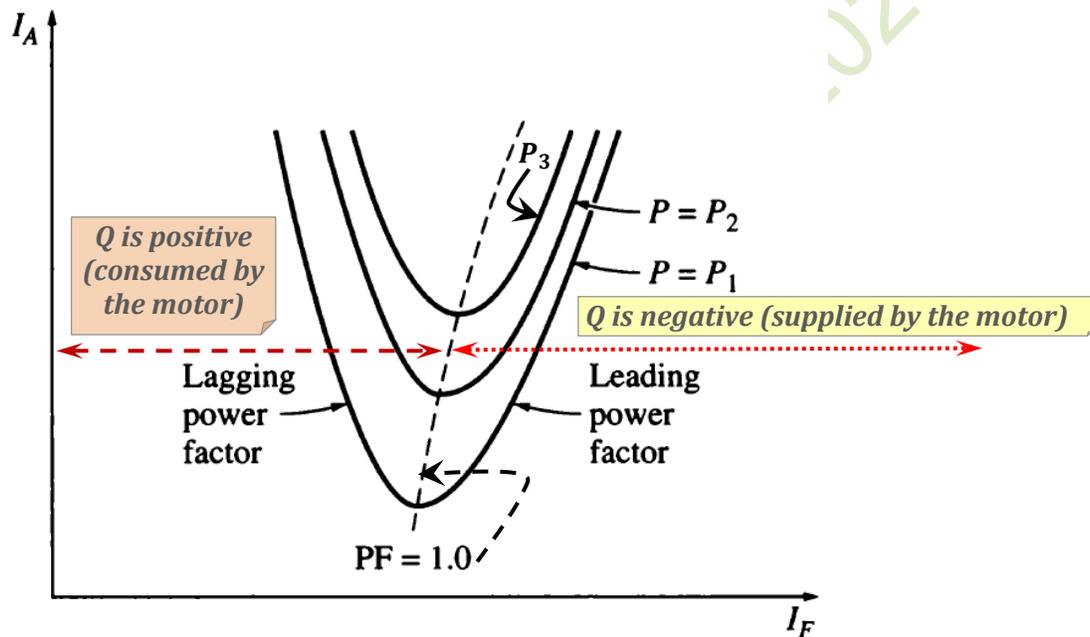
- A change in the field current does not affect the shaft speed, and since the load attached to the shaft is unchanged, the Real Power supplied to the load is unchanged, too. But, the magnitude of E_A will change by changing the field current, and hence the flux. However, $E_A \sin \delta$ and $I_A \cos \theta$ should remain constant, because the power is constant.
- The phasor diagram for an initially lagging power factor operation, case "1", of a Synchronous motor, whose field current is being increased, is shown in the Figure below.



- As the field current is increased, the power factor become unity in case "2", and then the power factor becomes leading in case "3", and more and more leading in case "4".
- As E_A increases, I_A decreases and then increases again.
- **Notes:**
 1. At low values of E_A , I_A is lagging and the motor is considered as an inductive load consuming Reactive Power.
 2. As E_A increases, I_A lines up with V_ϕ and has its minimum value, where the motor is acting as a resistive load; at a unity power factor.
 3. Increasing E_A further, causes I_A to become leading and the motor to behave as a capacitive load supplying Reactive Power to the system.

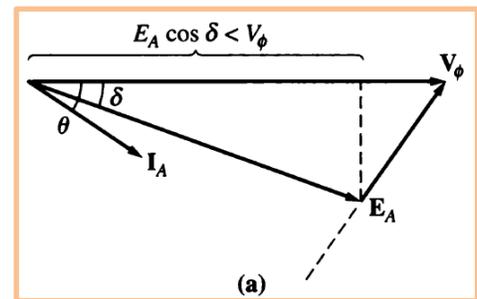
Synchronous Motor V-Curves

- It was shown that the minimum Armature current occurs at a unity power factor.
- At a constant motor Real power, controlling the field current controls the Reactive Power consumed or supplied by the motor.
- The relationship between the Armature current (I_A) versus the field current (I_f) at different values of Real Power is characterized by the plots shown in the Figure below. These plots are called **V-curves** of the Synchronous motor. Note that, $P_3 > P_2 > P_1$.



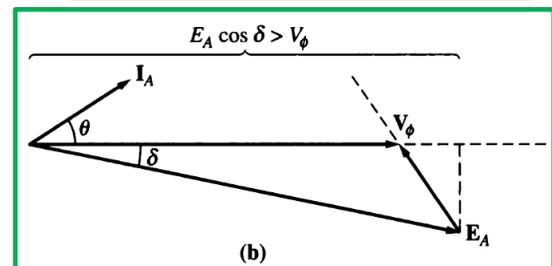
- Figure (a) next shows a lagging power factor operation, where the Reactive Power is consumed by the motor, and the motor is described as **under excited**;

$$E_A \cos \delta < V_\phi$$



- Figure (b) next shows a leading power factor operation, where the Reactive Power is supplied by the motor, and the motor is described as **overexcited**;

$$E_A \cos \delta > V_\phi$$



The Synchronous Motor and Power Factor Correction

- When the Synchronous motor is **overexcited**, it operates at a **leading power factor** **supplying Reactive Power** to nearby loads improving the overall (combined) power factor of the system.
- Improving the power factor, **reduces the power losses in transmission lines**, and hence improves the efficiency of the system, and consequently reducing the system cost; transmission lines of smaller cross-sectional areas are needed for the same transmitted Real power.
- Also, when overexcited, the **Pull-out torque of the Synchronous motor is increased**, reducing the chances of exceeding it; the **stability is improved!**
- However, over excitation needs larger field currents, which causes **significant rotor heating**.

Synchronous Condenser or Synchronous Capacitor

A **Synchronous Condenser (Capacitor)** is a Synchronous motor operated **at no load and overexcited** to supply Reactive Power. It has no shaft that comes out of the frame and hence no load can be connected to it. It was **used historically for Power Factor Correction**.

Since the Real power is zero; $E_A \sin \delta = I_A \cos \theta = 0$. Indicating that, for any given value of E_A or I_A , $\delta = 0$ and $\theta = \mp 90^\circ$. Therefore, the phasor diagram for the Synchronous Condenser is shown below.

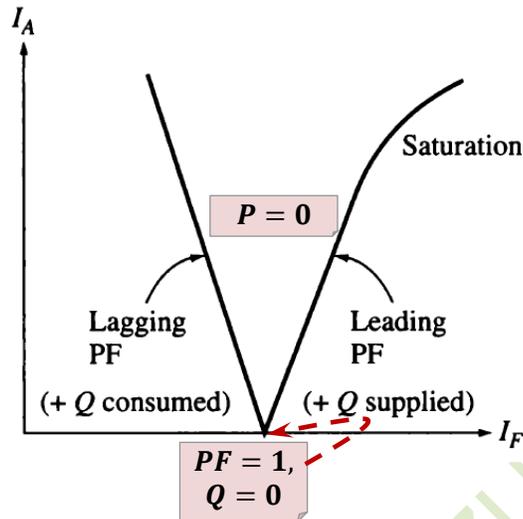
Applying KVL for the Synchronous Condenser yields:

$$\overline{V}_\phi = \overline{E}_A + jX_S \overline{I}_A$$

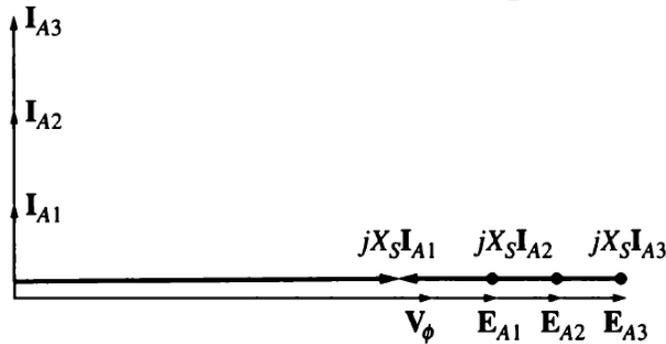


The above phasor diagram of a Synchronous Condenser at no load is similar to the phasor diagram of a passive capacitor, where \overline{I}_A leads \overline{V}_ϕ ; the **motor looks like a large capacitor**.

The **V-curve of a Synchronous Condenser** is shown in the Figure below.



The corresponding phasor diagram for different values of Reactive Power supplied is shown below.



Notes:

1. At a unity power factor and no load, $I_A = 0, P = Q = 0$, and $\overline{V_\phi} = \overline{E_A}$
2. In case "3", the largest amount of Reactive Power is supplied to the power system!
3. At a leading power factor, I_A has a value and $\theta = +90^\circ$
4. At a lagging power factor, I_A has a value and $\theta = -90^\circ$
5. **Power Factor Improvement (Correction)** can be achieved by:
 - a. **Synchronous Condensers**; Historical approach
 - b. **Conventional Static Capacitors**, which are more economical to buy than Synchronous Condensers
 - c. **Power Electronic Converters**; **Wide-spreading recently!**

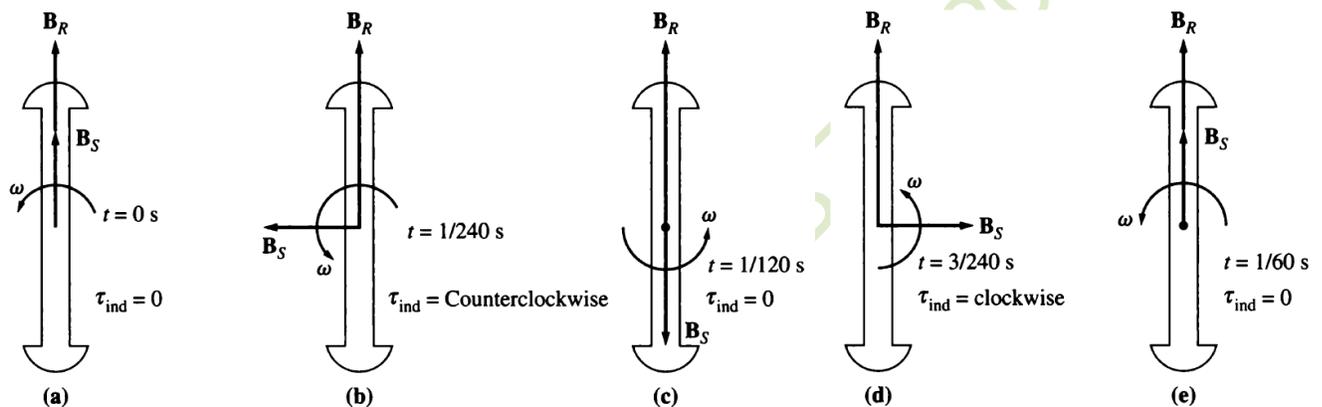
Starting Synchronous Motors

- Assume that a power of **60Hz** is applied to a two-pole Synchronous motor, which is stationary at $t = 0s$.

- Recall, the induced torque is:

$$\tau_{ind} = k\overline{B_R} \times \overline{B_S}$$

- The magnetic fields, induced torque and its direction during parts of a cycle of the AC current are shown in the Figures below.



- During one electrical cycle ($\frac{1}{60}s$; $16.667ms$), the torque was initially zero, then has a value in the counter clockwise direction when ($0 < t < \frac{1}{2 \times 60}$), it becomes zero again at $t = \frac{1}{2 \times 60}$, while it has a value in the clockwise direction during the period ($\frac{1}{2 \times 60} < t < \frac{1}{60}$), then it is zero at $t = \frac{1}{60}$; the average torque over a complete cycle is zero.

- Therefore, the motor **vibrates heavily** with each electrical cycle and **finally overheats**.

- The **Synchronous motor (as it is, without modifications)** has no starting torque, because the rotor field is stationary and the stator field moves very fast; **the average torque over one complete cycle is zero**.

Approaches Used for Starting a Synchronous Motor

- 1) **Reducing the frequency of electric power**, and thus the speed of the stator magnetic field (B_s), to allow the rotor to accelerate and B_R to lockup with the stator during one half cycle of magnetic field rotation.

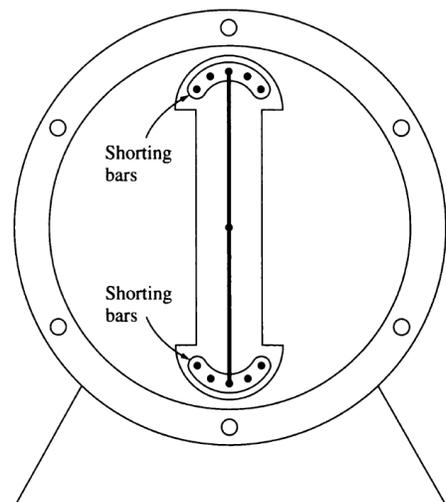
By **reducing the electrical frequency (f_e)**, the stator magnetic field rotates at a low enough speed **allowing the rotor to accelerate** and lock in with the stator magnetic field. The speed can then be increased to operating speed by gradually increasing f_e up to its normal value.

But, $E_A = k\phi\omega$, so at low speeds, E_A is small. Thus, the input voltage V_ϕ **should be also reduced accordingly** to keep the Armature current to safe levels.

This can be achieved by employing a **Variable Voltage Variable Frequency (VVVF) drive**, which varies the voltage linearly with frequency; a Voltage Boost at low frequencies to compensate for $R_A I_A$ voltage drop is needed! For example, Cycloconverter, Inverters,...

- 2) Using an **external Prime Mover** to accelerate the Synchronous motor up to synchronous speed, **going through paralleling procedure** and bringing the machine on the line as a Synchronous generator, **then turning-off or disconnecting the Prime Mover** will make the Synchronous machine a motor.
- 3) Using **Damper Windings or Amortisseur Windings** \Rightarrow It starts like a 3 ϕ Induction Motor.

- **Amortisseur Windings:** are special **bars laid into notches carved in the face of a Synchronous motor's rotor and then shorted out on each end by a large shorting ring (bar).**
- As B_s sweeps the rotor bars, it induces voltages in the rotor, these voltages allow a current flow from top/bottom bars to bottom/top bars, which produces B_r that interacts with B_s and produces a torque. This **torque will be in the same direction during a complete cycle, so the motor will speed up.**
- The torque developed by **Amortisseur Windings speed up slow machines and slow down fast machines; dampen load variations;** hence **Damper Windings!**



Example 5–1. A 208-V, 45-hp, 0.8-PF-leading, Δ -connected, 60-Hz synchronous machine has a synchronous reactance of 2.5Ω and a negligible armature resistance. Its friction and windage losses are 1.5 kW, and its core losses are 1.0 kW. Initially, the shaft is supplying a 15-hp load, and the motor's power factor is 0.80 leading.

- Sketch the phasor diagram of this motor, and find the values of \mathbf{I}_A , I_L , and \mathbf{E}_A .
- Assume that the shaft load is now increased to 30 hp. Sketch the behavior of the phasor diagram in response to this change.
- Find \mathbf{I}_A , I_L , and \mathbf{E}_A after the load change. What is the new motor power factor?

Solution

- Initially, the motor's output power is 15 hp. This corresponds to an output of

$$P_{\text{out}} = (15 \text{ hp})(0.746 \text{ kW/hp}) = 11.19 \text{ kW}$$

Therefore, the electric power supplied to the machine is

$$\begin{aligned} P_{\text{in}} &= P_{\text{out}} + P_{\text{mech loss}} + P_{\text{core loss}} + P_{\text{elec loss}} \\ &= 11.19 \text{ kW} + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} = 13.69 \text{ kW} \end{aligned}$$

Since the motor's power factor is 0.80 leading, the resulting line current flow is

$$\begin{aligned} I_L &= \frac{P_{\text{in}}}{\sqrt{3} V_T \cos \theta} \\ &= \frac{13.69 \text{ kW}}{\sqrt{3}(208 \text{ V})(0.80)} = 47.5 \text{ A} \end{aligned}$$

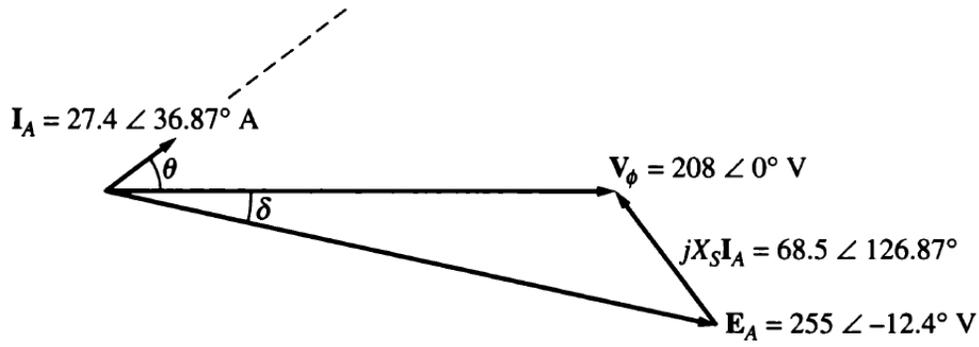
and the armature current is $I_L/\sqrt{3}$, with 0.8 leading power factor, which gives the result

$$\mathbf{I}_A = 27.4 \angle 36.87^\circ \text{ A}$$

To find \mathbf{E}_A , apply Kirchhoff's voltage law [Equation (5–2)]:

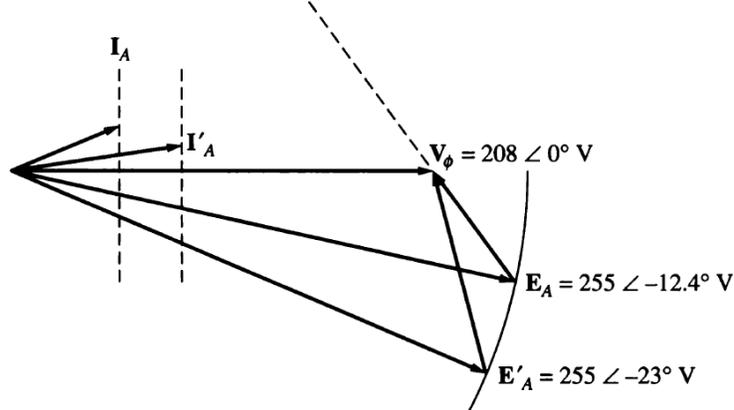
$$\begin{aligned} \mathbf{E}_A &= \mathbf{V}_\phi - jX_s \mathbf{I}_A \\ &= 208 \angle 0^\circ \text{ V} - (j2.5 \Omega)(27.4 \angle 36.87^\circ \text{ A}) \\ &= 208 \angle 0^\circ \text{ V} - 68.5 \angle 126.87^\circ \text{ V} \\ &= 249.1 - j54.8 \text{ V} = 255 \angle -12.4^\circ \text{ V} \end{aligned}$$

The resulting phasor diagram is shown in Figure 5–7a.



(a)

(b) As the power on the shaft is increased to 30 hp, the shaft slows momentarily, and the internal generated voltage E_A swings out to a larger angle δ while maintaining a constant magnitude. The resulting phasor diagram is shown in Figure 5-7b.



(b)

(c) After the load changes, the electric input power of the machine becomes

$$\begin{aligned}
 P_{\text{in}} &= P_{\text{out}} + P_{\text{mech loss}} + P_{\text{core loss}} + P_{\text{elec loss}} \\
 &= (30 \text{ hp})(0.746 \text{ kW/hp}) + 1.5 \text{ kW} + 1.0 \text{ kW} + 0 \text{ kW} \\
 &= 24.88 \text{ kW}
 \end{aligned}$$

From the equation for power in terms of torque angle [Equation (4-20)], it is possible to find the magnitude of the angle δ (remember that the magnitude of \mathbf{E}_A is constant):

$$P = \frac{3V_\phi E_A \sin \delta}{X_S} \quad (4-20)$$

so

$$\begin{aligned} \delta &= \sin^{-1} \frac{X_S P}{3V_\phi E_A} \\ &= \sin^{-1} \frac{(2.5 \Omega)(24.88 \text{ kW})}{3(208 \text{ V})(255 \text{ V})} \\ &= \sin^{-1} 0.391 = 23^\circ \end{aligned}$$

The internal generated voltage thus becomes $\mathbf{E}_A = 255 \angle -23^\circ \text{ V}$. Therefore, \mathbf{I}_A will be given by

$$\begin{aligned} \mathbf{I}_A &= \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} \\ &= \frac{208 \angle 0^\circ \text{ V} - 255 \angle -23^\circ \text{ V}}{j2.5 \Omega} \\ &= \frac{103.1 \angle 105^\circ \text{ V}}{j2.5 \Omega} = 41.2 \angle 15^\circ \text{ A} \end{aligned}$$

and I_L will become

$$I_L = \sqrt{3} I_A = 71.4 \text{ A}$$

The final power factor will be $\cos(-15^\circ)$ or 0.966 leading.

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PART VI

Induction (Asynchronous) Motors

- ➔ In Induction machines, the rotor voltages, which produce rotor currents, and hence the rotor magnetic field, are induced in the rotor windings rather than being physically connected by wires.
- ➔ No DC field current is required to run the machine.
- ➔ Induction motors have Amortisseur windings only!
- ➔ Induction motors are singly excited machine.



Induction Motor's Construction

A) Stator

It is the stationary part and is composed of three sets of windings distributed in the stator slots and displaced 120° (electrical) in space. Its stator is the same as that of a Synchronous machine. However, the rotor is different.



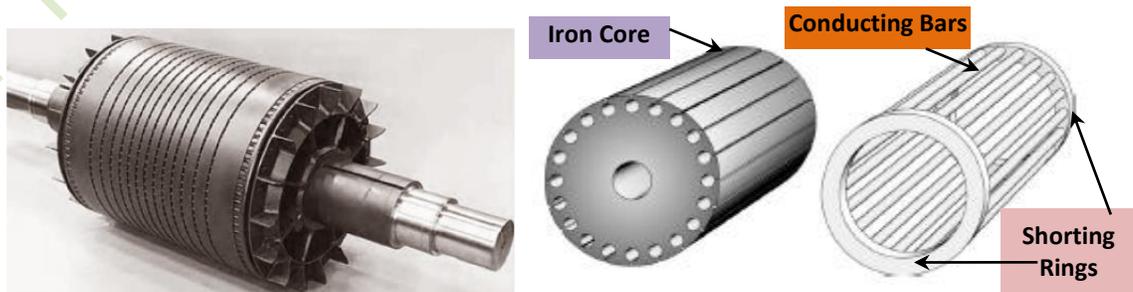
B) Rotor

It is the rotating part and consists of a stack of insulated laminations. There are **two types** of Induction motors according to rotor's construction:

1. Squirrel-Cage Induction Motor:

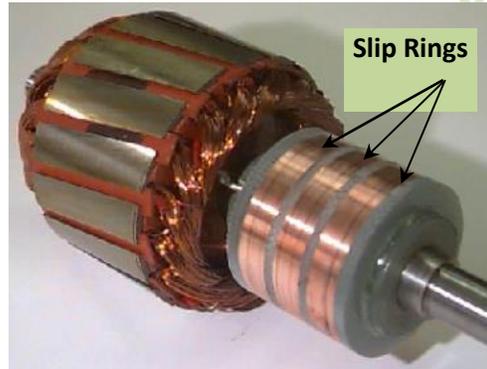
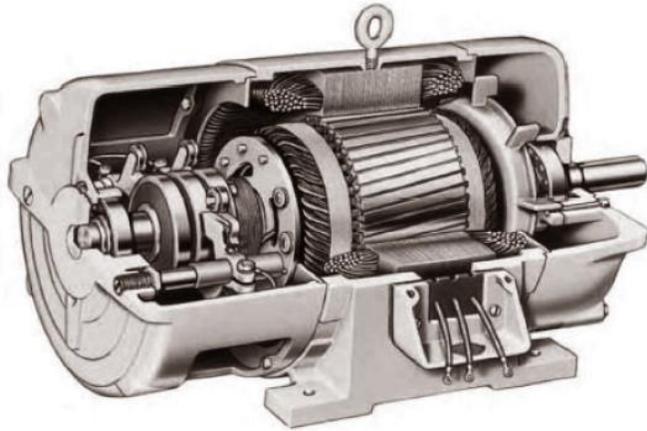
It consists of a series of conducting bars laid into slots carved in the face of the rotor and shorted at either end by large shorting rings.

The conductors are carved in an iron core, as illustrated in the Figure below.

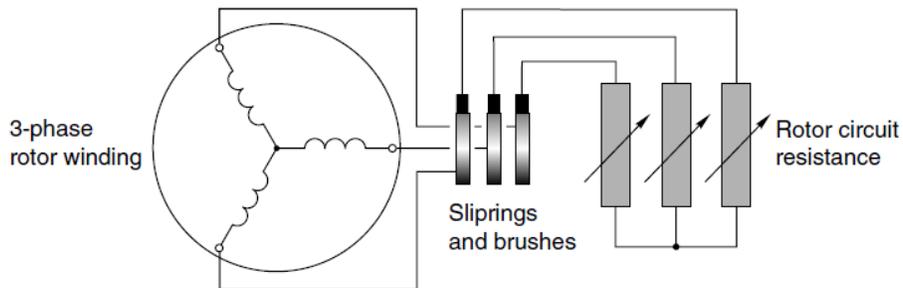


2. Wound Rotor Induction Motor:

It has three sets of three phase windings that are mirror images of stator windings. The rotor windings are usually Y-connected, with their terminals tied to Slip Rings on the rotor shaft, and shorted via Brushes.



Hence, the rotor currents can be examined and three-phase external resistance can be inserted into the rotor circuit, as shown in the Figure below, to modify the torque-speed characteristic.



This type of motors is **more expensive** than Squirrel-Cage Motors, and **requires maintenance**. Therefore, it is **rarely used**.

Basic Induction Motor Concepts

- ◆ Considering the Squirrel-Cage rotor, and assuming that the stator windings are connected to three-phase voltages, which produce three-phase currents. As a result, a uniform rotating magnetic field (B_s) is produced, as shown in Figure (a) next.

- ◆ The speed of rotation of the stator magnetic field (B_s) is:

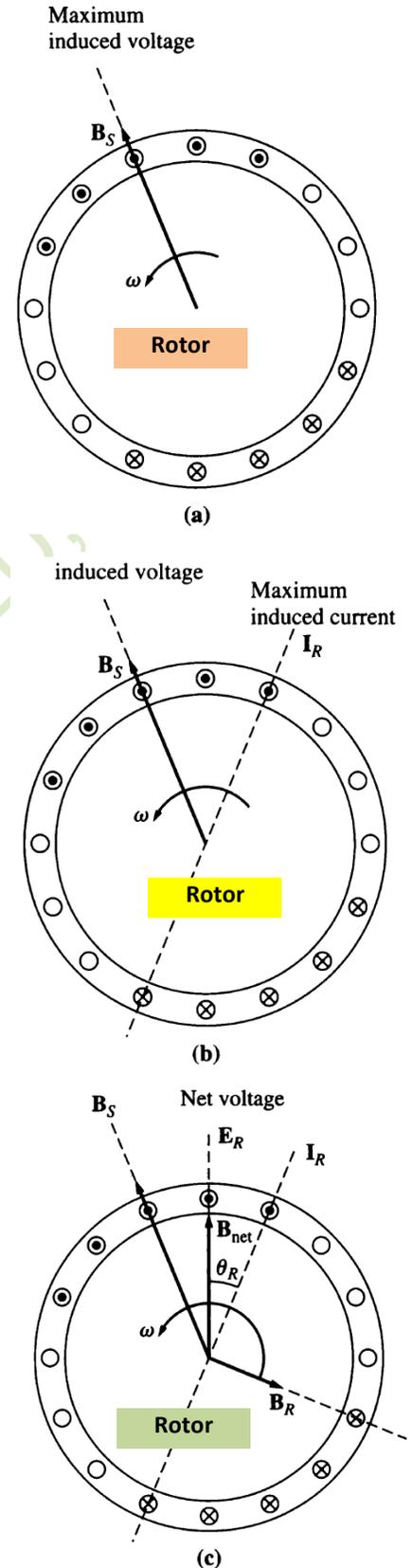
$$n_{sync} = \frac{120f_e}{P}$$

where, f_e is electrical frequency in Hz, P is number of poles, and n_{sync} is synchronous speed in rpm.

- ◆ The relative velocity of the upper rotor bars to the stator magnetic field (B_s) is to the right, and of the lower rotor bars is to left.
- ◆ The rotor conductors, whose relative velocity is \perp to B_s will have a maximum induced voltage;

$$e_{ind} = (\overline{V_{relative}} \times \overline{B_s}) \cdot \overline{L}$$

- ◆ The induced voltage in the upper bars is out of the page and in the lower bars is into the page.
- ◆ Since the rotor conductors are shorted via two shorting rings at either end of the conductors, **currents flow** in the rotor bars.
- ◆ Because the rotor is of inductive assembly, the **maximum induced rotor current (I_R) lags the maximum voltage**, as shown in Figure (b) next.
- ◆ Rotor currents produce a rotor magnetic field (B_R) lagging the maximum induced rotor current by 90° , as shown in Figure (c).
- ◆ The **rotor magnetic field** itself is also **rotating at the same speed of the stator magnetic field**, and induces a voltage in the rotor bars.
- ◆ The net voltage in the rotor conductors is E_R , and the net magnetic field in the airgap is: $\overline{B_{net}} = \overline{B_R} + \overline{B_s}$
- ◆ Since, $\tau_{ind} = k\overline{B_R} \times \overline{B_s}$, the resulting torque is counter clockwise, and the rotor accelerates in that direction.



- ◆ If the rotor were turning at synchronous speed, then the rotor bars would be stationary relative to the stator magnetic field (B_s), and there would be no induced voltage. Consequently, no rotor current and hence no B_R .
- ◆ With no B_R , the torque induced is zero and the rotor slows down as a result of friction. Induction **motor speeds up to near synchronous speed, but never reaches it.**
- ◆ **The magnetic fields B_s and B_R (also B_{net}) rotate together at synchronous speed (n_{sync}), whilst the rotor itself does not!**

The Concept of Rotor Slip

The induced voltages and currents in the rotor of an induction motor, therefore the performance, depends on the speed of the rotor relative to the magnetic fields.

To define the relative motion of the motor, the “slip” is defined.

Slip Speed: is the difference between the synchronous speed and the rotor mechanical speed.

$$n_{slip} = n_{sync} - n_m$$

where, n_{slip} is the slip speed of the machine

n_{sync} is the synchronous speed ($n_{sync} = \frac{120f_e}{P}$); the speed of magnetic fields.

n_m is the mechanical speed of the motor shaft (rotor)

The Slip (s): is the relative speed expressed in per unit or as a percentage of synchronous speed.

$$s = \frac{n_{slip}}{n_{sync}} (\times 100\%)$$

$$s = \frac{n_{sync} - n_m}{n_{sync}} (\times 100\%)$$

or in angular velocity,

$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} (\times 100\%)$$

If $s = 0$, then the rotor turns at synchronous speed

If $s = 1$, then the rotor is stationary or blocked; stall condition!

Then, the mechanical speed can be expressed in terms of the synchronous speed and the slip as:

$$n_m = (1 - s)n_{sync}$$

or

$$\omega_m = (1 - s)\omega_{sync}$$

Electrical Frequency on The Rotor

- ➔ An Induction motor works by inducing voltages and currents in the machine; thus it is sometimes called a **rotating transformer**.
- ➔ Like a transformer, the primary (stator) induces a voltage in the secondary (rotor), but unlike a transformer, the secondary (rotor) voltages' and currents' frequency is not necessarily the same as the primary's (stator's) frequency.
- ➔ If the rotor is locked (blocked) $\Rightarrow f_r = f_e$ and $s = 1$
- ➔ If the rotor turns at synchronous speed $\Rightarrow f_r = 0$ and $s = 0$
- ➔ For any other speed, the rotor frequency is directly proportional to the difference between the speed of the magnetic fields (n_{sync}) and the rotor speed (n_m); *i. e.*,

$$f_r = s f_e$$

$$f_r = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

$$f_r = \frac{n_{sync} - n_m}{\frac{120 f_e}{P}} f_e$$

$$f_r = \frac{P}{120} (n_{sync} - n_m)$$

Example 6–1. A 208-V, 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.

- What is the synchronous speed of this motor?
- What is the rotor speed of this motor at the rated load?
- What is the rotor frequency of this motor at the rated load?
- What is the shaft torque of this motor at the rated load?

Solution

- ◀ (a) The synchronous speed of this motor is

$$\begin{aligned} n_{sync} &= \frac{120 f_{se}}{P} \quad (6-1) \\ &= \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min} \end{aligned}$$

(b) The rotor speed of the motor is given by

$$\begin{aligned}n_m &= (1 - s)n_{\text{sync}} && (6-6) \\&= (1 - 0.05)(1800 \text{ r/min}) = 1710 \text{ r/min}\end{aligned}$$

(c) The rotor frequency of this motor is given by

$$f_{re} = sf_{se} = (0.05)(60 \text{ Hz}) = 3 \text{ Hz} \quad (6-8)$$

Alternatively, the frequency can be found from Equation (6-9):

$$\begin{aligned}f_{re} &= \frac{P}{120} (n_{\text{sync}} - n_m) && (6-9) \\&= \frac{4}{120} (1800 \text{ r/min} - 1710 \text{ r/min}) = 3 \text{ Hz}\end{aligned}$$

(d) The shaft load torque is given by

$$\begin{aligned}\tau_{\text{load}} &= \frac{P_{\text{out}}}{\omega_m} \\&= \frac{(10 \text{ hp})(746 \text{ W/hp})}{(1710 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min}/60 \text{ s})} = 41.7 \text{ N} \cdot \text{m}\end{aligned}$$

The shaft load torque in English units is given by Equation (1-17):

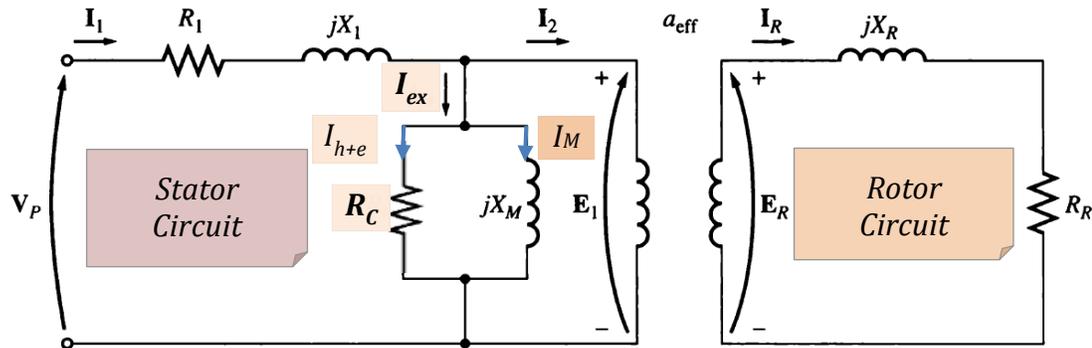
$$\tau_{\text{load}} = \frac{5252P}{n}$$

where τ is in pound-feet, P is in horsepower, and n_m is in revolutions per minute. Therefore,

$$\tau_{\text{load}} = \frac{5252(10 \text{ hp})}{1710 \text{ r/min}} = 30.7 \text{ lb} \cdot \text{ft}$$

The Equivalent Circuit of an Induction Motor

The per-phase equivalent circuit of the Induction motor is shown in the Figure below. It is very similar to that of a transformer, but with its secondary terminals short circuited. Hence, an **Induction motor** is called a **rotating transformer**.

