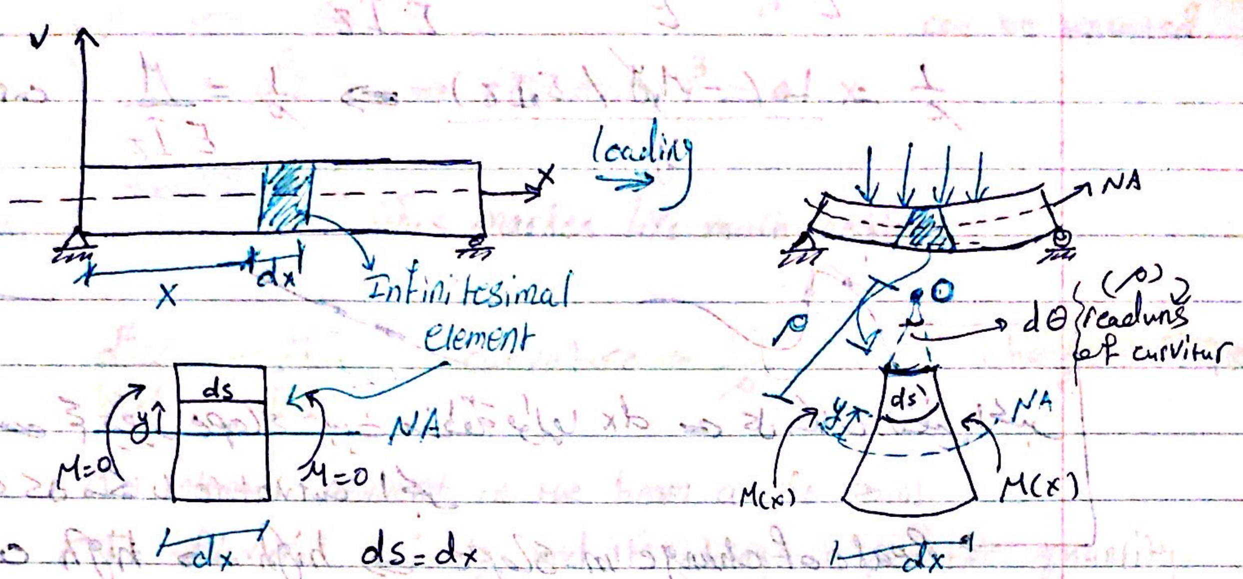
Elastic Beam theory and double integration method :-

Consider the Following beam, before and after loading :-



Before deformation . After deformation

NA :- Line between the comp. and Ten. in beam which dosent deformed after loading . لا يتغير طوله (م) منتصف البيم

⇒ No elongation nor shortening. $\sigma = 0$

Above the NA there ~~are~~ is shortening.] in Positive

belowe the NA there is elongation.] momente.

⇒ negative moment ⇒ ~~negative~~ opposite.

$dx = \rho \, d\theta$ "The infinisimal element is subjected to normal strain (ε)"

$ds' = (\rho - y) \, d\theta$.

$$\overset{\text{strain}}{\varepsilon_x(y)} = \frac{ds' - ds}{ds} = \frac{(\rho - y)\cancel{d\theta} - \rho \, d\theta}{\rho \, \cancel{d\theta}} = \frac{-y}{\rho}$$

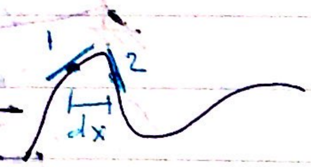
$$\frac{1}{\rho} = -\frac{\varepsilon_x(y)}{y}$$

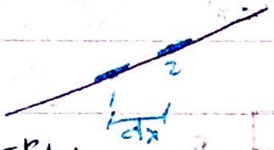
⇒ For linear-elastic behavior $\sigma_x = E \, \varepsilon_x$ "Hooke's law"

and homogenous beam $\sigma_x = -\frac{My}{I_z}$ "Flexure formula"

$$\mathcal{E}_x = \frac{\sigma_x}{E} = \frac{\left(\frac{-My}{I_z}\right)}{E} = \frac{-My}{EI_z}$$

$$\frac{1}{\rho} = \frac{-(-M \times \cancel{y} / E \times I_z)}{\cancel{y}} \Rightarrow \boxed{\frac{1}{\rho} = \frac{M}{EI_z}}$$ curvature.





حسب حجم تغير slope بين منطقة طولها dx ← كل مكان التغير اكبر

← كل مكان curvature اكبر

Reate of change in slope ⇒ high ⟶ high curvature

Reate of change in slope = 0 ⇒ No curvature.

$\frac{1}{\rho}$:- Curvature : is the amount by which a curve deviates from being a straight line.

E : Moduluse of Elasticity

I : Moment of inertiae of the beam about (NA)

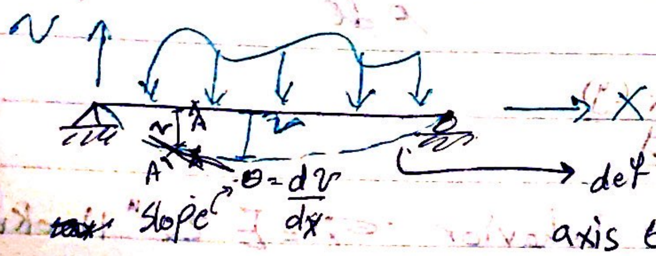
$$\frac{1}{\rho} = \frac{d^2v/dx^2}{\sqrt[2]{\left(1+\left(\frac{dv}{dx}\right)^2\right)^3}} = \frac{M}{EI}$$

اذا كانت لدينا علاقة رياضية تبع ال X

⟸ non-linear differential equation

v = vertical deflection of the beam ⇒ المطلوب

$\theta = \frac{dv}{dx}$ = slope of the tangent line to the deformed beam (elastic curve)



Elastic curve :- deflection curve of the longitudinal axis that passes through centroid of each cross-sectional area of the beam

For linear-elastic behavior → deformation is very small

$\theta = \frac{dv}{dx} << 1$ $\theta^2 = \left(\frac{dv}{dx}\right)^2 \approx 0$ can be neglected.

$\frac{1}{\rho} = \frac{d^2v}{dx^2}$ $\Rightarrow \sqrt{(1+(dv/dx)^2)^3} \approx 1$ x

linear 2nd order. This markes life much easier.

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \text{curvature} = \frac{1}{\rho} = \text{rate of change in slope}$$

M = internal moment in the beam at the point.

EI = flexural rigidity and it's always a positive quantity.

صعوبة ثني ال member

— Method :-

a) Calculate support reactions, if necessary.

b) Write the expression of the internal bending moment M(x)

c) Write the boundary and continuity conditions

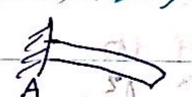
d) $v = \int\int \frac{M}{EI}\, dx$, $\theta = \frac{dv}{dx} = \int \frac{M}{EI}\, dx$

E) determine the constants of integration using the conditions specified in partic.

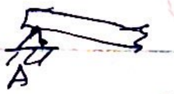
— Boundary conditions

لا يوجد به اي انكسار عند A (strait)

Roller $v_A = 0$, $M_A = 0$



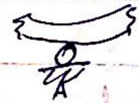
Fixed $v_A = 0$, $\theta_A = 0$



Pin $v_A = 0$, $M_A = 0$

Free end

$M_A = 0$, $\text{shear}_A = 0$



Roller $v_A = 0$

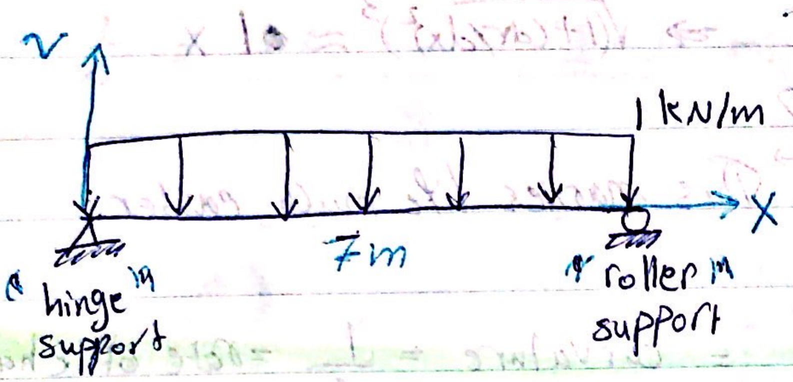
Internal pin or hinge

$M_A = 0$

def. يوجد


Pin $v_A = 0$

Example 8- Use the double integration method to determine the location and magnitude of maximum deflection for beam AB.



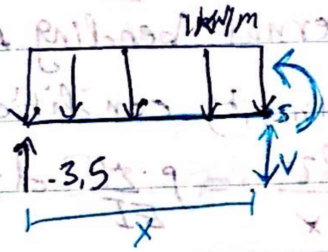
EI = Constant

Solution

ⓐ Determine the reactions at A and B :-

$A_y = 3.5kN$, $B_y = 3.5kN$

ⓑ write the internal moment as a function of x :-



$$+\circlearrowleft \Sigma M_s = 0 = 0 = M + 1(x)\frac{(x)}{2} - 3.5x = 0$$

$$\boxed{M = \frac{-x^2}{2} + 3.5x}$$

$$0 \leq x < 7m$$

note :- M is represnted by single function

deflected then ⟹ θ, v each will have single function

المومنت ممثلة بمعادلة واحدة بسيطة فيكون θ , v لهم معادلة واحدة بسيطة

⟹ No continuity conditions لأنه المومنت معادلة واحدة

ⓒ $\frac{d^2v}{dx^2} = \frac{M}{EI} \Rightarrow EI\left(\frac{d^2v}{dx^2}\right) = M$

$$\Rightarrow EI\left(\frac{d^2v}{dx^2}\right) = -\frac{x^2}{2} + 3.5x$$

$$\Rightarrow EI \int \frac{d^2v}{dx^2} = \int \left(\frac{-x^2}{2} + 3.5x\right)dx \Rightarrow$$

$$EI \frac{dv}{dx} = \frac{-x^3}{6} + \frac{3.5}{2}x^2 + C_1$$

$$EI \boxed{\int \frac{dv}{dx}} = \int\left(\frac{-x^3}{6} + \frac{3.5}{2}x^2 + C_1\right)dx$$

slope

$$EI\, \boxed{v} = \frac{-x^4}{24} + \frac{3.5x^3}{6} + \boxed{C_1}x + \boxed{C_2}$$

deflection

ⓓ boundary conditions :-

— Hinge support at A :- deflection ($v_A=0$) at $x=0$

— Roller support at B :- deflection ($v_B=0$) at $x=7m$

$$\text{At } x=0 \Rightarrow EI(0) = 0+0+0+C_2 \Rightarrow \boxed{C_2=0}$$

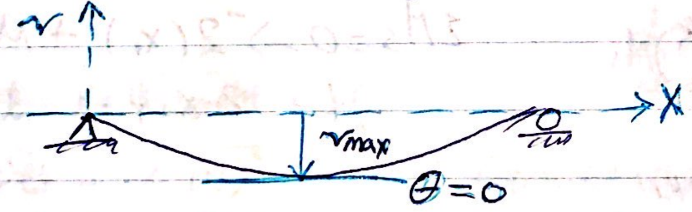
$$\text{At } x=7 \quad EI(0) = \frac{-(7)^4}{24} + \frac{3.5(7)^3}{6} + 7C_1$$

$$\Rightarrow \boxed{C_1 = \frac{-343}{24}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{EI}\left[\frac{-x^3}{6} + \frac{3.5}{2}x^2 - \frac{343}{24}\right]$$ "slope" radian

$$\Rightarrow v = \frac{1}{EI}\left[\frac{-x^4}{24} + \frac{3.5}{6}x^3 - \frac{343}{24}x\right]$$ "deflection" meter

ⓔ location and magnitude of the max. deflection :-



at the horizontal tangent at slope=0

⇒ The slope is zero at the max. deflection

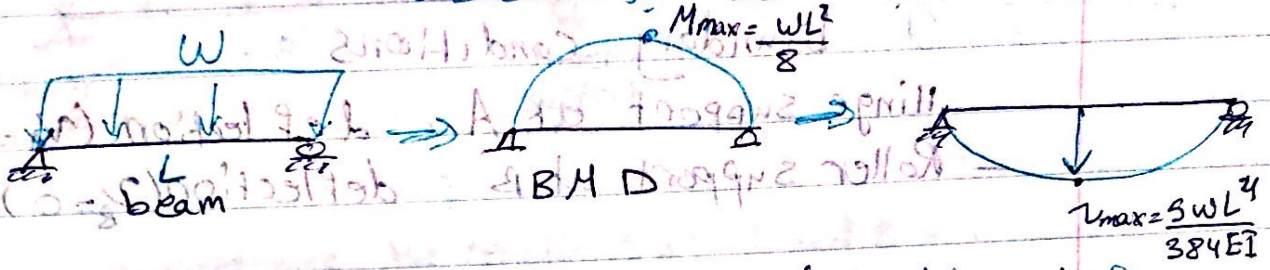
$$\frac{dv}{dx} = 0 \qquad \frac{-x^2}{6} + \frac{3.5}{2}x^2 - \frac{343}{24} = 0$$

$$\Rightarrow \boxed{x = 3.5m}$$

$$v_{max} = v\Big|_{x=3.5m} = \frac{1}{EI}\left[-\frac{3.5^4}{24} + \frac{3.5(3.5)^3}{6} - \frac{343}{24}(3.5)\right]$$

$$= \frac{-12005}{384EI} \text{ m } (\downarrow) \qquad v_{max} = \frac{5wL^4}{384EI}$$

*حالة تتكرر كثيراً للحفظ -

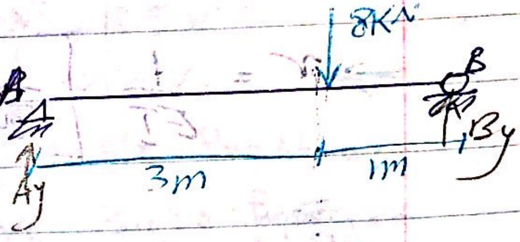


* ليس بالضرورة يكون max deflection عند max. moment.

where $M = 0 \Rightarrow$ curvature $= 0 \Rightarrow$ ~~not~~ deflection $\neq 0$

لا اذا كان مستقيم بيبقا مستقيم ولكن ممكن انه يتحرك مع ملاحظة حسب ال support

Example 8- For the beam shown, Calculate the maximum deflection?

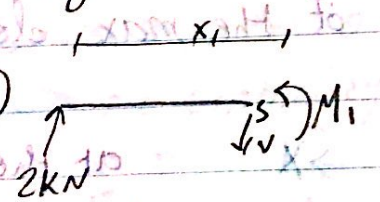


① $\Sigma M_A = 0 \Rightarrow -8(3) + B_y(4)$

$\Rightarrow B_y = 6\text{KN}$

$A_y = 2\text{KN}$

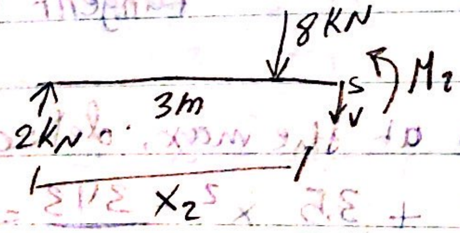
②



$\Sigma M_s = 0 \Rightarrow -2(x_1) + M_1 = 0$

$\boxed{M_1 = 2x_1}$

$0 < x < 3$



$\Sigma M_s = 0 \Rightarrow -2x_2 + 8(x_2 - 3) + M_2$

$\boxed{M_2 = 24 - 6x_2}$

$3 \leq x \leq 4$

③ A) $EI\frac{d^2v}{dx^2} = M_1$

$$EI\int \frac{d^2v}{dx^2} = \int 2x_1 \Longrightarrow \left[EI \overset{\text{slope}}{\frac{dv}{dx}} = x_1^2 + C_1 \right] \int$$

$$\boxed{EI\, v_1 = \frac{x_1^3}{3} + C_1 x_1 + C_2}$$

at $x=0$:- $0 = 0 + 0 + C_2 \Rightarrow \boxed{C_2 = 0}$

$$* \; EI\, v = \frac{x_1^3}{3} + C_1 x_1$$

$$\frac{dv}{dx} = x_1^2 + C_1$$

b) $EI \frac{d^2v}{dx^2} = M_2 \Rightarrow EI \int \frac{d^2v}{dx^2} = \int 24 - 6x_2 \, dx$

$$\Rightarrow \left[EI \left(\frac{dv}{dx} \right) = 24x_2 - 3x_2^2 + C_3 \right] \int$$

$$\Rightarrow \boxed{EI\, v_2 = 12x_2^2 - x_2^3 + C_3 x_2 + C_4}$$

* Boundary condition :- at A and B $\quad v_1\Big|_{x_1=0} = 0 \;,\; v_2\Big|_{x_2=4m} = 0$

* continuity condition :- at c $\quad v_1\Big|_{x_1=3m} = v_2\Big|_{x_2=3m}$

$$\frac{dv_1}{dx_1}\Big|_{x_1=3m} = \frac{dv_2}{dx_2}\Big|_{x_2=3m}$$

at $x_2 = 4 \Rightarrow v_2 = 0$

$$0 = 12(4)^2 - (4)^3 + C_3(4) + C_4$$

$$-128 = 4C_3 + C_4$$

at $x_1 = x_2 = 3 \Rightarrow \frac{(3)^{\cancel{3}\,2}}{\cancel{3}} + 3C_1 = 12(3)^2 - (3)^3 + 3C_3 + C_4$

$$9 + 3C_1 = 108 - 27 + 3C_3 + C_4$$

$$\left.\begin{aligned} 3C_1 - 3C_3 - \cancel{C_4} &= 72 \\ 4C_3 + \cancel{C_4} &= -128 \end{aligned}\right] +$$

$$\Rightarrow \boxed{C_3 + 3C_1 = -56}$$

+

$$\left.\frac{dv_1}{dx_1}\right|_{x_1=3} = \left.\frac{dv_2}{dx_2}\right|_{x_2=3}$$

$$24(3) - 3(3)^2 + C_3 = (3)^2 + C_1$$

$$45 + C_3 = 9 + C_1$$

$$\Rightarrow \boxed{C_1 - C_3 = 36}$$

$$\frac{4C_1 = -20}{4} \Rightarrow C_1 = -5,\ C_3 = -41,\ C_4 = 36$$

$$0 \leq x_1 \leq 3\,m$$

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left[x_1^2 - 5\right],\quad v_1 = \frac{1}{EI}\left[\frac{x_1^3}{3} - 5x_1\right]$$

$$3\,m \leq x_2 \leq 4\,m$$

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left[24x_2 - 3x_2^2 - 41\right]$$

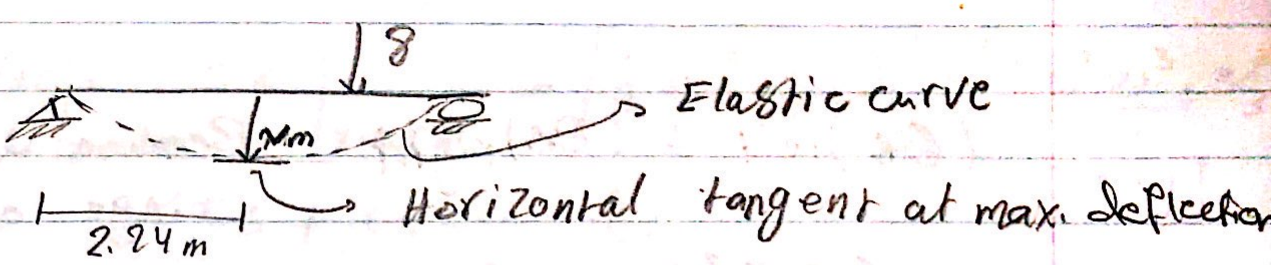
$$v_2 = \frac{1}{EI}\left[-x_2^3 + 12x_2^2 - 41x_2 - 36\right]$$

Assume max. deflection is within part AC:-

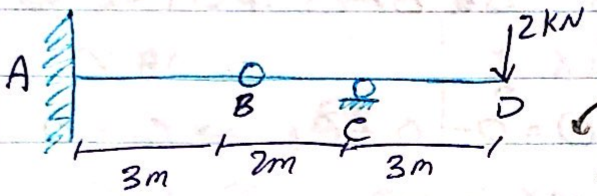
$$\frac{dv_1}{dx_1} = 0 :- \quad x_1^2 - 5 = 0 \Rightarrow x_1 = \sqrt{5} = 2.24 < 3\,m$$

$$v_{max} = \left.v_1\right|_{x_1=2.24} = \frac{1}{EI}\left[\frac{(2.236)^3}{3} - 5(2.236)\right]$$

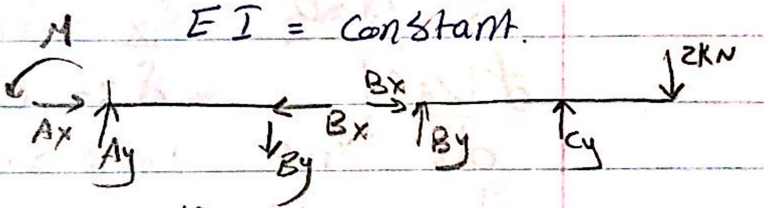
$$= \frac{-7.453\,m}{EI}\ (\downarrow)$$



Examples– Beam ABCD Shown has a fixed support at A an internal hinge at B, a roller support at C, and a free end at D. Determine the deflection and slope of points B, D?



EI = constant.



(2) $\Sigma F_x = 0 \Rightarrow B_x = 0 \Rightarrow A_x = 0$

$\Sigma M_B = 0 \Rightarrow C_y(2) - 2(5) \Rightarrow \boxed{C_y = 5KN}$

$\Sigma F_y = 0 \Rightarrow \boxed{B_y = -3KN \downarrow}$

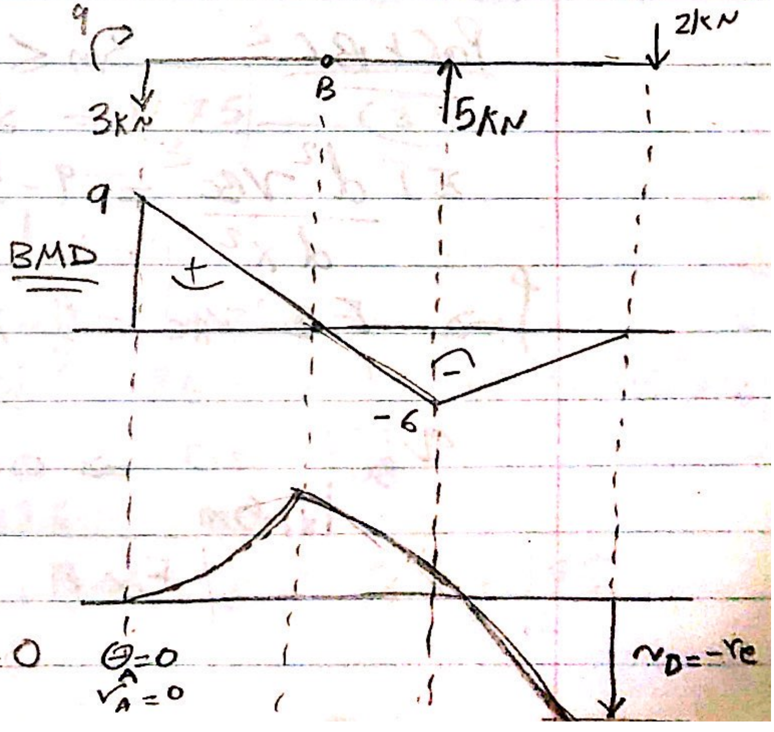
(1) $\Sigma F_y = 0 \Rightarrow 3 + A_y = 0 \Rightarrow \boxed{A_y = -3KN \downarrow}$

$\Sigma M_A = 0 \Rightarrow 3(3) + M \Rightarrow \boxed{M = -9KNm \curvearrowright}$

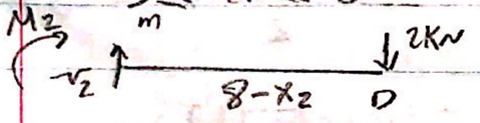
$0 < x_1 \leq 5m$

$\Sigma M_s = 0 : M_1 + 3x_1 - 9 = 0$

$M_1 = 9 - 3x_1$



$5 \leq x < 8m$



$\Sigma M_s = 0 \Rightarrow -M_2 - 2(8 - x_2) = 0$

$M_2 = 2x_2 - 16$

$(\theta_B)_{left} \neq (\theta_B)_{right}$ Because the internal hinge at $\underline{\underline{B}}$

$0 \leqslant x_1 \leqslant 3m$

$$EI \frac{d^2 v_{AB}}{dx_1^2} = 9-3x_1 \xrightarrow{\int} EI \frac{dv_{AB}}{dx_1} = 9x_1 - \frac{3}{2}x_1^2 + C_1$$

$$\xrightarrow{\int} EI\, v_{AB} = \frac{9}{2}x_1^2 - \frac{1}{2}x_1^3 + C_1x_1 + C_2$$

$$v_{AB}\Big|_{x_1=0} = 0 \quad 8\text{:-}\ 0 = 0 - 0 + 0 + C_2 \Rightarrow C_2 = 0$$

$$\frac{dv_{AB}}{dx}\Big|_{x_1=0} = 0 \quad 8\text{:-}\ 0 = 0 - 0 + C_1 \rightarrow C_1 = 0$$

$$\frac{dv_{AB}}{dx_1} = \frac{1}{EI}\left[9x_1 - \frac{3}{2}x_1^2\right]$$

$0 \leq x_1 \leq \boxed{3m}$ not to 5m → there is a discontinuity at B because of the Internal Hinge

$$v_{AB} = \frac{1}{EI}\left[\frac{9}{2}x_1^2 - \frac{1}{2}x_1^3\right]$$

Part BC $\quad 3m < x_1 \leqslant 5m$

$$EI \frac{d^2 v_{BC}}{dx_1^2} = 9-3x_1 \xrightarrow{\int} EI \frac{dv_{AC}}{dx_1} = 9x_1 - \frac{3}{2}x_1^2 + C_3$$

$$\xrightarrow{\int} EI\, v_{BC} = \frac{9}{2}x_1^2 - \frac{x_1^2}{2} + C_3x_1 + C_4$$

$$v_{BC}\Big|_{x_1=5m} = 0 \rightarrow 0 = \frac{9}{2}(5)^2 - \frac{(5)^3}{2} + 5C_3 + C_4$$

$$\boxed{5C_3 + C_4 = -50} \text{ — ①}$$

$$v_{AC}\Big|_{x_1=3m} = v_{BC}\Big|_{x_1=3m} \rightarrow \left[\frac{9}{2}(3)^2 - \frac{1}{2}(3^3)\right] = \left[\frac{9}{2}(3)^2 - \frac{(3)^3}{2} + 3C_3 + C_4\right]$$

$$\boxed{3C_3 + C_4 = 0} — ②$$

From equations ① and ② :- $C_3 = -25$

$C_4 = 75$

$$\frac{dv_{BC}}{dx_1} = \frac{1}{EI}\left[9x_1 - \frac{3}{2}x_1^2 - 25\right] \quad 3m \leq x \leq 5m$$

$$v_{BC} = \frac{1}{EI}\left[\frac{9}{2}x_1^2 - \frac{x_1^3}{2} - 25x_1 + 75\right]$$

Part CD $\quad 5m \leq x_2 \leq 8m$

$$EI\frac{d^2v_{CD}}{dx_2^2} = 2x_2 - 16 \longrightarrow \int$$

$$EI\frac{dv_{CD}}{dx_2} = x_2^2 - 16x_2 + C_5$$

$$\int \longrightarrow EI\,v_{CD} = -8x_2^2 + \frac{x_2^3}{3} + C_5x_2 + C_6$$

$$v_{CD}\Big|_{x_2=5m} = 0 :- \; 0 = -8(5)^2 + \frac{(5)^3}{3} + 5C_5 + C_6$$

$$\boxed{5C_5 + C_6 = \frac{475}{3}} — ③$$

$$\frac{dv_{BC}}{dx_2}\Big|_{x_1=5m} = \frac{dv_{CD}}{dx_2}\Big|_{x_2=5m}$$

$$\frac{1}{EI}\left[9(5) - \frac{3(5)^2}{2} - 25\right] = \frac{1}{EI}\left[-16(5) + (5)^2 + C_5\right]$$

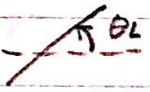
$$\rightarrow \boxed{C_5 = 37.5}$$

use equation ③ to find C_6 : $C_6 = \frac{-175}{6}$

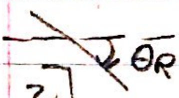
$$\frac{dv_{CD}}{dx_2} = \frac{1}{EI}\left[x_2^2 - 16x_2 + 37.5\right] \qquad 5m \leq x \leq 8m$$

$$v_{CD} = \frac{1}{EI}\left[-8x_2^2 + \frac{x_2^3}{3} + 37.5x_2 - \frac{175}{6}\right]$$

$$(\theta_B)_{left} = \left.\frac{dv_{AB}}{dx_1}\right|_{x_1=3m} = \left[\frac{9(3) - \frac{3}{2}(3)^2}{EI}\right] = \frac{27}{2EI}\text{ rad} \circlearrowleft$$



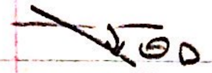
$$(\theta_B)_{Right} = \left.\frac{dv_{BC}}{dx_1}\right|_{x_1=3} = \left[\frac{9(3) - \frac{3}{2}(3)^2 - 25}{EI}\right] = \frac{-23}{2EI}\text{ rad} \circlearrowright$$



$$v_B = v_{AB}\Big|_{x_1=3m} = v_{BC}\Big|_{x_1=3m} = \left[\frac{\frac{9}{2}(3)^2 - \frac{1}{2}(3)^2}{EI}\right]$$

$$= \frac{27}{EI}\ m\ (\uparrow)$$

$$\theta_D = \left.\frac{dv_{CD}}{dx_2}\right|_{x_2=8m} = \left[\frac{-16(8) + (8)^2 + 37.5}{EI}\right] = \frac{53}{2EI}\text{ rad}$$



$$v_D = v_{CD}\Big|_{x_2=8m} = \left[\frac{-8(8)^2 + \frac{(8)^3}{3} + 37.5(8) - \frac{175}{6}}{EI}\right]$$

$$= \frac{-141}{2EI}\ m\ (\downarrow)$$

The max. deflection is at point D wich the free end

- Moment - Area Theorems :-

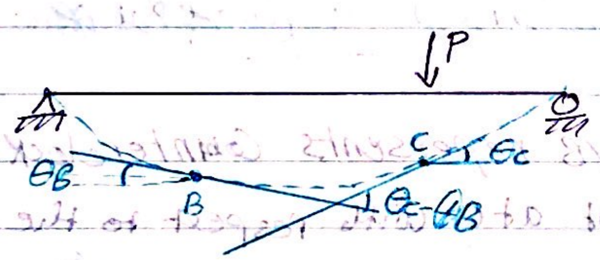
First moment - area theorem

The change in the slope of a beam between two points is equal to the area under the curvature diagram between these two points.

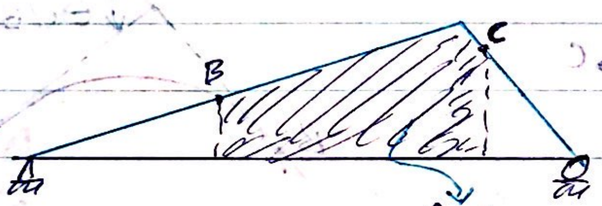
$$\frac{d^2v}{dx^2} = \frac{M}{EI} \longrightarrow \frac{d\theta}{dx} = \frac{M}{EI} \longrightarrow \int_{\theta_B}^{\theta_C} d\theta = \int_{x_B}^{x_C} \frac{M}{EI}\, dx$$

$$\theta_C - \theta_B = \int_{x_B}^{x_C} \frac{M}{EI}\, dx \longrightarrow \boxed{\theta_{C/B} = \int_{x_B}^{x_C} \frac{M}{EI}\, dx}$$

Area under curvature diagram between point B and C.



deflection Shape

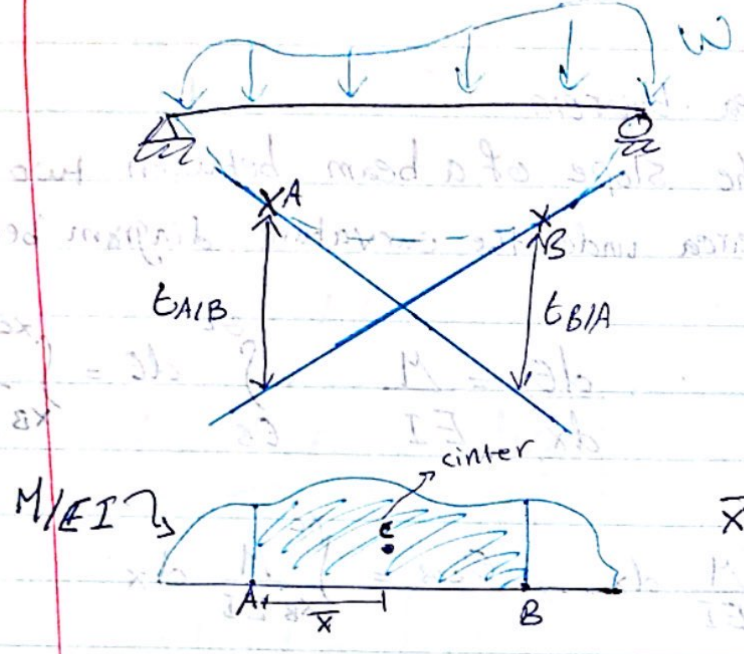


curvature diagram (M/EI)

Area = $\theta_c - \theta_B = \theta_{c/B}$

Second moment - area theorem

The vertical distance between the tangent at a point (A) on the elastic curve and the tangent extended from another point (B) equals the moment of the area under the curvature (M/EI) diagram between these two points (A and B). This moment is calculated about the point A where the vertical distance ($\delta_{A/B}$) is to be determined.