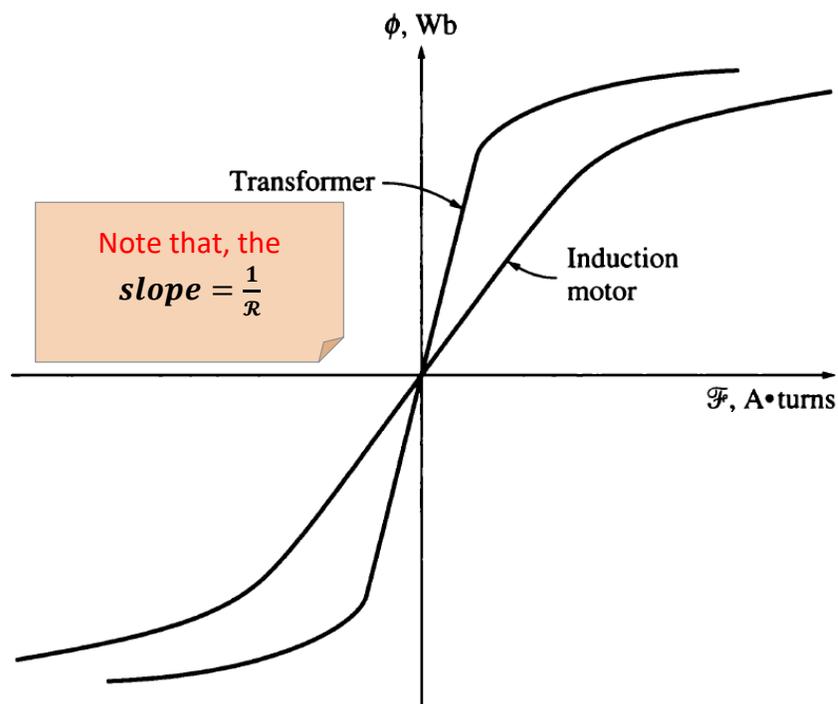


where,

- V_P : is the rms value of the phase voltage applied to stator windings ($v_p(t) = \sqrt{2} V_P \sin \omega t$)
- R_1 : the stator windings resistance
- R_R : the rotor bars (windings) resistance
- R_C : the core losses resistance (a resistance accounting for eddy current and hysteresis losses)
- X_M : the magnetizing inductance
- X_1 : the stator leakage reactance
- X_R : the rotor leakage reactance

- The airgap greatly increases the reluctance of the flux path, reducing the coupling between the primary (stator) and the secondary (rotor) windings.
- The higher the reluctance caused by the airgap is, **the higher the magnetization current (i_m) needed to obtain a given flux will be**; $\mathcal{F} = \mathcal{R}\phi$ or $Ni_M = \mathcal{R}\phi$
- **X_M has a much smaller value than it would be in an ordinary transformer**, to allow such a large magnetization current to flow.

- A comparison between the magnetization curves of an Induction motor and an ordinary transformer is shown in the Figure below.



- a_{eff} is the effective turns' ratio, which couples the voltages E_1 and E_R on the ideal transformer. It can be easily determined in a Wound-Rotor Induction motor.
- The rotor frequency effect must be included in the circuit model!

Rotor Circuit Model

- In general, the greater the relative motion between the rotor and the magnetic fields is, the greater the resulting rotor voltages (E_R) and frequency of rotor currents and voltages (f_r) will be.
- When the rotor is locked (blocked) or stationary, the slip, $s = 1$, the largest magnitude of the rotor voltage (E_{R0}) occurs, and the frequency of rotor currents and voltages f_r equals f_e .
- When the rotor is moving at synchronous speed; $s = 0$, the smallest rotor voltage is induced (**0V**) and the smallest frequency of rotor currents and voltages occur (**0Hz**).

- At any other slip, the rotor induced voltage is:

$$E_R = sE_{R0}$$

and the frequency of rotor currents and voltages is:

$$f_r = sf_e$$

- In other words, the **magnitude and frequency of the voltage induced in the rotor are directly proportional to the slip of the rotor.**

- The rotor resistance (R_R) is constant (neglecting skin effect) and is independent of the slip.

- The rotor reactance (X_R) depends on the inductance of the rotor windings (bars) and the frequency of the currents and voltages in the rotor, and is given by:

$$X_R = \omega_r L_R = 2\pi f_r L_R$$

$$X_R = 2\pi s f_e L_R$$

$$X_R = s(2\pi f_e L_R)$$

$$X_R = sX_{R0}$$

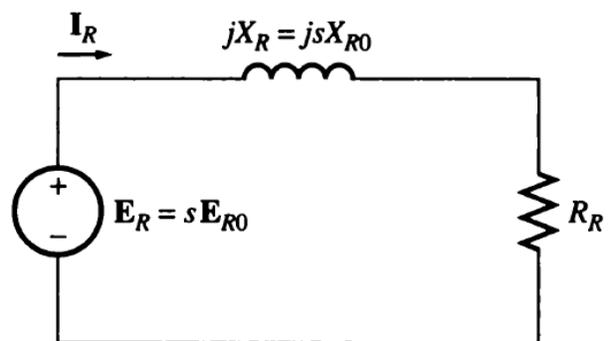
where X_{R0} is the blocked rotor reactance

- Recall the equivalent rotor circuit, shown in the Figure next, to calculate the rotor current;

$$\bar{I}_R = \frac{s\bar{E}_{R0}}{R_R + jsX_{R0}}$$

Dividing numerator and denominator by s yields:

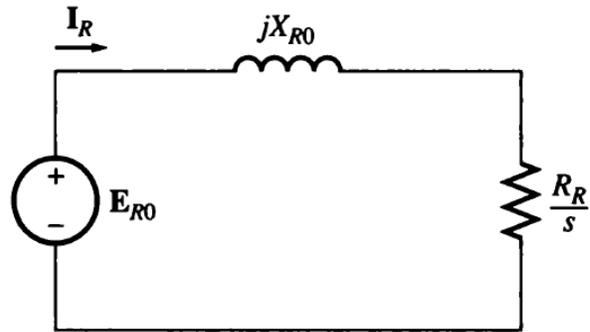
$$\bar{I}_R = \frac{\bar{E}_{R0}}{\frac{R_R}{s} + jX_{R0}}$$



- Therefore, the equivalent rotor impedance is:

$$Z_{Req} = \frac{R_R}{s} + jX_{R0}$$

- Thus, with all the effects of rotor slip (frequency) concentrated in the rotor resistance, the equivalent rotor circuit can be redrawn as shown in the Figure next.



Dr. M. Abu-Khaizaran, BZU, 2024

The Rotor Current Plot

- At a small slip, $\frac{R_R}{s} \gg X_{R0}$, the rotor current varies linearly with s ;

$$\bar{I}_R = \frac{\bar{E}_{R0}}{\frac{R_R}{s}}$$

or

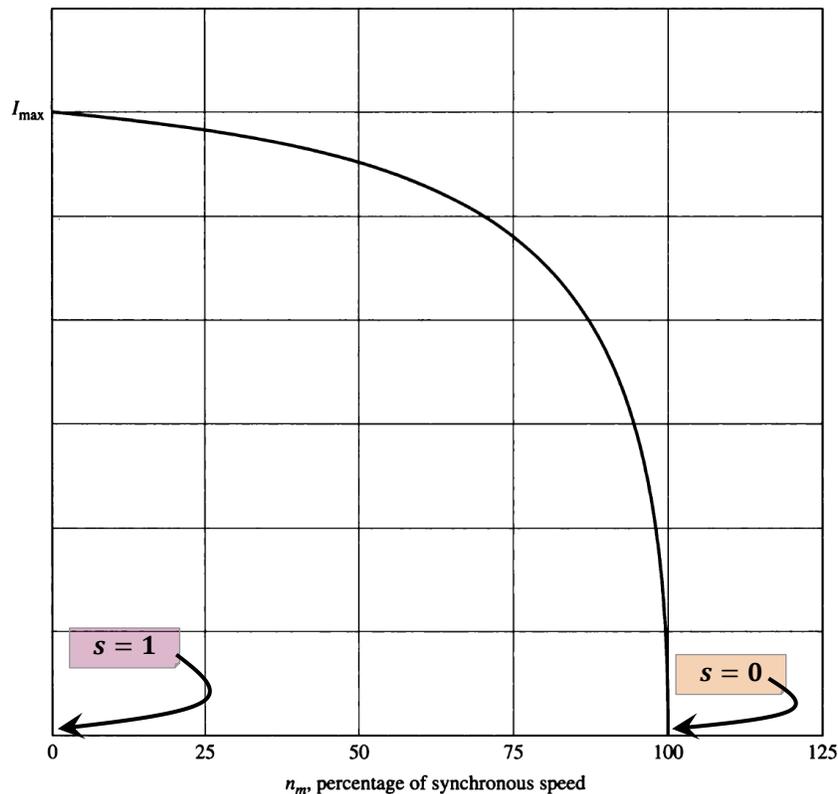
$$\bar{I}_R = \frac{\bar{E}_{R0}}{R_R} s$$

- Note that at $s = 0$, the rotor current is zero and the motor is operating at no-load; $P_{out} = 0$.

- At a large slip ($s \cong 1$), $X_{R0} \gg \frac{R_R}{s}$, the rotor current approaches a steady state value;

$$\bar{I}_R = \frac{\bar{E}_{R0}}{R_R + jX_{R0}}$$

- A plot of the rotor current as a function of rotor speed, or percentage of synchronous speed, is shown in the Figure below.



The Final Equivalent Circuit of an Induction Motor

The referred rotor voltage, $E_1 = E_R'$, is:

$$\overline{E_1} = a_{eff} \overline{E_{R0}}$$

The referred rotor current is:

$$\overline{I_2} = \frac{\overline{I_R}}{a_{eff}}$$

The referred rotor impedance is:

$$Z_2 = a_{eff}^2 \left(\frac{R_R}{s} + jX_{R0} \right)$$

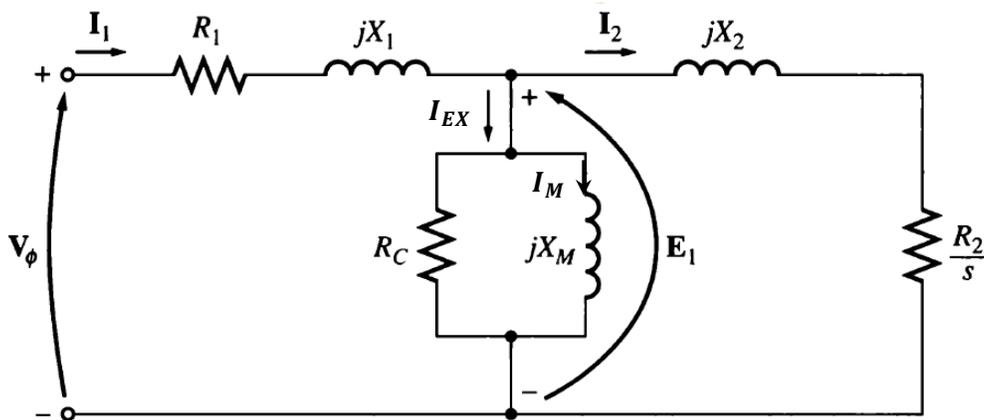
Let

$$R_2 = a_{eff}^2 R_R$$

and

$$X_2 = a_{eff}^2 X_{R0}$$

Then, the **final per-phase equivalent circuit of an Induction motor** is:

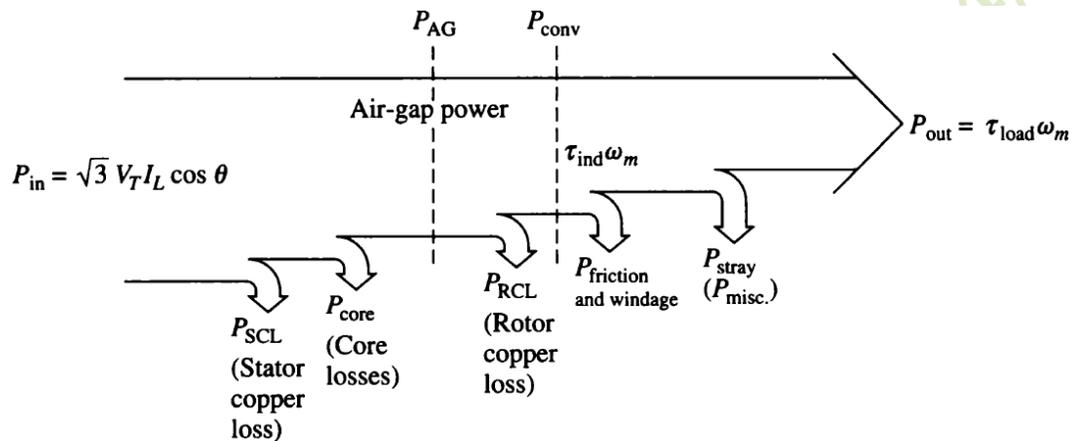


It is very similar to the transformer's equivalent circuit, but its "secondary" is short circuited and the secondary resistance is dependent on the slip!

Power and Torque in Induction Motors

Power Losses and Power-Flow Diagram

As mentioned before, an Induction motor can be described as a **rotating transformer, whose secondary winding is short circuited**. Therefore, no electrical output power exists; instead the output power is mechanical. The power flow diagram of an Induction motor is:



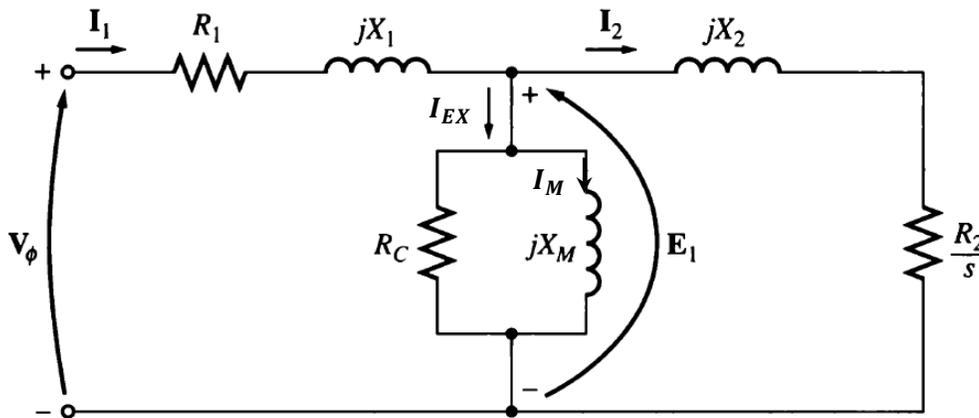
The core losses come partially from the stator and rotor circuits, but most of it comes from the stator circuit.

If the core power losses are given by a value (X Watts), then they are added to, or lumped with, the mechanical losses.

The rotational losses include Friction, Windage and stray losses.

Power and Torque in an Induction Motor

Recall the per-phase equivalent circuit of an Induction motor shown in the Figure below,



The input current is:

$$\bar{I}_1 = \frac{\bar{V}_\phi}{Z_{eq}}$$

where, the **input impedance** (Z_{eq}) is:

$$Z_{eq} = R_1 + jX_1 + \frac{1}{G_C - jB_M + \frac{1}{\frac{R_2}{s} + jX_2}}$$

where, $G_C = \frac{1}{R_C}$ is the conductance

$B_M = \frac{1}{X_M}$ is the susceptance

The **input power** is:

$$P_{in} = 3V_\phi I_1 \cos \theta$$

or $P_{in} = \sqrt{3}V_T I_L \cos \theta$

where, θ is the angle between the stator input current (\bar{I}_1) and input phase voltage (\bar{V}_ϕ)

The **stator copper power loss** is:

$$P_{SCL} = 3I_1^2 R_1$$

The **core power losses** are:

$$P_{Core} = 3E_1^2 G_C$$

The **airgap power** is the power crossing the airgap from the stator to the rotor, and it is:

$$P_{AG} = P_{in} - P_{SCL} - P_{Core}$$

also,
$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

The **actual rotor copper power loss** is:

$$P_{RCL} = 3I_R^2 R_R$$

This power loss should not change when the rotor is referred to stator side; i.e.,

$$P_{RCL} = 3I_2^2 R_2$$

where, $R_2 = a_{eff}^2 R_R$

Therefore, the rotor copper power loss can be related to the airgap power by:

$$P_{RCL} = sP_{AG}$$

Note that **the rotor power loss is directly proportional to the slip!**

The **converted power** or **developed mechanical power** is:

$$P_{Conv} = P_{AG} - P_{RCL}$$

$$P_{Conv} = 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$P_{Conv} = 3I_2^2 R_2 \left(\frac{1}{s} - 1 \right)$$

The converted power can be rewritten as:

$$P_{Conv} = P_{AG} - P_{RCL}$$

$$P_{Conv} = P_{AG} - sP_{AG}$$

$$P_{Conv} = (1 - s)P_{AG}$$

Note also that, the larger the rotor slip is, the smaller the converted power will be!

The rotor power loss can be separated from the converted power, recalling:

$$P_{Conv} = P_{AG} - P_{RCL}$$

$$P_{Conv} = 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

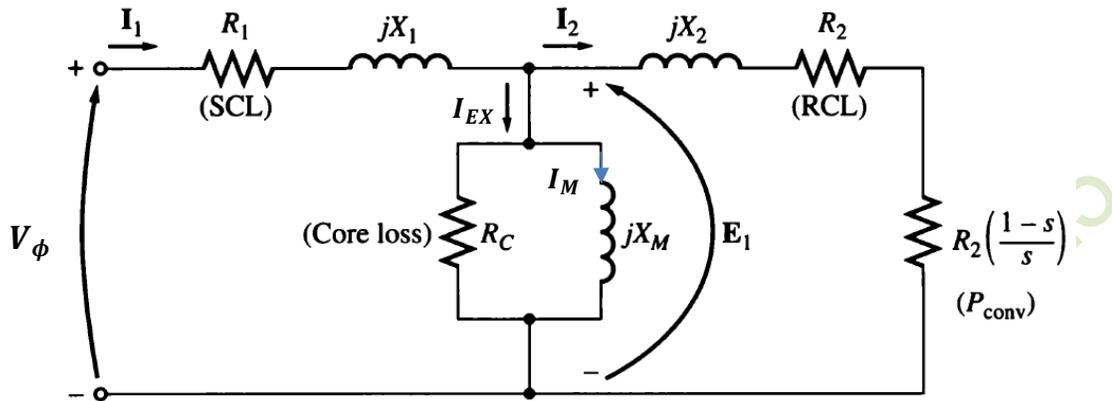
$$3I_2^2 R_{Conv} = 3I_2^2 \frac{R_2}{s} - 3I_2^2 R_2$$

$$R_{Conv} = \frac{R_2}{s} - R_2$$

Hence, the **converted resistor, which represents the converted power**, can be written as:

$$R_{Conv} = R_2 \left(\frac{1-s}{s} \right)$$

Thus, the per-phase equivalent circuit of an Induction motor can be redrawn as shown in the Figure below.



The output power is:

$$P_{out} = P_{Conv} - P_{F\&W} - P_{stray}$$

The induced (developed) torque is:

$$\tau_{ind} = \frac{P_{Conv}}{\omega_m}$$

$$\tau_{ind} = \frac{(1-s)P_{AG}}{(1-s)\omega_{sync}}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

The load torque is:

$$\tau_{load} = \frac{P_{out}}{\omega_m}$$

The actual torque available at the terminals (shaft) of the motor differs from the induced torque by an amount equivalent to Friction, Windage and stray torques losses on the machine.

Thus, the **loss torque**, which accounts for the aforementioned mechanical losses, is:

$$\tau_{loss} = \tau_{ind} - \tau_{load}$$

Example 6–2. A 480-V, 60-Hz, 50-hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction and windage losses are 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

- (a) The air-gap power P_{AG}
- (b) The power converted P_{conv}
- (c) The output power P_{out}
- (d) The efficiency of the motor

Solution

To answer these questions, refer to the power-flow diagram for an induction motor (Figure 6–13).

- (a) The air-gap power is just the input power minus the stator I^2R losses and core losses. The input power is given by

$$P_{in} = \sqrt{3}V_T I_L \cos \theta$$

$$= \sqrt{3}(480 \text{ V})(60 \text{ A})(0.85) = 42.4 \text{ kW}$$

From the power-flow diagram, the air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

$$= 42.4 \text{ kW} - 2 \text{ kW} - 1.8 \text{ kW} = 38.6 \text{ kW}$$

- (b) From the power-flow diagram, the power converted from electrical to mechanical form is

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 38.6 \text{ kW} - 700 \text{ W} = 37.9 \text{ kW}$$

- (c) From the power-flow diagram, the output power is given by

$$P_{out} = P_{conv} - P_{F\&W} - P_{misc}$$

$$= 37.9 \text{ kW} - 600 \text{ W} - 0 \text{ W} = 37.3 \text{ kW}$$

or, in horsepower,

$$P_{out} = (37.3 \text{ kW}) \frac{1 \text{ hp}}{0.746 \text{ kW}} = 50 \text{ hp}$$

- (d) Therefore, the induction motor's efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{37.3 \text{ kW}}{42.4 \text{ kW}} \times 100\% = 88\%$$

Example 6–3. A 460-V, 25-hp, 60-Hz, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

$$\begin{aligned} R_1 &= 0.641 \, \Omega & R_2 &= 0.332 \, \Omega \\ X_1 &= 1.106 \, \Omega & X_2 &= 0.464 \, \Omega & X_M &= 26.3 \, \Omega \end{aligned}$$

The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

- (a) Speed
- (b) Stator current
- (c) Power factor
- (d) P_{conv} and P_{out}
- (e) τ_{ind} and τ_{load}
- (f) Efficiency

Solution

The per-phase equivalent circuit of this motor is shown in Figure 6–12, and the power-flow diagram is shown in Figure 6–13. Since the core losses are lumped together with the friction and windage losses and the stray losses, they will be treated like the mechanical losses and be subtracted after P_{conv} in the power-flow diagram.

- (a) The synchronous speed is

$$n_{\text{sync}} = \frac{120 f_{se}}{P} = \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min}$$

or
$$\omega_{\text{sync}} = (1800 \text{ r/min}) \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 188.5 \text{ rad/s}$$

The rotor's mechanical shaft speed is

$$\begin{aligned} n_m &= (1 - s)n_{\text{sync}} \\ &= (1 - 0.022)(1800 \text{ r/min}) = 1760 \text{ r/min} \end{aligned}$$

or
$$\begin{aligned} \omega_m &= (1 - s)\omega_{\text{sync}} \\ &= (1 - 0.022)(188.5 \text{ rad/s}) = 184.4 \text{ rad/s} \end{aligned}$$

- (b) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$\begin{aligned} Z_2 &= \frac{R_2}{s} + jX_2 \\ &= \frac{0.332}{0.022} + j0.464 \\ &= 15.09 + j0.464 \, \Omega = 15.10 \angle 1.76^\circ \, \Omega \end{aligned}$$

The combined magnetization plus rotor impedance is given by

$$\begin{aligned} Z_f &= \frac{1}{1/jX_M + 1/Z_2} \\ &= \frac{1}{-j0.038 + 0.0662\angle -1.76^\circ} \\ &= \frac{1}{0.0773\angle -31.1^\circ} = 12.94\angle 31.1^\circ \Omega \end{aligned}$$

Therefore, the total impedance is

$$\begin{aligned} Z_{\text{tot}} &= Z_{\text{stat}} + Z_f \\ &= 0.641 + j1.106 + 12.94\angle 31.1^\circ \Omega \\ &= 11.72 + j7.79 = 14.07\angle 33.6^\circ \Omega \end{aligned}$$

The resulting stator current is

$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{V}_\phi}{Z_{\text{tot}}} \\ &= \frac{266\angle 0^\circ \text{ V}}{14.07\angle 33.6^\circ \Omega} = 18.88\angle -33.6^\circ \text{ A} \end{aligned}$$

(c) The power motor power factor is

$$\text{PF} = \cos 33.6^\circ = 0.833 \quad \text{lagging}$$

(d) The input power to this motor is

$$\begin{aligned} P_{\text{in}} &= \sqrt{3}V_T I_L \cos \theta \\ &= \sqrt{3}(460 \text{ V})(18.88 \text{ A})(0.833) = 12,530 \text{ W} \end{aligned}$$

The stator copper losses in this machine are

$$\begin{aligned} P_{\text{SCL}} &= 3I_1^2 R_1 && (6-25) \\ &= 3(18.88 \text{ A})^2(0.641 \Omega) = 685 \text{ W} \end{aligned}$$

The air-gap power is given by

$$P_{\text{AG}} = P_{\text{in}} - P_{\text{SCL}} = 12,530 \text{ W} - 685 \text{ W} = 11,845 \text{ W}$$

Therefore, the power converted is

$$P_{\text{conv}} = (1 - s)P_{\text{AG}} = (1 - 0.022)(11,845 \text{ W}) = 11,585 \text{ W}$$

The power P_{out} is given by

$$\begin{aligned} P_{\text{out}} &= P_{\text{conv}} - P_{\text{rot}} = 11,585 \text{ W} - 1100 \text{ W} = 10,485 \text{ W} \\ &= 10,485 \text{ W} \left(\frac{1 \text{ hp}}{746 \text{ W}} \right) = 14.1 \text{ hp} \end{aligned}$$

(e) The induced torque is given by

$$\begin{aligned}\tau_{\text{ind}} &= \frac{P_{\text{AG}}}{\omega_{\text{sync}}} \\ &= \frac{11,845 \text{ W}}{188.5 \text{ rad/s}} = 62.8 \text{ N} \cdot \text{m}\end{aligned}$$

and the output torque is given by

$$\begin{aligned}\tau_{\text{load}} &= \frac{P_{\text{out}}}{\omega_m} \\ &= \frac{10,485 \text{ W}}{184.4 \text{ rad/s}} = 56.9 \text{ N} \cdot \text{m}\end{aligned}$$

(In English units, these torques are 46.3 and 41.9 lb-ft, respectively.)

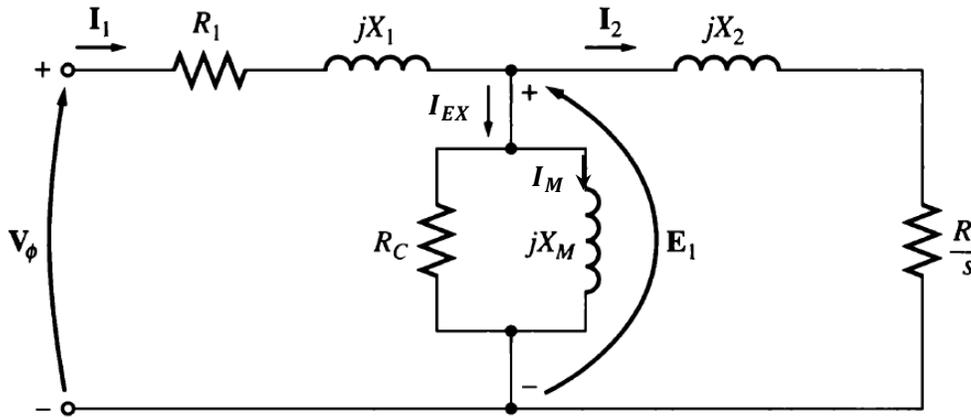
(f) The motor's efficiency at this operating condition is

$$\begin{aligned}\eta &= \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% \\ &= \frac{10,485 \text{ W}}{12,530 \text{ W}} \times 100\% = 83.7\%\end{aligned}$$

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Induction Motor Torque-Speed Characteristics

Recall the per-phase equivalent circuit of an Induction motor, shown in the Figure below,

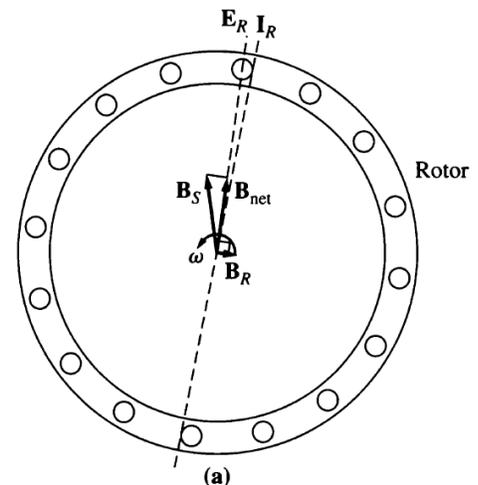


- ▣ X_M corresponds to the mutual flux linking the stator and rotor windings.
- ▣ The net magnetic field (B_{net}) is proportional to the magnetizing current (I_M), which is directly proportional to E_1 .
- ▣ If the voltage drop in the stator circuit is very small, E_1 may be assumed constant, as V_ϕ is. Hence, I_M and B_{net} are constants.
- ▣ **At no-load**, the slip is very small, the relative motion is also very small. Therefore, the rotor induced voltage (E_R) and the resulting rotor current (I_R) and rotor frequency (f_r) are all small.
- ▣ Because f_r is small, the rotor reactance is small, almost zero, and the maximum rotor current (I_R) is almost in phase with the rotor voltage (E_R).
- ▣ The rotor current produces a small rotor magnetic field (B_R), at an angle just slightly greater than 90° behind B_{net} .
- ▣ The magnetic fields, the rotor currents and voltages are shown in the Figure next.
- ▣ Since B_R is small, the stator current (I_1) must supply most of B_{net} , so I_1 is large even at no-load.
- ▣ The induced torque is:

$$\tau_{ind} = k \overline{B_R} X \overline{B_{net}}$$

$$\tau_{ind} = k B_R B_{net} \sin \delta; \text{ C.C.W.}$$

- ▣ Note that, at no-load B_R is small and so is the torque; the torque is just large enough to overcome the motor's rotational losses.



Loaded Motor:

- ✓ As the load torque on the motor shaft increases, the slip increases. Therefore, the rotor mechanical speed decreases, increasing the relative motion between the rotor and the magnetic fields. Consequently, the rotor induced voltage (E_R) and the resulting rotor current (I_R) and rotor frequency (f_r) all increase. The result is a larger rotor magnetic field (B_R). In brief,

$$\text{as } \tau_{load} \uparrow \Rightarrow s \uparrow \Rightarrow \omega_m \downarrow \Rightarrow \text{Relative motion} \uparrow \Rightarrow E_R \uparrow \text{ and } f_r \uparrow \Rightarrow I_R \uparrow \Rightarrow B_R \uparrow$$

- ✓ The rotor circuit becomes more inductive, because the increase in slip results in an increase in rotor frequency, thus an increase in rotor reactance; i.e.,

$$\text{as } s \uparrow \Rightarrow f_r \uparrow \Rightarrow X_R \uparrow$$

- ✓ Consequently, the rotor current (I_R) lags the rotor voltage (E_R) by a larger angle (θ_R), as seen in the Figure next.

- ✓ Also, the angle between $\overline{B_{net}}$ (or $\overline{E_R}$) and $\overline{I_R}$ increases.

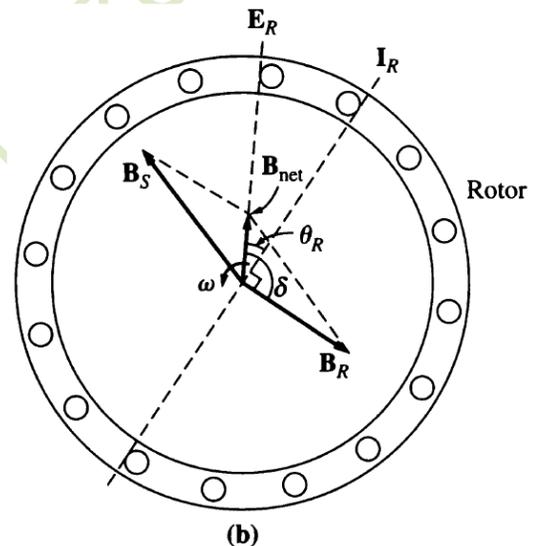
- ✓ The torque angle (δ) between $\overline{B_R}$ and $\overline{B_{net}}$ increases too!

- ✓ Recall, the induced torque,

$$\tau_{ind} = k B_R B_{net} \sin \delta; \quad \text{C.C.W.}$$

- ✓ Note that the increase in B_R tends to increase the induced torque, and an increase in the torque angle ($\delta > 90^\circ$) tends to decrease the torque induced. However, the overall effect is an increase in induced torque to supply the load torque.

- ✓ The peak or **Pullout torque** is reached when $\sin \delta$ decreases as much as B_R increases with increasing load torque. At this point, a further increase in load torque decreases the induced torque and the motor stops. The current drawn during halt condition will be as much as the starting current; very dangerous on the motor!



Torque-Speed Characteristic Plot of an Induction Motor

To plot the torque, the effect of varying the slip on each term on the plot will be examined. Recall,

$$\tau_{ind} = k B_R B_{net} \sin \delta$$

1. The rotor current, and hence rotor magnetic field (B_R), are linearly proportional to the slip (s) at low slip values, and they are almost constant at large slip values.

At low slip values, as $s \uparrow \Rightarrow I_R \uparrow \Rightarrow B_R \uparrow$

2. The net magnetic field (B_{net}) is proportional to the applied voltage (E_1). Since, E_1 is constant, the net magnetic field (B_{net}) is also constant regardless of the slip (s).

3. To explore the effect of the sine term ($\sin \delta$), recall that,

$$\delta = \theta_R + 90$$

where, θ_R is the power factor angle of the rotor, and it is the angle between $\overline{I_R}$ and $\overline{E_R}$.