Elastic curve

$$t_{AB} = \bar{x} \int_{x_A}^{x_B} \frac{M}{EI}\, dx$$

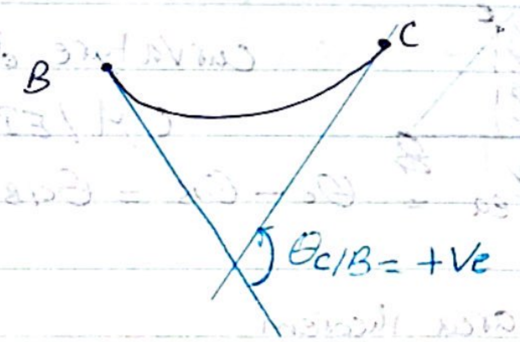
Area under M/EI diagram

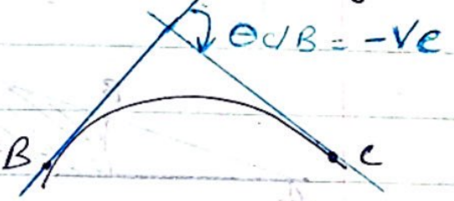
$\bar{x}$:- is the distance from A to the centroid of the area under M/IE between A and B

$t_{A/B} \neq t_{B/A}$

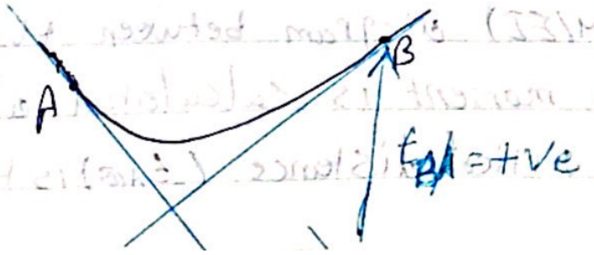
* ارجعيل السلايدات *

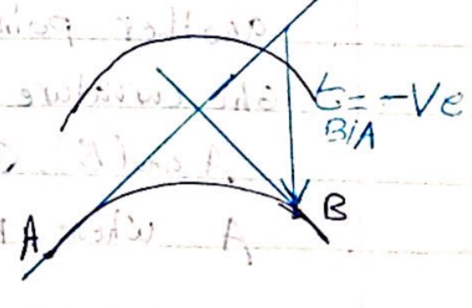
* A positive $\theta_{C/B}$ repersents Counterclockwise rotation of tangent at C with respect to the tangent at A



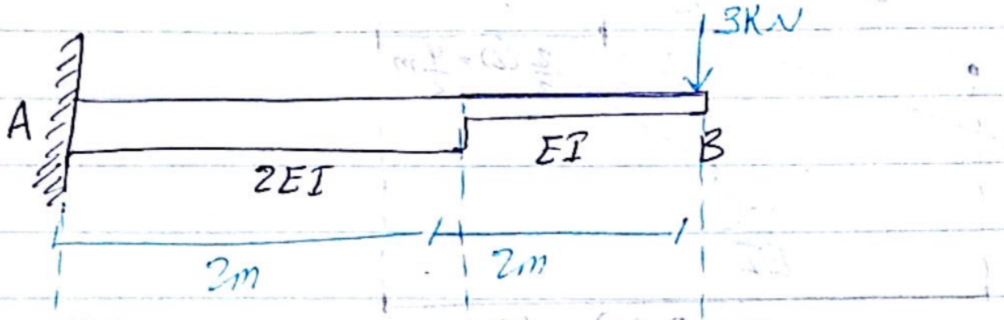


* A positive $t_{B/A}$ indicates that point B on the elastic curve lies above the extended tangent from point A.

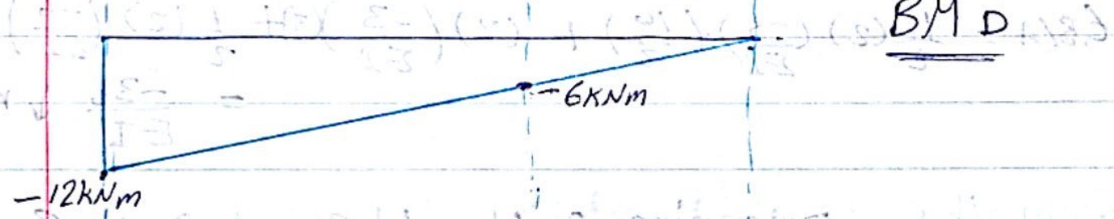


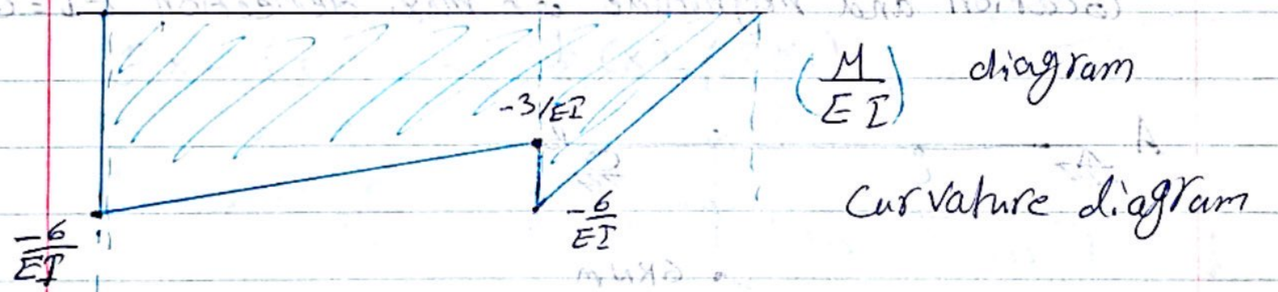


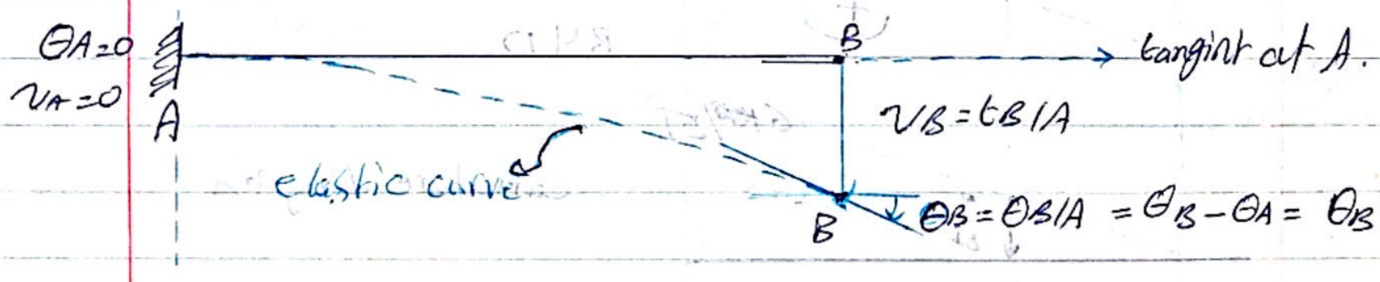
Example :- Beam AB is non prismatic and subjected to the load shown. Determine slope and deflection at point B on the elastic curve ?



Solution :-



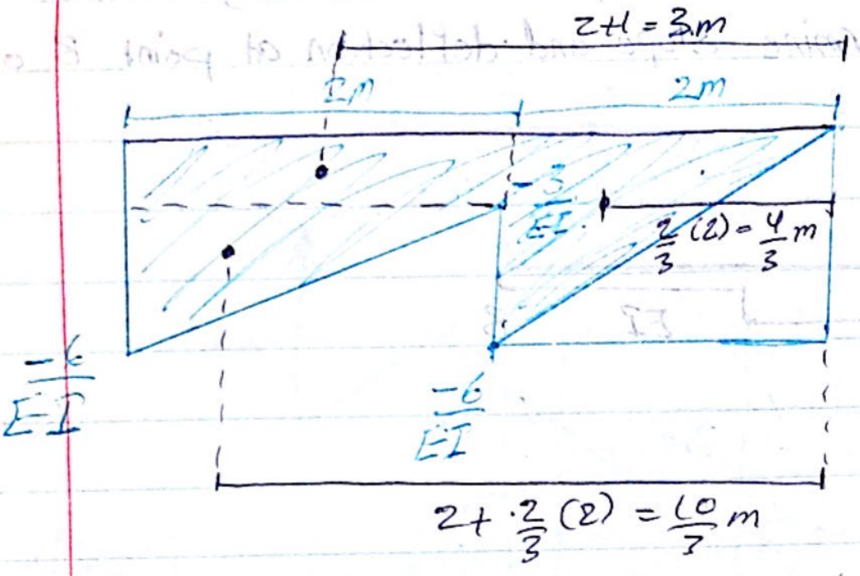




$$\theta_{B/A} = \text{Area under } \frac{M}{EI} \text{ diagram from "A" to "B"}$$

$$\theta_B = \frac{1}{2}\left(\frac{-6}{EI} + \frac{-3}{EI}\right)(2) + \frac{1}{2}\left(\frac{-6}{EI}\right)(2) = \frac{-15}{EI} \text{ rad}$$

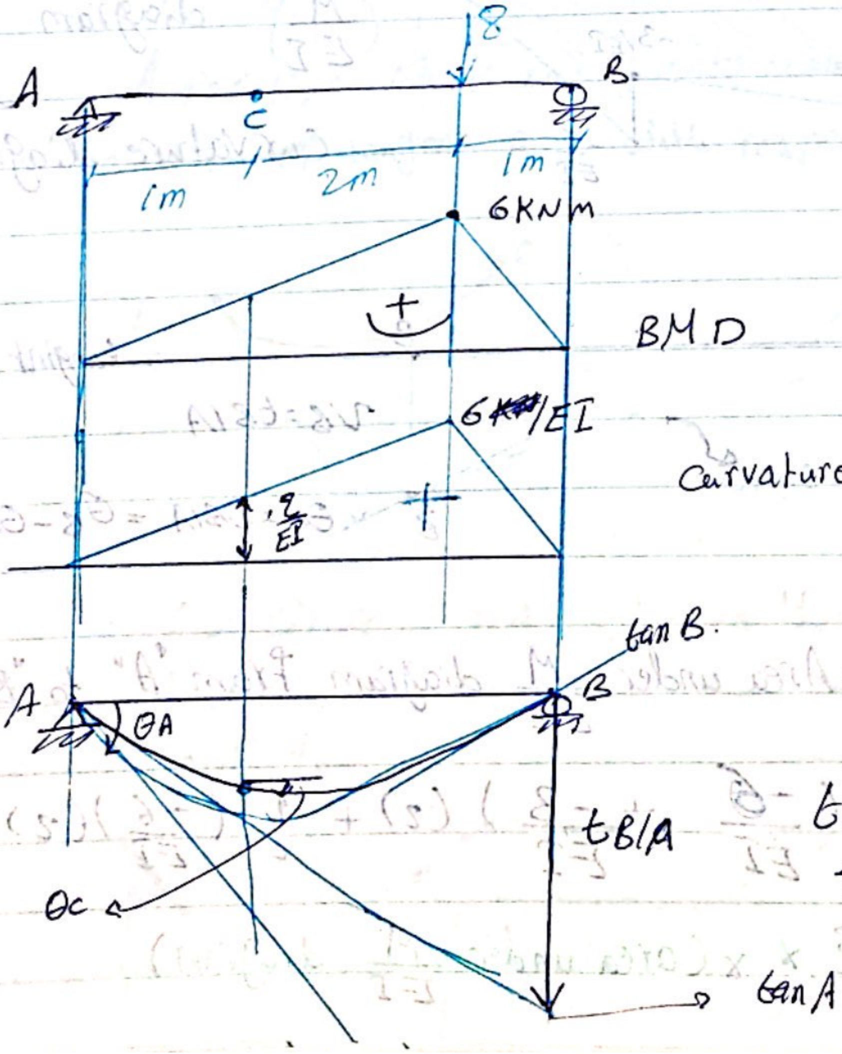
$$t_{B/A} = \Sigma\, \bar{x} \times (\text{area under } \frac{M}{EI} \text{ diagram}).$$



$$t_{B/A} = \frac{1}{2}(2)\left(\frac{-3}{EI}\right)\cdot\left(\frac{10}{3}\right) + (2)\left(\frac{-3}{EI}\right)(3) + \frac{1}{2}(2)\left(\frac{-6}{EI}\right)\left(\frac{4}{3}\right)$$

$$= \frac{-36}{EI} \downarrow m$$

Example 8 - Determine (a) the slope at point C, (b) location and magnitude of max. deflection EI = constant



$$\theta_A = \frac{-t_{B/A}}{L_{AB}} \circlearrowright$$

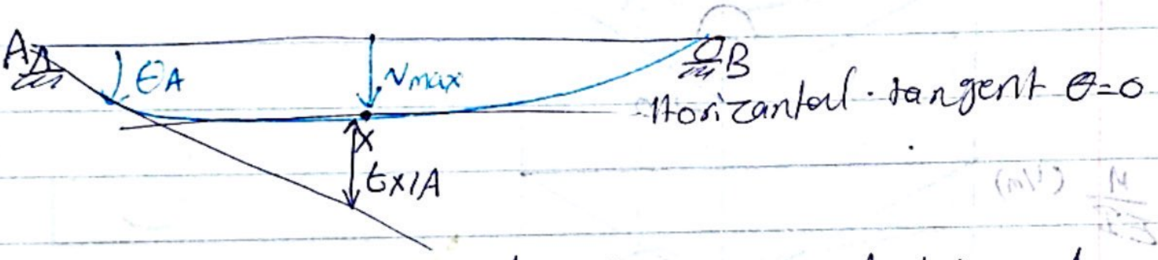
$$t_{B/A} = \frac{1}{2}(6/EI)(1)\left(\frac{2}{3}\right) + \frac{1}{2}\left(\frac{6}{EI}\right)(3)(2) = \frac{20}{EI}$$

$$\theta_A = -\frac{20/EI}{4} = \frac{-5}{EI} \text{ rad} \circlearrowright$$

$\theta_{C/A}$ = Area between A and C under $\frac{M}{EI}$ diagram

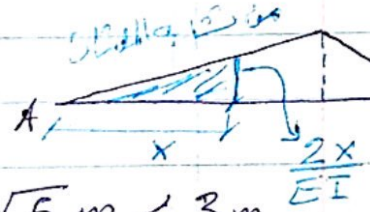
$$\theta_C - \left(\frac{-5}{EI}\right) = +\frac{1}{2}(1)\left(\frac{2}{EI}\right) \longrightarrow \theta_C = \frac{-4}{EI} \text{ rad.}$$

max. deflaction ⟶ in slope = 0

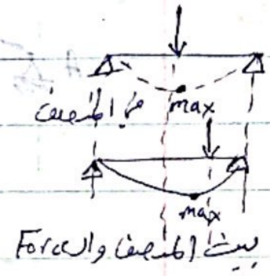


$\theta_X - \theta_A$ = Area under $\frac{M}{EI}$ diagram between A and x

$$\theta_X - \left(\frac{-5}{EI}\right) = \frac{1}{2}(x)\left(\frac{2x}{EI}\right)$$



$$\theta_X = 0 \therefore -\frac{5}{EI} = \frac{x^2}{EI} \longrightarrow X = \sqrt{5}\, m < 3m$$



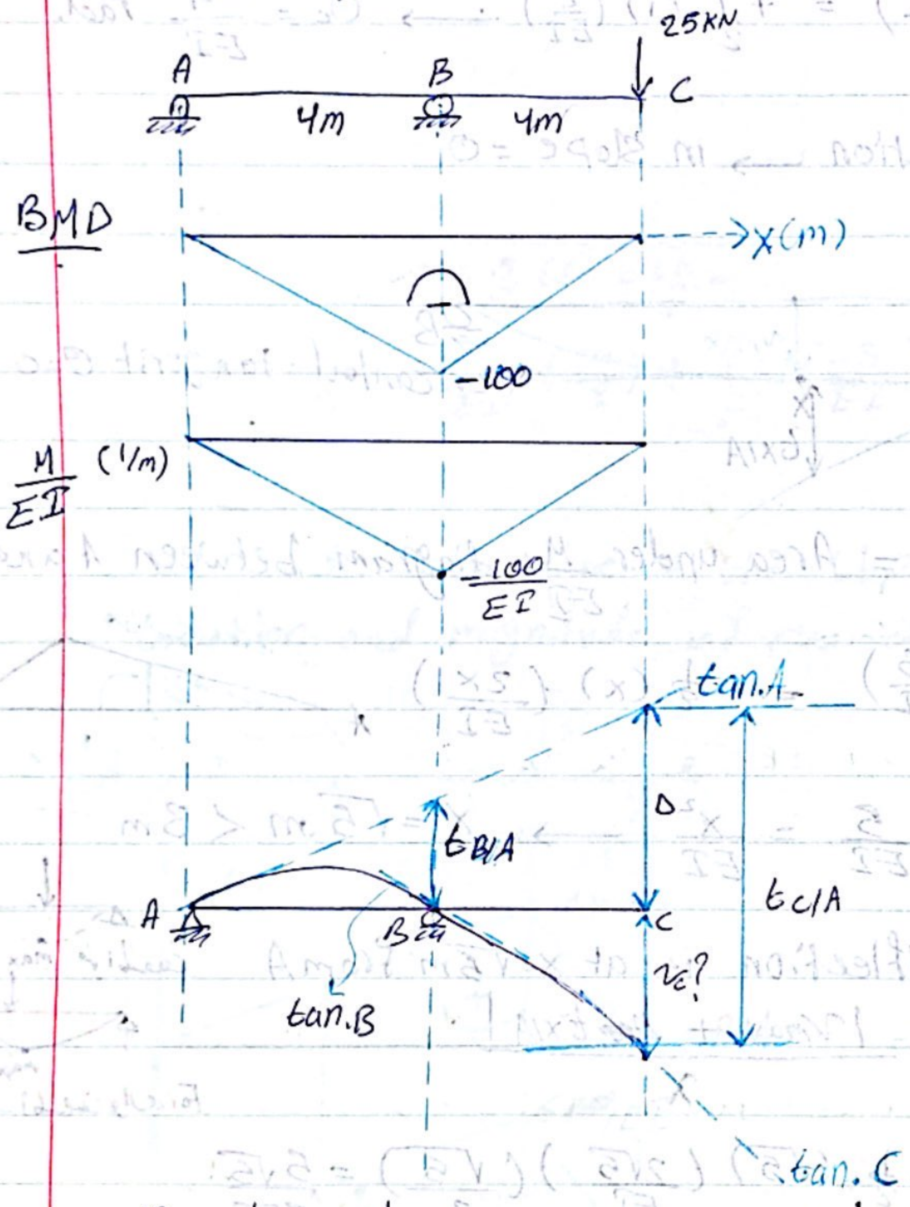
The max. diflection is at $x = \sqrt{5}m$ from A

$$|\theta_A| = \frac{|V_{max} + \cancel{Bu}\, t_{X/A}|}{X}$$

$$t_{X/A} = \frac{1}{2}(\sqrt{5})\left(\frac{2\sqrt{5}}{EI}\right)\left(\frac{\sqrt{5}}{3}\right) = \frac{5\sqrt{5}}{3EI}$$

$$\theta_A = \frac{5}{EI} = \frac{\overset{Area}{V_{max}} + \frac{5\sqrt{5}}{3EI}}{\sqrt{5}} \longrightarrow V_{max} = \frac{7.453}{EI}\ m\,(\downarrow)$$

Example: Determine the displacment at point C for the steel over hanging beam shown. Use $E_{st} = 200\,GPa$, $I = 50 \times 10^6\,mm^4$.



$$v_C = \frac{1600}{EI} - \frac{1600}{3EI}$$

$$= \frac{3200\ kN\,m^3}{EI\,(3)}$$

$$= 0.1067\,m\,(\downarrow)$$

$$\theta_A = \frac{|t_{B/A}|}{L_{AB}} = \frac{\Delta}{L_{AC}} \longrightarrow \Delta = \left(\frac{L_{AC}}{L_{AB}}\right)\cdot|t_{B/A}|$$

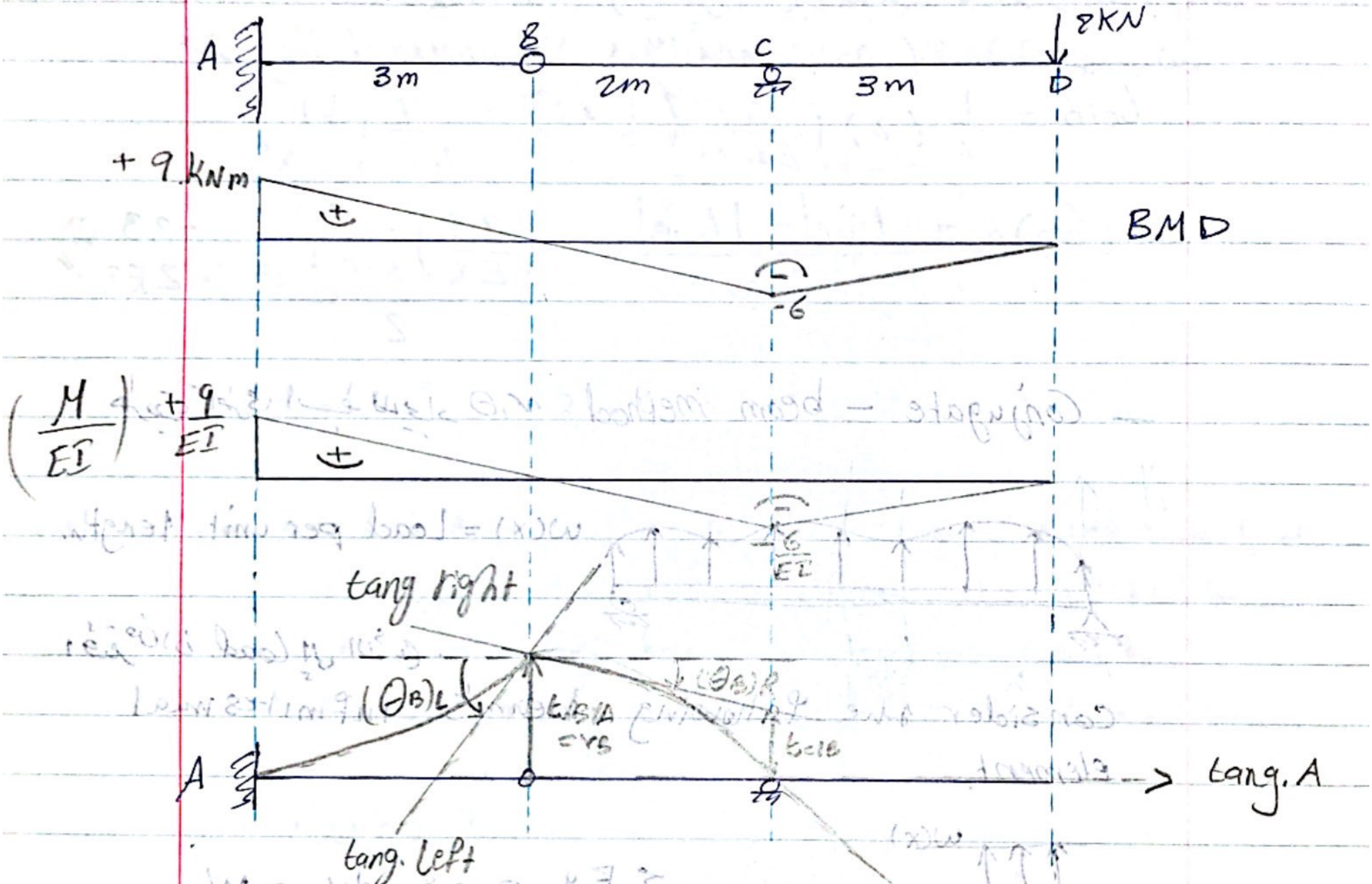
$$v_C = |t_{C/A}| - \Delta$$

$$t_{B/A} = \frac{1}{2}(4)\left(\frac{-100}{EI}\right)\left(\frac{1}{3} \times 4\right) = -\frac{800}{3EI}$$

$$\Delta = \left(\frac{8}{4}\right)\cdot\left(\frac{800}{3EI}\right) = \frac{1600}{3EI}$$

$$t_{C/A} = \frac{1}{2}\cdot(8)\left(\frac{-100}{EI}\right)(4) = -\frac{1600}{EI}$$

Example 8- Beam ABCD shown has a fixed support at A, an internal hinge at B, a roller support at C, and a free end at D. Determine the slope and deflection of point B. EI const.



$(\theta_B)_L - \theta_A$ = Area under $\frac{M}{EI}$ between A and B

$$(\theta_B)_L - 0 = \frac{1}{2}(3)\left(\frac{9}{EI}\right) \longrightarrow (\theta_B)_L = \frac{27}{2EI} \text{ rad} \circlearrowleft$$

$$v_B = t_{B/A} = \frac{1}{2}(3)\left(\frac{9}{EI}\right)\left(\frac{2}{3} \times 3\right) = \frac{27}{EI} \text{ m } (\uparrow)$$

For $(\theta_B)_R$ moment area theorms cannot be used between A and the point just right of point B, because of the discontinuity at B "internal hinge"

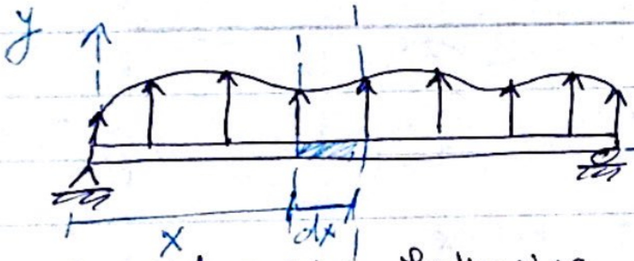
$$(\theta_B)_R - \theta_A \neq \int_A^B \frac{M}{EI}\, dx$$

يوجد انكسار عند B و $(\theta_B)_R$ سلوب لنقطة واقعة على جزء BCD
θ_A سلوب لنقطة AB ويعتبروا قطعتين موصولان ولكن انكسار
عند internal Hinge (no contenuity ؛)

$$t_{C/B} = \frac{1}{2}(2)\left(\frac{-6}{EI}\right)\left(\frac{1}{3}\times 2\right) = \frac{-4}{EI} (\downarrow)$$

$$(\theta_B)_R = \frac{|v_B| - |t_{C/B}|}{L_{BC}} = \frac{\left(\frac{27}{EI}\right)-\left(\frac{4}{EI}\right)}{2} = \frac{-23}{2EI} \text{ rad} \circlearrowright$$

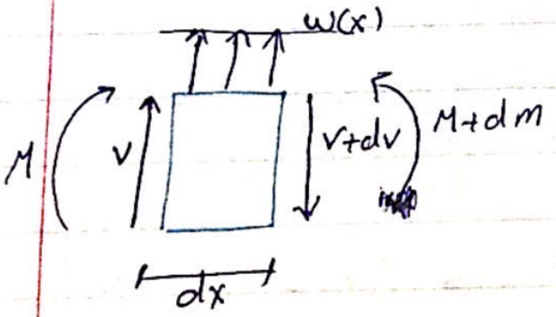
— Conjugate – beam method ع v,θ طريقة اخرى اسهل للايجاد



$w(x)$ = Load per unit length.

افترض ان load إلى الأعلى.

Consider the following beam's infinitesimal element.



$$\Sigma F_y = 0 \Rightarrow \boxed{\frac{dV}{dx} = W} \quad (I)$$

$$\Sigma M_o = 0 \Rightarrow \boxed{\frac{dM}{dx} = V}$$

From elastic beam theory we know

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad \text{and} \quad \theta = \frac{dy}{dx}$$

$$\boxed{\frac{d\theta}{dx} = \frac{M}{EI}} \quad , \quad \boxed{\frac{dy}{dx} = \theta}$$

(II)

(I) and (II)
نوجد مع بعض علاقة

The basis of this method comes from the similarity of eq. 1 and eq. 2 to equ. 3 and eq. 4. Here the shear V compares with the slope θ, the moment M compares with the displacment y, and the external load W with the M/EI diagram

"Load": W $\longleftrightarrow$ $\frac{M}{EI}$ "Curvature"

conject beam: "Shear" V $\longleftrightarrow$ θ "slope" Real beam

"Moment" M $\longleftrightarrow$ y "deflection"

M, V, w وهمي عليه Beam $\longleftarrow$ y, θ, $\frac{M}{EI}$ حقيقي عليه Beam لدينا

load of conj. beam $\rightarrow$ curvature of Real beam

Shear of conj. beam $\rightarrow$ slope of Real beam

Moment of conj. beam $\rightarrow$ deflection of Real beam

* Theorem I :-

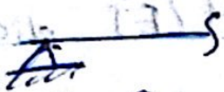
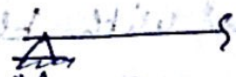
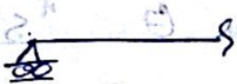
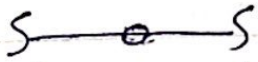
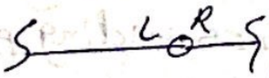
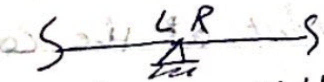
The slope at a point in the real beam is numerially equal to the shear at the corresponding point in the conjugate beam.

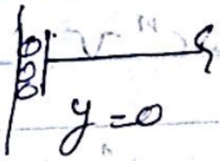
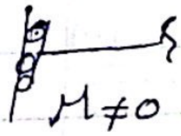
[illegible]

* Theorem II :-

The displacement of a point in the real beam is num. equal to the moment at the corresponding point in the conjugate beam.

11. Conjugate beam Supports :

	Real beam	Conjugate beam
Hinge support	 $y = 0$ $\theta \neq 0$	 $M = 0$ $V \neq 0$
Roller support	 $y = 0$ $\theta \neq 0$	 $M = 0$ $V \neq 0$
Fixed support	 $y = 0$ $\theta = 0$	 $M = 0$ Free $V = 0$ end
Free end	 $y \neq 0$ $\theta \neq 0$	 $M \neq 0$ Fixed $V \neq 0$ Support
Middle support	 $y = 0$ $\theta \neq 0$	 $M = 0$ Middle $V \neq 0$ Hinge
Middle hinge	 $y \neq 0$ $\theta \neq 0$ $\theta L \neq \theta R$ discontinute	 $M \neq 0$ Middle $V \neq 0$ Support $VL \neq VR$ discontinue.

Real beam	Conjugte beam
 $y = 0$ $\theta L \neq \theta R$	 $M = 0$ $VL \neq VR$
 $y = 0$ $\theta = 0$	 $M \neq 0$ $V = 0$

ارجع للسيدا [illegible]
امثلة

- ✱ IF a beam is determinate, its conjugate beam will be determinate.
- ✱ IF a beam is indeterminde, it's conjugate beam will be unstable.

The procedure for conjugate beam method :-

① Draw BMD for real beam, and then the $\frac{M}{EI}$ diagram.

② Draw the conjugate beam. This beam has the same length as the real beam and has the corresponding supports shown above.