Thus,

$$\sin \delta = \sin(\theta_R + 90) =$$

$\cos \theta_R$

But,

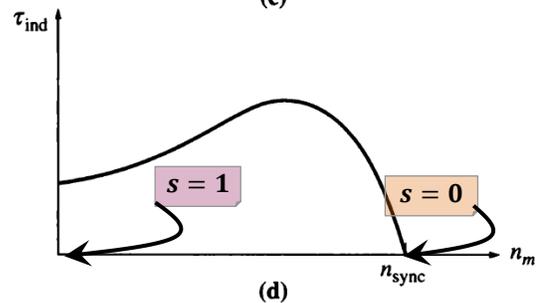
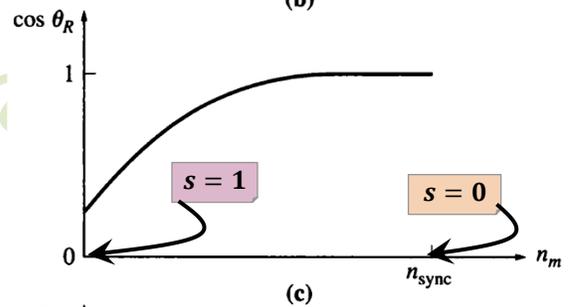
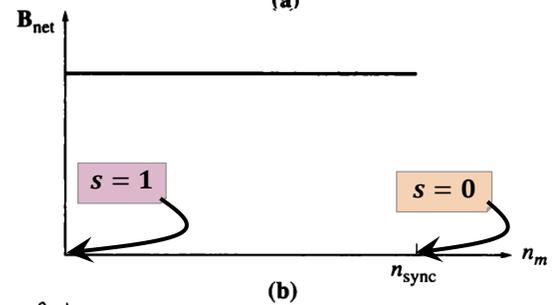
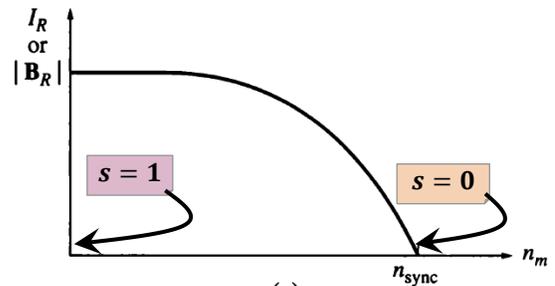
$$\theta_R = \tan^{-1} \frac{X_R}{R_R} = \tan^{-1} \frac{sX_{R0}}{R_R}$$

The rotor power factor, therefore, is:

$$\cos \theta_R = \cos \left(\tan^{-1} \frac{sX_{R0}}{R_R} \right)$$

$\cos \theta_R$ is almost unity at low slip values and decreases as s increases, as shown in Figure (c) above.

The overall effects of varying the slip on all terms are combined to produce a plot of the induced torque versus speed as shown in Figure (d) above.

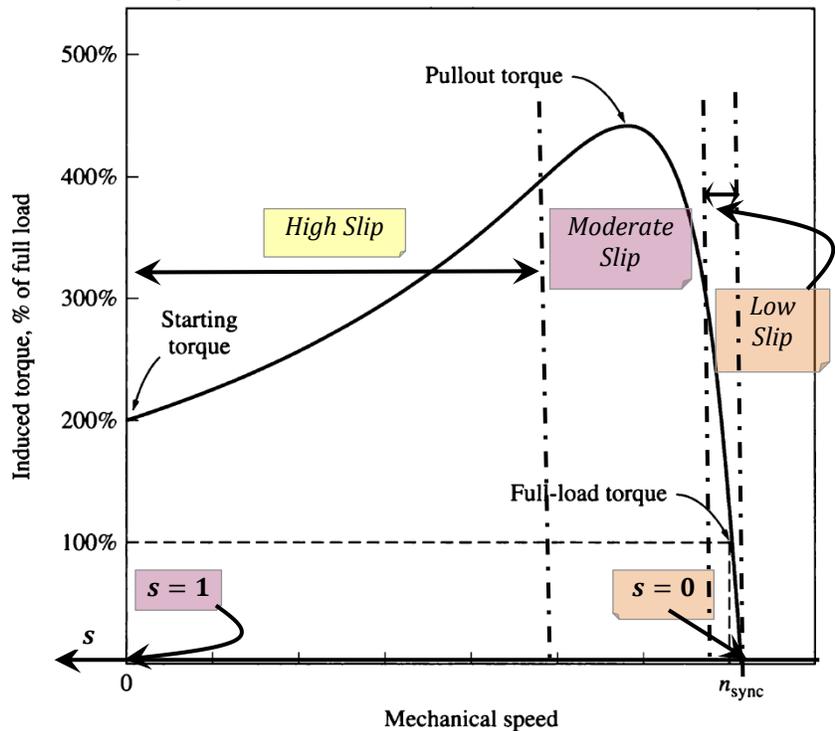


Characteristics of Torque-Speed Curves of an Induction Motor

The characteristic curve can be divided into 3 regions:

I. Low Slip Region

- The motor slip increases linearly with load.
- The motor speed decreases linearly with load.
- Rotor current increases linearly with slip.
- X_R is negligible.
- $\cos \theta_R \approx 1$
- It is the normal steady state operating region.



II. Moderate Slip Region

- X_R is of the same order of magnitude as R_R .
- I_R does not increase as rapidly as before.
- The rotor power factor starts to drop.
- The **peak (Pullout) torque** occurs at the point where, for an incremental increase in the load, the increase in the rotor current is exactly balanced by the decrease in the rotor power factor.

III. High Slip Region

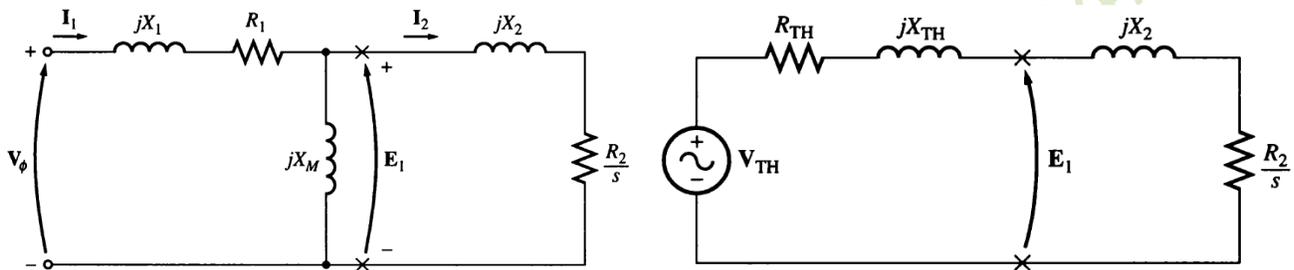
- The induced torque decreases with increasing the load (increasing the slip), because the increase in rotor current is overshadowed by the decrease in the rotor power factor.
- This is unstable region.
- Typically, Pullout torque is 200% to 300% of full (rated) load torque.
- The starting torque is 150% to 200%. Therefore, the Induction motor can start with full load attached to its shaft.

Derivation of The Induction Motor Induced Torque Equation

The induced (developed) torque is:

$$\tau_{ind} = \frac{P_{Conv}}{\omega_m} = \frac{P_{AG}}{\omega_{sync}} = \frac{3I_2^2 \frac{R_2}{s}}{\omega_{sync}}$$

and can be expressed in terms of motor parameters, by finding the Thevenin's Equivalent circuit (righthand Figure) below, and then deriving the rotor current.



Assuming that $X_M \gg X_1$ and $X_M + X_1 \gg R_1$ then,

$$R_{TH} = R_1 \left(\frac{X_M}{X_M + X_1} \right)^2$$

$$X_{TH} \approx X_1$$

$$V_{TH} = \frac{X_M}{X_M + X_1} V_\phi$$

Then, the induced torque can be derived as:

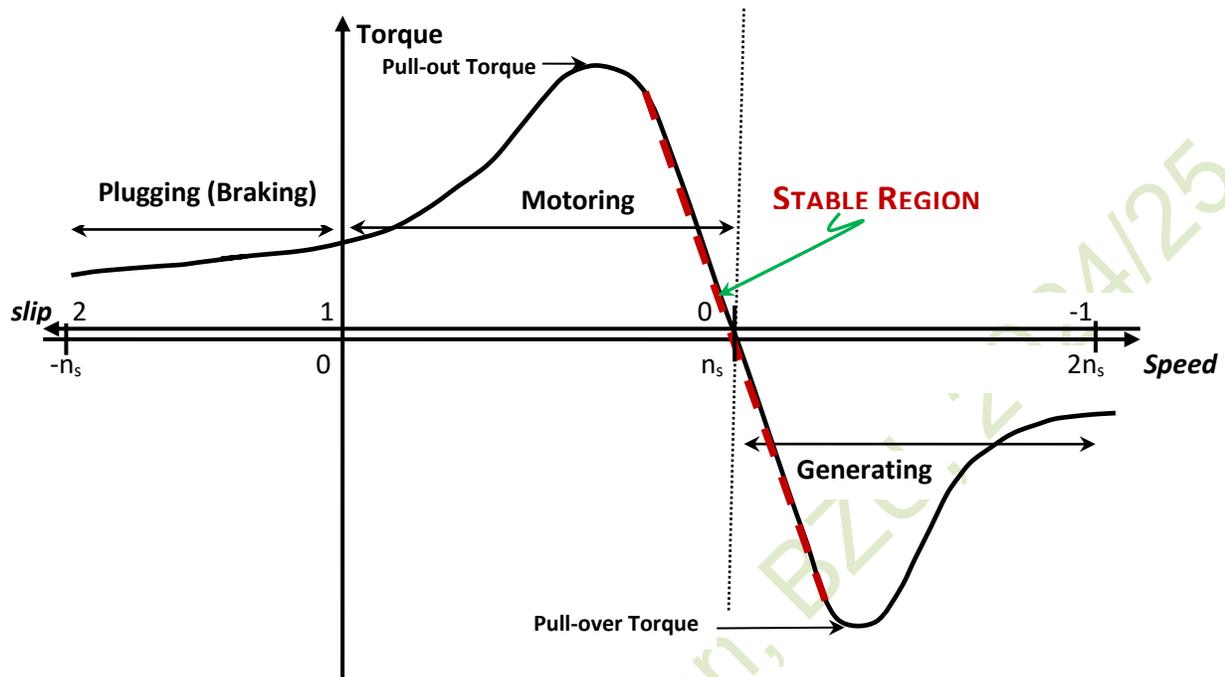
$$\tau_{ind} = \frac{3I_2^2 R_2/s}{\omega_{sync}}$$

$$\tau_{ind} = \frac{3|V_{TH}|^2 R_2/s}{\omega_{sync} \left[\left(R_{TH} + \frac{R_2}{s} \right)^2 + (X_{TH} + X_2)^2 \right]}$$

Exercise:

Use Matlab/Simulink Environment to plot the induced torque versus speed/slip for a slip in the range [-1.0001 to 2.0001]; avoid division by a slip of 0; use a small step change; e.g. $\Delta s = 0.005$!

A plot of the torque equation as a function of speed (or slip) is shown in the Figure below.



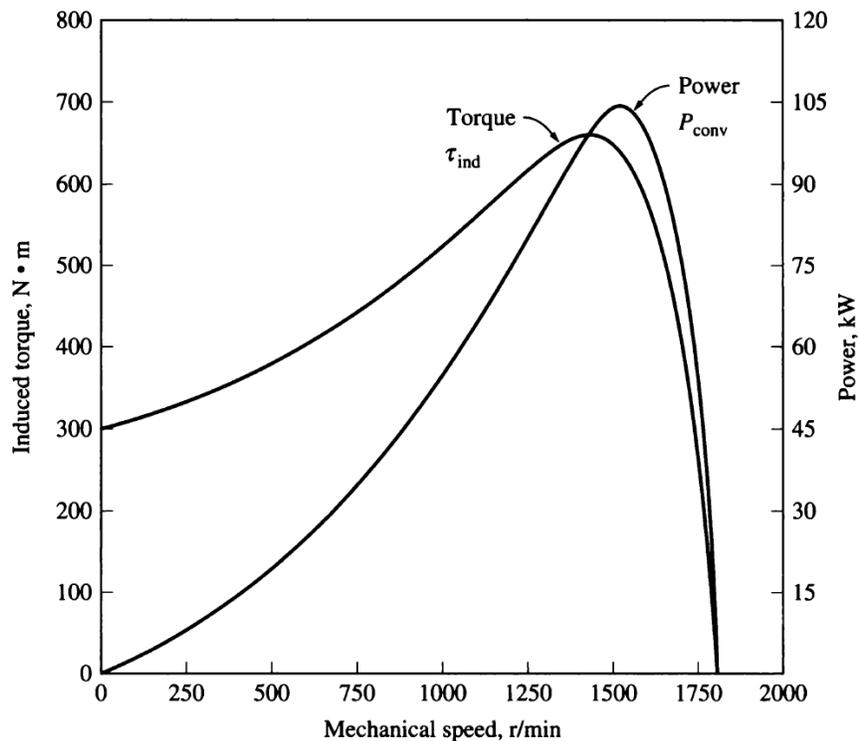
Comments on The Induction Motor Torque-Speed Curves

1. $\tau_{ind} = 0$ at synchronous speed, also E_R , I_R and B_R are all zero.
2. Torque–speed characteristic is **nearly linear** between no-load and full load. In this region, the rotor resistance is much larger than the rotor reactance. Therefore, I_R , B_R and τ_{ind} increase linearly with increasing slip.
3. The maximum possible torque (Pullout torque) is 2 to 3 times the full (rated) load torque.
4. The **starting torque** is slightly **higher than its full load** torque.
5. By inspecting the torque equation, **the torque (for a given slip) varies as the square of the applied voltage.** This is a useful fact for speed control.

6. If the rotor is driven faster than the synchronous speed, then the direction of τ_{ind} reverses and the machine operates as an Induction (asynchronous) generator.
7. If the motor is turning backward relative to the direction of magnetic fields, τ_{ind} will stop the machine very rapidly and will try to rotate it in the other direction.

This can be achieved by switching (swapping) any two stator phases. Stopping the motor rapidly is called **Plugging**.

8. The peak converted power occurs at different speed of that where the peak torque occurs as shown in the Figure below.



The Maximum (Pullout) Torque in an Induction Motor

➔ Since, $\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$, τ_{max} occurs when P_{AG} is maximum.

➔ But, the P_{AG} is consumed in $\frac{R_2}{s}$, so from the Maximum Power Transfer Theorem, the magnitude of the source impedance should equal $\frac{R_2}{s}$, (because the angle of the load impedance is fixed).

➔ The source impedance is,

$$Z_{source} = R_{TH} + j(X_{TH} + X_2)$$

➔ Therefore, the Maximum Power Transfer is satisfied when,

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

➔ Solving for the slip at maximum torque yields:

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

➔ s_{max} is directly proportional to the referred rotor resistance (R_2).

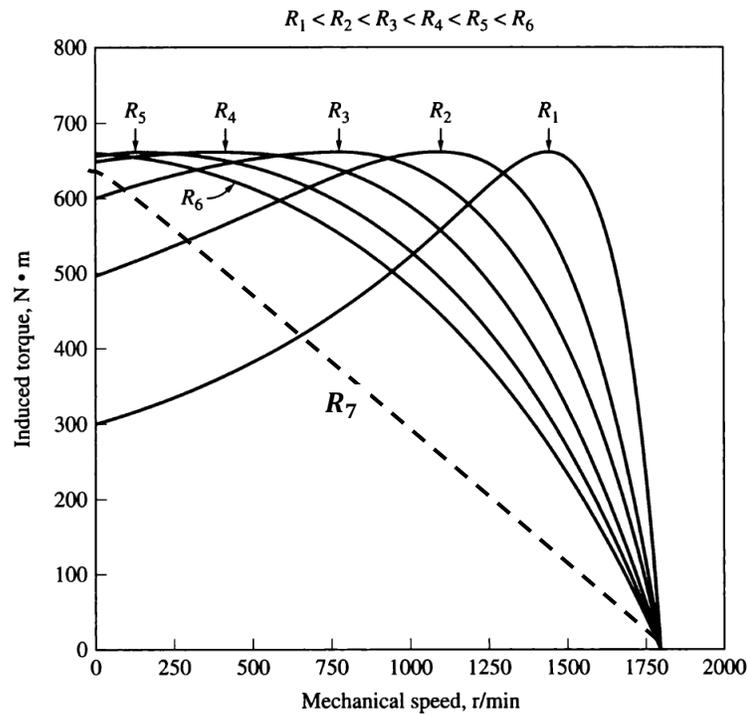
➔ Inserting the expression for s_{max} in the torque equation yields:

$$\tau_{max} = \frac{3|V_{TH}|^2}{2\omega_{sync} \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2} \right]}$$

➔ τ_{max} is directly proportional to $|V_{TH}|^2$ and is inversely proportional to the stator impedance and the rotor reactance.

➔ τ_{max} is independent of the referred rotor resistance (R_2).

➔ The effect of increasing the rotor resistance is demonstrated in the Figure below.



➔ In a Wound-Rotor Induction Motor, the **starting torque** may be increased, to start up heavy loads, by inserting WYE-connected three-phase resistors in series with the Brushes and Slip Rings. Once the motor has started, the external resistors can be removed off the rotor circuit (by shorting them out).

➔ Note that, the starting torque increases with increasing the rotor resistance up to a particular value of rotor resistance (R_6), greater than which, the starting torque starts decreasing again as is the case with R_7 , which is greater than R_6 !

➔ Increasing the rotor resistance (or addition of the external rotor resistors) increases the motor slip, reduces the motor efficiency, decreases the motor speed, and increases the speed regulation.

Trends in Induction Motor Design

- ◆ The fundamental ideas behind the Induction motor were developed during the late 1880's by **Nicola Tesla**.
- ◆ However, Induction motors were not **commercially available** until 1896, after the development of two and three phase power systems and rotating magnetic fields.
- ◆ Between then and 1970's, efforts were concerted on improving the quality of steels, casting techniques, insulation and construction features of Induction motors. These trends resulted in a smaller size and reduction in construction costs. For example, a modern 100hp is of the same size as 7.5 hp of 1897.
- ◆ However, these efforts did not lead to improvements in the motor efficiency.
- ◆ Since 1973, after the climb in oil prices and electricity, the efficiency and the life time operating cost of the machine have become more and more important than the initial installation cost.

Techniques Used to Improve Induction Motor's Efficiency

1. Reducing the **stator copper losses** by using more copper in the stator windings.
2. Increasing the rotor and stator core length to **reduce the magnetic flux density** in the airgap of the machine. This reduces magnetic saturation of the machine and decreases core losses.
3. Using **more steel** in the stator to allow a great amount of heat transfer out of the motor, which reduces the operating temperature. The **rotor's fans** are redesigned to reduce Windage losses. Fannies at the motor body also improve the heat transfer.
4. Using a **special high-grade electrical steel** with low Hysteresis losses in the stator.
5. Using steel that has a **high internal resistivity** and which is made of thin lamination to reduce eddy current losses.
6. Designing the rotor carefully to produce **a uniform airgap**, and hence to reduce the stray losses.

Starting Induction Motors

- They do not have starting problems like Synchronous motors. However, their starting current is high.
- In Wound-Rotor Induction motors, inserting external rotor resistors can reduce the starting current. By doing so, the starting torque may be increased.
- In Squirrel-Cage Induction motors, the starting current depends on the motor's power and the effective rotor resistance at starting conditions.
- All Squirrel-Cage motors have a **Starting Code Letter** on the name plate that sets the limits on the amount of starting current drawn by the motor. **National Electrical Manufacturers Association (NEMA)** Starting Code Letters are shown in the Table below.

Nominal code letter	Locked rotor, kVA/hp	Nominal code letter	Locked rotor, kVA/hp
A	0–3.15	L	9.00–10.00
B	3.15–3.55	M	10.00–11.00
C	3.55–4.00	N	11.20–12.50
D	4.00–4.50	P	12.50–14.00
E	4.50–5.00	R	14.00–16.00
F	5.00–5.60	S	16.00–18.00
G	5.60–6.30	T	18.00–20.00
H	6.30–7.10	U	20.00–22.40
J	7.10–8.00	V	22.40 and up
K	8.00–9.00		

- The apparent power at starting can be expressed as:

$$S_{start} = \text{Rated Motor's Horse Power} \times \text{Code Letter Factor}$$

- Then, the starting current can be estimated from the equation:

$$I_{L,start} = \frac{S_{start}}{\sqrt{3}V_L}$$

Example 6–7. What is the starting current of a 15-hp, 208-V, code-letter-F, three-phase induction motor?

Solution

According to Figure 6–34, the maximum kilovoltamperes per horsepower is 5.6. Therefore, the maximum starting kilovoltamperes of this motor is

$$S_{start} = (15 \text{ hp})(5.6) = 84 \text{ kVA}$$

The starting current is thus

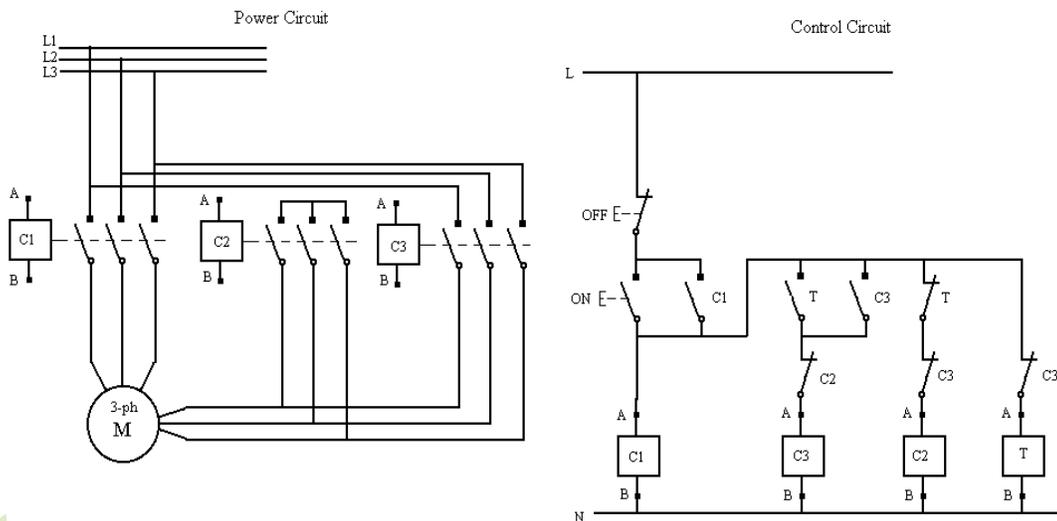
$$I_L = \frac{S_{start}}{\sqrt{3}V_T} = \frac{84 \text{ kVA}}{\sqrt{3}(208 \text{ V})} = 233 \text{ A}$$

Reducing The Starting Current

1) Inserting extra inductors or resistors in the power line (old fashion and rarely used). To be explained later!

2) Star-Delta or WYE-Delta (Y/Δ) Starter

- This method is appropriate for Induction motors designed to operate normally with “Delta” connection; in other words, for 400V supply, motors assigned Y/Δ and rated as **692/400V are appropriate** for this method, whilst motors whose rated voltage **400/230V are not appropriate for this method.**
- Connecting the motor as a “Star” during starting-up, will reduce **the stator phase voltage and current by $\frac{1}{\sqrt{3}}$** ; the supply **(line) current will be reduced by $\frac{1}{3}$.**
- Also, by this method, the starting (and full load) **torque is reduced by $\frac{1}{3}$** , because $\tau_{ind} \propto V_1^2$.
- Note that, both connections have the same Ampere/torque rating.
- The power and control circuits for a Y/Δ Starter, employing contactors, are shown in the Figure below.



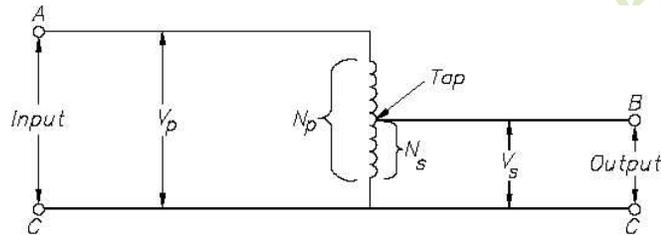
- Initially, contactors C_1 and C_2 are closed, when motor is up to speed and after a preset time, C_2 is opened and C_3 is closed (C_1 is still closed).

3) Auto-Transformer

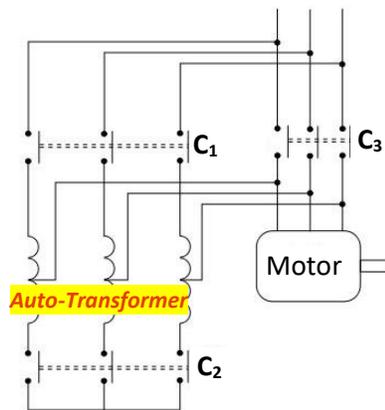
Reducing the motor terminal voltage by using Auto-Transformers to step down the voltage (note that torque decreases with $|V_{TH}|^2$).

Recall, the Auto-Transformer voltage relationship,

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = a_T$$



A schematic of an Auto-Transformer starter is shown in the Figure below.



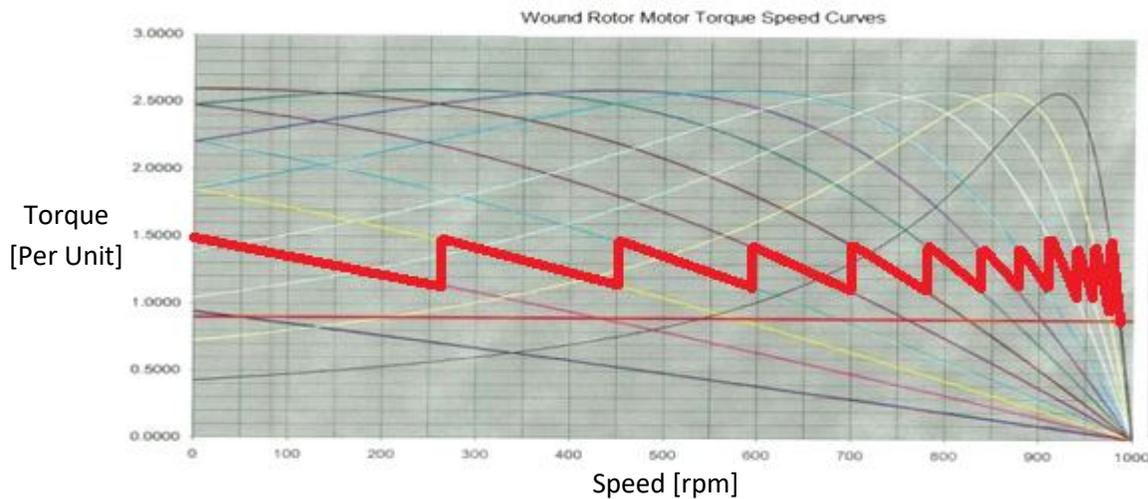
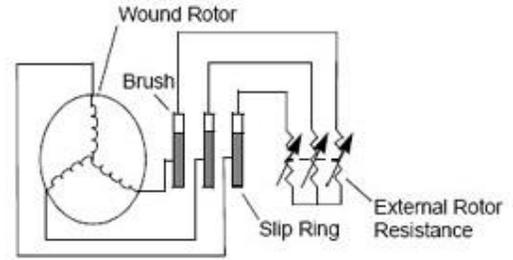
- ◆ The contactors in the Figure are normally open contacts.
- ◆ The sequence of switching the contactors in a “closed transition fashion” is summarized as:

$$C_1(\text{on}) \xrightarrow{t_0} C_2(\text{on}) \xrightarrow{t_1} C_2(\text{off}) \xrightarrow{t_2} C_3(\text{on}) \xrightarrow{t_3} C_1(\text{off})$$

- ◆ The secondary to primary turns ratio is: $a_T = \frac{N_S}{N_P}$, such that $a_T < 1$.
- ◆ The motor's phase voltage and stator phase current are reduced by $a_T \left(= \frac{N_S}{N_P} \right)$; about 65%. However, the starting torque is reduced by a_T^2 .

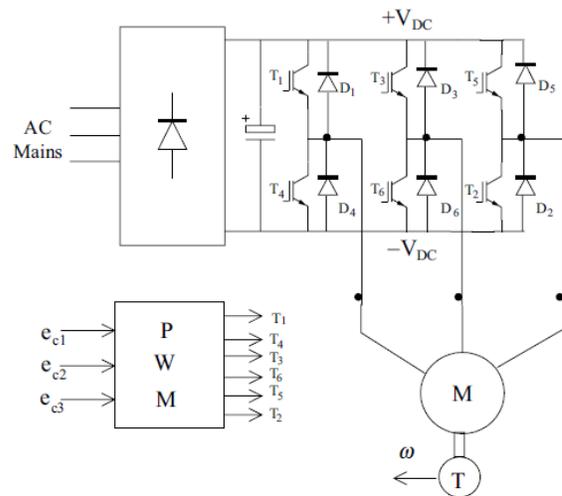
4) Rotor Resistance Starters

- ☐ Three-phase External Resistors are connected in series with the rotor windings of a Wound-Rotor Induction motor via the Slip Rings and Brushes, as shown next.
- ☐ The effective rotor resistance increases, and consequently the starting current decreases.
- ☐ The resistor is **removed gradually or in steps** as the motor has started, following the torque-speed trajectory shown in the Figure below.



5) Variable Voltage–Variable Frequency (VVVF) Drive

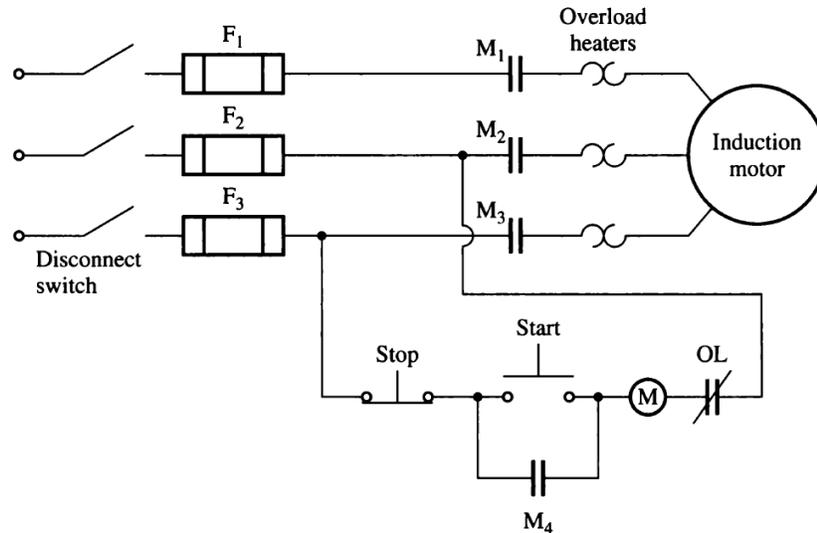
- 🌈 The output voltage and frequency of a three-phase Current Source Inverter (CSI), Voltage Source Inverter (VSI), which is shown in the Figure next, or Cycloconverter can be adjusted to control the starting characteristics.
- 🌈 At starting, the voltage and frequency are reduced in proportionality (**$\frac{V_\phi}{f_e}$ is kept constant**), and then increased gradually as the motor starts. Hence, it is called Variable-Voltage Variable-Frequency (VVVF) Drive.



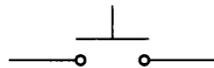
Induction Motor Starting Circuits

I. Basic Magnetic Induction Motor Starter Circuit

A typical Across-the-Line Magnetic Induction Motor Starter Circuit is shown in the Figure below.



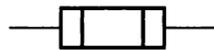
Disconnect switch



Push button; push to close



Push button; push to open



Fuse

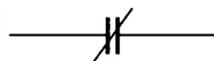


Relay coil; contacts change state when the coil energizes



Normally open

Contact open when coil deenergized

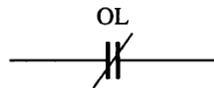


Normally shut

Contact shut when coil deenergized



Overload heater



Overload contact; opens when the heater gets too warm

Notes:

- Normally closed (shut) overload contacts, open when heaters get too hot.
- **M₄** shorts the start-button allowing the operator to release it without disconnecting power from the **M** coil.

The Basic Starter Circuit Built-in Protective Features are:

1. Short Circuit Protection

It is achieved by the Fuses which blow up when very high currents occur (greater than the full load current; the fuse operating current equals the full load current multiplied by a Fusing Factor).

2. Overload Protection

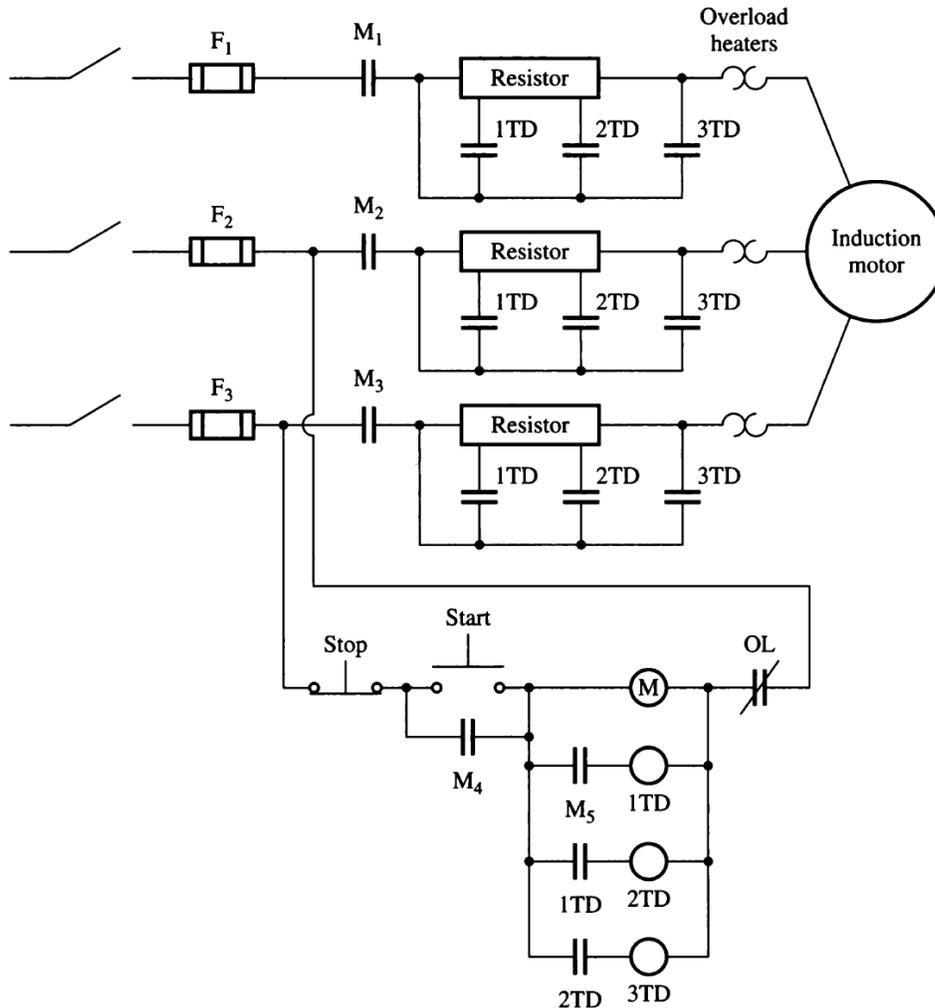
When the temperature of the heater rises far enough, the **OL** contacts open de-energizing the **M** relay, which in turn opens the normally open **M** contacts and removing power from the motor.

3. Under Voltage Protection

If the line voltage applied to the motor falls below a certain value, the voltage applied to the **M** relay will also fall and the relay will be de-energized. The **M** contacts then open removing power from the motor terminals.

II. A Modified Magnetic Induction Motor Starter Circuit

To reduce the starting current, the basic magnetic circuit is modified by adding a series resistor in each line. Each resistor will be shorted out in steps controlled by time delay relays (**1TD, 2TD, and 3TD**).