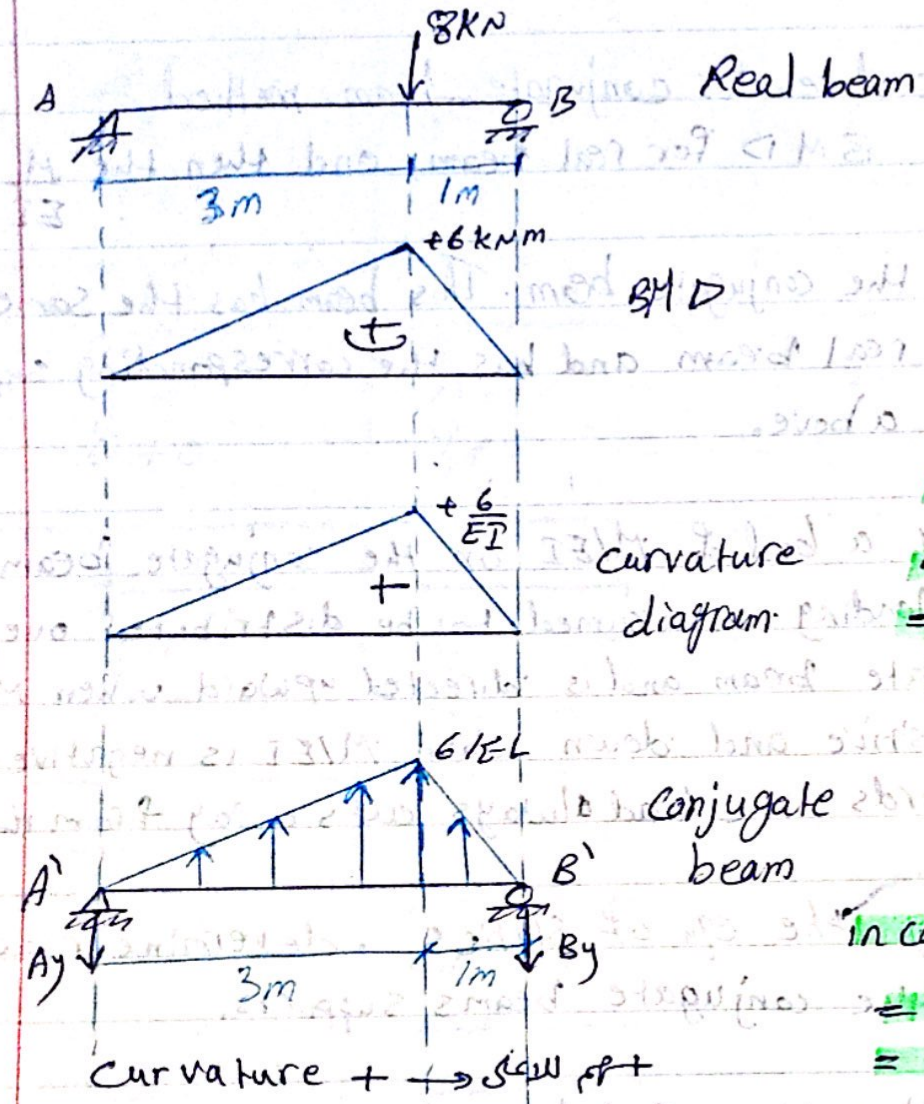
③ Apply a load of M/EI on the conjugate beam. This loading is assumed to be distributed over the conjugate beam and is directed upward when M/EI is positive and down when M/EI is negative. In other words, the load always acts away from the beam.

④ Using the equ of statics, determine reactions at the conjugate beams supports.

⑤ Section the conj. beam where the θ and y of the real beam are to be determined. At the section, show V (shear) and M (moment) equal to θ and y, for the real beam.

In particular, if these values (θ, y) are positive the slope is counterclockwise and the displacement is upward.

Example 8- For the beam shown, determine ⓐ Slope at A and B, ⓑ The max. deflection ; EI = constant



reactions Ay , By on CB = slope at point A , B in Real Beam

max deflection in conjugate beam = max moment = where shear zero

Curvature + → شكل of +

$+\circlearrowleft \Sigma M_{A'} = 0 : -4B_y + \frac{1}{2}(3)(\frac{6}{EI})(\frac{2}{3} \times 3) + (\frac{1}{2})(1)(\frac{6}{EI})(3 + \frac{1}{3} \times 1)$

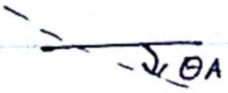
$\Rightarrow B_y = \frac{7}{EI}$ ⇒ sine convention

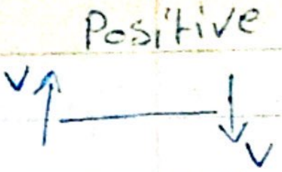
$+\uparrow \Sigma F_y = 0 : -A_y - \frac{7}{EI} + (\frac{1}{2})(4)(\frac{6}{EI}) = 0$

$\Rightarrow A_y = \frac{+5}{EI}$ ⇒ عكس sine convention

Slope at point A on real beam = Shear at A' on Conj.b.

$V_{A'} = \frac{-5}{EI}$ rad ↻

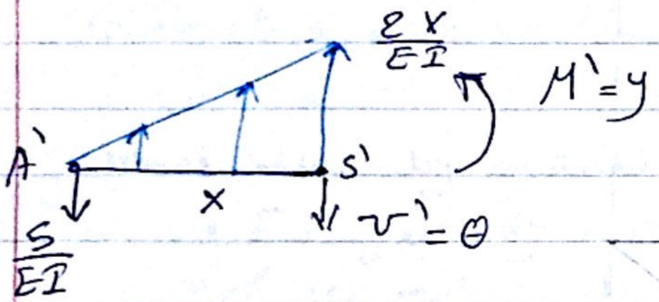




Slope at point B on real beam = Shear at B' on C.B

$\theta_B = V_{B'} = \frac{+7}{EI}$ rad ↺

* For the max. deflection :-



$+\uparrow \sum F_y = 0 : \frac{-5}{EI} + \frac{1}{2}(x)\frac{(2x)}{EI} - \theta = 0$

$$\Rightarrow \boxed{\theta = \frac{x^2}{EI} - \frac{5}{EI}}$$

$\Rightarrow \theta = 0$ (for max deflection) $\rightarrow x^2 - 5 = 0$

$x = \sqrt{5}$

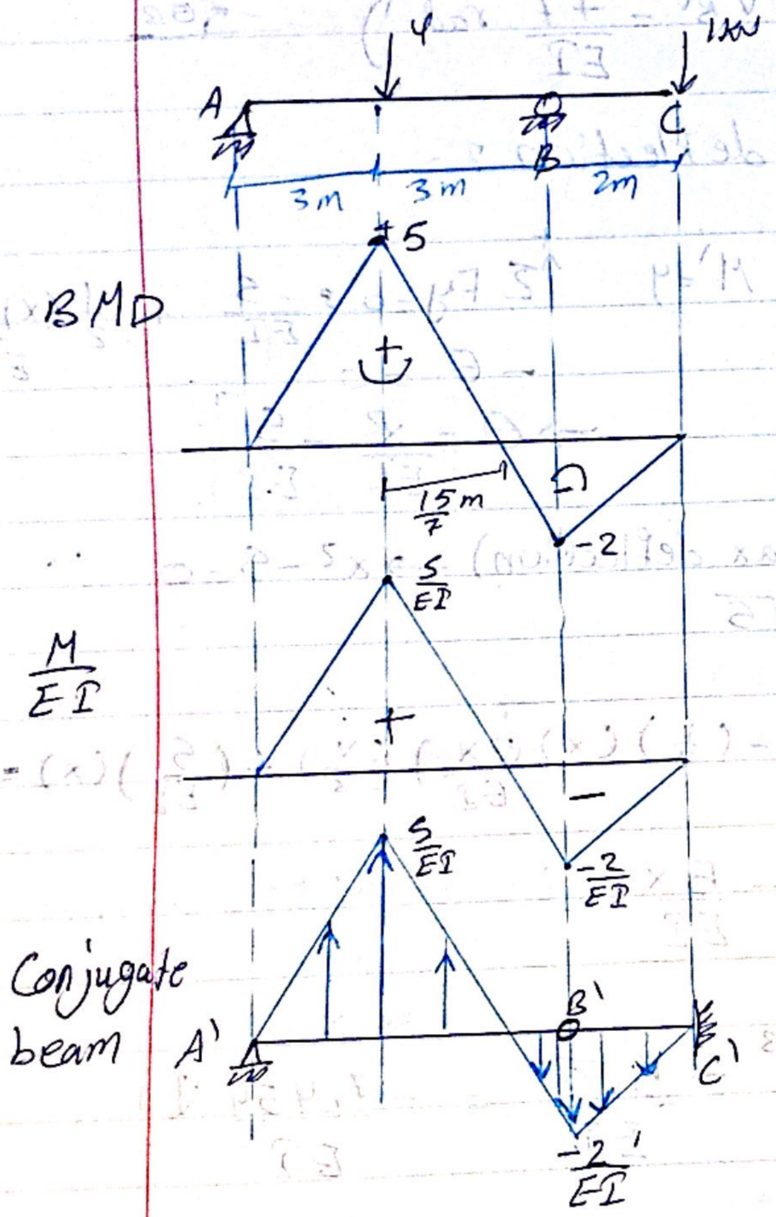
$+\circlearrowleft \sum M_{S'} = 0 :- +y - (\frac{1}{2})(x)\frac{(2x)}{EI}(\frac{x}{3}) + (\frac{5}{EI})(x) = 0$

$\Rightarrow y = \frac{x^3}{3EI} - \frac{5x}{EI}$ This is the equ. of the elastic curve for $0 \leq x \leq 3m$

at $x = \sqrt{5}\,m$

$\Rightarrow y = \frac{(\sqrt{5})^3}{3EI} - \frac{5\sqrt{5}}{EI} = -\frac{7.454}{EI}(\downarrow)$

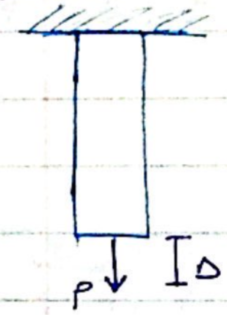
Example :- Beam ABC shown has a hinge support at A and a roller support at B. The beam is prismatic and homogeneous (EI = constant) : Determine the slope and deflection at C ?


4
1kN
A
B
C
3m
3m
2m
5
BMD
+
$\frac{15}{7}$m
-2
$\frac{5}{EI}$
$\frac{M}{EI}$
+
−
$\frac{5}{EI}$
$\frac{-2}{EI}$
Conjugate beam
A'
B'
C'
$\frac{-2}{EI}$

— Energy Methods :- slope , deflo ستخدام الطاقة لحساب

External work and strain energy :- Define the work caused by an external Force and Couple moment and show how to express this work in terms of a body's strain energy.

Work of a force :- $U_e = \int_0^{\Delta} F\,dx$ (Δ desplacment) Scalar quantity

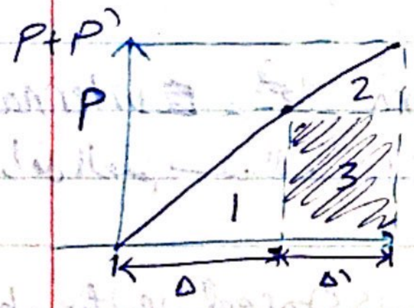


— work done by an axial force applied :-

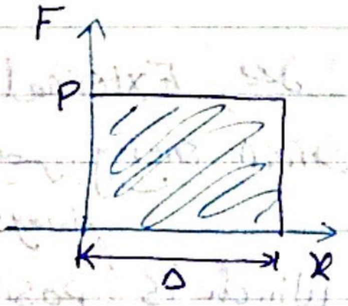
- F is gradually applied (From 0 to P).
- if the material has a linear elastic response then $F = \frac{P}{\Delta} x$ من تعويض

$$U_{ext.} = \int_0^{\Delta} F\,dx = \int_0^{\Delta} \frac{P}{\Delta} x = \frac{P}{\Delta} \left(\frac{\Delta^2}{2}\right) = \frac{1}{2} P\Delta$$

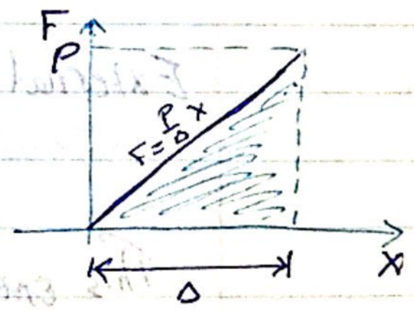
which is the shaded area under the line ($F = \frac{P}{\Delta} x$)



add P` to the body after P (gradualy)



not gradualy Force applayed. suden
Work = P X = PΔ



Load displacement diagram = Work.

1 :- work done by applyed Force P and desplacment Δ

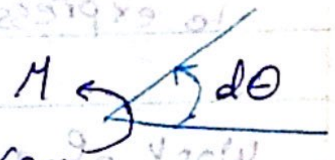
2 :- work done by applyed Force P` and desplacment Δ`

3 :- work done by Force P and desplacment Δ`

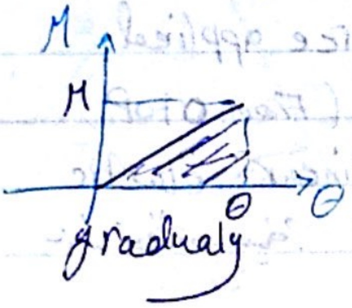
* Work of a Couple moment :- تعمل دوران $\underline{d\theta}$

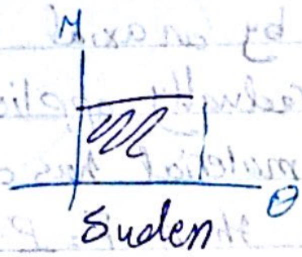
work done $\Rightarrow dU_{ex} = M\,d\theta$

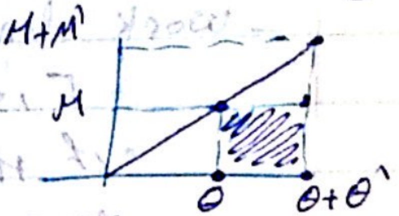
IF the total angle of rotation is θ^{rad}, the work becomes $U_e = \int_0^{\theta} M\,d\theta$



– Moment is gradually applied from 0 to M, then the work done is $U_e = \frac{1}{2} M\theta$







* Strain Energy :-

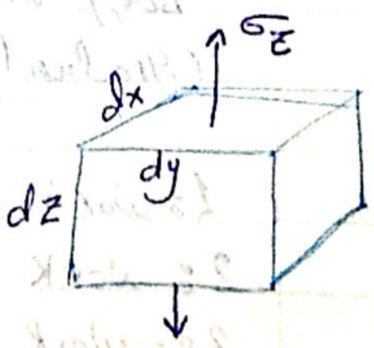
External load $\xrightarrow{\text{يعمل}}$ External Work $\xrightarrow{\text{يتحول}}$ internal work يحفظ داخل الجسم بسبب التشوهات و يسمى (Strain energy).

This energy, which is always positive, is stored in the body and is caused by the action of either normal or shear stress.

– Normal stress :-

$\sigma_z = \frac{dF_z}{dA} \Rightarrow dF_z = \sigma_z\, dA$

$\Rightarrow dF_z = \sigma_z\, dx\, \underline{dy}$



Volume element.

z التغير با $d\Delta z = \varepsilon_z\, dz$

σ_z :- internal stress

The work done by dF_z is therefore $dU_{in} = \frac{1}{2} dF_z d\Delta_z$
$= \frac{1}{2} [\sigma_x \, dx \, dy] \varepsilon_z \, dz$. ($d$Volume $= dx\,dy\,dz$)

$$\Rightarrow \quad dU_{in} = \frac{1}{2} \sigma_z \varepsilon_z \, dV$$ F_z gradualy applyed.