1TD, 2TD, and 3TD are time delay relay; when their coil is energized, there is a preset time delay before their contacts close. The starting resistors are shorted out in **3 steps**.

This modified starter circuit has the following Protective features:

1. Short Circuit Protection
2. Overload Protection
3. Under Voltage Protection
4. Reducing the starting current by inserting three-phase resistors in series with each line during the starting, and then removing parts of each resistor in steps (3 steps for resistors overall removal).

Speed Control Induction Motor

Recall, the synchronous speed is:

$$n_{sync} = \frac{120f_e}{p}$$

and the mechanical speed is:

$$n_m = (1 - s)n_{sync}$$

Speed is controlled by:

1) Pole Changing

a- The Method of Consequent Poles

- This method changes the number of poles by a ratio of 2:1 via a simple switching operation.
- It is simple to implement in Squirrel-Cage, but in a Wound-Rotor the rotor windings must be rearranged.
- By this method, the speed can be changed by a factor 2:1.

b- Multiple Stator Windings Method

It employs multiple stator windings with different number of poles. One set only is energized at a time. It increases cost and weight of the motor.

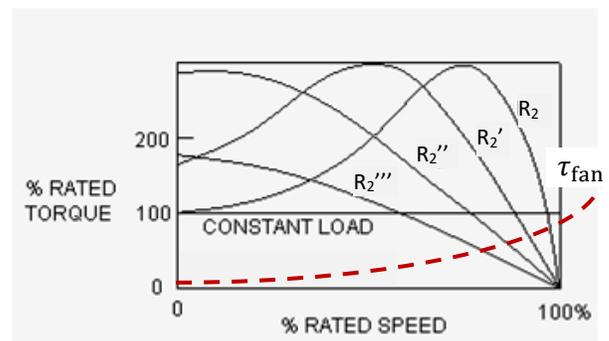
2) Changing the Rotor Resistance

- ➔ Historically, added rotor resistance method was used widely for Induction motors' speed control.
- ➔ Inserting external resistors to the rotor circuit changes torque-speed characteristic, reduces the starting current, may increase the starting torque, but reduces the efficiency (increasing slip; since $P_{RCL} = sP_{AG}$, the rotor losses are increased!).

- ➔ The Figure next, shows the torque-speed curves for an Induction motor with different values of added rotor resistance, such that:

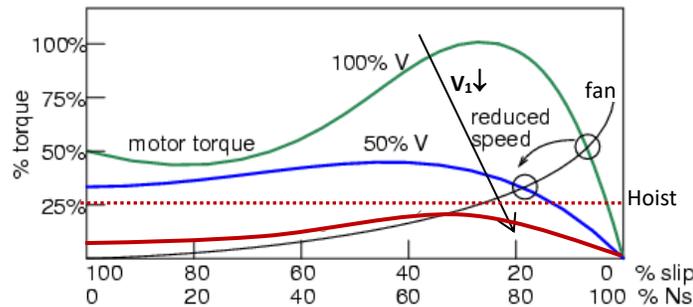
$$R_2''' > R_2'' > R_2' > R_2$$

- ➔ As the rotor resistance increases, the motor speed decreases.
- ➔ At no-load the motor still runs at the synchronous speed.



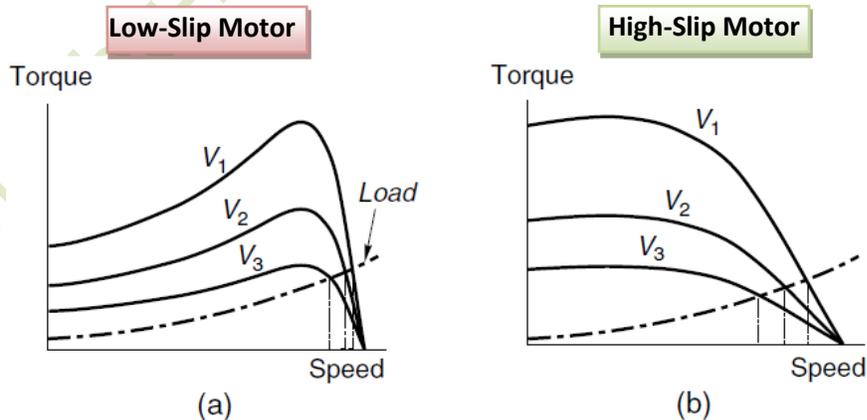
3) Changing the Line Voltage

- This method is used for small motor driving fans.
- The method relies on the fact that, decreasing the applied voltage reduces the induced torque ($\tau_{ind} \propto V_{TH}^2$). Consequently, the speed decreases.



- The difficulty is **loss of torque at reduced voltages**.
- **This method is suitable for high slip motors. On the other hand, it is not suitable for low resistance rotors (low slip motors).**
- The Figures below show that the speed control range by this method is restricted in a low slip motor (Figure (a)) compared to that in a high slip motor (Figure (b)). Note that the stator voltages are related as:

$$V_1 > V_2 > V_3$$



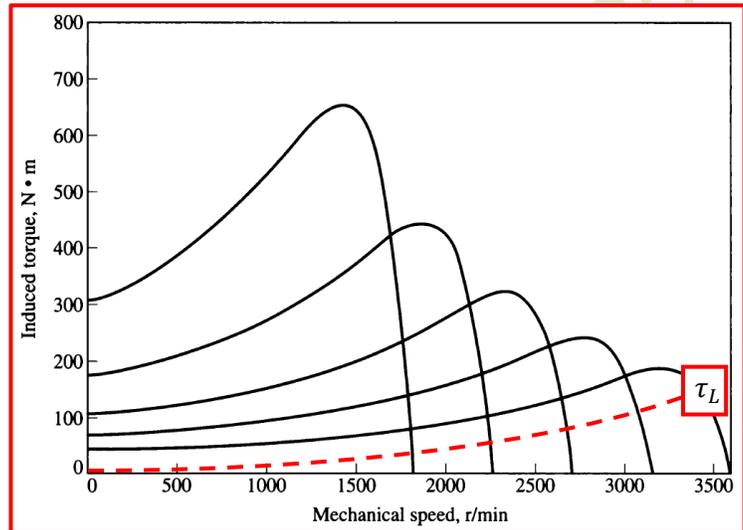
- However, the **efficiency is reduced** by this method to $\sim(1 - s)$.

4) Changing The Line Frequency

The frequency may be varied by employing a Cycloconverters, Inverters,...

- By changing the electrical frequency, the no-load speed on the torque-speed characteristic curve changes. The speed may be changed in the range: $0.05n_{base}$ to $2n_{base}$

- Above the rated frequency, the voltage is held at rated value, resulting in a flux weakened machine. The Figure next shows the characteristics for 3000rpm motor.



- Below the rated frequency, if the terminal voltage stays at its rated value, excessive magnetization

current flows, because the machine may become over fluxed. The machine flux ($\phi(t)$) is related to the stator voltage and the angular speed (frequency) by:

$$\phi(t) = \frac{\sqrt{2}V_1}{N_s\omega} \cos \omega t$$

where, N_s is the number of stator windings' turns.

- This may cause saturation to the Induction motor core!
- Hence, V_1 should be reduced with f_e "derating"! \Rightarrow The motor power should also be decreased to avoid overheating the stator windings.

5) Variable Voltage Variable Frequency Drives (VVVF)

- It is clear that the most efficient and robust speed control is based on adjusting the supply voltage and frequency using a Power Electronic AC-to-AC Controller or DC-to-AC Inverter.
- In a Variable-Voltage Variable-Frequency (VVVF) Induction Motor Drive, employ **constant V/f** control below rated speed. *make $V_1 \propto \omega$* ; i.e.,

$$V_1 = k\omega$$

where, the constant $k = \frac{V_{rated}}{\omega_{rated}}$

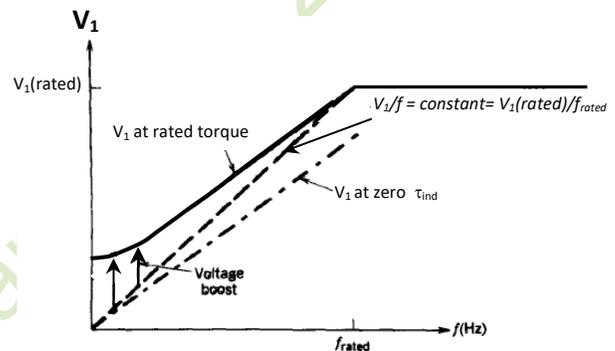
- But, at low frequencies (below 10Hz), the controller should control for R_1 loss somehow! A **Voltage Boost** is needed to compensate for the voltage drop across R_1 at low frequencies.

$$V_1 = k\omega + constant$$

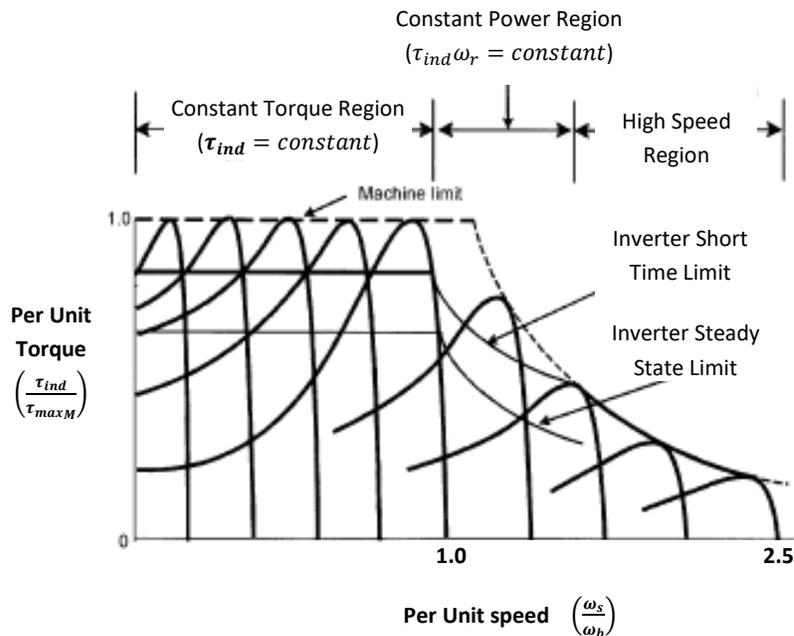
- The magnitude of the Voltage Boost is dependent on the motor load (current) and the stator resistance, as seen in the Figure next.

Therefore,

$$V_1 = k\omega + (I_2 + I_m)R_1$$

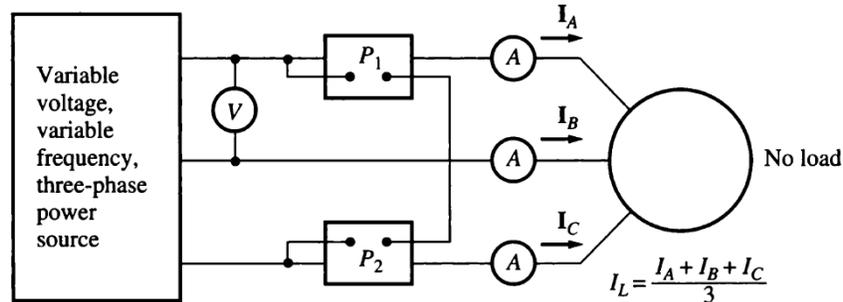


- Above rated frequency, the voltage is held at its rated value.



Determining Induction Motor Equivalent Circuit Parameters

- i. **No Load Test** \Rightarrow analogous to open circuit test in a transformer
 - a. Connect the meters as shown in the Figure below

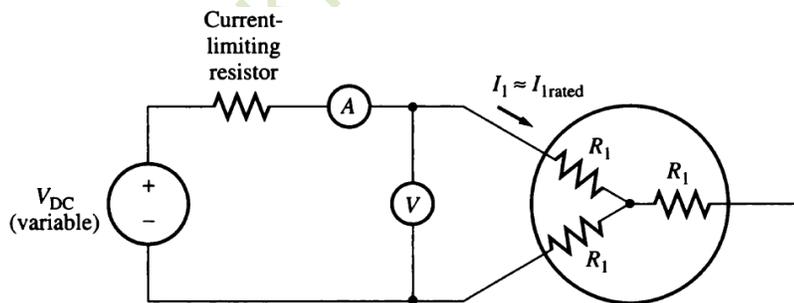


- b. Measure no-load (rotational) losses
- c. Measure the 3 line currents and find their average value
- d. The **rotational power losses**, and also the **excitation branch impedance** can be found;

$$|Z_{eq}| = \frac{V_{\phi}}{I_{1,nl}} \approx X_1 + X_M$$

ii. DC Test for Stator Resistance

- This test is used to measure the stator resistance (R_1). The effect of inductance is cancelled here.
- Adjust the DC voltage until rated current flows in the stator windings



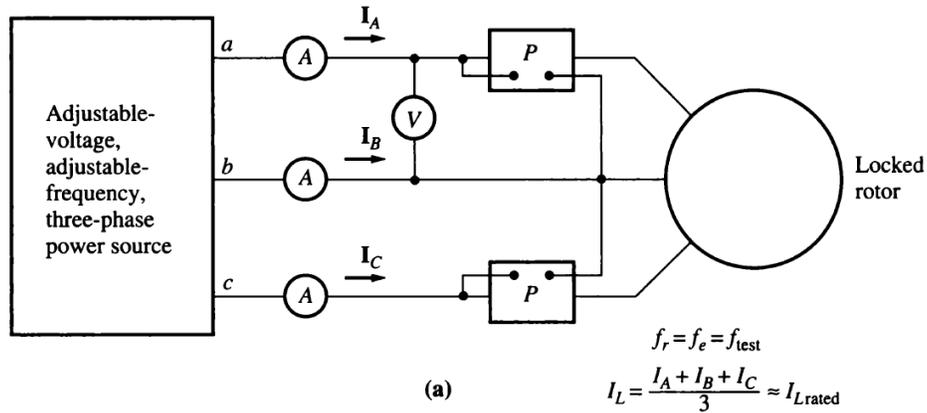
- Measure the DC voltage and current, by connecting the meters as shown in the Figure below,
- Then, the stator resistance (R_1) can be found as:

$$2R_1 = \frac{V_{DC}}{I_{DC}}$$

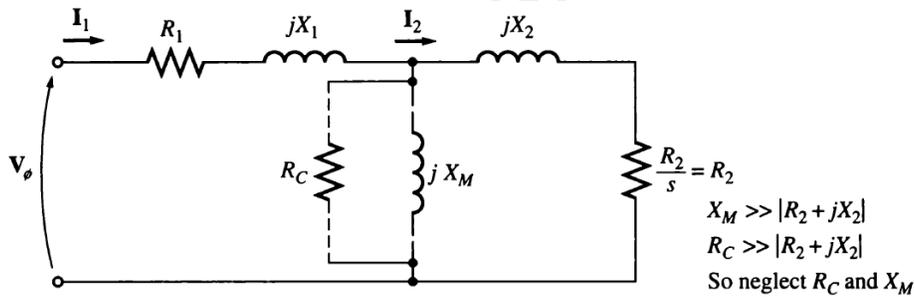
$$R_1 = \frac{V_{DC}}{2I_{DC}}$$

iii) **Locked (Blocked) Rotor Test** \Rightarrow analogous to short circuit test in a transformer

- ✓ Since the rotor normal operating frequency is low, and it differs from that at locked rotor condition, and as a compromise the **test source frequency is set to be 25% or less of the rated frequency.**
- ✓ The meters during this test are connected as shown in the Figure below.



- ✓ The excitation branch is neglected during this test.
- ✓ Adjust the AC test voltage **until the rated line current flows!**
- ✓ The equivalent circuit during this test is shown in the Figure below.



- ✓ Then calculate:

$$PF = \cos \theta = \frac{P_{in}}{\sqrt{3}V_T I_L}$$

$$|Z_{LR}| = \frac{V_\phi}{I_1} = \frac{V_T}{\sqrt{3}I_L}$$

$$Z_{LR} = R_{LR} + jX'_{LR}$$

$$= |Z_{LR}| \cos \theta + j|Z_{LR}| \sin \theta$$

$$X'_{LR} = X'_1 + X'_2$$

$$R_{LR} = R_1 + R_2$$

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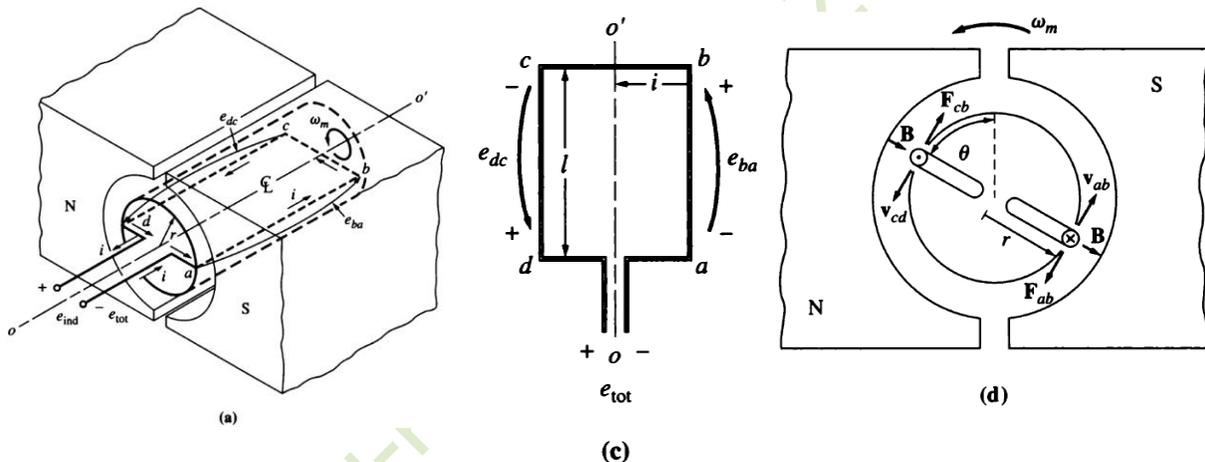
PART VII

DC Machinery Fundamentals

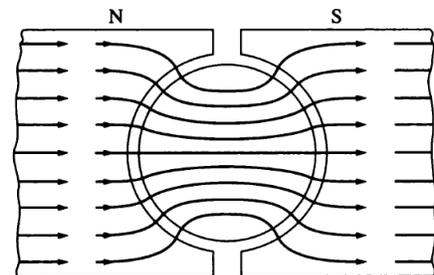
- ➔ Most of DC machines are like AC machines in that they have AC voltages and currents within them.
- ➔ A DC machine has a DC output only because it has a **commutator and brushes**, which convert the internal AC voltage into a DC voltage.

A Simple Rotating Loop between Curved Pole Faces

A simple loop was placed between curved pole faces as shown in Figure (a) below. The top view and front view of the loop are shown in Figure (c) and Figure (d), respectively.



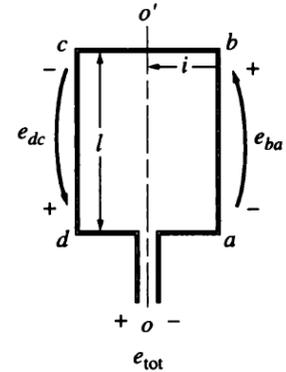
- ➔ To minimize the reluctance of the flux path through the machine, the magnetic flux must take the shortest path through the airgap between the pole face and the rotor surface.
- ➔ Thus, the **magnetic flux is perpendicular to the rotor surface**, as shown in the Figure next.
- ➔ The airgap is uniform \Rightarrow The reluctance is the same under the pole faces \Rightarrow The magnetic flux density (B) is the same (constant) everywhere under the pole faces.
- ➔ The loop is turned at an angular velocity (ω) around the axis $o-o'$.
- ➔ The tangential speed is: $v = \omega r$, where ' r ' is the radius of the loop.



- ➔ Each segment of the loop will have an induced voltage in it;

$$e_{ind} = (\vec{v} \times \vec{B}) \cdot \vec{l}$$

- ➔ The total induced voltage in the loop (e_{tot}) is the summation of the induced voltages in individual segments. The individual segments are:



- Segment 'ba':

$$e_{ba} = \begin{cases} vBl & \text{positive into the page, under pole face} \\ 0 & \text{beyond pole edges} \end{cases}$$

- Segment 'cb':

$(\vec{v} \times \vec{B})$ is either into the page or out of the page, whilst \vec{l} is in the plane of the page; i.e., $(\vec{v} \times \vec{B}) \perp \vec{l}$.

Therefore,

$$(\vec{v} \times \vec{B}) \cdot \vec{l} = 0.$$

$$\therefore e_{cb} = 0$$

- Segment 'dc':

$$e_{dc} = \begin{cases} vBl & \text{positive out of the page, under pole face} \\ 0 & \text{beyond pole edges} \end{cases}$$

- Segment 'ad':

$(\vec{v} \times \vec{B})$ is either into the page or out of the page, whilst \vec{l} is in the plane of the page. Therefore,

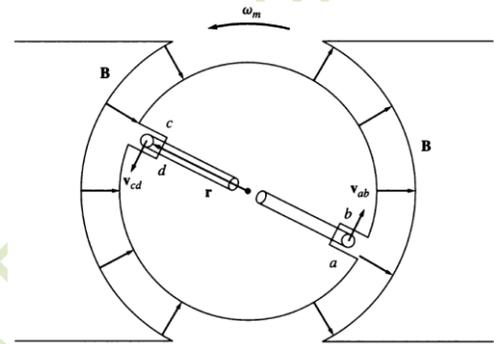
$$(\vec{v} \times \vec{B}) \cdot \vec{l} = 0.$$

$$\therefore e_{ad} = 0$$

- ➔ The total induced voltage, therefore, is:

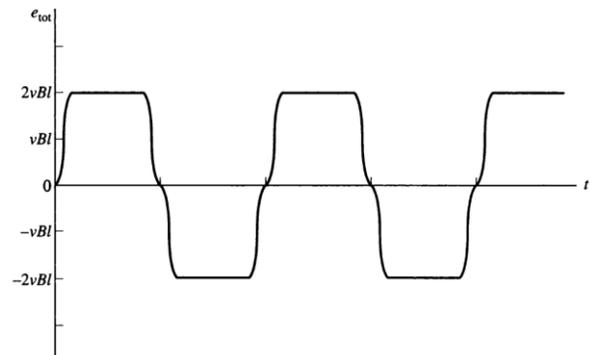
$$e_{tot} = e_{ba} + e_{cb} + e_{dc} + e_{ad}$$

$$\Rightarrow e_{tot} = \begin{cases} 2vBl & \text{under pole face, with the polarity shown in the Figure above} \\ 0 & \text{beyond pole edges} \end{cases}$$



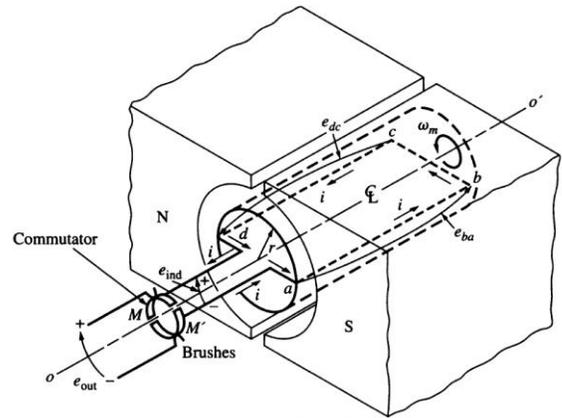
- ➔ When the loop rotates 180° , the direction of voltage reverses, but the magnitude remains constant.

- ➔ The resulting voltage as a function of time is shown in the Figure next.

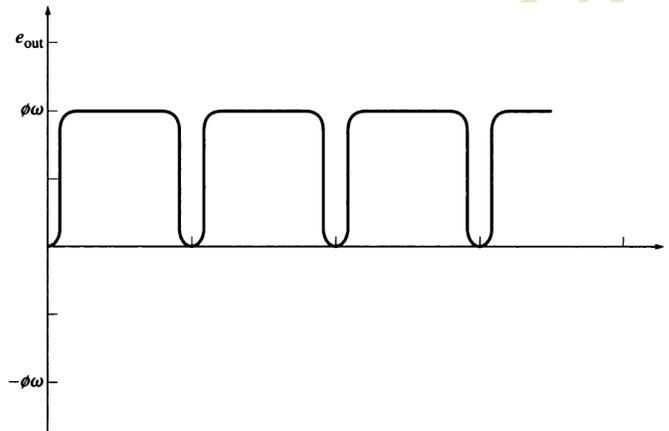


Getting DC Voltage out of The Rotating Loop

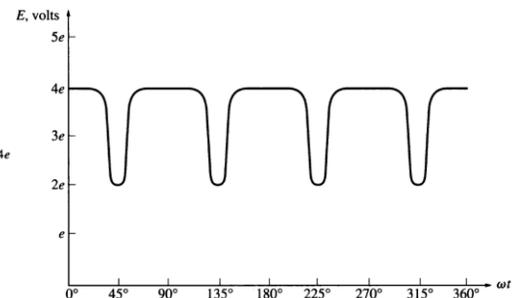
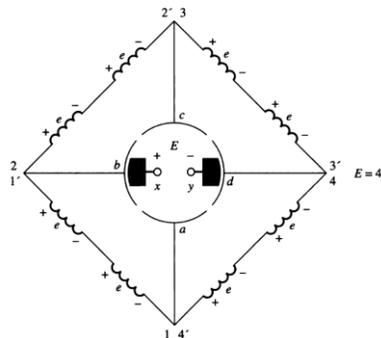
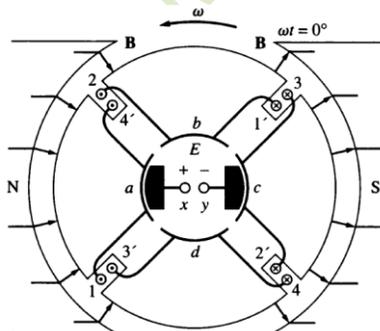
- ➔ This can be achieved by commutator segments and brushes, which act as a **mechanical rectifier**.
- ➔ **Commutator Segments:** are two (or more) semicircular conducting segments encircling the shaft and insulated from it. Each segment is connected to an end of the loop, and these segments are insulated from each other, as shown in the Figure next.



- ➔ **Brushes:** are blocks of Graphite-like Carbon-Compounds with good conductivity and low friction.
- ➔ Every time the voltage switches direction, the contacts (brushes) also switch connections, and the output voltage at the terminals of the brushes is built up in the same way as shown in the Figure below.



- ➔ When the voltage is zero, the brushes short circuit two commutator segments.
- ➔ If the machine has **four loops**, as shown in the Figures below, then the output voltage will be as illustrated.



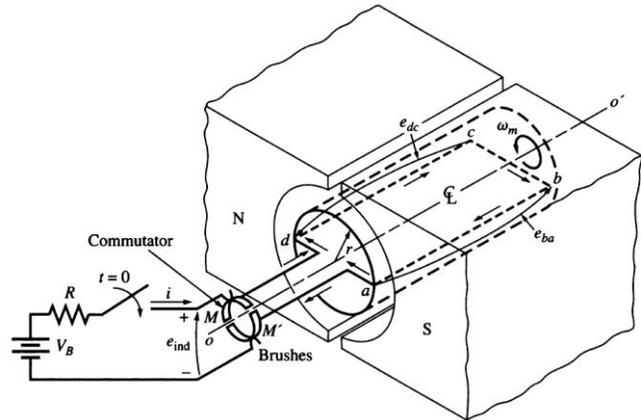
The Induced Torque in The Rotating Loop

Now, if the loop was supplied by a DC current, as shown in the Figure next, then it can be shown that the torque induced on the loop is:

$$\tau_{ind} = \begin{cases} \frac{2}{\pi} \phi i & \text{under pole face} \\ 0 & \text{beyond pole edges} \end{cases}$$

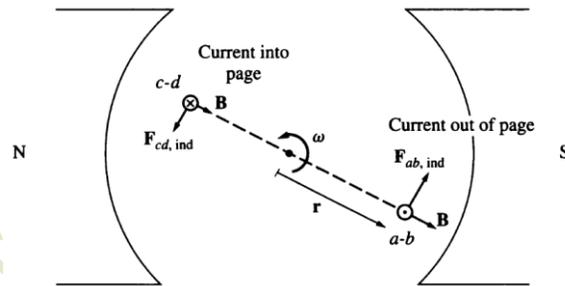
or

$$\tau_{ind} = \begin{cases} k\phi i & \text{under pole face} \\ 0 & \text{beyond pole edges} \end{cases}$$



i.e.

$$\tau_{ind} = k\phi i$$



The torque depends on the flux in the machine, the current in the machine's loop and on the construction of the machine.

Problems with Commutation in Real Machines

The commutation process is not as simple in practice as it seems in theory, because two major effects occur in the real world to disturb it:

- 1) Armature Reaction
- 2) $L \frac{di}{dt}$ Voltages

1) Armature Reaction

A) Arcing and Sparking

◆ Define, the **Neutral Plane** as, the plane within the machine where the velocity of the rotor conductors (wires) is exactly parallel to the magnetic flux lines, so e_{ind} in the conductors in the Neutral Plane is exactly zero. As illustrated in the Figures next.

◆ When a load is connected to the terminals of the armature (rotor) windings of the machine, a current will flow in the armature windings. The current flow produces a magnetic field of its own, which distorts the original magnetic field from the machine's poles.

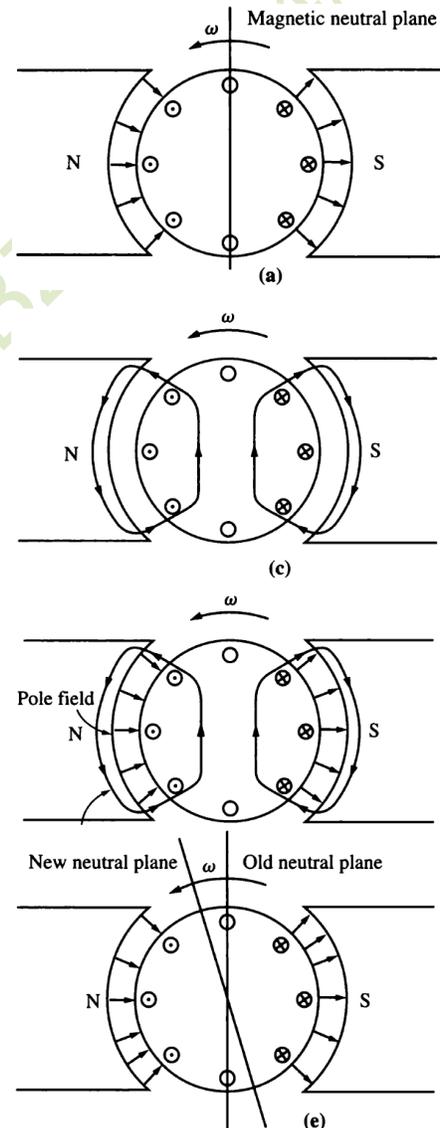
◆ **Armature Reaction** is the distortion of the original magnetic field (flux) in the machine as the load increases.

◆ For the generator, shown in the Figure next, the Neutral Plane has shifted in the direction of motion.

◆ The amount of shift depends on the amount of rotor current, hence on the load of the machine.

◆ In general, the Neutral Plane shifts in the direction of motion for a generator and opposite to the direction of motion for a motor.

◆ The brushes must short out commutator segments when the voltage across them is zero.



- ◆ If the brushes are set to short out conductors in the vertical plane, the voltage between the segments is zero unless the machine is loaded.
- ◆ When the machine is loaded, the Neutral Plane shifts, and the brushes short out commutator segments with a finite voltage across them.
- ◆ This results in a circulating current flowing between the shorted segments and large sparks at the brushes when the current path is interrupted as the brush leaves a segment. **Arcing and Sparking** at the brushes occur.
- ◆ This **reduces the life time of brushes** and **increases the machine's cost**.
- ◆ Note that, under heavy loads, it is possible to induce voltages in the coils experiencing the increased flux density to break down the ionized air between commutator segments. An arc may be established between the brushes, which is called **Flash Over**.

Flux Weakening

■ A second major problem caused by Armature Reaction is **Flux Weakening**.

■ Most machines operate at flux densities near the saturation point.

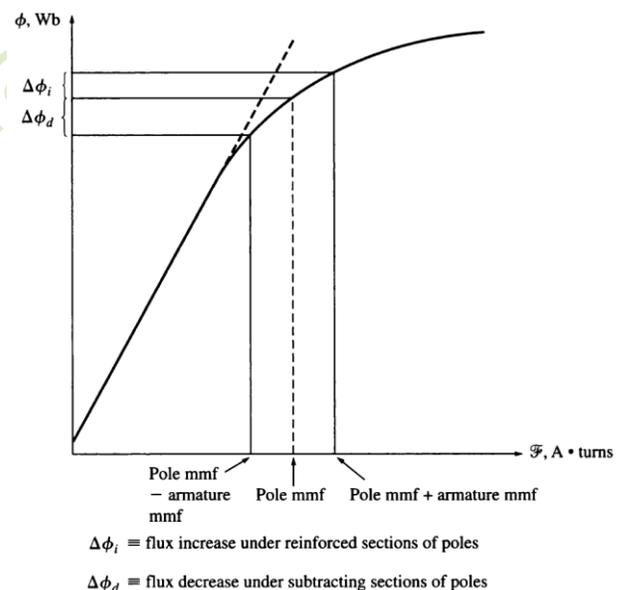
■ At locations on the pole, where the rotor magnetomotive force adds to the pole magnetomotive force, only a small increase in the flux occurs.

■ But, at locations on the pole surface, where the rotor magnetomotive force subtracts from the pole magnetomotive force, there is a larger decrease in flux.

■ The net result is the **total average flux under the entire pole face is decreased**.

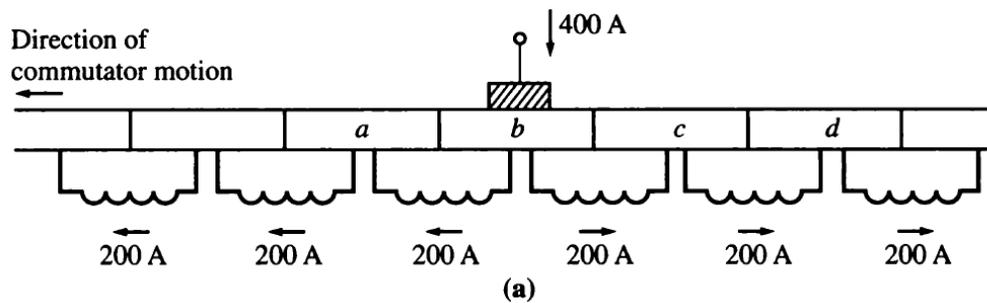
■ **In generators**, the effect of flux weakening is reducing the voltage applied across the load.

■ **In motors**, the effect of flux weakening is more serious. When the flux is decreased, the speed increases ($\omega = \frac{e_{ind}}{k\phi}$). And increasing the speed of the motor can increase its load, resulting in more and more flux weakening. Therefore, as a result of flux weakening, the machine may reach a **run-away** condition and may **destroy itself**.



2) $L \frac{di}{dt}$ Voltages

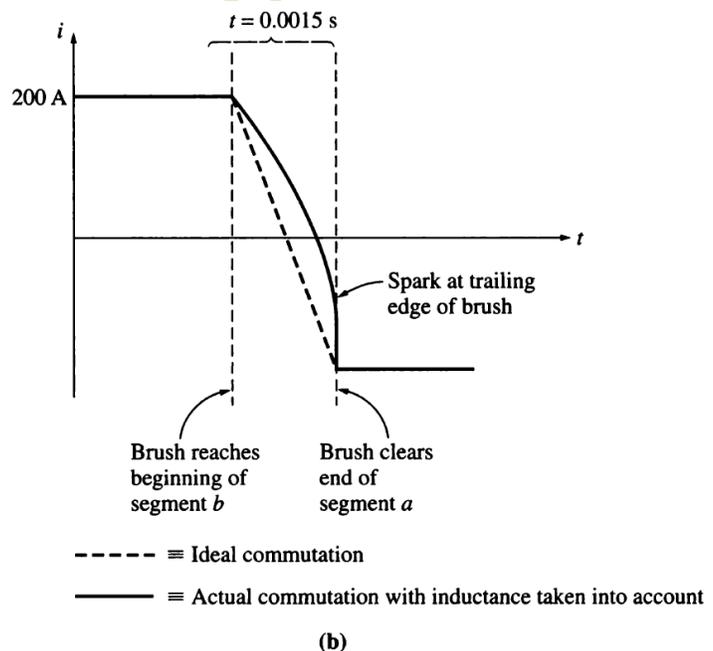
Assuming that initially the currents in the brushes and the coils are as shown in the Figure below.



✚ When the commutator segment is shorted out, the current through that segment must reverse.

✚ The time for current reversal depends on ω and number of commutator segments.

✚ A tiny inductance in the loop will induce a large inductive kick ($L \frac{di}{dt}$) in the shorted commutator segment.



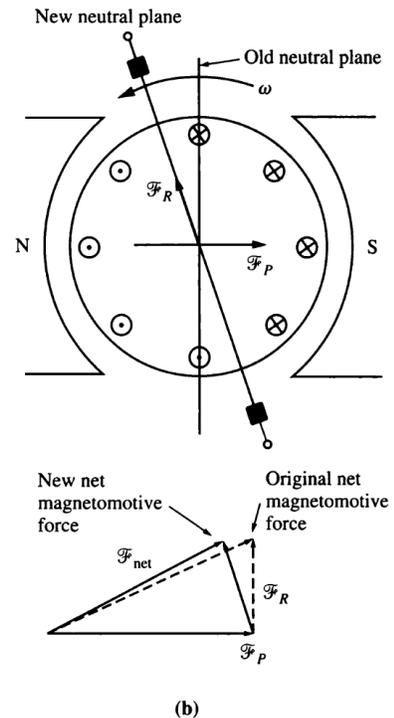
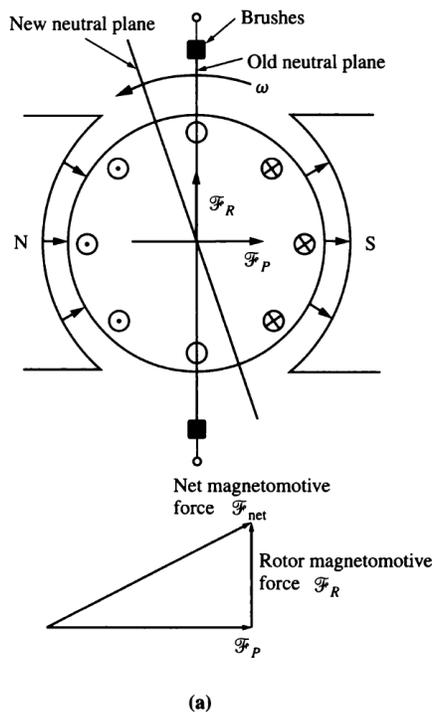
✚ This large induced voltage ($L \frac{di}{dt}$) causes sparking at the brushes of the machine.

Solutions to Problems with Commutation

- I) Brush Shifting
- II) Commutating Poles or Inter-Poles
- III) Compensating Windings

I) Brush Shifting

- ✚ Shifting the brushes with the Neutral Plane shift to stop sparking, but there are several problems associated with it.
 - a. The Neutral Plane moves with every change in the load.
 - b. The Neutral Plane shift changes direction when the machine goes from motor operation to generator operation or vice versa.
 - c. It aggravates the flux weakening effect of the armature reaction of the machine because:
 1. The rotor magnetomotive force after brush shifting has a vector component that opposes the magnetomotive force from the poles, as illustrated in the Figures below.

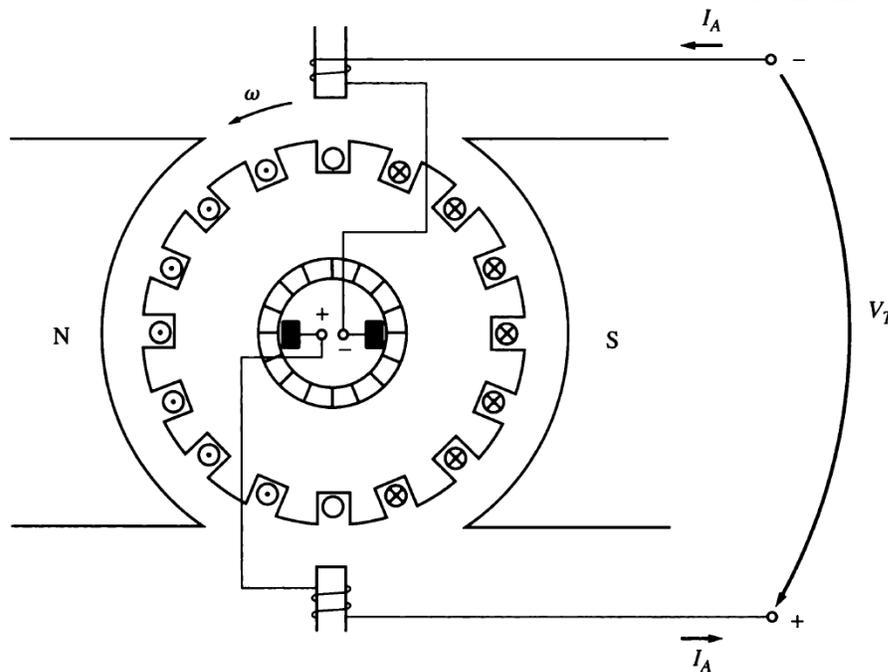


2. The change in the armature current distribution causes the flux to bunch up even more at the saturated parts of the pole faces.

However, sparks will occur at no-load, unless one keeps shifting brushes with every load change!

II) Commutating Poles or Inter-Poles

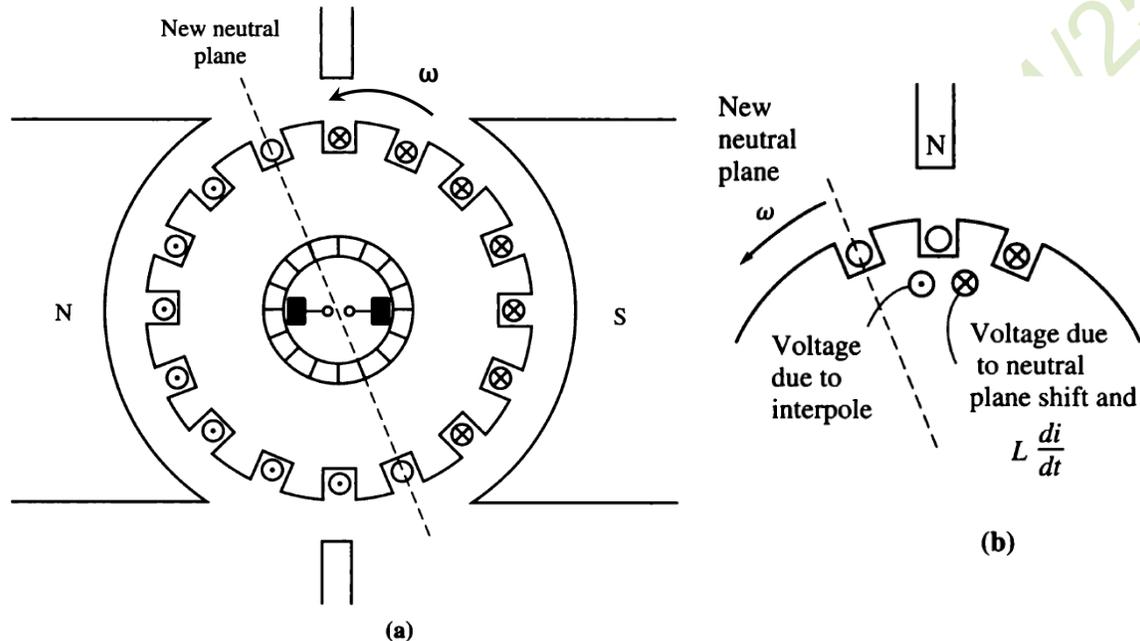
- ➔ They are small poles placed midway between the main poles. They are located directly over the conductors being commutated. They provide a flux such that the voltage induced in the coils undergoing commutation is cancelled; No sparking on the brushes.
- ➔ These poles are so small such that the armature reaction under the main poles, and, therefore the flux weakening, are not affected.
- ➔ The Inter-Pole windings are connected in series with the windings on the rotor, as shown in the Figure below; i.e., the load (rotor) current, I_A passes also in the Inter-Poles' windings.



- ➔ As the load increases, the rotor current increases, the magnitude of the Neutral Plane shift and the size of the $L \frac{di}{dt}$ effects increase. Both of these effects increase the voltage in the conductors undergoing commutation.
- ➔ On the other hand, the Inter-Poles flux increases too, producing a larger voltage, than before, that opposes the voltage due to the Neutral Plane shift and the $L \frac{di}{dt}$ effect.
- ➔ However, the **Flux Weakening effect still exists!**

Polarity of Inter-Poles' Flux

- a) In a generator, the Inter-Poles must be of the same polarity as the next upcoming main pole, because the Neutral Plane shifts in the direction of motion; i.e., conductors undergoing commutation have the same polarity of voltage as that of the conductors under the main pole they just left.

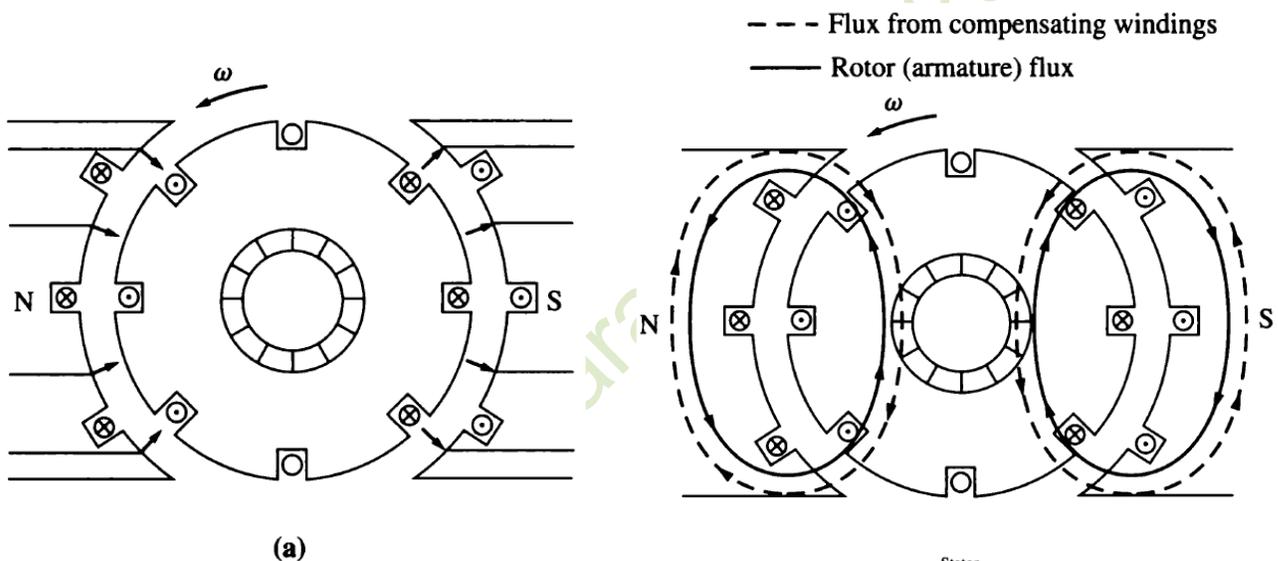


- b) In a motor, the Inter-Poles must be of the same polarity as the previous main pole, because the Neutral Plane shifts opposite to the direction of rotation; i.e., conductors undergoing commutation have the same polarity of voltage as that of the conductors under the main pole they are approaching.

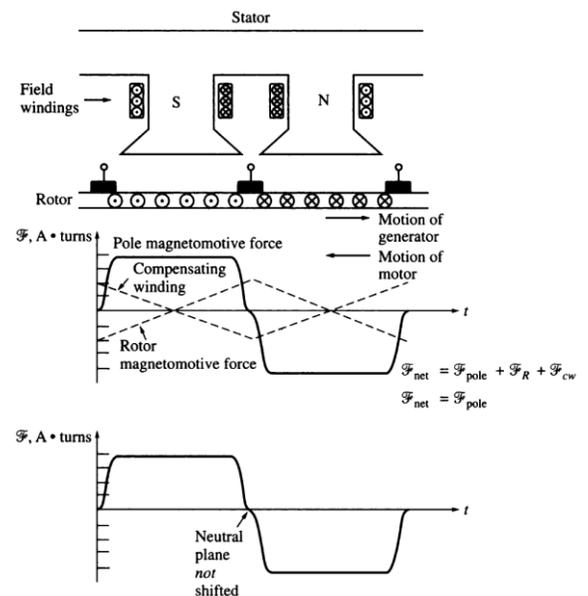
➔ Inter-Poles do not fix the Flux Weakening problem, but they fix the problem due to the Neutral Plane shift and the $L \frac{di}{dt}$ effect.

III) Compensating Windings

- This method completely cancels the armature reaction effect and thus **eliminates both Neutral Plane shift and Flux Weakening effect.**
- The Compensating windings are placed in slots carved in the faces of the stator poles parallel to the rotor conductors, and are connected in series with the rotor windings, so whenever the load on the rotor changes, the current in the Compensating windings changes too.
- The Compensating windings are designed such that their flux equals and opposes the rotor flux, as shown in the Figure below. So they cancel each other and no Neutral Plane shift with load. i.e., **No Neutral Plane shift and no Flux Weakening problem.**



- The effect of Compensating windings and rotor windings on magnetomotive forces is shown in the Figure next.
- **Compensating windings do not cancel $L \frac{di}{dt}$ effect**, so the machine must have Inter-Poles.
- A disadvantage is that the Compensating windings **are expensive.**
- **To cancel all commutation effects, large machines have both Inter-Poles and Compensating windings.**



In Summary:

The Internally Generated voltage is:

$$E_A = k\phi\omega$$

The induced torque is:

$$\tau_{ind} = k\phi I_A$$

where, I_A is the armature current.

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PART VIII

DC Motors and Generators

DC motors were very common for many reasons:

- DC power systems are still common in cars, vehicles, aircrafts...
- Before the widespread of power electronics, DC motors were appropriate for speed control applications.

Speed Regulation: is a measure of the shape of torque-speed characteristics of a motor. The Speed Regulation (SR) is defined as:

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100\%$$

or

$$SR = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

where, ω_{nl} is the no-load speed in rad/s

ω_{fl} is the full load speed in rad/s

n_{nl} is the no-load speed in revolutions per minute [rpm]

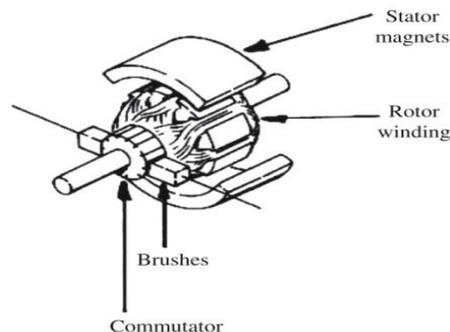
n_{fl} is the full load speed in revolutions per minute [rpm]

Notes:

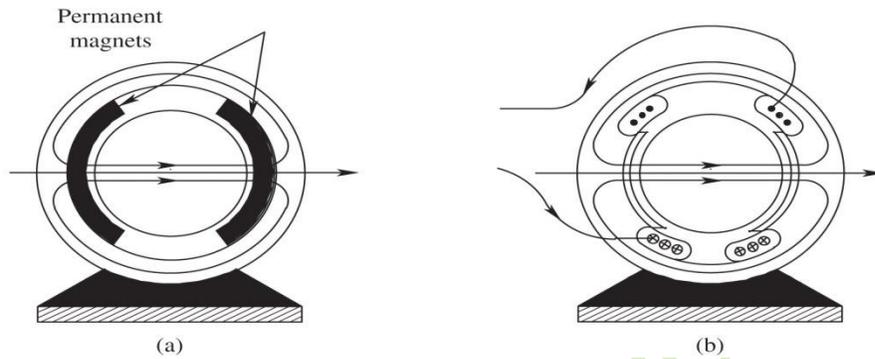
- If the SR is positive, the speed drops with increasing the mechanical load.
- If the SR is negative, the speed increases with increasing the mechanical load.
- DC motors are compared to each other by their Speed Regulation.

Construction of DC Motors

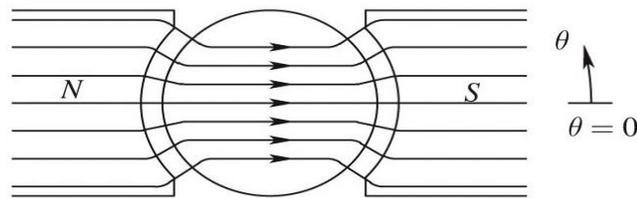
- A DC machine consists mainly of a stator, a rotor, a commutator and brushes, as illustrated in the Figure below.



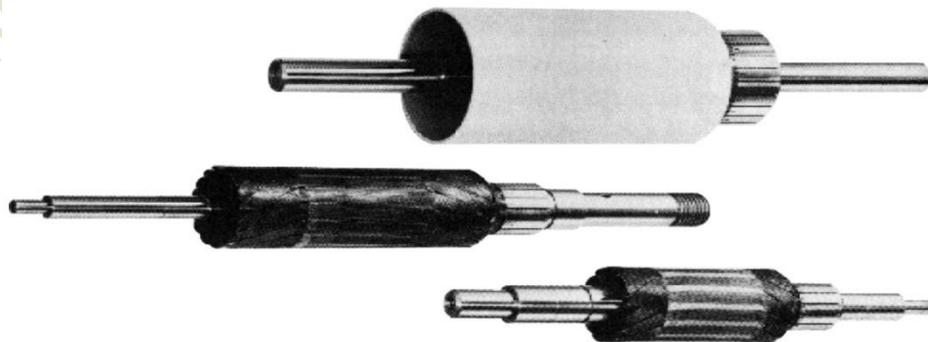
- The field flux is established at the stator either by means of a permanent magnet (Figure (a) below) such that the magnetic field flux (ϕ_f) is constant, or by means of field windings (Figure (b) below), where the magnetic field flux, neglecting saturation, is directly proportional to the field current; i.e., $\phi_f = K_f I_f$.



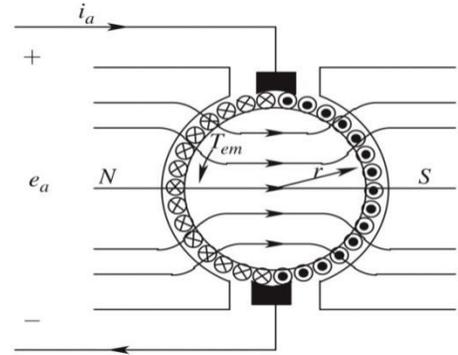
- The stator is designed to have curved pole faces, such that the magnetic field flux lines are perpendicular to the airgap and to the rotor surface, as shown in the Figure below.



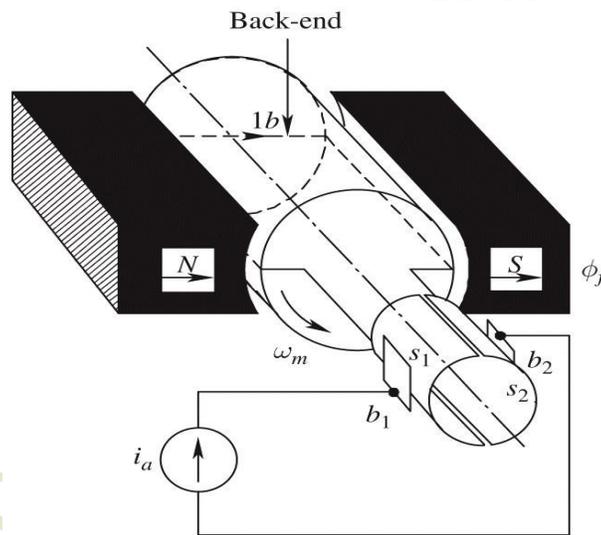
- The rotor carries the armature windings in its slots, and it handles the electrical power. Samples of rotors are shown in the Figures below.



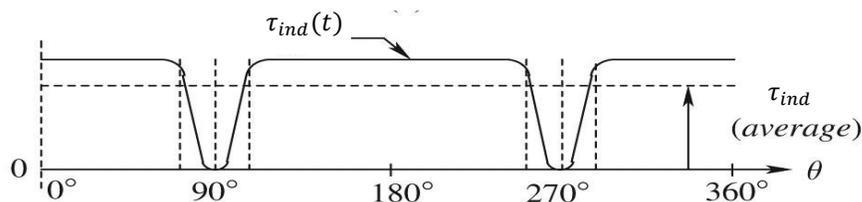
- When the rotor conductors have a DC current in the presence of a magnetic field, an induced torque will be developed, as illustrated in the Figure next.



- The induced torque developed is bidirectional and has a quasi-square wave shape.
- However, the rotor is connected to the DC source by the commutator and brushes, as shown in the Figure below, which reverse the direction of armature current for every half cycle. The commutator and brushes also unify the induced torque direction in the DC motor.

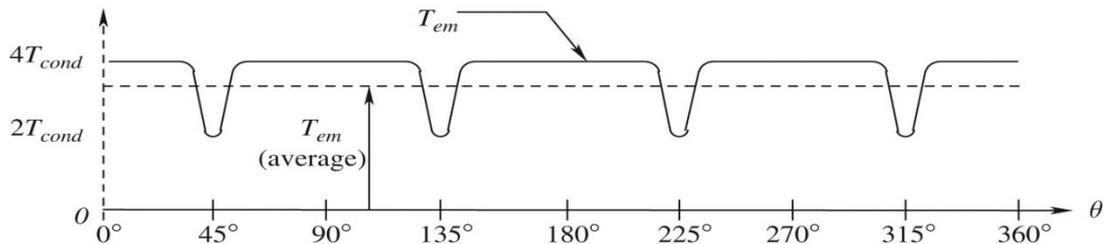


- The Figure below shows the unified torque due to the effect of commutator and brushes.



- The induced torque of the DC motor consists of an average value and a ripple, as was illustrated in the above Figure.

- ➔ The torque ripple may be reduced by increasing the number of commutator segments; for 4 commutator segments, the induced torque of a DC motor is shown in the Figure below. Note that the frequency of the ripple is also increased.

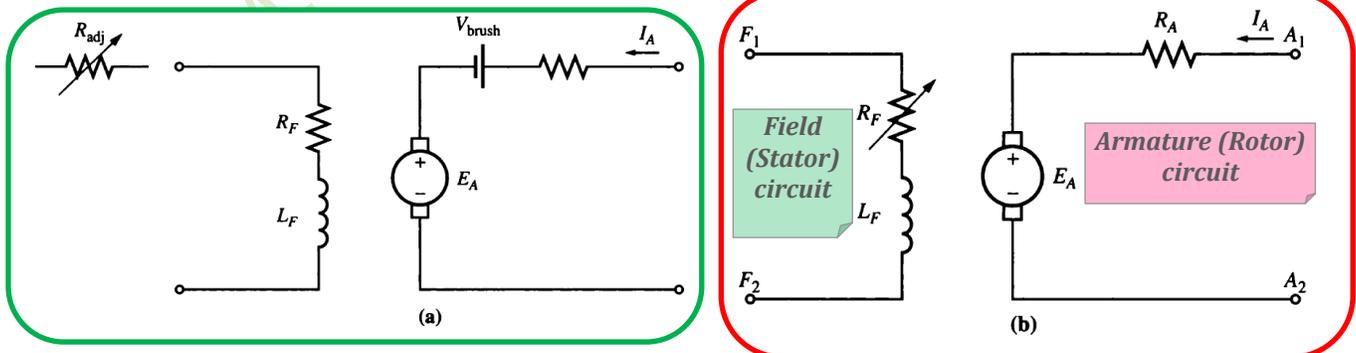


Types of DC Motors

- 1) Separately Excited DC Motor
- 2) Shunt DC Motor
- 3) Series DC Motor
- 4) Compounded DC Motor
- 5) Permanent Magnet DC (PMDC) Motor

Equivalent Circuit of a DC Motor

The equivalent circuit of a DC motor is shown in Figure (a) below, whilst a simplified equivalent circuit is shown in Figure (b).



- ◆ The armature circuit is represented by a voltage source (E_A) and a resistor (R_A), which represent the Thevenin's equivalent circuit of the rotor structure; including rotor windings, Inter-Poles, and/or Compensating windings.
- ◆ V_{brush} : is the brushes' voltage drop and is opposite to the current flow.
- ◆ L_F & R_F : represent the field coil's inductance and resistance, respectively, which produces flux in the machine.
- ◆ R_{adj} : is an external variable resistor used to control the amount of field current (flux) of the field circuit.
- ◆ In a DC motor, the field flux is:

$$\phi = K_f I_F$$

where, K_f : is the field constant of proportionality, neglecting flux saturation

I_F : is the field current.

- ▶ The internally generated voltage is:

$$E_A = k\phi\omega$$

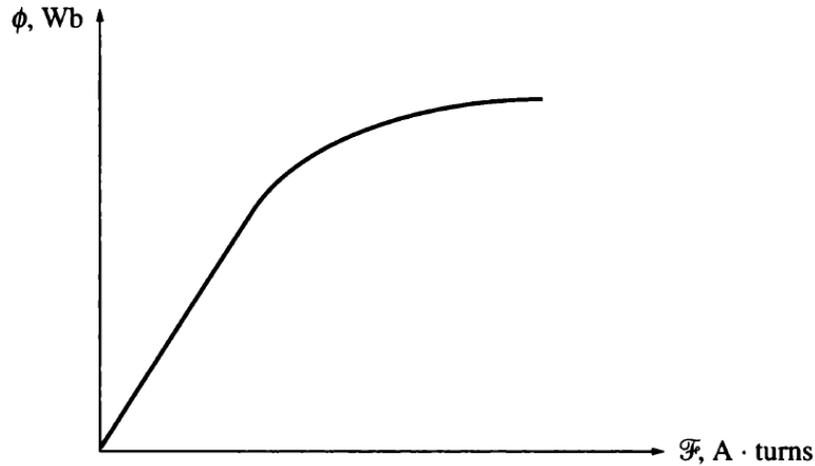
- ▶ A current-carrying conductor in the presence of a magnetic field has a torque induced on it. The induced torque developed on the armature conductors is:

$$\tau_{ind} = k\phi I_A$$

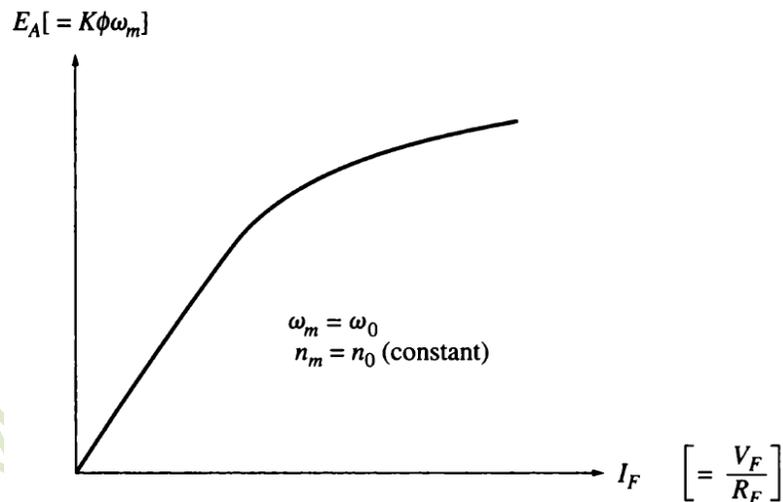
I_A : is the armature current.

The Magnetization Curve of a DC Machine

The field current produces a field magnetomotive force; $\mathcal{F} = N_f I_F$. This force produces a flux in the machine in accordance with its magnetization curve, shown in the Figure below.



Since I_F is directly proportional to \mathcal{F} , and E_A is directly proportional to flux (ϕ), it is customary to present the magnetization curve as a plot of E_A versus I_F for a given speed ω_o , as shown in the Figure below.



To get the maximum possible power/weight out of the machine, most motors and generators are designed to operate near the knee of the saturation curve (saturation point).

1 & 2) Separately Excited DC Motor and Shunt DC Motor

The Equivalent circuit of **a Separately Excited DC motor** is shown in the Figure below.

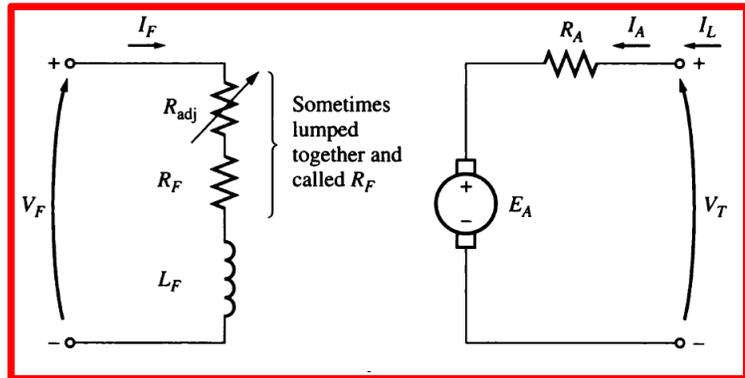
- ▶ Applying KVL to the armature circuit of a Separately Excited DC motor yields:

$$V_T = E_A + R_A I_A$$

- ▶ The field current is:

$$I_F = \frac{V_F}{R_F + R_{adj}}$$

- ▶ The line current equals armature current; $I_L = I_A$.



The Equivalent Circuit of **a Shunt DC motor** is shown in the Figure below.

- Applying KVL to the armature circuit of a DC motor yields:

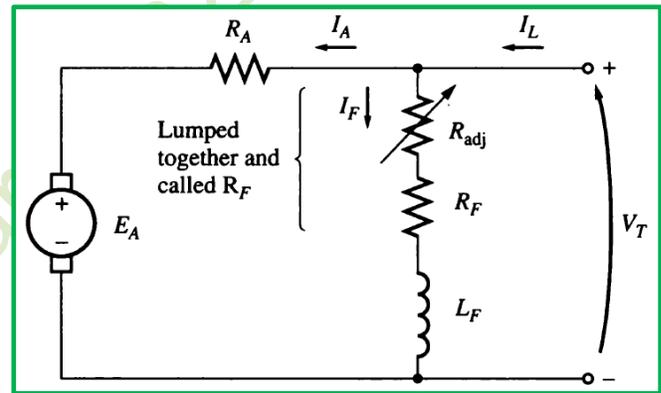
$$V_T = E_A + R_A I_A$$

- The field current is:

$$I_F = \frac{V_T}{R_F + R_{adj}}$$

- The line current is the total current;

$$I_L = I_A + I_F$$



Terminal Characteristic of a Shunt DC Motor

- ▶ Terminal characteristic is a plot of the machine's output quantities versus each other. For a motor, it is a plot of torque versus speed.

- ▶ as $\tau_{load} \uparrow \Rightarrow \omega \downarrow \Rightarrow E_A (= k\phi\omega) \downarrow \Rightarrow I_A (= \frac{V_T - E_A}{R_A}) \uparrow \Rightarrow \tau_{ind} (= k\phi I_A) \uparrow$

and finally the induced torque equals the load torque, but at a lower mechanical speed.

- ▶ Applying KVL,

$$V_T = E_A + R_A I_A$$

- ▶ But, the internally generated voltage is:

$$E_A = k\phi\omega$$

Therefore, $V_T = k\phi\omega + R_A I_A$

- ▶ Recall, the induced torque equation;

$$\tau_{ind} = k\phi I_A$$

$$\Rightarrow I_A = \frac{\tau_{ind}}{k\phi}$$

- ▶ Substituting for the armature current in the voltage equation yields:

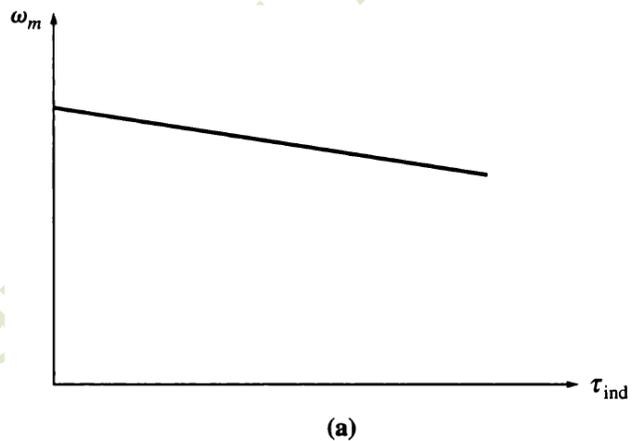
$$V_T = k\phi\omega + R_A \frac{\tau_{ind}}{k\phi}$$

- ▶ Rearranging for the mechanical speed yields:

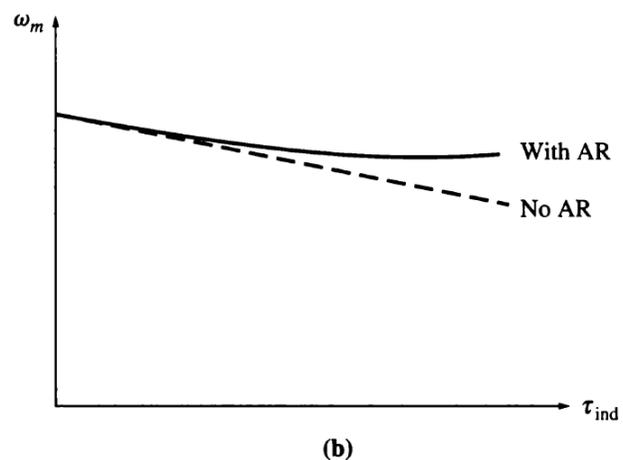
$$\omega = \frac{V_T}{k\phi} - \frac{R_A}{(k\phi)^2} \tau_{ind}$$

- ▶ Note that, if a Shunt (or a Separately Excited) DC motor was running and suddenly its field flux falls to zero, "Run Away" condition is reached, and the motor may damage itself.