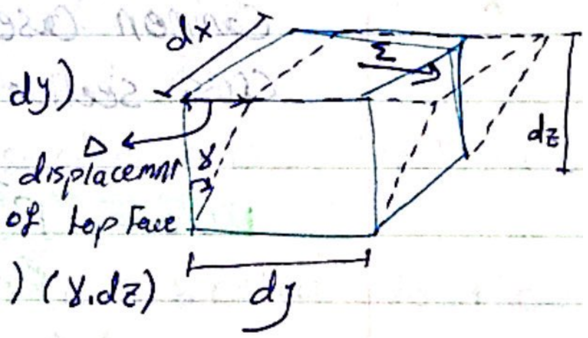
internal

In general, if the body is subjected to a uniaxial normal stress σ, the ~~ex~~ strain energy is $U_i = \int \frac{\sigma \varepsilon}{2} dV$ للجسم كامل

Also, Assuming linear-elastic behavior

$\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E} \Rightarrow U_i = \int \frac{\sigma^2}{2E} dv$

- Shear stress :-



The Force on top Face : $dF = \tau (dx\,dy)$

displacement of top Face $= \Delta$
$= \gamma \cdot dz$

$dU_{in} = \frac{1}{2} dF \Delta = \frac{1}{2} (\tau \cdot dx \cdot dy)(\gamma \cdot dz)$

$\Rightarrow (dV = dx \cdot dy)$

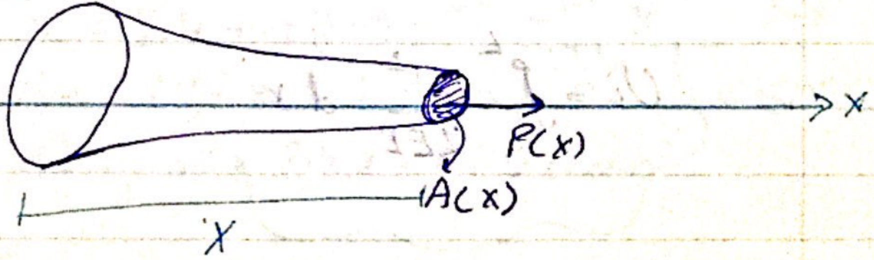
$$dU_i = \frac{1}{2} \tau \gamma \, dv$$

The strain energy stored in abody is therefore :

sub. $\left(\gamma = \frac{\tau}{G}\right) \Rightarrow U_i = \int \frac{\tau \cdot \gamma}{2} dv \Rightarrow U_i = \int \frac{\tau^2}{2G} dV$

* Elastic strain Energy for Various Types of loading :-

Axial load :- non prismatic



at distance x : $\sigma_x = \frac{P(x)}{A(x)}$ and $dV = A(x)\,dx$

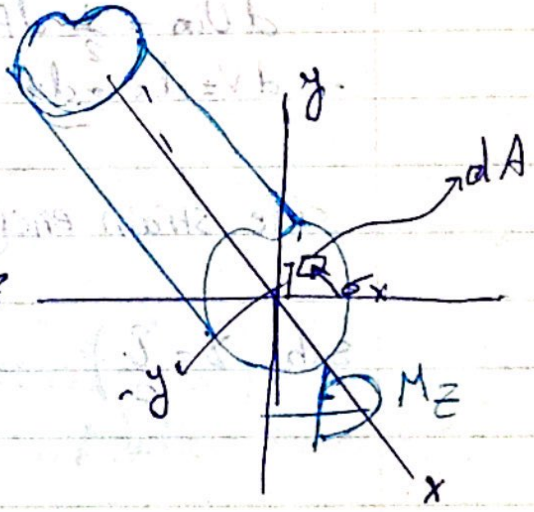
$$U_i = \int_V \frac{\sigma_x^2}{2E}\,dV = \int_0^L \frac{[P(x)]^2}{2[A(x)]^2 E} A(x)\,dx$$

$$\boxed{U_i = \int_0^L \frac{[P(x)]^2}{2A(x)E}\,dx}$$

Common case of a prismatic bar of constant cross-section area A, Length L, and axial load P

$U_i = \frac{P^2 L}{2AE}$ For prismatic body shape

Bending Moment :-



$$U_i = \int \frac{\sigma^2}{2EI}\,dV$$

$$= \int \frac{\left(\frac{My}{I}\right)^2}{2EI}\,dA\,dx$$

$$= \int_0^L \frac{M^2}{2EI^2} \cancel{\left(\int y^2\,dA\right)} dx$$

$\sigma_x = \frac{M_z}{I_z} y$

$dV = dA\,dx$

$I = \int y^2\,dA$

$$\boxed{U_i = \int_0^L \frac{M^2}{2EI}\,dx}$$

Transverse shear :-

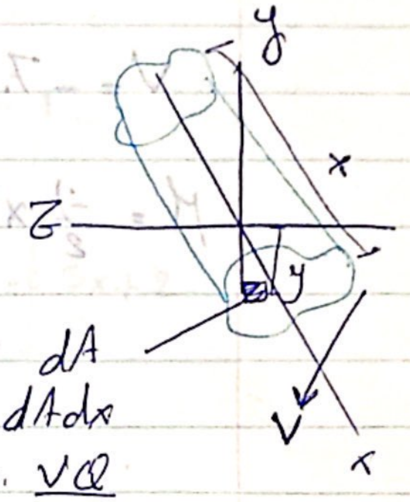
$$U_i = \int_v \frac{\tau^2 dv}{2G} = \int \frac{1}{2G} \left(\frac{VQ}{It}\right)^2 dA\, dx$$

$$U_i = \int_0^L \frac{V^2}{2GI^2} \left(\int \frac{Q^2}{t^2} dA\right) dx$$

$$\boxed{U_i = \int_0^L \frac{f_s V^2 dx}{2GA}}$$

$$f_s = \frac{A}{I^2} \int \frac{Q^2}{t^2} dA$$

يعتمد على اطوال المقطع فقط



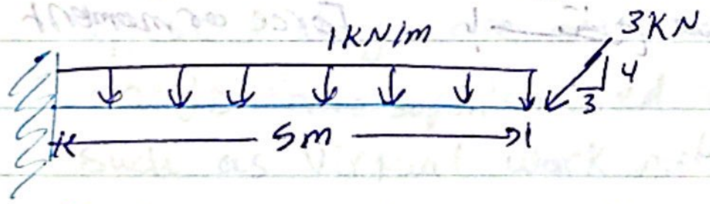
$dv = dA\,dx$

$\tau = \frac{VQ}{It}$

f_s is a dimensionless number that is unique for each cross-section area. (shap factor)

لكن سيكون هناك fs معطاة

Example :- The cantilever beam is subjected to the loads shown, and has rectangular cross section. Determine the total strain energy stored in the beam.

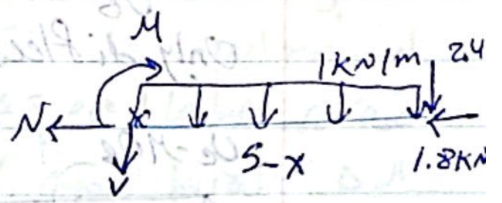


Solution :- make a section cut at a distance x and determine the internal loading :-

$\Sigma F_x = 0$: $N = -1.8$ kN

$+\uparrow \Sigma F_y = 0$: $-V - 1(5-x) - 2.4 = 0$

$-V = 7.4 + x$ (kN)



$\circlearrowleft^{+} M = 0$: $-M - \frac{1}{2}(5-x)^2 - 2.4(5-x) = 0$ $\quad 0 \leq x \leq 5$

$M = -\frac{1}{2}x^2 + 7.4x - 24.5$ (kN.m)

$$N = -1.8\,kN \longrightarrow (U_i)_N = \frac{P^2 L}{2AE} = \frac{(-1.8)^2(5)}{2AE} \text{ "+v"}$$

$$V = -7.4 + x \longrightarrow (U_i)_V = \int_0^L \frac{(\frac{6}{5})V^2}{2GA}dx = \int_0^5 \frac{(\frac{6}{5})(-7.4+x)^2}{2GA}$$

$$M = \frac{-1}{2}x^2 + 7.4x - 24.5 \longrightarrow (U_i)_M = \int_0^L \frac{M^2}{2EI}dx$$

$$= \frac{\int_0^{5m}\left[-\frac{x^2}{2} + 7.4x - 24.5\right]^2 dx}{2EI}$$

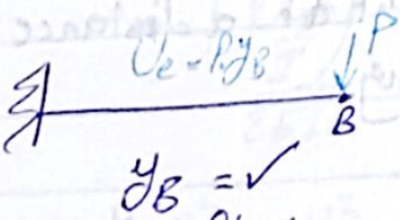
$$(U_i)_{total} = (U_i)_N + (U_i)_V + (U_i)_M$$

★ Work and Energy principle. "Real work - real energy"

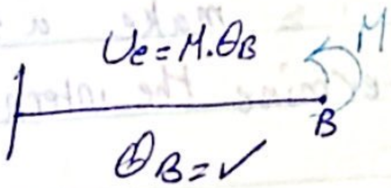
$$U_{ex} = U_{in}$$

عندما يكون لدينا single Force or moment نستطيع ايجاد θ و y عند نقطة التأثير باستخدام هذه الطريقة

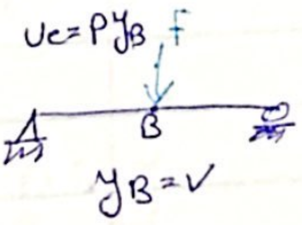
For example :-

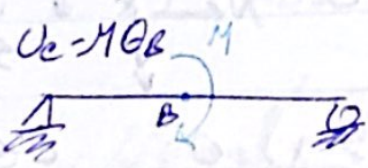


Only diflection at B

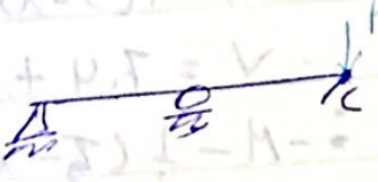


Only slope at B.

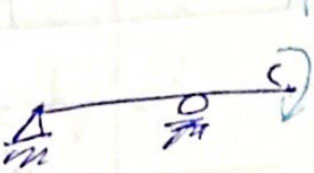




$\theta_B = ✓$

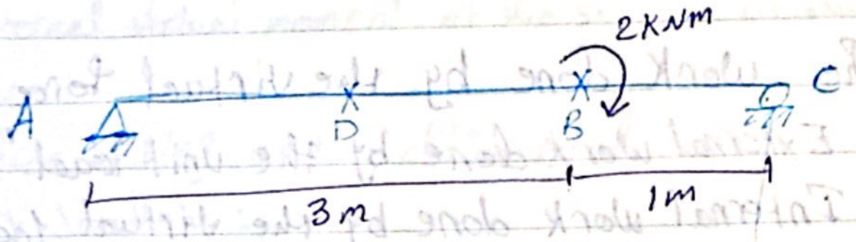


$y_c = ✓$

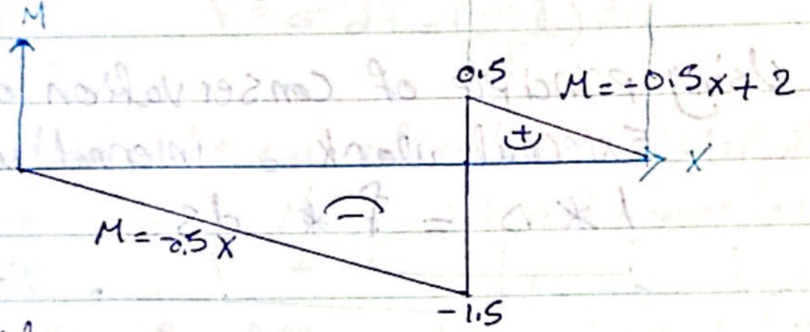


$\theta_c = ✓$

Example:- Determine the slope at point B for the beam ABC shown below. EI = constant.



Solution:-



$$U_e = U_i$$

$$\frac{1}{2} M_B \theta_B = \int_0^L \frac{M^2}{2EI} dx \Longrightarrow \frac{1}{2}(2)\theta_B = \int_0^3 \frac{(-0.5x)^2}{2EI} dx$$

$$+ \int_3^4 \frac{(0.5x+2)^2 dx}{2EI} = \frac{9}{8EI} + \frac{1}{24EI} = \frac{7}{6EI} \text{ rad} \curvearrowright$$

principle of work and energy was used to determine θ_B only suppose that you are interested in determing θ_D, y_D, or y_B more sophisticated energy methods are needed such as virtual work method, and Castiglin's method.

* Virtual work method "Unit - Load method":-

Δ: External displacement caused by real load

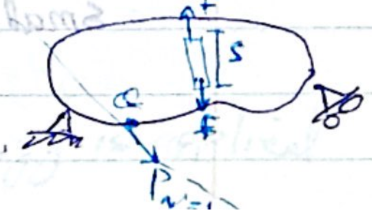
لنستطيع ايجاد Δ عند النقطة Q نفترض ان هناك

قوة P_v مقدارها 1 موجودة على النقطة وبنفس اتجاه Δ

ds: internal def. caused by real lod Δ الـ

P_v = 1 = External virtual unit load

f = Internal virtual load.



The work done by the virtual force are follows

- External work done by the unit load "P_v" = $P_v * \Delta$
- Internal work done by the virtual load f = $f * ds$

Using principle of conservation of energy :-

External work = internal work

$$1 * \Delta = f * ds$$

Virtual load

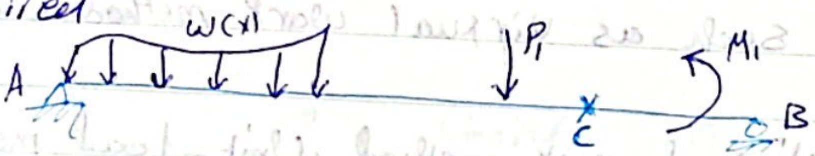
Real displacement

- To obtain the slope at a point on a structure :-

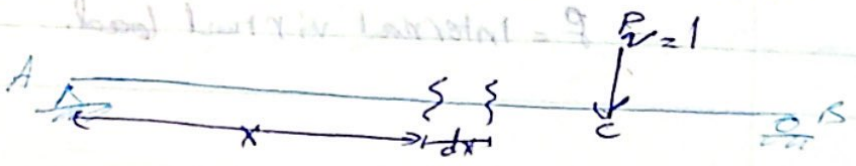
$$1 * \theta = f * ds$$

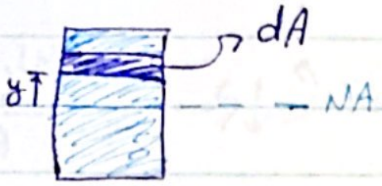
- Virtual work Formulation for the Deflection and slope of Beams and Frams :-

The deflection at point C due to external loads is required



$\boxed{I}$ removing all the real loads (w ; P_1 , M_1) and applied a virtual unit load $P_v = 1$ will cause elementary forces and deformations to develop in the member, and a small deflection to occure at C, as follows :-





نريد معرفة تأثير V.load على البيم :-

اخذنا مقطع منه :- سترس $\sigma' = \frac{m y}{I}$ من Pv

m: internal virtual moment at the section. at distance x

The force acting on the differential area due to the virtual unit load is:

$$f = \sigma' dA = \left(\frac{m y}{I}\right) dA$$

The stress due to the external (real) loads (w, P, M) on the beam is :- $\sigma = \frac{M y}{I}$ internal moment in the beam.

The deformation of a differential beam length dx at a distance x from A is :- $\delta = \varepsilon\, dx = \left(\frac{\sigma}{E}\right) dx = \left(\frac{M y}{E I}\right) dx$

The work done by the force f :

$$dU_i = f\delta = \left(\frac{m y}{I}\right) dA * \left(\frac{M y}{E I}\right) dx = \left(\frac{M m y^2}{E I^2}\right) dA\, dx$$

$$\int dU_i = \int \left[\int_A \left(\frac{M m\ y^2}{E I^2}\right) dA\right] dx = \int \left(\frac{M m}{E I^2} \underbrace{\int_A y^2 dA}_{I}\right) dx$$

$U_i = \int \frac{M m}{E I} dx$ internal work done by the total force.

$$\boxed{U_i = \int_0^L \left(\frac{M m}{E I}\right) dx}$$

The external work done by the virtual load

$U_e = 1 * \Delta$

The principle of conservation of energy is applied to obtain the expression of the deflection at any point in a beam or frame :-

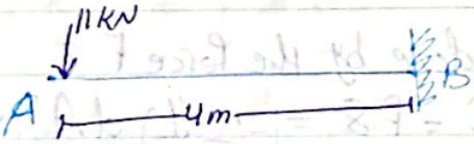
$$U_e = U_i$$

$$1 * \Delta = \int_0^{l} \frac{Mm}{EI} dx$$

$$\boxed{\Delta = \int_0^{L} \left(\frac{Mm}{EI}\right) dx}$$

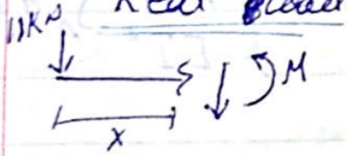
computation of the slope at point $\theta = \int_0^{L} \left(\frac{Mm}{EI}\right) dx$

Example :- Determine slope and deflection of point A of the cantilever beam shown. EI constant.



Solution :-

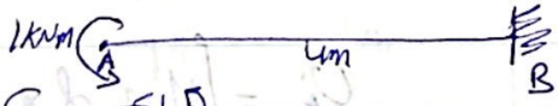
Real load

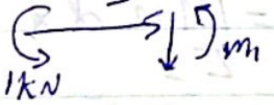


$M = -11x \quad 0 \le x \le 4m$

Virtual load

apply one unit moment at A





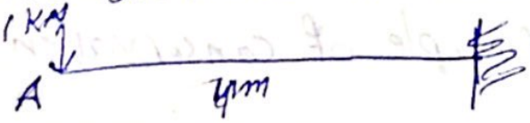
$m_1 = -1 \quad 0 < x < 4m$

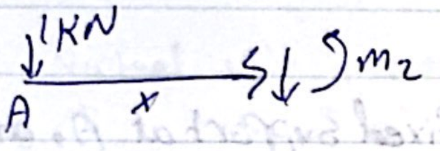
$$\theta A = \int_0^{L} \frac{M m_1}{EI} dx = \int_0^{4} \frac{(-11x)(-1)}{EI} dx = \frac{+88}{EI} \text{ rad}$$

[illegible] + تكون بنفس اتجاه ال load الذي وضعته

وإذا كانت سالبة تكون بعكس ال unit load الذي وضعته.