- ▶ Note that, the speed varies linearly with torque if other terms are constants, as depicted in the Figure next.



- ▶ If the motor has armature reaction effect, then as the load increases, the Flux Weakening effects reduce its average flux, which will increase the speed at any given load over the speed it would run at without armature reaction. This phenomenon is illustrated in the Figure next.



- ▶ If the motor has Compensating windings, there

will be no Flux Weakening problem, and the flux stays constant.

- ▶ Since $\tau_{ind} = k\phi I_A$, the direction of rotation is reversed by switching the terminals either of the field or the armature windings.

Speed Control of a Shunt DC Motor (or a Separately Excited DC Motor)

Recall, the speed equation:

$$\omega = \frac{V_T}{k\phi} - \frac{R_A}{(k\phi)^2} \tau_{ind}$$

By inspecting the above equation, one can deduce that the speed can be controlled by:

- I. Adjusting the field flux via the field resistance (R_F)
- II. Adjusting the terminal voltage applied to the armature windings
- III. Inserting a resistor in series with the armature winding, but this method is less commonly used.

I. Changing the Field Resistance

➔ As $(R_F + R_{adj}) \uparrow \Rightarrow I_F \left(= \frac{V_F}{R_F + R_{adj}} \right) \downarrow \Rightarrow \phi \downarrow \Rightarrow E_A (= k\phi\omega) \downarrow \Rightarrow I_A \left(= \frac{V_T - E_A}{R_A} \right) \uparrow$

➔ But,

$$\tau_{ind} = k\phi I_A, \text{ where } \phi \downarrow \text{ and } I_A \uparrow$$

➔ However, over the motor's normal operating torque range, the increase in armature current predominates over the decrease in the flux, and hence the **induced torque rises**; i.e.,

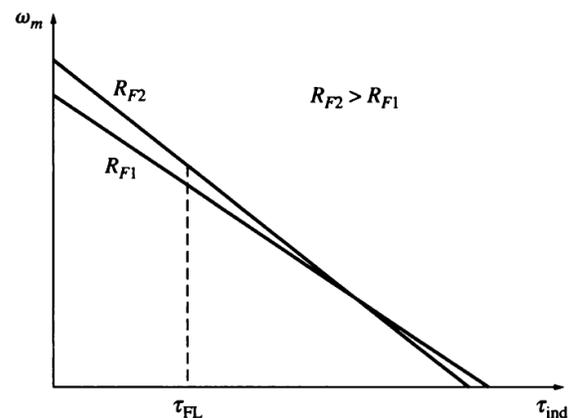
$$\tau_{ind} = k\phi \downarrow I_A \uparrow$$

➔ And since the induced torque becomes greater than the load torque ($\tau_{ind} > \tau_{load}$), the motor speeds up; $\omega \uparrow$.

➔ As $\omega \uparrow \Rightarrow E_A (= k\phi\omega) \uparrow \Rightarrow I_A \left(= \frac{V_T - E_A}{R_A} \right) \downarrow \Rightarrow \tau_{ind} (= k\phi I_A) \downarrow$, until it equals the load torque, but at a higher speed than the original speed.

➔ This method is used for speed control **above the rated speed**, and is called **Flux Weakening method**.

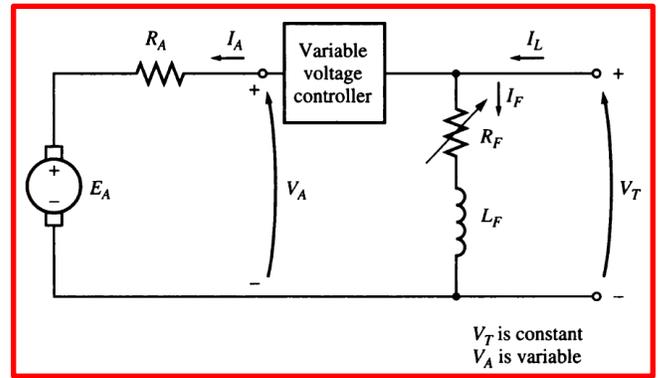
➔ The effect of varying the field resistance on the torque-speed characteristic of a Shunt DC motor is illustrated in the Figure next.



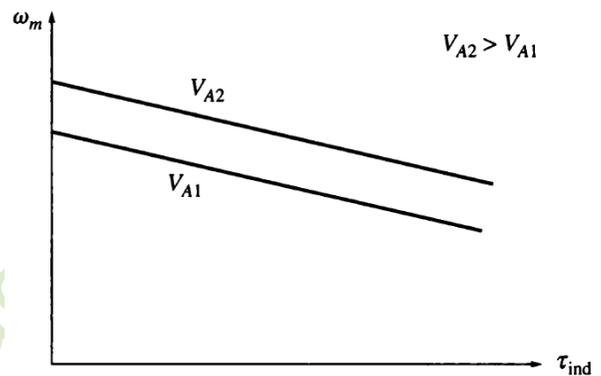
➔ Note that, in a Separately Excited DC motor, the field flux may be varied by applying a variable voltage source to the field circuit instead of adding an adjustable resistor to the field circuit.

II. Varying the Armature Voltage

- This method is used for speed control **below rated speed**.
- The field voltage (current) is kept constant, but the armature voltage (V_A) is varied as illustrated in the Figure next.

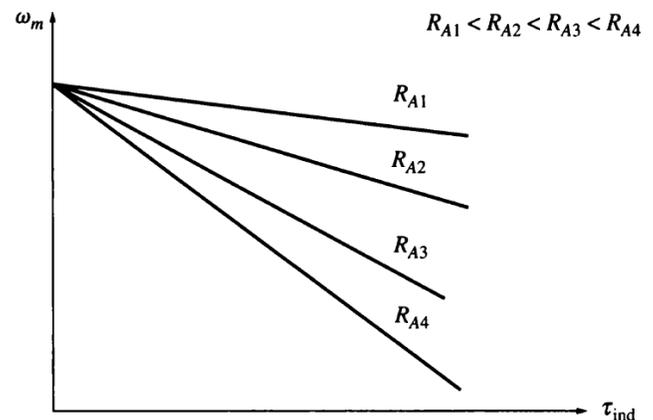


- As $V_A \downarrow \Rightarrow I_A \left(= \frac{V_A - E_A}{R_A} \right) \downarrow \Rightarrow \tau_{ind} (= k\phi I_A) \downarrow$
- The induced torque decreases and becomes less than the load torque ($\tau_{ind} < \tau_{load}$), thus the motor slows down; $\omega \downarrow$.
- But, as $\omega \downarrow \Rightarrow E_A \downarrow \Rightarrow I_A \uparrow \Rightarrow \tau_{ind} \uparrow$, and becomes equal to the load torque, but at a lower speed than the original speed.
- The effect of varying the armature voltage (V_A) on the torque-speed characteristic of a Shunt DC motor is illustrated in the Figure next.



III. Inserting a Resistor in Series with the Armature Circuit

- ✚ Inserting a series resistor with the armature windings, increases the slope of motor's torque-speed characteristic. Therefore, the **speed regulation becomes worse!**
- ✚ The effect of increasing the armature resistor on the motor's torque-speed characteristic is illustrated in the Figure next.
- ✚ The added resistor **increases the losses**, so this method is **rarely used**.
- ✚ This method is used for applications, where the motor spends almost most of its time operating at full load speed, or in applications too inexpensive to justify a better form of speed control.



Example 8–1. A 50-hp, 250-V, 1200 r/min dc shunt motor with compensating windings has an armature resistance (including the brushes, compensating windings, and interpoles) of 0.06Ω . Its field circuit has a total resistance $R_{\text{adj}} + R_F$ of 50Ω , which produces a *no-load* speed of 1200 r/min. There are 1200 turns per pole on the shunt field winding (see Figure 8–7).

- Find the speed of this motor when its input current is 100 A.
- Find the speed of this motor when its input current is 200 A.
- Find the speed of this motor when its input current is 300 A.
- Plot the torque–speed characteristic of this motor.

Solution

The internal generated voltage of a dc machine with its speed expressed in revolutions per minute is given by

$$E_A = K' \phi n_m \quad (7-41)$$

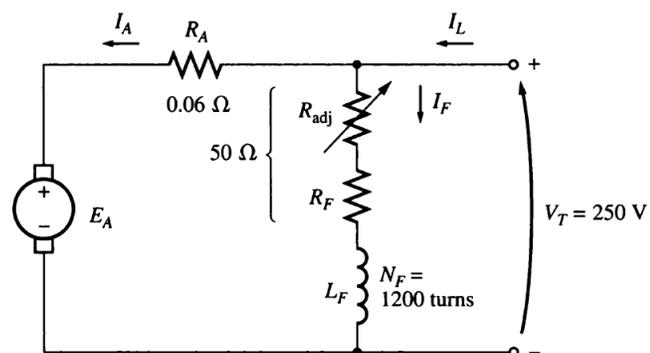
Since the field current in the machine is constant (because V_T and the field resistance are both constant), and since there are no armature reaction effects, *the flux in this motor is constant*. The relationship between the speeds and internal generated voltages of the motor at two different load conditions is thus

$$\frac{E_{A2}}{E_{A1}} = \frac{K' \phi n_{m2}}{K' \phi n_{m1}} \quad (8-8)$$

The constant K' cancels, since it is a constant for any given machine, and the flux ϕ cancels as described above. Therefore,

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} \quad (8-9)$$

The equivalent circuit of the motor is shown in the Figure next.



At no load, the armature current is zero, so $E_{A1} = V_T = 250 \text{ V}$, while the speed $n_{m1} = 1200 \text{ r/min}$. If we can calculate the internal generated voltage at any other load, it will be possible to determine the motor speed at that load from Equation (8–9).

(a) If $I_L = 100$ A, then the armature current in the motor is

$$\begin{aligned} I_A &= I_L - I_F = I_L - \frac{V_T}{R_F} \\ &= 100 \text{ A} - \frac{250 \text{ V}}{50 \Omega} = 95 \text{ A} \end{aligned}$$

Therefore, E_A at this load will be

$$\begin{aligned} E_A &= V_T - I_A R_A \\ &= 250 \text{ V} - (95 \text{ A})(0.06 \Omega) = 244.3 \text{ V} \end{aligned}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{244.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} = 1173 \text{ r/min}$$

(b) If $I_L = 200$ A, then the armature current in the motor is

$$I_A = 200 \text{ A} - \frac{250 \text{ V}}{50 \Omega} = 195 \text{ A}$$

Therefore, E_A at this load will be

$$\begin{aligned} E_A &= V_T - I_A R_A \\ &= 250 \text{ V} - (195 \text{ A})(0.06 \Omega) = 238.3 \text{ V} \end{aligned}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{238.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} = 1144 \text{ r/min}$$

(c) If $I_L = 300$ A, then the armature current in the motor is

$$\begin{aligned} I_A &= I_L - I_F = I_L - \frac{V_T}{R_F} \\ &= 300 \text{ A} - \frac{250 \text{ V}}{50 \Omega} = 295 \text{ A} \end{aligned}$$

Therefore, E_A at this load will be

$$\begin{aligned} E_A &= V_T - I_A R_A \\ &= 250 \text{ V} - (295 \text{ A})(0.06 \Omega) = 232.3 \text{ V} \end{aligned}$$

The resulting speed of the motor is

$$n_{m2} = \frac{E_{A2}}{E_{A1}} n_{m1} = \frac{232.3 \text{ V}}{250 \text{ V}} 1200 \text{ r/min} = 1115 \text{ r/min}$$

(d) To plot the output characteristic of this motor, it is necessary to find the torque corresponding to each value of speed. At no load, the induced torque τ_{ind} is clearly zero. The induced torque for any other load can be found from the fact that power converted in a dc motor is

$$P_{\text{conv}} = E_A I_A = \tau_{\text{ind}} \omega_m \quad (7-55, 7-56)$$

From this equation, the induced torque in a motor is

$$\tau_{\text{ind}} = \frac{E_A I_A}{\omega_m} \quad (8-10)$$

Therefore, the induced torque when $I_L = 100$ A is

$$\tau_{\text{ind}} = \frac{(244.3 \text{ V})(95 \text{ A})}{(1173 \text{ r/min})(1 \text{ min}/60\text{s})(2\pi \text{ rad/r})} = 190 \text{ N} \cdot \text{m}$$

The induced torque when $I_L = 200$ A is

$$\tau_{\text{ind}} = \frac{(238.3 \text{ V})(195 \text{ A})}{(1144 \text{ r/min})(1 \text{ min}/60\text{s})(2\pi \text{ rad/r})} = 388 \text{ N} \cdot \text{m}$$

The induced torque when $I_L = 300$ A is

$$\tau_{\text{ind}} = \frac{(232.3 \text{ V})(295 \text{ A})}{(1115 \text{ r/min})(1 \text{ min}/60\text{s})(2\pi \text{ rad/r})} = 587 \text{ N} \cdot \text{m}$$

The resulting torque–speed characteristic for this motor is plotted in Figure 8–8.

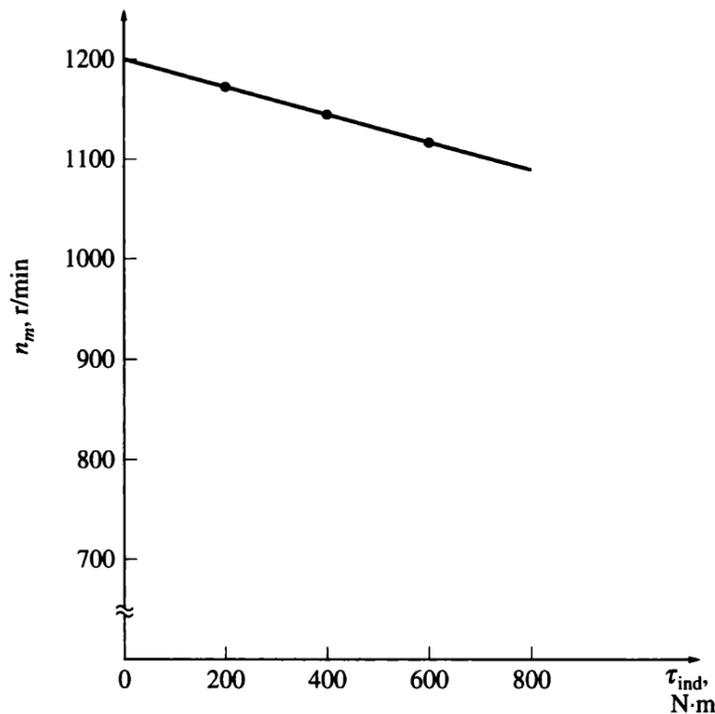


FIGURE 8–8

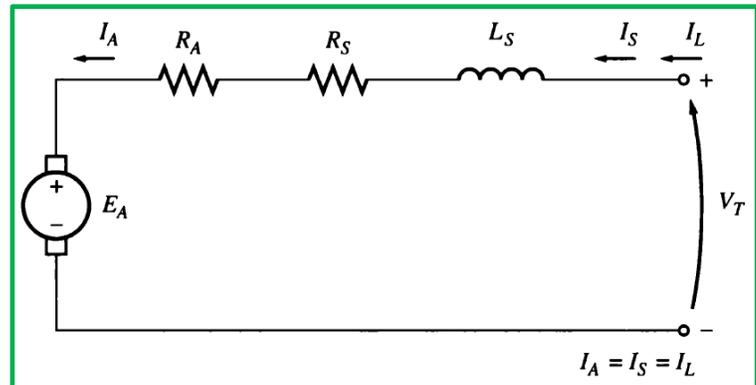
3) The Series DC Motor

- The field winding consists of a few turns connected in series with the armature circuit, as shown in the Figure next.

- At steady state, applying KVL to the equivalent circuit yields:

$$V_T = E_A + I_A(R_A + R_S)$$

- Note that, $I_A = I_S = I_L$



Induced Torque in a Series DC Motor

- The flux is directly proportional to the armature current, at least until saturation is reached; i.e.,

$$\phi = CI_A$$

where, C is a constant of proportionality.

- As the load on the motor increases, the flux increases too. An increase in the flux causes a decrease in speed; the **Series DC Motor has a sharply drooping torque-speed characteristic.**

- The induced torque is:

$$\tau_{ind} = k\phi I_A$$

- Substituting for the flux yields:

$$\tau_{ind} = kCI_A^2$$

where, C is a constant of proportionality.

- Note that, the torque is proportional to the square of the armature current.
- Thus, **the Series DC Motor gives more torque/Ampere than any other type of DC motors.**
- Therefore, it is used in applications requiring **very high starting torques**; e.g., starter motors in cars, elevator motors, trains and tractor motors in locomotives,...

The Terminal Characteristic of a Series DC Motor

Assuming a linear magnetizing curve, the flux is related to armature current by: $\phi = CI_A$

Applying KVL to the equivalent circuit yields:

$$V_T = E_A + I_A(R_A + R_S)$$

But, the induced torque was derived as:

$$\tau_{ind} = kCI_A^2 \Rightarrow I_A = \sqrt{\frac{\tau_{ind}}{kC}}$$

Thus, the flux can be expressed as:

$$\phi = CI_A \Rightarrow \phi = C\sqrt{\frac{\tau_{ind}}{kC}} \Rightarrow \phi = \sqrt{\frac{C}{k}}\sqrt{\tau_{ind}}$$

Also, the internally generated voltage is: $E_A = k\phi\omega$

Substituting for the armature current and internally generated voltage in the KVL equation yields:

$$V_T = k\phi\omega + \sqrt{\frac{\tau_{ind}}{kC}}(R_A + R_S)$$

Substituting for the flux results in:

$$V_T = k\sqrt{\frac{C}{k}}\sqrt{\tau_{ind}}\omega + \frac{1}{\sqrt{kC}}\sqrt{\tau_{ind}}(R_A + R_S)$$

Solving for ω yields: $\omega = \frac{V_T}{\sqrt{kC}\sqrt{\tau_{ind}}} - \frac{R_A + R_S}{kC}$

- For unsaturated Series DC motor, the speed of the motor varies as the reciprocal of the square root of torque induced.
- The torque-speed characteristic can be drawn as shown in the Figure next.

○ The starting torque is: $\tau_{start} = \frac{kCV_T^2}{(R_A + R_S)^2}$

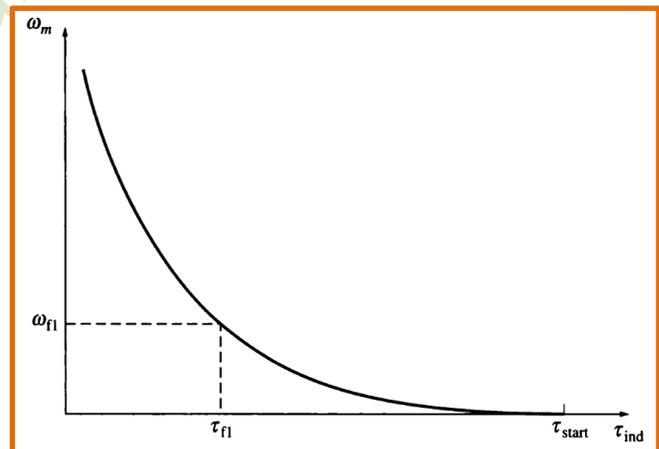
- One disadvantage of the Series DC motor is that,

when the torque on the motor goes to zero, **its speed goes to infinity (theoretically!).**

- In practice, the torque on the shaft never goes to zero, because there are mechanical, core and stray losses that must be overcome.

- If no other load is connected to the motor, the motor may turn fast enough to seriously damage itself.

So, the **Series DC motor must never be completely unloaded or never be connected to a load by a belt or any other mechanism that could break.**

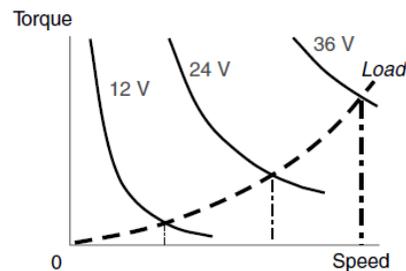
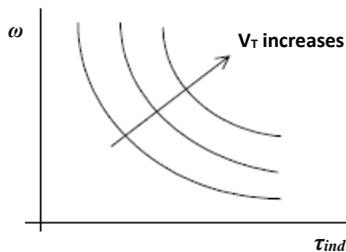


Methods for Speed Control of a Series DC Motor

Recall,
$$\omega = \frac{V_T}{\sqrt{kC} \sqrt{\tau_{ind}}} - \frac{R_A + R_S}{kC}$$

I. Changing the Terminal Voltage

Varying the terminal voltage controls the speed of the Series DC motor. As the terminal voltage is increased, the speed of the motor increases for any given torque, as shown in the Figures below. Examples of the effect of the terminal voltage control are also illustrated. This can be achieved by SCR-based control circuit.

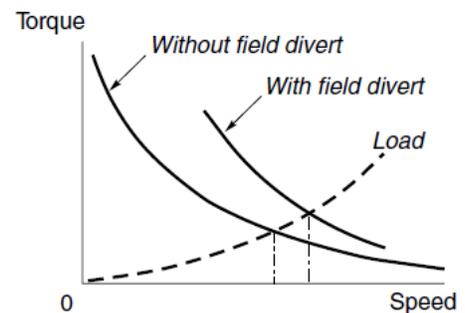


II. Inserting a Resistor in the Motor Circuit

Inserting a resistor in series with the motor may be used for decreasing the speed, but it **wastes power**. It is used only for intermittent periods during start ups of some motors. However, it is rarely used.

III. Using a Field Divert Resistor

A resistor can be connected in parallel with the field winding to increase the armature current. Hence, the induced torque increases for the same speed (the flux will be reduced, but not as much as the increase in the armature current). In other words, the speed increases for the same induced torque, as illustrated in the Figure next. ⇒ **Field Weakening!**



This method is inefficient (but as much as method II) because the power is wasted in the resistor, but it is simple and cheap to implement.

The **direction of rotation** is reversed by switching the terminals of the field or armature windings.

Example 8-5. Figure 8-20 shows a 250-V series dc motor with compensating windings, and a total series resistance $R_A + R_S$ of 0.08Ω . The series field consists of 25 turns per pole, with the magnetization curve shown in Figure 8-22.

- (a) Find the speed and induced torque of this motor for when its armature current is 50 A.

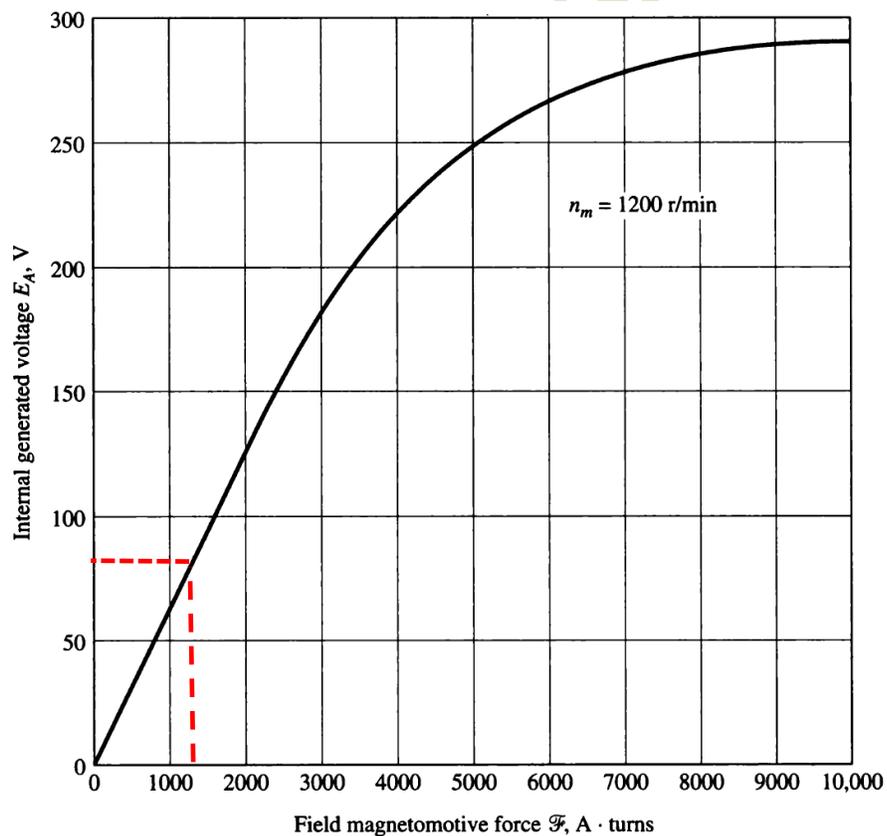
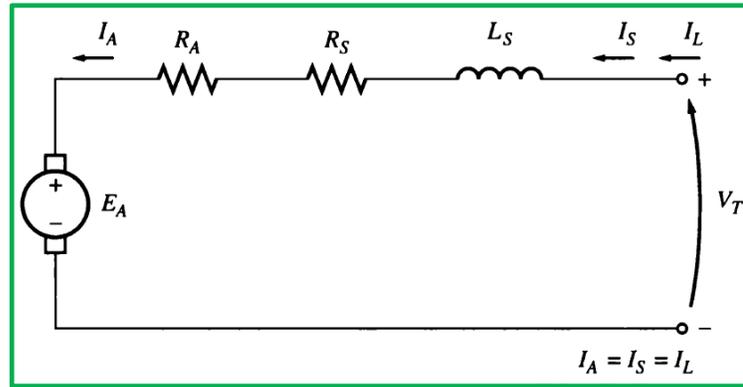


FIGURE 8-22

Solution

(a) To analyze the behavior of a series motor with saturation, pick points along the operating curve and find the torque and speed for each point. Notice that the magnetization curve is given in units of magnetomotive force (ampere-turns) versus E_A for a speed of 1200 r/min, so calculated E_A values must be compared to the equivalent values at 1200 r/min to determine the actual motor speed.

For $I_A = 50$ A,

$$E_A = V_T - I_A(R_A + R_S) = 250 \text{ V} - (50\text{A})(0.08 \Omega) = 246 \text{ V}$$

Since $I_A = I_F = 50$ A, the magnetomotive force is

$$\mathcal{F} = NI = (25 \text{ turns})(50 \text{ A}) = 1250 \text{ A} \cdot \text{turns}$$

From the magnetization curve at $\mathcal{F} = 1250 \text{ A} \cdot \text{turns}$, $E_{A0} = 80$ V. To get the correct speed of the motor, remember that, from Equation (8–13),

$$\begin{aligned} n_m &= \frac{E_A}{E_{A0}} n_0 \\ &= \frac{246 \text{ V}}{80 \text{ V}} \mathbf{1200\text{rpm}} = 3690 \text{ r/min} \end{aligned}$$

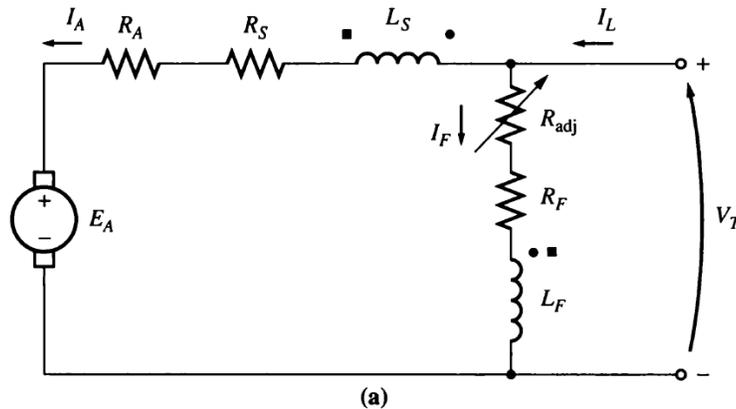
To find the induced torque supplied by the motor at that speed, recall that $P_{\text{conv}} = E_A I_A = \tau_{\text{ind}} \omega_m$. Therefore,

$$\begin{aligned} \tau_{\text{ind}} &= \frac{E_A I_A}{\omega_m} \\ &= \frac{(246 \text{ V})(50 \text{ A})}{(3690 \text{ r/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/r})} = 31.8 \text{ N} \cdot \text{m} \end{aligned}$$

4) The Compounded DC Motor

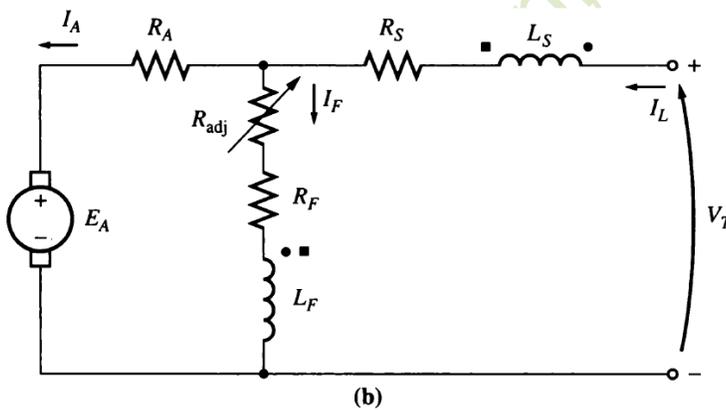
- ◆ A Compounded DC motor has both shunt (parallel) and series field windings.
- ◆ Two possible connections of the equivalent circuits of a Compounded DC motor, based on the configuration of these field windings, are shown in the Figures below:

A) Long-Shunt Connection



- Cumulatively compounded
- Differentially compounded

B) Short-Shunt Connection



- ◆ If the current flows in the dotted end, it produces a positive magnetomotive force (as in a Transformer), and vice versa.
- ◆ If the currents flow into the dotted end in both field coils, then the resulting magnetomotive forces add to produce a larger total magnetomotive force, hence producing a **Cumulatively Compounded DC motor**.
- ◆ If one current flows into the dotted end of one field coil, and the other current out of the dotted end of the other field coil, then the resulting magnetomotive forces subtract to produce a smaller magnetomotive force. Hence, producing a **Differentially Compounded DC motor**.

- ◆ For a long shunt Compounded DC motor, the KVL yields:

$$V_T = E_A + I_A(R_A + R_S)$$

- ◆ The currents are related by:

$$I_A = I_L - I_F$$

But, the field current is:

$$I_F = \frac{V_T}{R_F + R_{adj}}$$

- ◆ The net magnetomotive force in a Compounded DC motor is given by:

$$\mathcal{F}_{net} = \mathcal{F}_F \mp \mathcal{F}_{SE} - \mathcal{F}_{AR}$$

where, \mathcal{F}_F is the shunt field magnetomotive force

\mathcal{F}_{SE} is the series field magnetomotive force

\mathcal{F}_{AR} is the armature reaction magnetomotive force, as the machine flux will be reduced with each increase in load.

Positive sign is associated with **Cumulatively Compounded** DC motor.

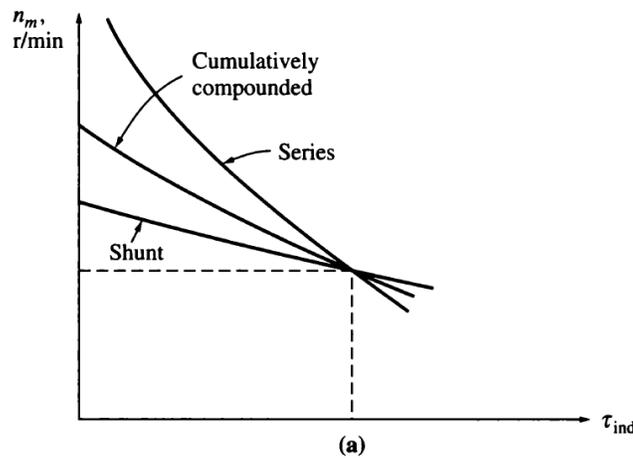
Negative sign is associated with **Differentially Compounded** DC motor.

- ◆ The effective shunt field current in a Compounded DC motor is obtained by dividing the previous equation by N_F as:

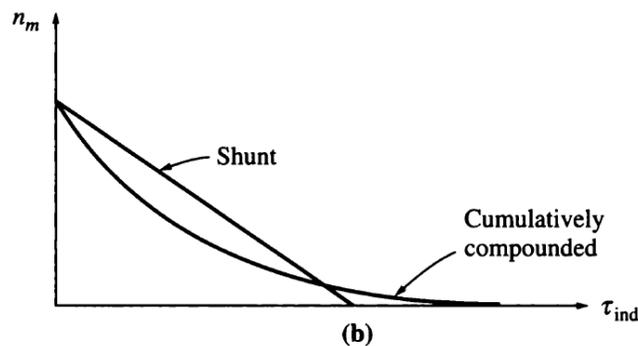
$$I_F^* = I_F \mp \frac{N_{SE} I_A}{N_F} - \frac{\mathcal{F}_{AR}}{N_F}$$

- ◆ A Cumulatively Compounded DC motor combines the best features of both the Shunt and the Series DC motors; like a Series DC motor, **it has extra torque for starting**. And like a Shunt DC motor, **it does not over speed at no load**. It is manufactured based on a compromise between both types of DC motors.

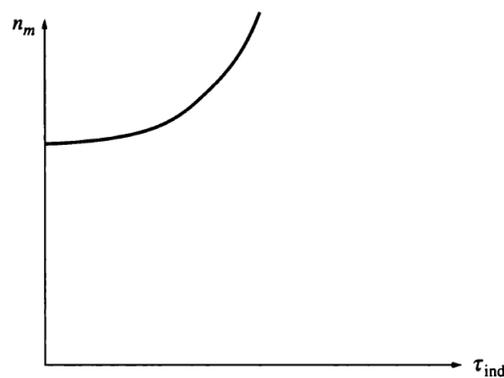
- ◆ The torque-speed characteristic of a Cumulatively Compounded DC motor, compared to a Series and a Shunt DC motors of the same full load ratings, is shown in the Figure (a) below.



- ◆ Figure (b) below shows the torque-speed characteristic of a Cumulatively Compounded DC motor, compared to a Shunt DC motor of the same no-load speed.



- ◆ **Differentially Compounded DC motors are unstable motors and tend to run away. Therefore, they are unsuitable for any application!** The Figure below illustrates the torque-speed characteristic of a Differentially Compounded DC motor.



DC Motor Control Circuits

Control circuits of a DC motor must have the following features:

- i. Protecting the motor against damage from high starting current
- ii. Protecting the motor against damage from long term overloads
- iii. Protecting the motor against damage due to short circuits
- iv. Providing a convenient manner for controlling speed

DC Motor Starting Problems

- At starting, E_A is zero and R_A is very low ($\sim 3\text{-}6\%$ per unit), resulting in a very high starting current (~ 20 times the rated (full) load current).
- This starting current can be reduced by inserting a starting resistor in series with the armature winding to limit the current flow, until E_A builds up to do the limiting.
- The starting resistor is removed gradually, in order to limit the current, but not to reduce it to a too low value, which may prevent the fast acceleration (resulting in not enough accelerating torque).

Types of DC Motor Starters:

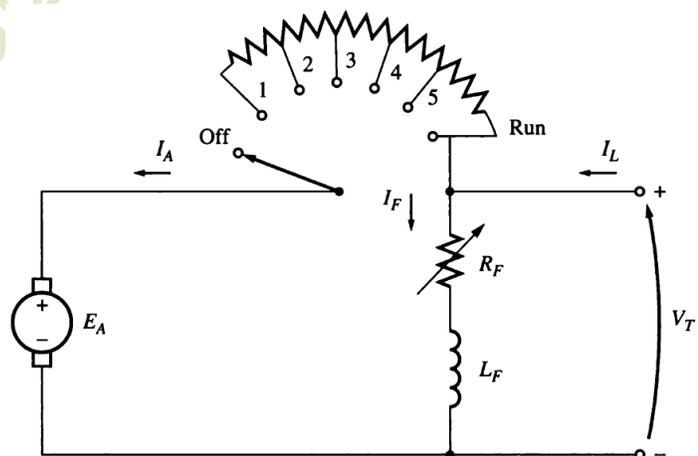
a. Manual DC Motor Starter

✚ It is an old DC Motor Starter using a continuous starting resistor, which is gradually cut out of the circuit by a person moving its handle, as shown in the Figure next.

✚ The starting depends on the person moving the handle.

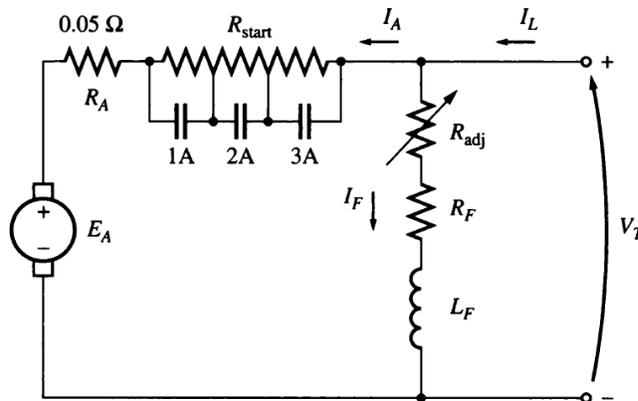
✚ The handle should **not** be moved **too quickly to prevent the resulting current from being too large.**

✚ The handle should **not** be moved **too slowly to prevent burning up** the resistor.



b. Automated DC Motor Starter

❖ The equivalent circuit of an Automated DC Motor Starter is shown in the Figure below.



❖ In this method, parts of the resistor are removed sequentially, within appropriate times.

❖ The contactors; "1A, 2A, and 3A" are closed at proper and preset times.

❖ Initially, the number and size of resistors should be chosen properly. Then, the timing for resistors' removal is set and adjusted.

c. Practical DC Motor Starters

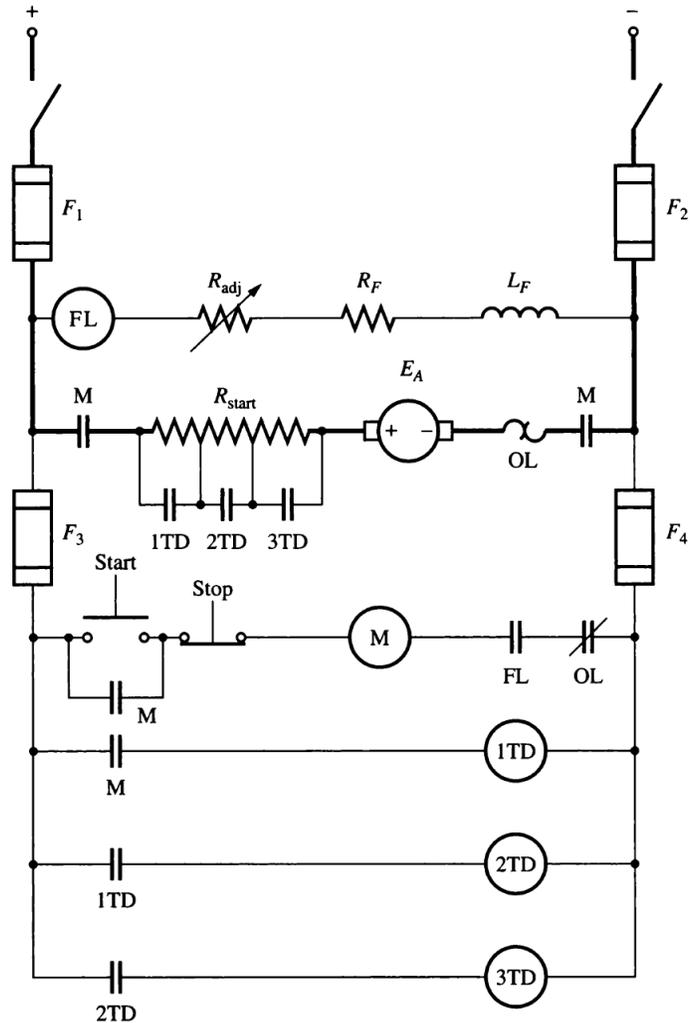
i. DC Motor Starter with Time-Delay Relays

- The circuit, shown in the Figure next, provides **overload protection**, which is achieved by overload (OL) heater and contact, and provides **short circuit protection**, which is achieved by Fuses; F_1 , F_2 , F_3 , and F_4 .

- 'Start' and 'Stop' are push buttons, which are normally open and closed, respectively.

- Note that, "M" is a contactor with 4 normally open contacts.

- The **Field Loss relay (FL) prevents the motor from running away** if the field was suddenly lost while the motor is running. It de-energizes the 'M' contactor.



- The **starting resistors** are cut in 3 steps, and **are controlled by** the **time-delay relays** "1TD, 2TD and 3TD".
- Since, the starting time and acceleration of the motor depend on the motor loading and inertia**, the time-delay relays has to be adjusted, with every change in load, so that the speed of resistors' removal is consistent with the motor gained speed and acceleration.

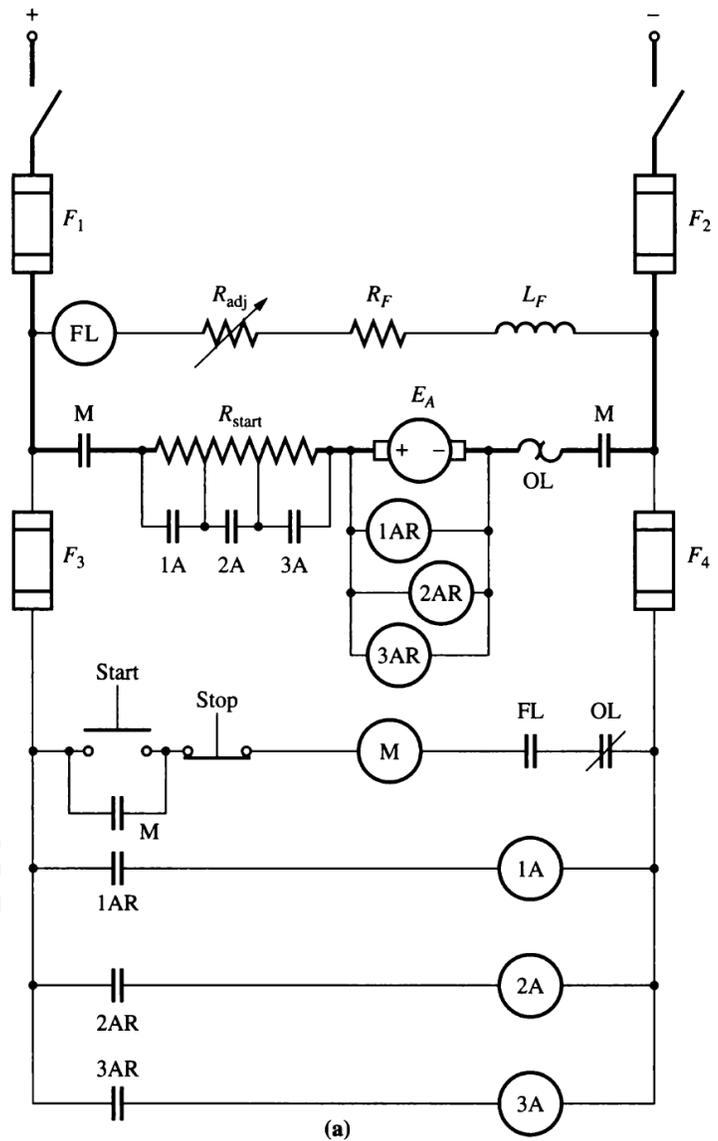
ii. **DC Motor Starter with Armature-Voltage Sensing Relays**

- The starting circuit, shown in the Figure next, has a series of voltage sensing relays (1AR, 2AR and 3AR), which sense the value of the armature voltage (E_A) in the motor and **cut out a part of the starting resistor as E_A rises to a preset value** when the motor speeds up.

- In this circuit, **the starting resistor is cut in 3 steps, and is controlled by the armature-voltage sensing relays**; 1AR, 2AR and 3AR.

- Note that, "1A, 2A and 3A" are contactors with normally open contacts.

- Here, the **starting time and resistor's removal are both dependent on the loading** of the motor, which is better!



- Also, this circuit provides overload protection, which is achieved by overload (OL) heater and contact, and provides short circuit protection, which is achieved by Fuses; F_1 , F_2 , F_3 , and F_4 .

Dr. M. Abu-Khaizaran, BZU, 2024/25

PART IX

Single Phase and Special Purpose Motors

In most homes and small businesses, motors must be run from a single phase power source.

Types of Single Phase and Special Purpose Motors:

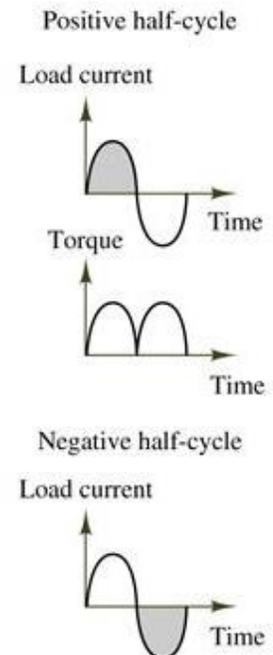
- 1) Universal Motors \Rightarrow Extension to a Series DC Motor
- 2) Single Phase Induction Motors
- 3) Reluctance Motors
- 4) Hysteresis Motors
- 5) Stepper Motors
- 6) Brushless DC Motors

1) Universal Motor

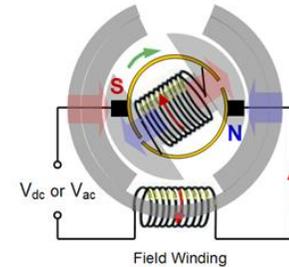
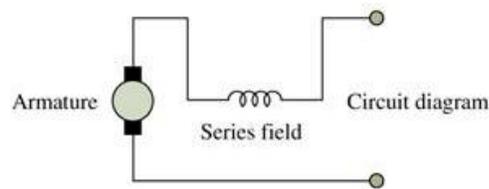
- ❖ It is the simplest way to design an AC motor.
- ❖ Recall, the induced torque of a DC motor is:

$$\tau_{ind} = k\phi I_A$$

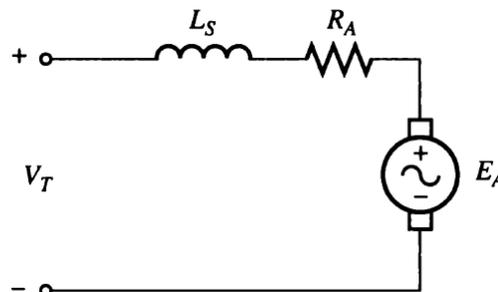
- ❖ If the polarity of the voltage applied to a Shunt or a Series DC motor is reversed, both the direction of the field flux and the direction of the armature current reverse, and, therefore, the resulting induced torque continues in the same direction as before.
- ❖ Therefore, it should be possible to achieve a pulsating, but unidirectional torque from a DC motor connected to an AC power source.
- ❖ Such a design is **practically possible only for Series DC motor**, since the armature current and the field current must reverse at the same time; because, $\phi = CI_A$ and $\tau_{ind} = kCI_A^2$.
- ❖ For a Shunt DC motor, the very **high field inductance tends to delay the reversal of the field current, compared to the armature current**, and thus to unacceptably reduce the average induced torque of the motor.



- ❖ To reduce the core losses, the field poles and the **stator frames** of a Series DC motor are **laminated**. If so, the motor is called **Universal motor**, since **it can be run from either an AC or a DC source**.
- ❖ Therefore, **a Universal motor is, basically, a Series DC motor run from an AC source, with its core laminated!** The armature and field circuits are connected in series with each other, as illustrated in the Figures below.



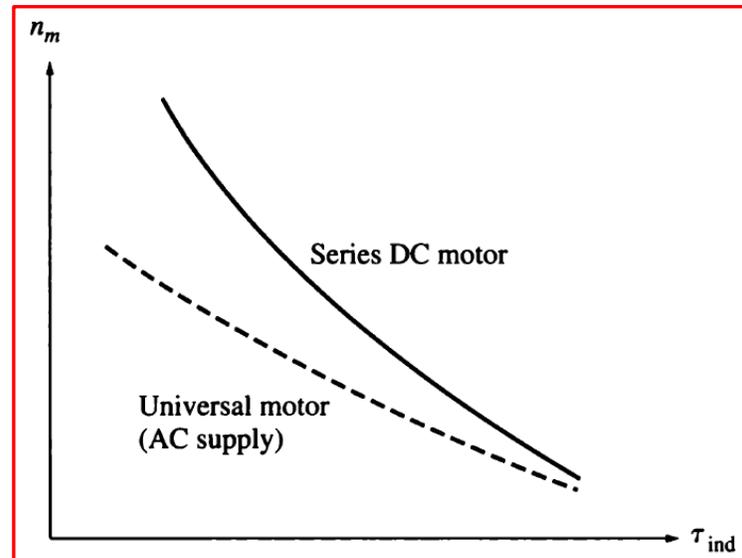
- ❖ The equivalent circuit of a Universal motor is shown in the Figure below.



- ❖ When the motor is running from an AC source, the **commutation will be much poorer** than it would be with a DC source.
- ❖ The **extra sparking** at the brushes **is caused by the transformer action** inducing voltages in the coils undergoing commutation.
- ❖ These sparks significantly shorten the brushes' life and can be a source of radio-frequency interference in certain environments.

Torque-Speed Characteristic of a Universal Motor

A comparison of the torque-speed characteristic of a Universal motor when operated from either AC or DC power supplies is depicted in the Figure below.



The Universal motor torque-speed characteristic is different from that of the same motor operating from a DC source, because:

- a. At 50 or 60Hz, the armature and the field reactances are large causing a significant voltage drop across them, and reducing E_A for a given input voltage, during AC operation, than it is during DC operation.

Since, $E_A = k\phi\omega$, the motor is slower for a given armature current and induced torque on an AC current than it would be on a DC current; $P_{out} = (E_A \downarrow I_A)$ becomes smaller $\Rightarrow \omega = \frac{P_{out} \downarrow}{\tau_{ind} \downarrow}$

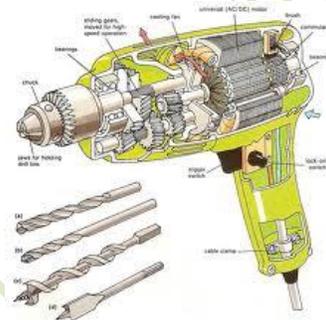
- b. For AC operation, the peak voltage; $V_{peak} = \sqrt{2}V_{rms}$

So the magnetic saturation could occur near the peak current in the machine. This saturation could significantly lower the rms flux of the motor for a given current level, tending to reduce the machine's induced torque.

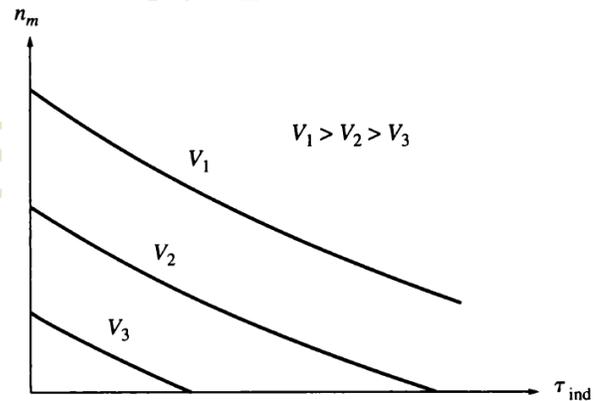
But, a decrease in flux increases the speed of a DC machine, so this effect may partially offset the speed decrease caused by the first effect (or part (a)).

Applications of a Universal Motor

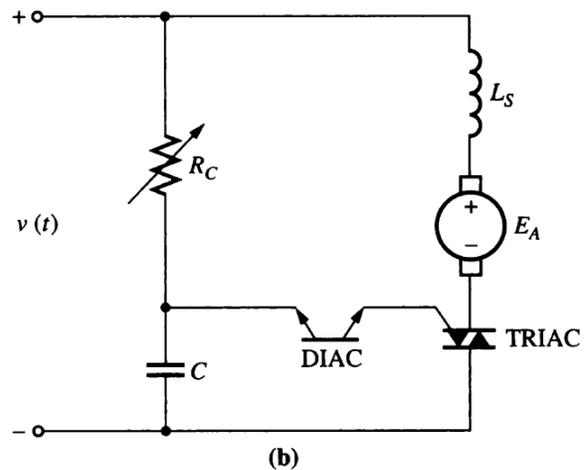
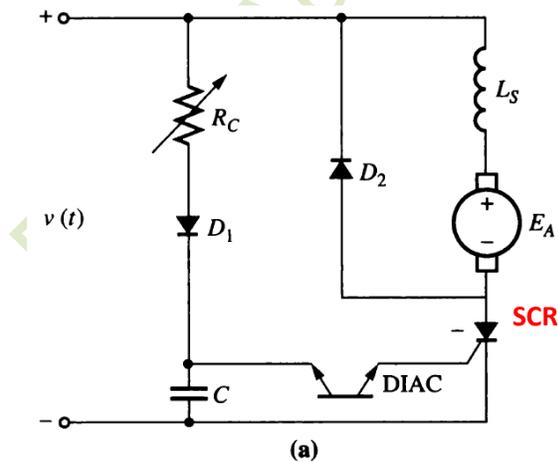
- It has a sharply drooping torque-speed characteristic. Therefore, it is not suitable for constant speed applications.
- It is compact and gives **more torque/Ampere** than any other single phase motor.
- It is used where light weight applications and high torque are important.
- Typical Applications: **vacuum cleaners, drills, hand held and similar portable tools and kitchen appliances.**



- The speed control of a Universal motor can be **achieved by varying the rms terminal voltage** of the motor. The Figure next shows the torque-speed characteristics for different values of input voltage.



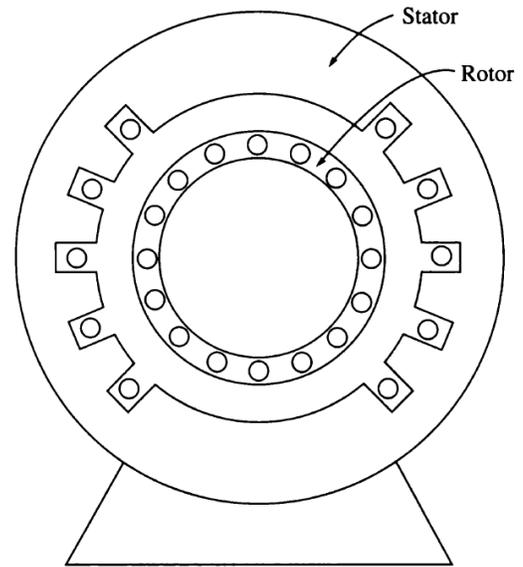
- Typical half wave (Figure (a)) and full wave (Figure (b)) control circuits are shown below.



2) Single Phase Induction Motors

Construction and Operation:

- It has a stator and a (Squirrel Cage) rotor.
- The rotor is a **squirrel cage type**, as illustrated in the Figure next, and is the same as that of a three-phase Induction motor.
- The stator has a single phase (main) winding fed from a single phase AC source.
- Applying a single phase current to the stator main winding produces a **stator magnetic field, which pulsates on the same axis and does not rotate.**

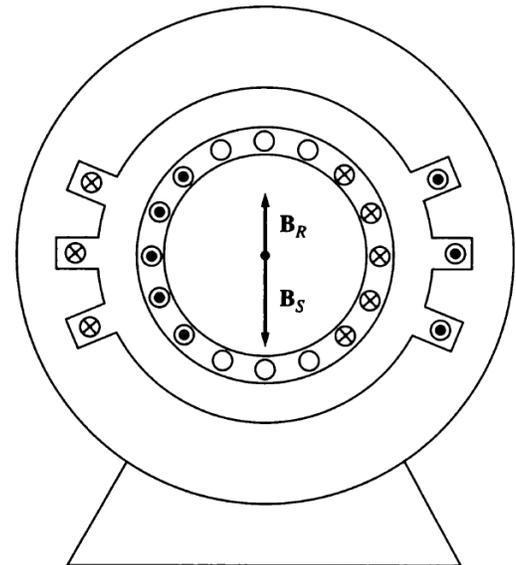


- Since the stator magnetic field does not rotate, there is no relative motion between the stator field and rotor bars. Therefore, **no induced voltage** in the rotor bars due to relative motion \Rightarrow **No induced currents due to relative motion** \Rightarrow **No induced torque.**
- However, the stator magnetic field (B_s) induces, by induction (Transformer action; $\frac{d\phi}{dt}$), voltages in the rotor bars. Since, these bars are shorted from either end, rotor currents flow, which in turn produce a rotor magnetic field (B_r) opposing (lined up with) the original magnetic field which produced it in the first place (Lenz' Law), as depicted in the Figure next.
- Thus, the induced torque is:

$$\tau_{ind} = k B_r X B_s$$

$$\tau_{ind} = k B_r B_s \sin 180^\circ$$

$$\tau_{ind} = 0$$

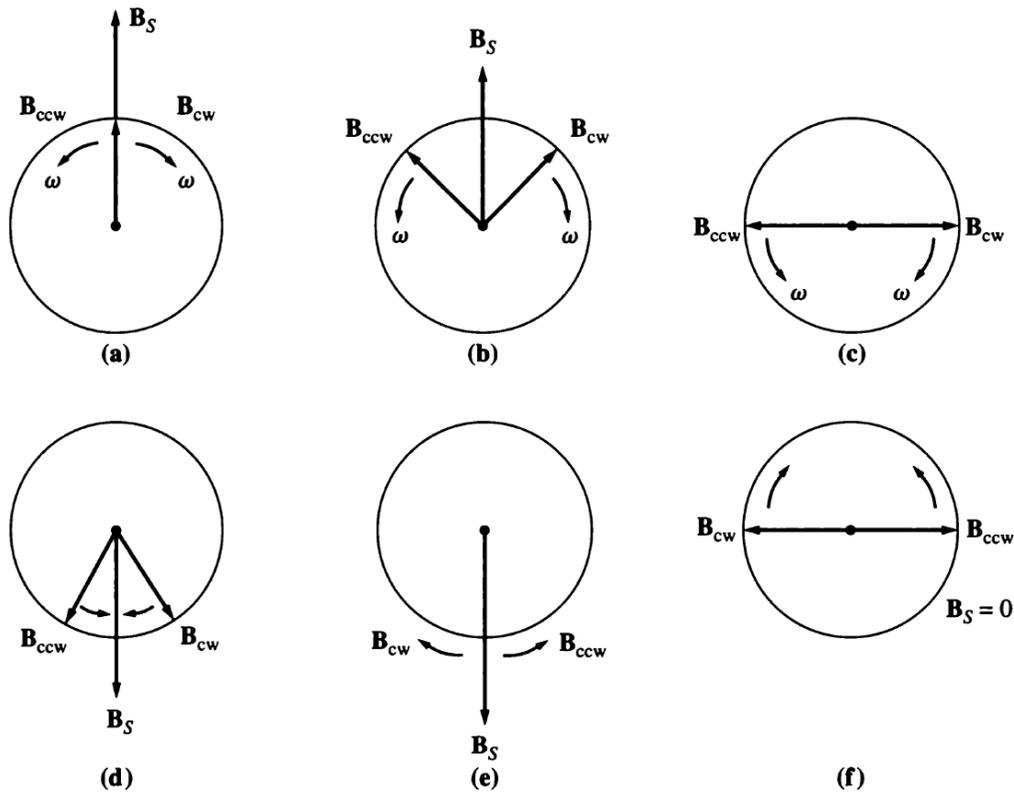


Therefore, the Single Phase Induction motor, without modifications on its construction, has **no rotating magnetic fields, and hence no starting torque.**

- At stall condition, the motor looks like a transformer with a short circuit on its secondary winding.
- However, **once the rotor begins to turn, an induced torque will be induced on it.**

Double-Revolving Field Theory

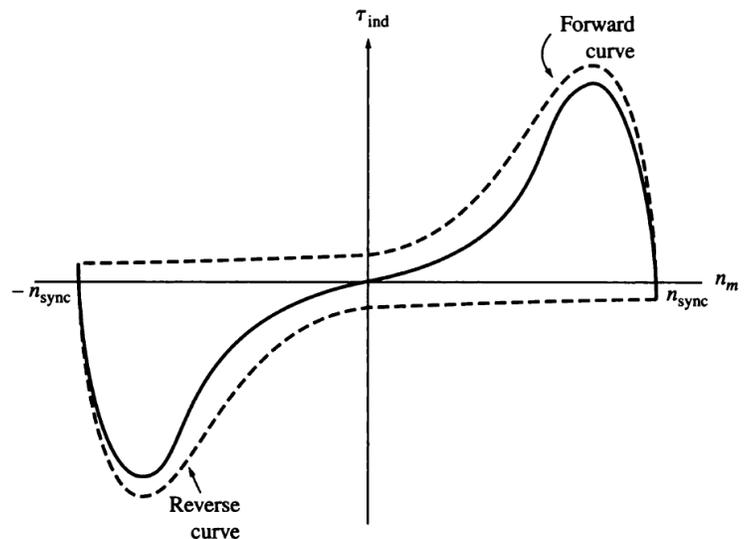
A stationary pulsating magnetic field can be resolved into **two rotating magnetic fields**, each of equal magnitude, but **rotating in opposite direction**, as illustrated in the Figures below. At any time, the vector sum of the two magnetic fields lies on the vertical axis.



The induction motor responds to each magnetic field separately, and the net torque in the machine is the sum of torques due to each of the two magnetic fields, as depicted in the Figure next.

The solid line in the Figure represents the actual torque in the machine.

If the rotor was turning, there are torque pulsations at twice the electric frequency. \Rightarrow **It is a noisy motor.**



Starting Single Phase Induction Motors

Starting techniques differ in cost and amount of starting torque.

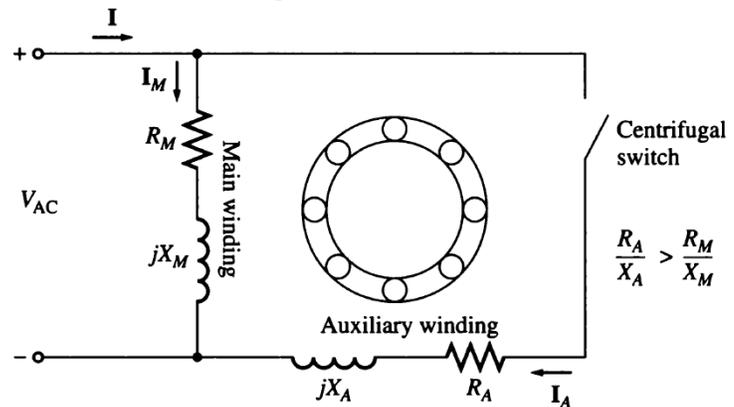
The common starting techniques are:

- I. Split Phase Winding
- II. Capacitor Type Winding
- III. Shaded Stator Poles

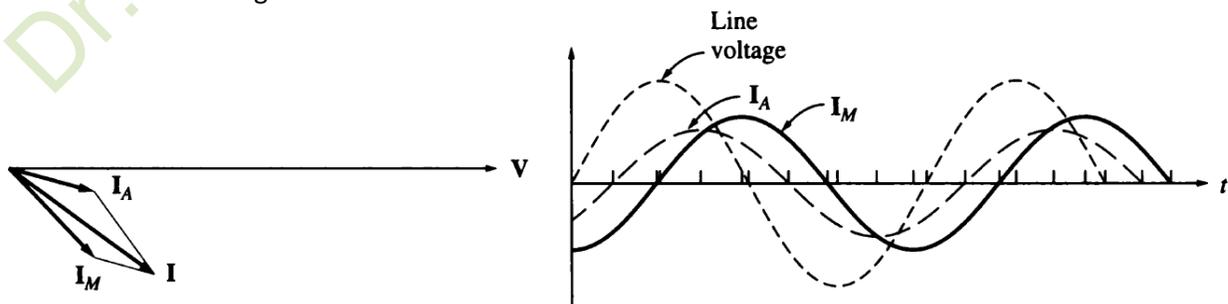
I. Split-Phase (Separate) Winding Induction Motor

- ◆ To generate a rotating magnetic field, and therefore a starting torque, an Auxiliary (Start) winding is added to stator, whose current is shifted from the current in the Main (Run) winding by an angle; typically 90° , but in this type of motor it is in the range: **30° to 45° !**
- ◆ The motor has two stator windings; a Main "M" (Run) stator winding and an Auxiliary "A" (Start) stator winding. These windings are set 90° electrical apart along the stator.

- ◆ The Auxiliary winding is designed to be switched out of the circuit at some preset speed by a centrifugal switch, connected as illustrated in the Figure next.

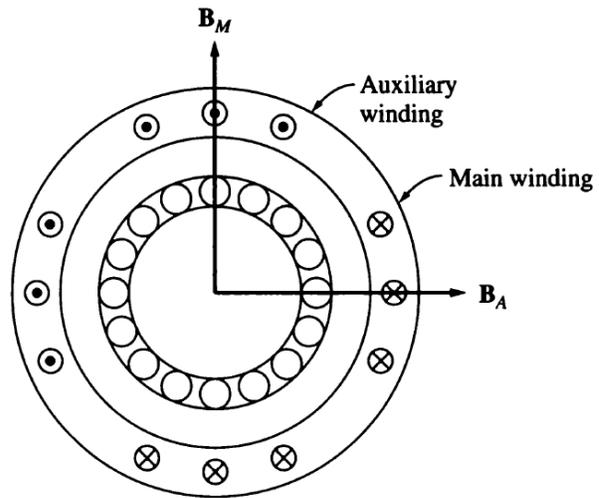


- ◆ The Auxiliary winding is designed to have a higher resistance/reactance ratio than that of the Main winding; $\frac{R_A}{X_A} > \frac{R_M}{X_M}$. This can be accomplished by using smaller wires for the Auxiliary winding.
- ◆ Consequently, the current in the Auxiliary winding leads the current in the Main winding by some angle as shown in the Figures below.



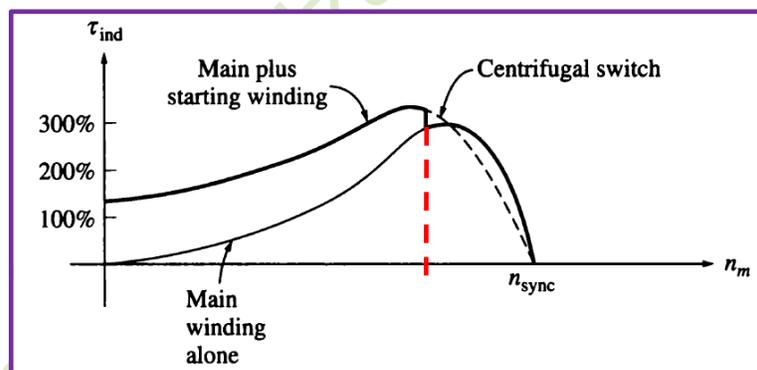
The Function of the Auxiliary Winding

- Since the current in the Auxiliary winding leads the current in the Main winding, the Auxiliary's magnetic field (B_A) peaks before the Main's magnetic field (B_M). The axis of action of each magnetic field is illustrated in the Figure next.
- Therefore, there is a **net counter-clockwise rotation in the magnetic field**.
- The Auxiliary winding makes one of the oppositely rotating stator magnetic fields larger than the other one, and therefore **a net stator rotating magnetic field results**. Hence, there is a relative motion between the rotor bars and net stator rotating magnetic field.
- Consequently, **a starting torque results**.



Torque-Speed Characteristic

- The torque-speed characteristic of a Split-phase motor is shown in the Figure below.

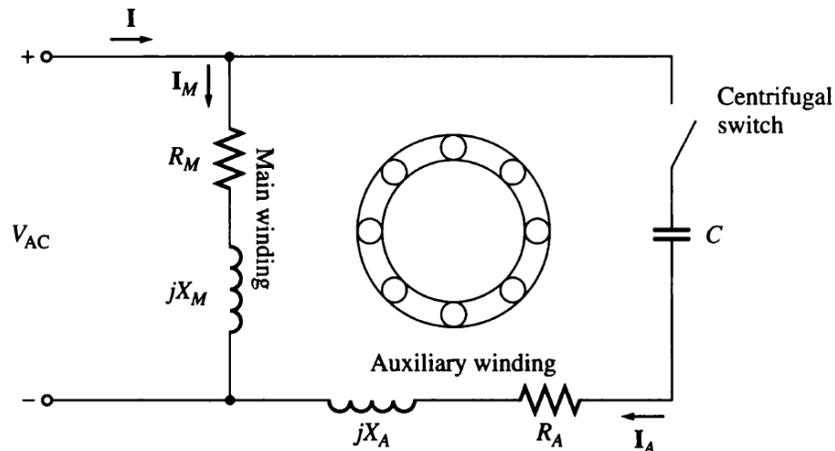


- The Split-phase motor has a moderate starting torque, because the angle between I_A and I_M is small; 30° to 45° , with a fairly low starting current!
- Split-phase motors are used in applications, which **do not require very high starting torques such as; fans, blowers, centrifugal pumps,...**
- They are inexpensive and available in the fractional Horse power range.
- The direction of rotation of the motor can be reversed by swapping the connections of either the Auxiliary or Main winding, while leaving the other winding's connections unchanged.**

II. Capacitor Type Winding Induction Motor

i. Capacitor-Start Induction Motor

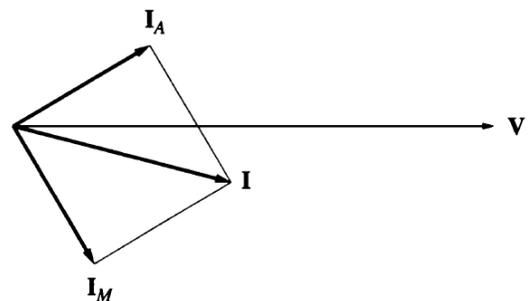
- In this motor, a capacitor is connected in series with the Auxiliary winding, as shown in the Figure below.
- It is more expensive than a Split Phase motor; it has a capacitor!