Apply one unit ~~load~~ Force at A to find V_A :-





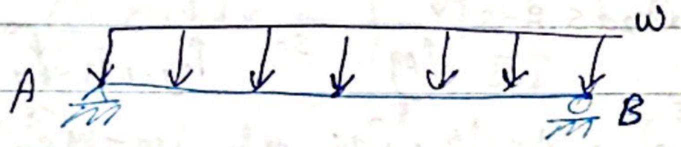
$$\Delta_A = \int_0^L \frac{M m_2}{EI}\,dx$$

$m_2 = -x \quad 0 \le x \le 4m$

$$= \int_0^4 \frac{(-4x)(-x)}{EI}\,dx \; \textcircled{+} \; \frac{704}{3EI} \downarrow$$

يتعامل مع إشارة ال unit force الـ W وليس .

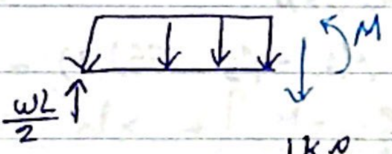
Example :- Determine mid-span deflection and end slopes of a simply supported beam shown. EI = constant.



Solution :-

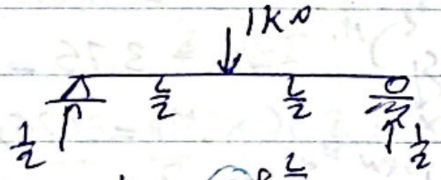
$0 \le x \le L$

Real load



$$M = \frac{wL}{2}x - \frac{wx^2}{2}$$

Virtual load



$m_1 = \frac{x}{2}$

$0 < x \le L/2$

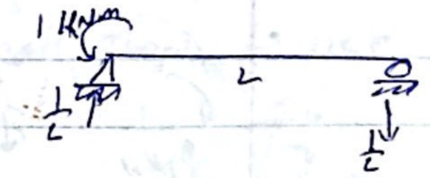
$$V_{mid\text{-}span} = \int \frac{M m_1}{EI}\,dx = \textcircled{2}\int_0^{\frac{L}{2}} \frac{\left(\frac{wLx}{2} - \frac{wx^2}{2}\right)\left(\frac{x}{2}\right)dx}{EI}$$

لانه يوجد تشابه بين 1 ـ 2

$$= \frac{2}{EI}\left(\frac{wL}{12}x^3 - \frac{wx^4}{16}\right)\Bigg|_0^{\frac{L}{2}} = \boxed{\frac{5wL^4}{384EI}}$$

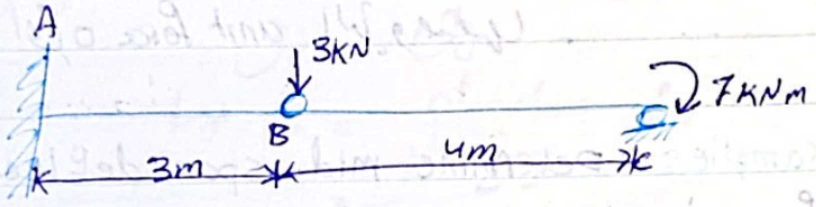
حفظ لهذا القانون

$$\Theta_A = \int \frac{M m_2}{EI}\,dx = \int_0^L \frac{\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)\left(\frac{x}{L} - 1\right)dx}{EI} \Leftarrow -\frac{wL^3}{24EI} \circlearrowright$$

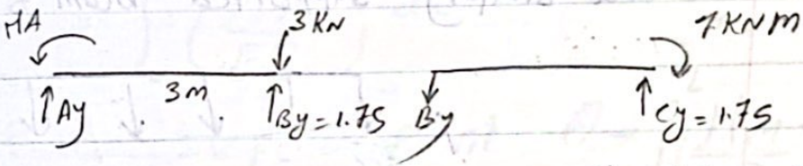


Due to symmetry $\Theta B = \Theta A = \frac{wL^3}{24EI} \circlearrowleft$

Example:- Beam ABC has a fixed support at A, an internal hinge at B, and a roller support at C. $EI = $ constant. Determine the deformations at point B.?



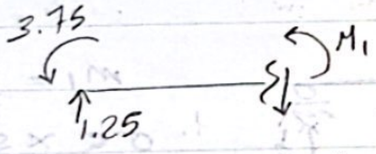
Real loads :-



$\Sigma F_y = 0 \Rightarrow A_y - 3 + 1.75 = 0$ $\qquad \Sigma M_B = 0 \Rightarrow -7 + 4C_y = 0$

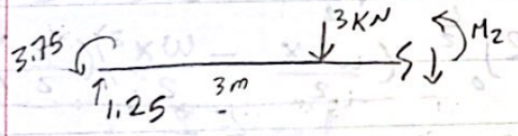
$A_y = 1.25 KN$ $\qquad C_y = 1.75.$

$\Sigma M_A = 0 \Rightarrow M_A - (3 - 1.75)(3) = 0 \Rightarrow M_A = 3.75 KNm$



$M_1 + 3.75 - 1.25 x_1 = 0$

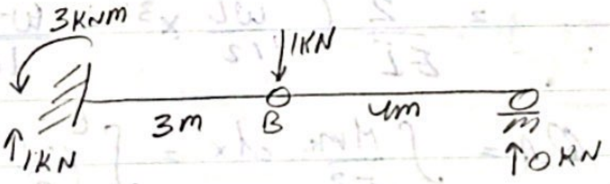
$M_1 = 1.25x - 3.75 \qquad 0 \leq x \leq 3m$

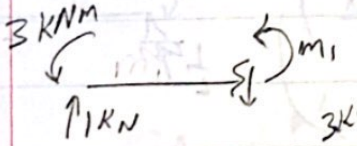


$M_2 + 3.75 - 1.25\,x + 3(x - 3)$

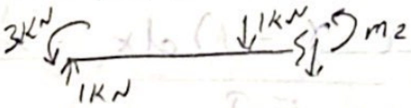
$M_2 = 5.25 - 2.75x \qquad 3 \leq x \leq 7m$

Virtual load :-





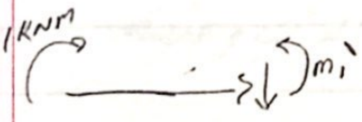
$m_1 + 3 - 1(x) \Rightarrow m_1 = x - 3 \qquad 0 < x \leq 3$



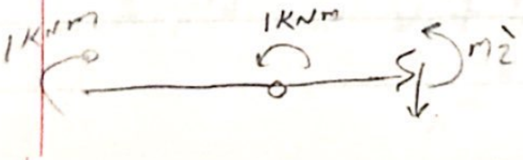
$m_2 + 3 - 1(x) + 1(x - 3) = 0$

$m_2 = 0$

Virtual unit moment just before the internal hinge :-

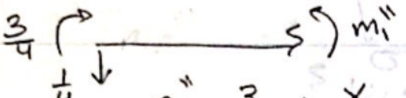


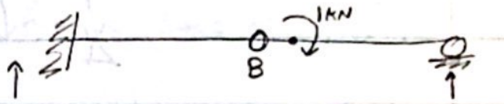
$m_1 - 1 = 0 \Rightarrow m_1 = 1 KN.m$



$m_2 - 1 + 1 \Rightarrow m_2 = 0$

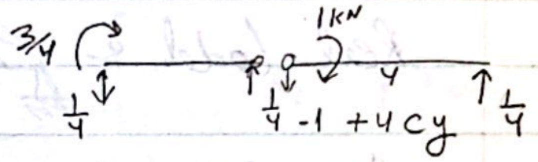
Virtual unit moment just after the internal hinge B :-

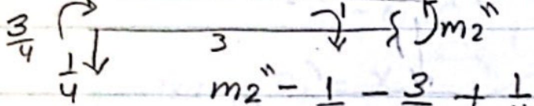




$m_1'' - \frac{3}{4} + \frac{x}{4}$

$$\boxed{m_1'' = \frac{3}{4} - \frac{x}{4}}$$





$m_2'' - 1 - \frac{3}{4} + \frac{1}{4}(x)$ $\quad M = \frac{1}{4}(3) \quad cy = \frac{1}{4}$

$$\boxed{m_2'' = \frac{7}{4} - \frac{1}{4}x}$$

$$V_B = \int_0^3 \frac{M_1 m_1}{EI} dx + \int_3^7 \cancel{\frac{M_2 m_2}{EI}}^{\,0} dx = \frac{^{+}45}{EI} \downarrow$$

$$(\theta_B)_L = \int_0^3 \frac{M_1 m_1'}{EI} dx + \int_3^7 \frac{M_2 m_2'}{EI} = \int_0^3 \frac{(1.25x - 3.75)(-1)dx}{EI}$$

$$\frac{\pm 45}{8EI} \curvearrowright$$

$$(\theta_B)_R = \int_0^3 \frac{M_1 m_1''}{EI} dx + \int_3^7 \frac{M_2 m_2''}{EI} dx = \int_0^3 \frac{(1.25 - 3.75)(\frac{3}{4} - \frac{x}{4})}{EI}$$

$$+ \int_3^7 \frac{(5.25 - 1.75x)(\frac{7}{4} - \frac{x}{4})dx}{EI} = -\frac{359}{48EI} \curvearrowleft$$

يمكننا رسم BMD لـ real and virtual واستخدام مساحات الجدول الذي بالسلايدة 23
لإيجاد slope and the deformation

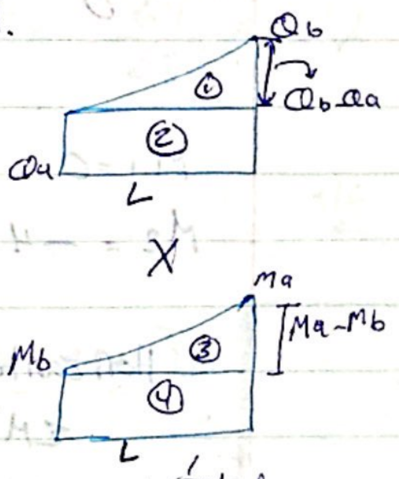
we can divied the shape to small shaps to use the table. trapezoid x trapezoid.

$$\overset{①\times③}{\frac{L}{6}(Ma - Mb)(Qb - Qa)} + \overset{①\times④}{}$$

$$\frac{L}{2}(Mb)(Qb - Qa) +$$

$$\overset{③\times②}{\frac{L}{2}(Ma - Mb)(Qa)} +$$

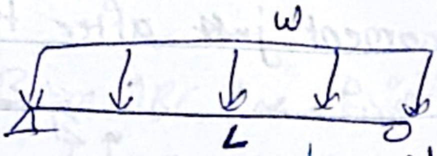
$$\underset{②\times④}{L(Mb)(Qa)}$$



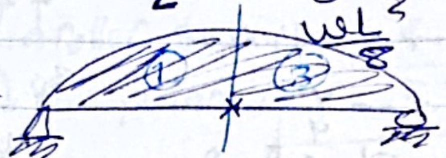
* بالعكس *

Find $v_{\frac{L}{2}}$, $\theta_{\frac{L}{2}}$

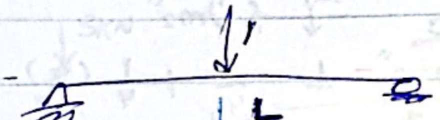
Example :-

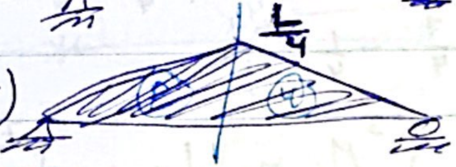


Real load :-



Virtual load :-

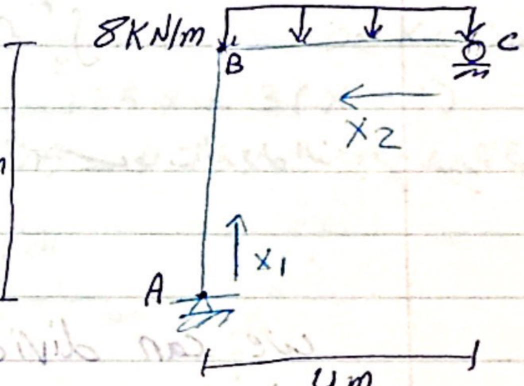




$$2\times \frac{5\left(\frac{L}{2}\right)\left(\frac{wL^2}{8}\right)\left(\frac{L}{4}\right)}{12}$$

$$\frac{5wL^4}{(4)(8)12\,EI} = \boxed{\frac{5wL^4}{384\,EI}}$$

Example :- Determine horizontal displacement of C and slope at A of a rigid-jointed plane frame shown. Both members of the frame have same flexural rigidity (EI).



$+\uparrow \Sigma F_x = 0 \Rightarrow A_x = 0$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow 4C_y - 8(4)(2) = 0$

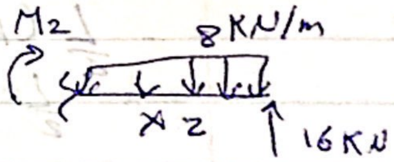
$C_y = 16\,KN$

$+\uparrow \Sigma F_y = 0 ; A_y + 16 - 8\times 4 = 0$

$A_y = 16\,KN$

$M_1 = 0 \quad 0 \le x_1 \le 6m$

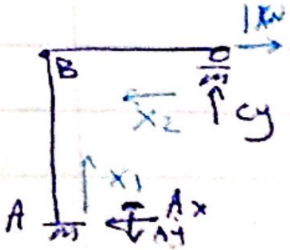
$M_2 = -4x_2^2 + 16x_2 \quad 0 \le x_2 \le 4m$



* Horizontal desplacment at C :-

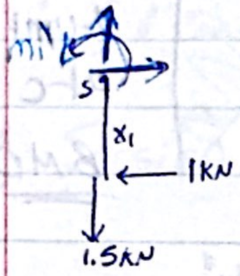
$\Sigma M_A = 0 ; 4C_y - 6(1) = 0$

$C_y = 1.5\,KN$



$\xrightarrow{+} \Sigma F_x = 0 : -A_x + 1 = 0 \Rightarrow A_x = 1\ KN$

$+\uparrow \Sigma F_y = 0 : -A_y + 1.5 = 0 \Rightarrow A_y = 1.5 KN$

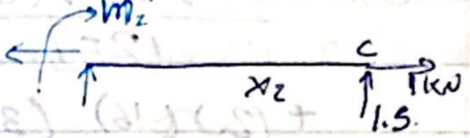


$\circlearrowleft \Sigma M_s = 0.$

$m_1' - x_1 = 0$

$m_1' = x_1$

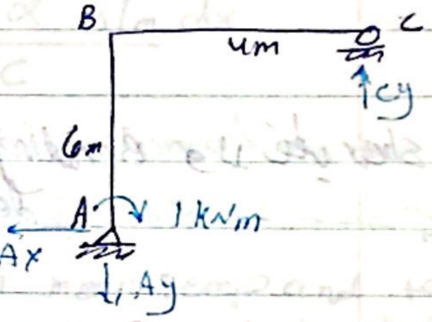
$0 \leq x_1 \leq 6m$



$m_2' = 1.5 x_2 \quad 0 < x_2 < 4m$

$$\Delta c = 0 + \int_0^4 \frac{(-4x_2^2 + 16x_2)(1.5x_2)}{EI} dx = \frac{+128}{EI} m\ (\rightarrow)$$

* Slope at A :-

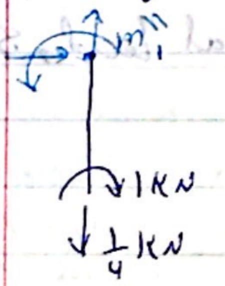


$+\circlearrowleft \Sigma M_A = 0 : -1 + 4C_y = 0$

$C_y = \frac{1}{4} KN$

$\xrightarrow{+} \Sigma F_x = 0 : A_x = 0$

$+\uparrow \Sigma F_y = 0 : -A_y + \frac{1}{4} = 0 \Rightarrow A_y = \frac{1}{4} KN$



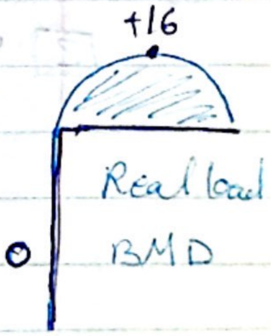
$m_1'' = 1\ KNm$

$0 \leq x_1 \leq 6m$

$m_2'' = x_2/4 \quad 0 \leq x_2 \leq 4m$

$$\theta_A = 0 + \int_0^4 \frac{(-4x_2^2 + 16x_2)(\frac{x_2}{4})}{EI} dx = \frac{+64}{3EI} \text{ rad} \circlearrowright$$

Or using graphical method:-

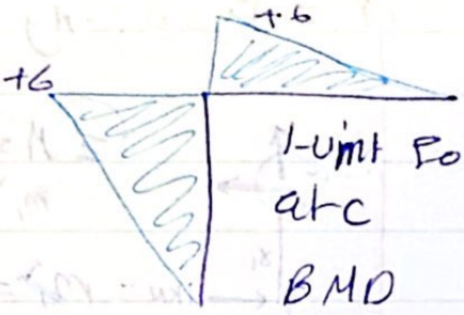


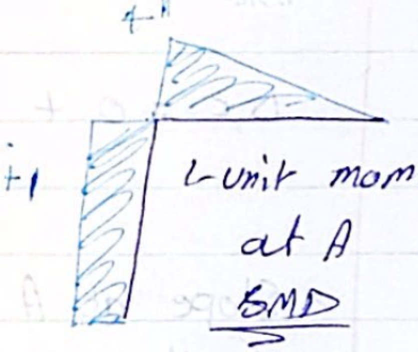
$$\Delta c = \frac{5(2)(16)(3)}{12EI} + \frac{2(16)(3(6)+5(3))}{12EI}$$

$$= \frac{+128\ m}{EI}\ (\rightarrow)$$

$$\theta_A = \frac{5(2)(16)(0.5)}{12EI} + \frac{(2)(16)(3\times1+5(0.5))}{12EI}$$

$$= \frac{+64}{3EI} \text{ rad } \circlearrowright$$





(كل الحل و المعادلات السابقة تعطي ال Bending moment ولا تعطي axial, shear, etc ---- deformation)

* Virtual strain Energy caused by Axial load, shear, Torsion, and Temperature :-

1. Axial load

$$\boxed{U_n = \frac{nNL}{AE}} \longrightarrow \text{real displacement}$$

عنا اوتعلمناه من الميكانيكس

n: internal virtual axial load caused by external virtual unit load
N: internal axial force in the member caused by the real loads.