- The capacitor size is selected such that:
 - a. The magnetomotive force of the starting current in the Auxiliary winding is EQUAL to the magnetomotive force of the starting current in the Main winding.
 - b. The phase angle of the current in the Auxiliary winding can be made to LEAD the current in the Main winding by 90° .

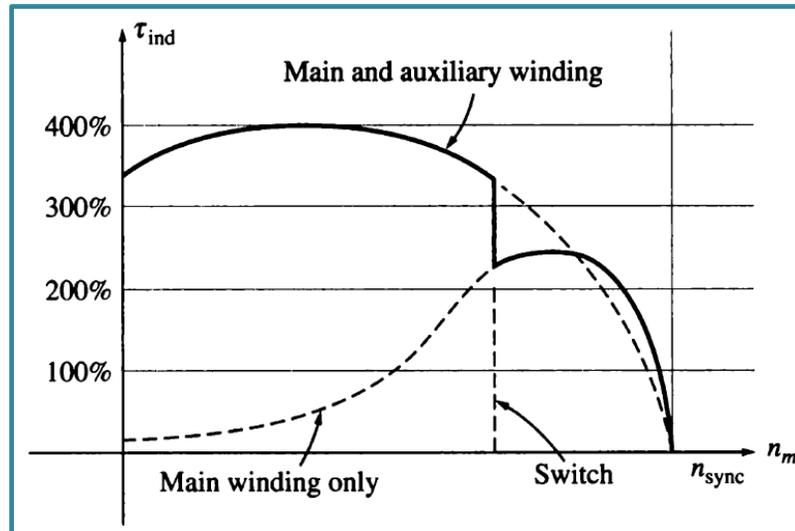
In summary: $mmf_A \approx mmf_M$, and I_A Leads I_M by 90°

- Since the two windings are physically separated by 90° , 90° phase difference in currents will yield a single uniform rotating stator magnetic field. The Capacitor-Start motor will behave as if it were started from three phase power source.



Torque-Speed Characteristic of a Capacitor-Start Motor:

The **starting torque** of the Capacitor-Start motor is very high and **may exceed 300% of the rated torque**, as illustrated in the Figure below.



Applications:

They are used in compressors, pumps, air conditioners, and other pieces of equipment that must start under load.

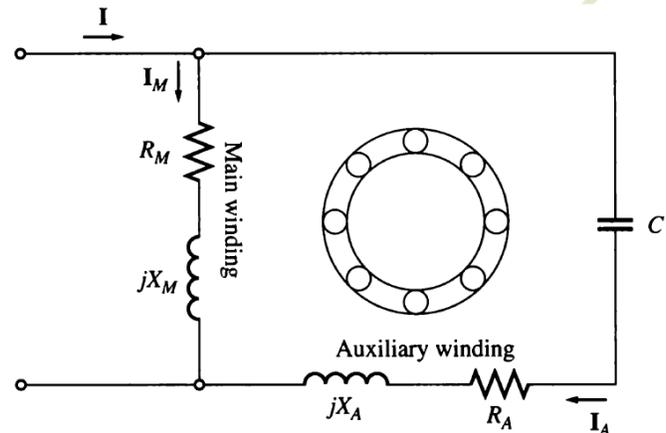
Reversing the Direction of Rotation:

The **direction of rotation** of the motor can be reversed by **switching the connections of either the Auxiliary or Main winding**, while leaving the other winding's connections unchanged.

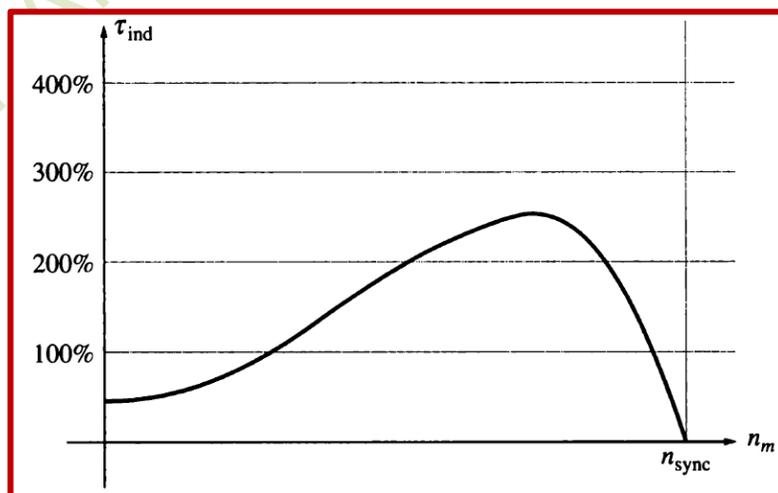
ii. Permanent Split-Capacitor Motor

- ✓ It is also called a Capacitor-Start-and-Run motor.
- ✓ A capacitor, of small size, is chosen such that the motor has a uniform rotating magnetic field at some specific load, and it will behave like a three-phase Induction motor at that point.

- ✓ The capacitor stays in series with the Auxiliary winding all the time, as shown in the Figure next.



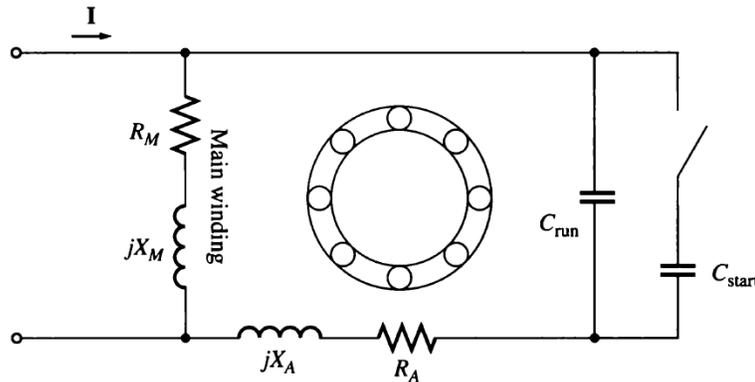
- ✓ At normal loads, it is more efficient, has a higher power factor, and has a smoother torque than an ordinary single phase Induction motor.
- ✓ It has less starting torque than a Capacitor-Start motor, because the capacitor is sized to balance the currents in the Main and the Auxiliary windings at normal conditions only.
- ✓ Since the starting current is much greater than the normal (rated) current, a capacitor that balances the phases (magnetomotive forces) under normal loads leaves them very unbalanced under starting conditions.
- ✓ The torque-speed characteristic of a Permanent Split-Capacitor motor is shown in the Figure below.



iii. Capacitor-Start, Capacitor-Run Motor

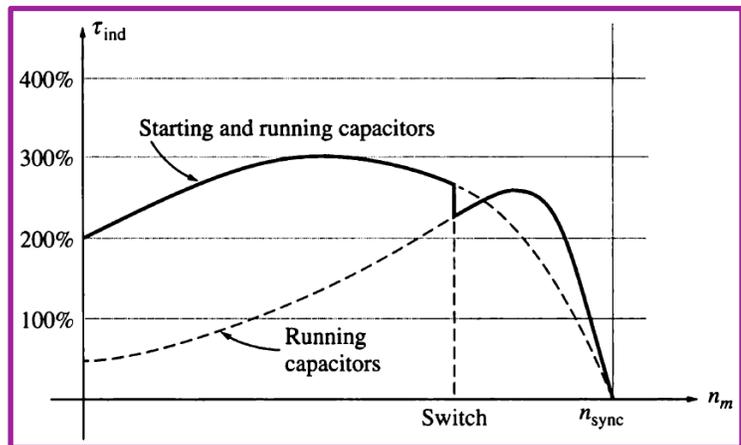
Two-Value-Capacitor Motor; it has two capacitors, as shown in the Figure below.

- ◆ It has the **largest possible starting torque and the best running conditions** amongst other single phase motors.



- ◆ A large capacitance ($C_{start} + C_{run}$) is present in the circuit during starting, which balances the currents in the Main and the Auxiliary windings during starting, yielding a very high starting torque.
- ◆ When the motor speeds up, the centrifugal switch disconnects the starting capacitor (C_{start}), leaving the main capacitor (C_{run}) in the circuit only.
- ◆ The permanent capacitor (C_{run}) is large enough to balance the currents at normal load conditions, so the motor operates efficiently with high starting torque and power factor.
- ◆ Note that, $C_{run} \sim 10 \text{ to } 20\% \text{ of } C_{start}$

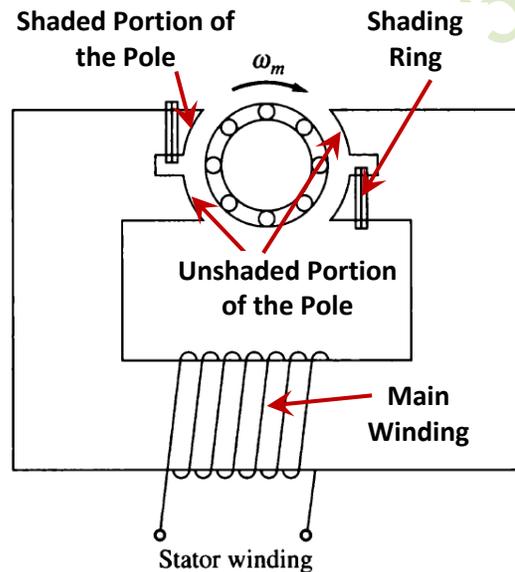
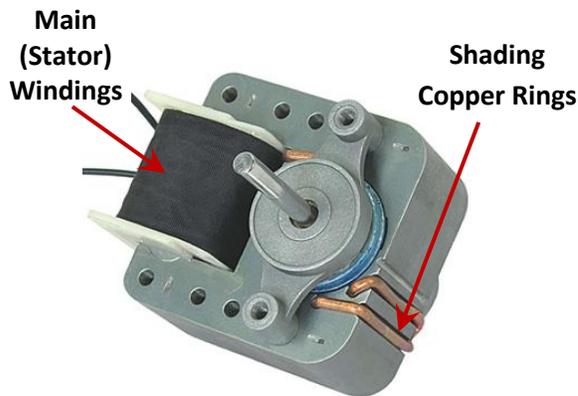
- ◆ **The Torque-Speed Characteristic** is shown in the Figure next.



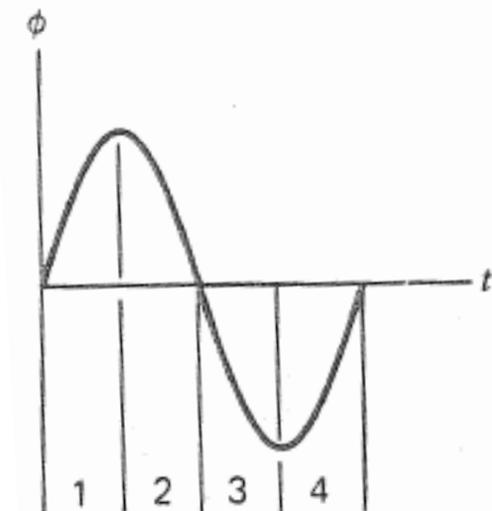
- ◆ The direction of rotation of the motor can be reversed by switching the connections of either the Auxiliary or Main winding, while leaving the other winding's connection unchanged.

III. Shaded Pole Motor

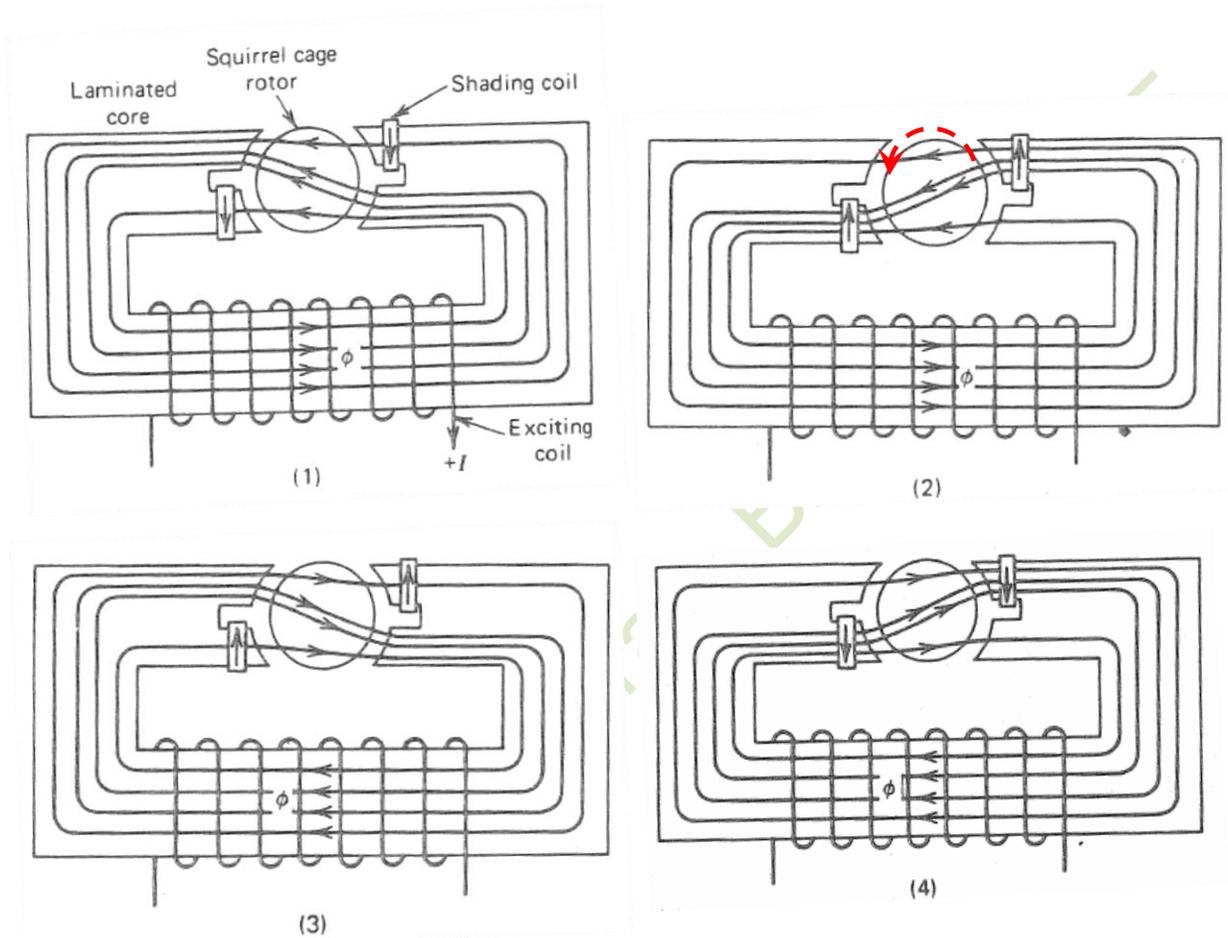
- A Shaded Pole motor has a Main (stator) winding only.
- Instead of having an Auxiliary winding, the motor has salient poles, such that one portion of each pole is surrounded by a short circuited coil (a copper ring or conductor) called a **Shading Ring (Coil)**, as illustrated in the Figures below.



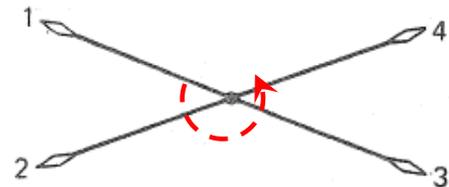
- If an AC voltage/current is applied to the main (stator) coil, it produces a flux in the core which is also sinusoidal as shown in the Figure next.
- Because the pole's flux varies, it induces a voltage and a current in the Shading Coils, which produces a flux that opposes the original flux (Lenz' Law!).



- This opposition retards the flux changes under the shaded portions of the coils, and therefore produces a slight imbalance between the two oppositely rotating magnetic fields, as illustrated in the Figures below.

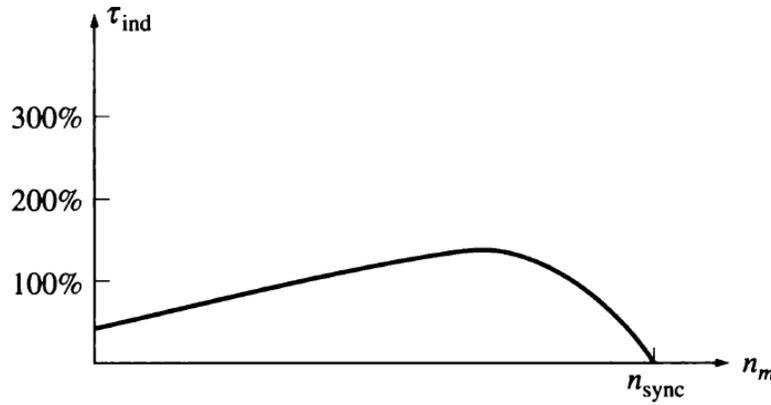


- The net rotation is in the direction from the unshaded portion to the shaded portion of the pole face, as illustrated in the Figures above and summarized in the Figure next.



- This produces a weak non uniform rotating magnetic field in the airgap.

- The **non uniform rotating magnetic field** passes in the rotor, producing a **weak Induction motor** action. The torque-speed characteristic of the Shaded Pole motor is shown in the Figure below. It has **less starting torque than any other type of Induction motor.**



- Thus, the Shaded pole motor is a small motor ($\sim 1/20$ hp) **with low starting torque and moderate rated torques.**
- Shaded pole motor is much **less efficient** and has a **much higher slip** than any other type of single phase Induction motor.
- It is the **simplest and cheapest motor** available.

Applications: **small fans, hair dryers, and washing machines pumps,...**

Reversing the Direction of Rotation:

The motor must be designed with two sets of Shading Rings (coils); one set is closed and the other set is opened. Otherwise, it may be reversed permanently by inverting the core.

Comparison between Single Phase Induction Motors

They can be ranked from the best to the worst in terms of the starting torque and the running characteristics:

- Capacitor-Start, Capacitor-Run Motor (**Best but most expensive**)
- Capacitor Start Motor
- Permanent Split Capacitor Motor
- Split Phase Motor
- Shaded Pole Motor (**cheapest**)

Speed Control of Single Phase Induction Motors

As in three-phase Induction motors, the synchronous speed is, also, given by:

$$n_{sync} = \frac{120f_e}{p}$$

and the mechanical speed is:

$$n_m = (1 - s)n_{sync}$$

Thus, the speed can be controlled in the same manner as the speed of a three-phase Induction motor.

However, the **methods used for speed control** are:

i. Varying the Frequency of the Voltage Applied to the Main Stator Winding

Varying the supply frequency is a rarely used method for speed control in such motors, as it is expensive!

ii. Changing the Number of Stator Poles

It is not practical in small motors!

iii. Varying the Magnitude of the Applied Voltage across the Terminals of the Stator Winding

For high slip motors, varying the effective voltage applied to the Main stator windings is a very appropriate method for speed control. This can be achieved by:

- a- Inserting **a resistor in series** with the stator Main winding. It is the cheapest method, but it **increases losses and reduces the efficiency** of the motor.
- b- **An SCR or a TRIAC** may be used to chop the AC voltage, but it **increases harmonics, noise, and vibrations**.
- c- **An Autotransformer** may be used to continually adjust the line voltage. It gives very smooth speed control, but **it is expensive**.

Speed Control of a Shaded Pole Motor

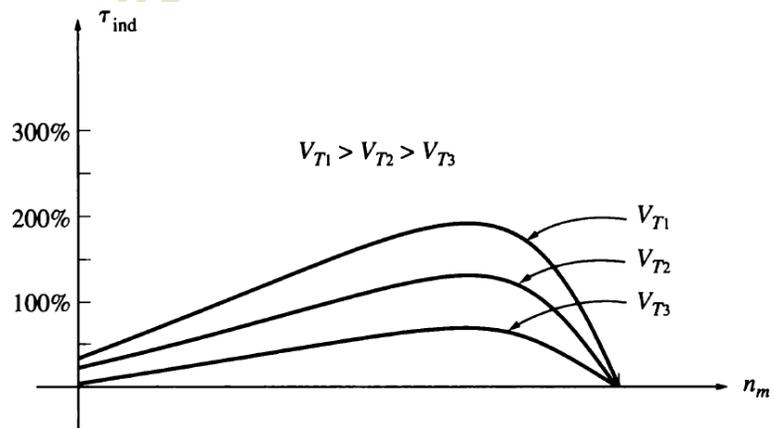
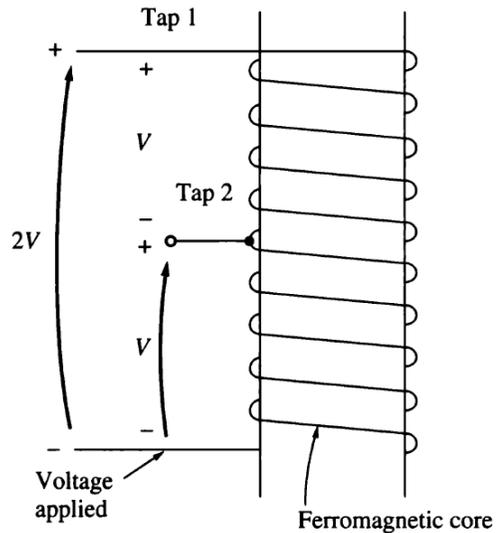
The stator (Main) winding may be used as an Autotransformer to vary the effective applied voltage; Taps are designed to be along the stator winding, as shown in the Figure next.

If a voltage (V) is applied across the entire winding ("Tap 1" with respect to the Common Tap), the Shaded Pole motor operates normally.

If the voltage (V) is applied to "Tap 2" at the center, with respect to the Common Tap, an identical voltage will be induced in the upper half by transformer action, and the total effective voltage, across the winding, will be twice the applied voltage ($2V$).

More than two Taps can be added to the stator winding, so that the motor can have more than two speeds, as it is the case in motors driving small fans.

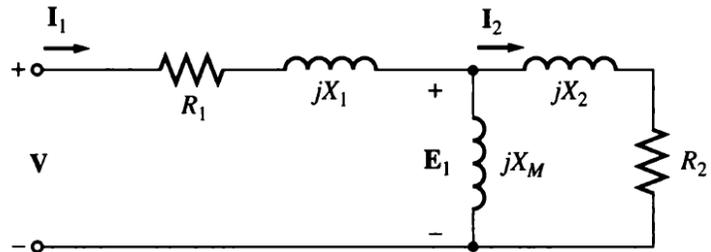
Therefore, the smaller the fraction of the total Main winding (coil) that the line voltage is applied across, the greater the total induced voltage will be across the whole winding, and the higher the speed will be for a given load, as illustrated in the Figure next.



This approach is used to control speed of a single-phase Induction motor driving fan or blower applications; it is very inexpensive, and what needed are Taps only on the Main winding.

The Circuit Model of a Single Phase Induction Motor

- ▶ The equivalent circuit may be examined based on Double Revolving Field Theory.
- ▶ The equivalent circuit of the Main winding of a single phase Induction motor when operating alone will be developed.
- ▶ At a stall condition, the motor appears as a single phase transformer with its secondary circuit shorted out. Its equivalent circuit is shown in the Figure next.



where,

R_1 : is the stator Main winding's resistance

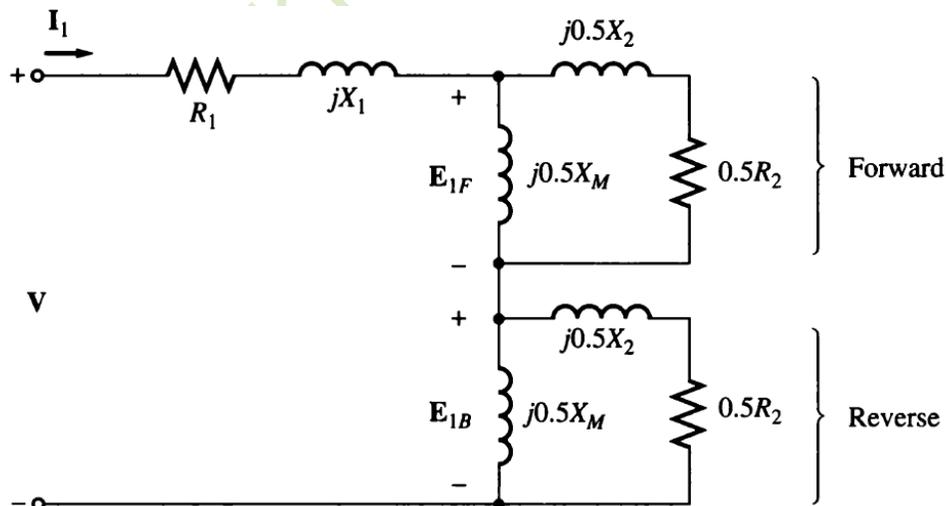
X_1 : is the stator Main winding's leakage reactance

X_M : is the magnetizing inductance

R_2 : is the referred rotor bar's (windings) resistance

X_2 : is the referred rotor bar's (windings) leakage reactance

- ▶ It is possible to split the rotor equivalent circuit into two sections, as illustrated in the Figure below; each one corresponds to the effects of one of the two oppositely revolving fields (Forward and Backward (Reverse)).



- ▶ Here, the core losses are added to the mechanical losses.

- Assuming that the motor has started to turn with the help of the Auxiliary winding, which was disconnected again after the motor has come up to a particular speed
- The effective rotor resistance depends on the relative motion, which varies depending on the magnetic fields (Forward or Backward).
- The Forward Magnetic Field rotates at n_{sync} .
- The Backward (Reverse) Magnetic Field rotates at $-n_{sync}$.
- For a Forward Magnetic Field, the per unit difference between the rotor speed and the speed of the magnetic field is "s".

- Therefore, the rotor resistance associated with the Forward Magnetic Field is:

$$\frac{0.5R_2}{s}$$

- Thus, the total per unit difference in speed (on a base of n_{sync}) between the Forward and the Reverse Magnetic Fields is "2"; i.e.,

$$\frac{n_{sync} - (-n_{sync})}{n_{sync}} = 2$$

- But, the rotor is turning at a speed slower than the Forward Magnetic Field by a per unit speed "s".

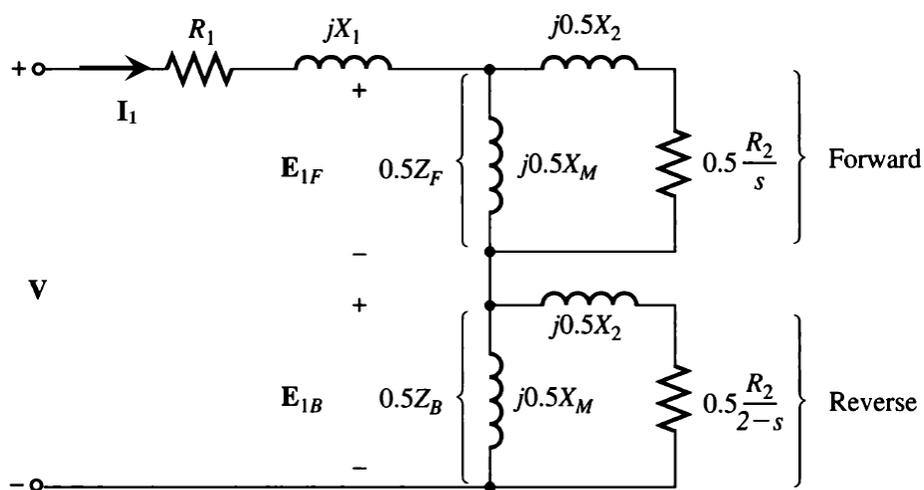
- Hence, the total per unit difference in speed between the rotor and the Reverse Magnetic Field is:

$$\frac{-n_{sync} - n_m}{-n_{sync}} = \frac{-n_{sync} - (1-s)n_{sync}}{-n_{sync}} = \frac{-1 - (1-s)}{-1} = 2 - s$$

- The effective rotor resistance in the part associated with the Reverse Magnetic Field is:

$$\frac{0.5R_2}{2-s}$$

The Final Equivalent Circuit of the Main Winding of a Single-Phase Induction Motor is:



Power and Torque in a Single-Phase Induction Motor:

The net power and torque of the machine are the difference between Forward and Backward components.

The **net Airgap Power** is:

$$P_{AG} = P_{AGF} - P_{AGB}$$

The **Forward Airgap Power** is:

$$P_{AGF} = I_1^2 (0.5R_F); R_F \text{ is the resistive part of the Forward impedance}$$

The **Backward Airgap Power** is:

$$P_{AGB} = I_1^2 (0.5R_B); R_B \text{ is the resistive part of the Backward impedance}$$

Therefore, the torque induced:

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

The rotor copper losses are:

$$P_{RCL} = P_{RCLF} + P_{RCLB}$$

where, $P_{RCLF} = sP_{AGF}$

$$P_{RCLB} = sP_{AGB}$$

The converted power is:

$$P_{conv} = \tau_{ind}(1-s)\omega_{sync}$$

$$\Rightarrow P_{conv} = (1-s)P_{AG}$$

Example 9-1. A ½-hp, 110-V, 60-Hz, six-pole, split-phase induction motor has the following impedances:

$$\begin{aligned} R_1 &= 1.52 \, \Omega & X_1 &= 2.10 \, \Omega & X_M &= 58.2 \, \Omega \\ R_2 &= 3.13 \, \Omega & X_2 &= 1.56 \, \Omega \end{aligned}$$

The core losses of this motor are 35 W, and the friction, windage, and stray losses are 16 W. The motor is operating at the rated voltage and frequency with its starting winding open, and the motor's slip is 5 percent. Find the following quantities in the motor at these conditions:

- (a) Speed in revolutions per minute
- (b) Stator current in amperes
- (c) Stator power factor
- (d) P_{in}
- (e) P_{AG}
- (f) P_{conv}
- (g) τ_{ind}
- (h) P_{out}
- (i) τ_{load}
- (j) Efficiency

Solution

The forward and reverse impedances of this motor at a slip of 5 percent are

$$\begin{aligned} Z_F &= R_F + jX_F = \frac{(R_2/s + jX_2)(jX_M)}{(R_2/s + jX_2) + jX_M} & (9-5) \\ &= \frac{(3.13 \, \Omega/0.05 + j1.56 \, \Omega)(j58.2 \, \Omega)}{(3.13 \, \Omega/0.05 + j1.56 \, \Omega) + j58.2 \, \Omega} \\ &= \frac{(62.6 \angle 1.43^\circ \, \Omega)(j58.2 \, \Omega)}{(62.6 \, \Omega + j1.56 \, \Omega) + j58.2 \, \Omega} \\ &= 39.9 \angle 50.5^\circ \, \Omega = 25.4 + j30.7 \, \Omega \end{aligned}$$

$$\begin{aligned} Z_B &= R_B + jX_B = \frac{[R_2/(2 - s) + jX_2](jX_M)}{[R_2/(2 - s) + jX_2] + jX_M} & (9-6) \\ &= \frac{(3.13 \, \Omega/1.95 + j1.56 \, \Omega)(j58.2 \, \Omega)}{(3.13 \, \Omega/1.95 + j1.56 \, \Omega) + j58.2 \, \Omega} \\ &= \frac{(2.24 \angle 44.2^\circ \, \Omega)(j58.2 \, \Omega)}{(1.61 \, \Omega + j1.56 \, \Omega) + j58.2 \, \Omega} \\ &= 2.18 \angle 45.9^\circ \, \Omega = 1.51 + j1.56 \, \Omega \end{aligned}$$

These values will be used to determine the motor current, power, and torque.

- (a) The synchronous speed of this motor is

$$n_{sync} = \frac{120f_{se}}{P} = \frac{120(60 \text{ Hz})}{6 \text{ pole}} = 1200 \text{ r/min}$$

Since the motor is operating at 5 percent slip, its mechanical speed is

$$n_m = (1 - s)n_{sync}$$

$$n_m = (1 - 0.05)(1200 \text{ r/min}) = 1140 \text{ r/min}$$

(b) The stator current in this motor is

$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{V}}{R_1 + jX_1 + 0.5Z_F + 0.5Z_B} & (9-7) \\ &= \frac{110\angle 0^\circ \text{ V}}{1.52 \Omega + j2.10 \Omega + 0.5(25.4 \Omega + j30.7 \Omega) + 0.5(1.51 \Omega + j1.56 \Omega)} \\ &= \frac{110\angle 0^\circ \text{ V}}{14.98 \Omega + j18.23 \Omega} = \frac{110\angle 0^\circ \text{ V}}{23.6\angle 50.6^\circ \Omega} = 4.66\angle -50.6^\circ \text{ A} \end{aligned}$$

(c) The stator power factor of this motor is

$$\text{PF} = \cos(-50.6^\circ) = 0.635 \text{ lagging}$$

(d) The input power to this motor is

$$\begin{aligned} P_{\text{in}} &= VI \cos \theta \\ &= (110 \text{ V})(4.66 \text{ A})(0.635) = 325 \text{ W} \end{aligned}$$

(e) The forward-wave air-gap power is

$$\begin{aligned} P_{\text{AG},F} &= I_1^2(0.5 R_F) & (9-8) \\ &= (4.66 \text{ A})^2(12.7 \Omega) = 275.8 \text{ W} \end{aligned}$$

and the reverse-wave air-gap power is

$$\begin{aligned} P_{\text{AG},B} &= I_1^2(0.5 R_B) & (9-9) \\ &= (4.66 \text{ A})^2(0.755 \Omega) = 16.4 \text{ W} \end{aligned}$$

Therefore, the total air-gap power of this motor is

$$\begin{aligned} P_{\text{AG}} &= P_{\text{AG},F} - P_{\text{AG},B} & (9-10) \\ &= 275.8 \text{ W} - 16.4 \text{ W} = 259.4 \text{ W} \end{aligned}$$

(f) The power converted from electrical to mechanical form is

$$\begin{aligned} P_{\text{conv}} &= (1 - s) P_{\text{AG}} & (9-17) \\ &= (1 - 0.05)(259.4 \text{ W}) = 246 \text{ W} \end{aligned}$$

(g) The induced torque in the motor is given by

$$\begin{aligned} \tau_{\text{ind}} &= \frac{P_{\text{AG}}}{\omega_{\text{sync}}} & (9-11) \\ &= \frac{259.4 \text{ W}}{(1200 \text{ r/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/r})} = 2.06 \text{ N} \cdot \text{m} \end{aligned}$$

(h) The output power is given by

$$\begin{aligned} P_{\text{out}} &= P_{\text{conv}} - P_{\text{rot}} = P_{\text{conv}} - P_{\text{core}} - P_{\text{mech}} - P_{\text{stray}} \\ &= 246 \text{ W} - 35 \text{ W} - 16 \text{ W} = 195 \text{ W} \end{aligned}$$

(i) The load torque of the motor is given by

$$\begin{aligned}\tau_{\text{load}} &= \frac{P_{\text{out}}}{\omega_m} \\ &= \frac{195 \text{ W}}{(1140 \text{ r/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/r})} = 1.63 \text{ N} \cdot \text{m}\end{aligned}$$

(j) Finally, the efficiency of the motor at these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{195 \text{ W}}{325 \text{ W}} \times 100\% = 60\%$$

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