2. Shear load

$$U_s = \int_0^L k\left(\frac{vV}{GA}\right)dx$$

v: internal virtual shear in the member, as a function of x
V: internal shear in the member, function of x (real load
K: form factor for the cross-section area

K = 1.2 for rectangular cross sections.

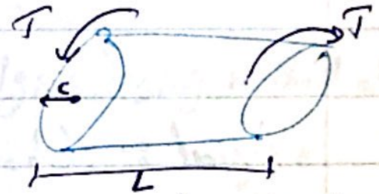
K = 10/9 for circular cross sections

K = 1 for wide-Flange and I-beams, A is area of web.

G: shear modulus of elasticity for the material.

3 Torsion

$$U_t = \frac{tTL}{GJ}$$



t: internal virtual torque by virtual unit load

T: " torque by real load.

J: polar moment of inertia $J = \frac{\pi c^4}{2}$ c: radius of C.S.

4 Temperature

$$U_{temp} = \int_0^L \frac{m \alpha \Delta T_m}{c} dx$$

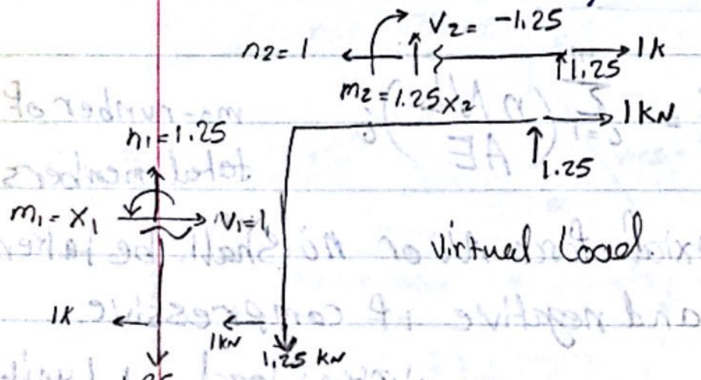
m: internal virtual moment, function of x.

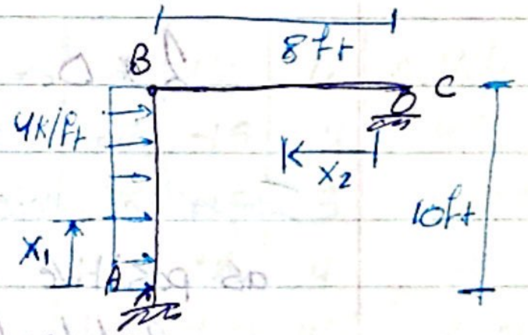
α: Coefficient of thermal expansion.

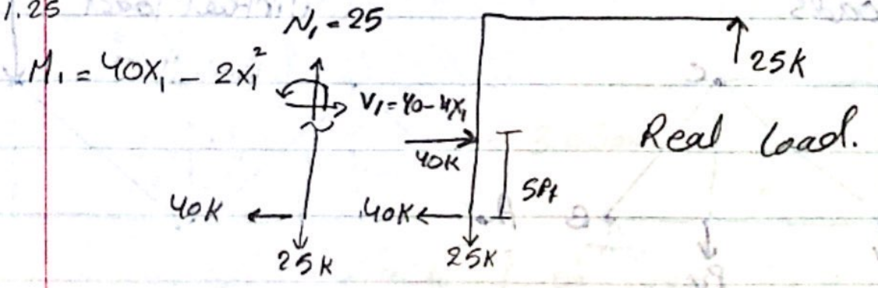
ΔT_m: temp. difference between mean temp and top or bottom bea

Example:- Determine the horizontal displacement of point C

$E = 29 \times 10^3$ ksi, $G = 12 \times 10^3$ ksi, $I = 600 in^4$, and $A = 80 in^2$







Bending :- $U_b = \int_0^L \frac{mM}{EI} dx = \frac{13666.7\,K^2 . ft^3}{EI}$

$$= \frac{13666.7\,K^2 . ft^3 \, (12^3 in^3 / 1 ft^3)}{29 \times 10^3 K/in^2 \, (600 in^4)} = 1.357\, in.K$$

Axial :- $U_a = \Sigma \frac{nNL}{AE} = \frac{1.25(25)(120)}{80(29 \times 10^3)} + \frac{1(0)(96)}{80[29(10)^3)}$

$$= 0.001616 in.K$$

Shear :- $U_s = \int_0^L K \cdot \left(\frac{vV}{GA}\right) dx = \int_0^{10} \frac{1.2(1)(40-4x_1)\,dx_1}{GA}$

$$+ \int_0^8 \frac{1.2(-1.25)(-25)}{GA} dx_2 = \frac{540.(12)}{12(10^3)80} = 0.00675 in.K$$

$1\,k \quad DC_h = 1.357 in.K + 0.001616 in.K + 0.00675 in.K$

$\Delta_{ch} = 1.37 in$

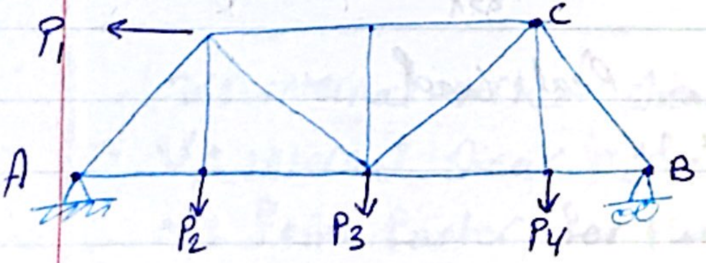
* Virtual work method _ Trusses :-

$$1 * D_c = \Sigma n_i (\partial L)_i = \sum_{i=1}^{m} \left(\frac{nNL}{AE}\right)_i$$

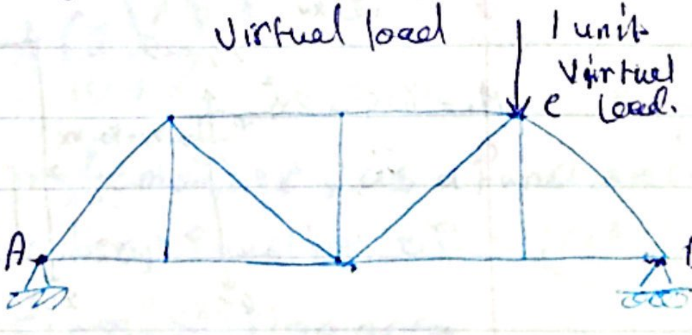
m :- number of total members.

Important note :- the axial force N_i or n_i shall be taken as positive if tensile and negative if compressive.

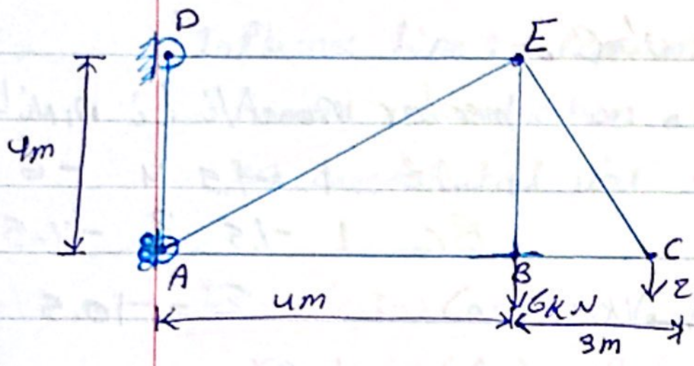
Real loads



Virtual load



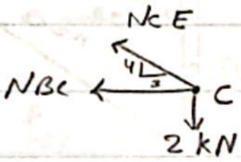
Example :- For steel truss shown, determine the vertical and horizontal displacement at joint C. Each member has a cross sectional area of $A = 350\,mm^2$ and $E = 200\,GPa$.



Solution :-

Real analysis using method of joint

$+\uparrow \Sigma F_y = 0$



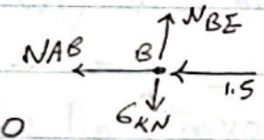
$-2 + \frac{4}{5} N_{CE} = 0$

$N_{CE} = 2.5\,KN\,(T)$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$-N_{BC} - \frac{3}{5}(2.5) = 0 \Rightarrow N_{BC} = -1.5\,KN$ (C)

$\overset{+}{\rightarrow} \Sigma F_x = 0$



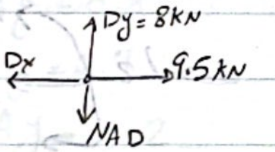
$-N_{AB} - 1.5 = 0$

$N_{AB} = -1.5\,kN$ (c)

$+\uparrow \Sigma F_y = 0$

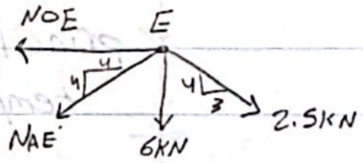
$-6 + N_{BE} = 0 \Rightarrow N_{BE} = 6\,KN$ (T)

$+\uparrow \Sigma F_y = 0$



$N_{AD} = 8\,KN$ (T)

$+\uparrow \Sigma F_y = 0$

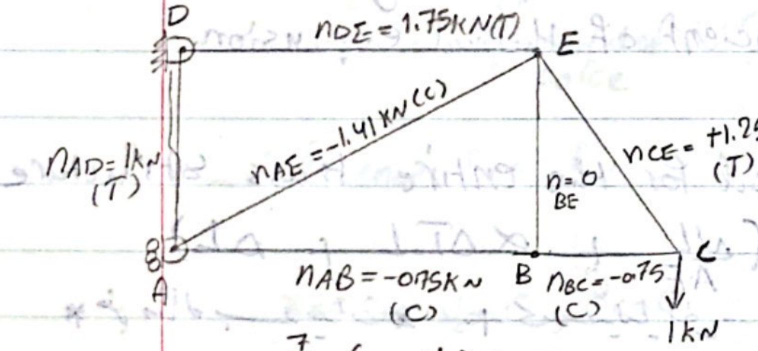


$-N_A\left(\frac{4}{4\sqrt{2}}\right)$

$-6 - 2.4\left(\frac{4}{5}\right) = 0$

$N_{AE} = -11.31\,kN$ (c)

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow -N_{DE} + 11.31\left(\frac{4}{4\sqrt{2}}\right) + 2.5\left(\frac{3}{5}\right) = 0 \Rightarrow N_{DE} = 9.5\,KN$ (T)



$$\Delta_{C_V} = \sum_{i=1}^{7} \left(\frac{nNL}{AE}\right)_i$$

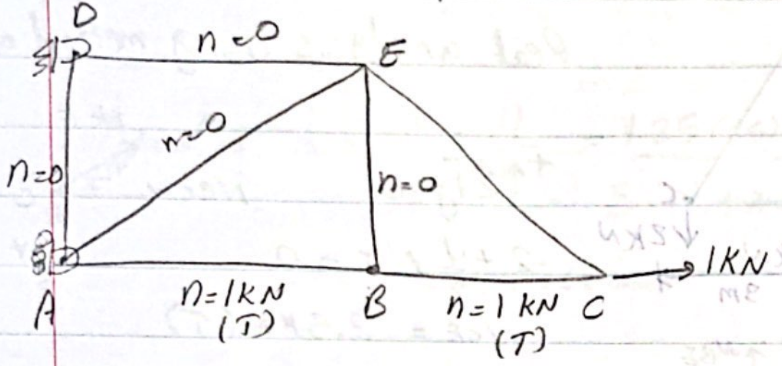
$$= \frac{212.21}{200\times10^6\,KPa}\left(\frac{1}{350\times10^{-6}}\right) = 3.03\,mm\,(\downarrow)$$

member	n_i	N_i	l_i	$n_i N_i l_i$
AB	-0.75	-1.5	4	4.5
BC	-0.75	-1.5	3	3.375
AD	+1	+8	4	32
AE	-1.41	-11.31	$4\sqrt{2}$	90.21
BA	0	+6	4	0
CE	1.25	+2.5	5	15.625
DE	1.75	+9.5	4	66.5

$\Sigma = 212.21$

virdical displacment of joint C is 3.03mm

Horizontal displacement at C :-



member	ni	Ni	Li	niNiLi
AB	1	-1.5	4	-6
BC	1	-1.5	3	-4.5
			Σ =	-10.5

$$\Delta C_h = \frac{-10.5}{200(10^6)(350\times10^{-6})} = -1.5\times10^{-4} = -0.15\,mm \quad (\longleftarrow)$$

* principle of virtual work for truss deflections due to temperature changes and fabrication errors

$U_i = \Sigma n_i (\Delta L)_i \rightarrow L_i$
- → From real load (N) ✓
- → Temp. chang خطأ
- → fabrication error مثلاً التصنيع

$\Delta L_i = \alpha_i \, \Delta T . L_i$ ⇒ Temp. chang.

$\alpha \rightarrow$ coefficient of thermal expansion.

The unit load method for the entire truss structure

$$\Delta = \Sigma n_i * \left(\frac{NL}{AE} + \alpha . \Delta T . L + \Delta L\right)$$

* حل مثال بسيط * + [illegible] -

اذا مقطع العرض أو المادة للـ member مختلفة نضيف E , A للجدول ونستخدمها لكل member لحال.

Influence Lines :- Live load or moving load → magnitude, → Location.

يجب الثبوت باسوأ وضعية لهذا الحمل المتغير.

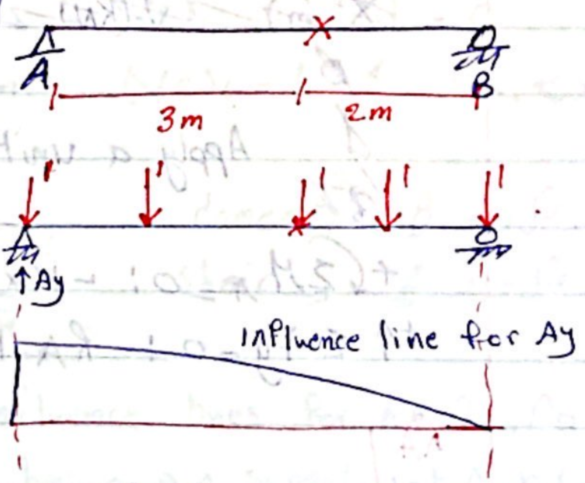
اريد نقطة معينة احسب عندها reaction, moment, interna force وكيف بتغيروا

نتيجة التغيير في موقع الحمل.

Influence line :- represents the variation of the reaction, shear, moment, or deflection at a specific point in a member as a ~~st~~ concentrated unit force moves over the member.

Example :- Construct the influence lines for the support reaction (Ay), shear force at C and bending moment at C ?

- Influence line for vertical reaction at A (Ay) :-



$x = 0 \Rightarrow Ay = 1$

$x = 1 \Rightarrow Ay = 0.8$

$x = 3 \Rightarrow Ay = 0.4$

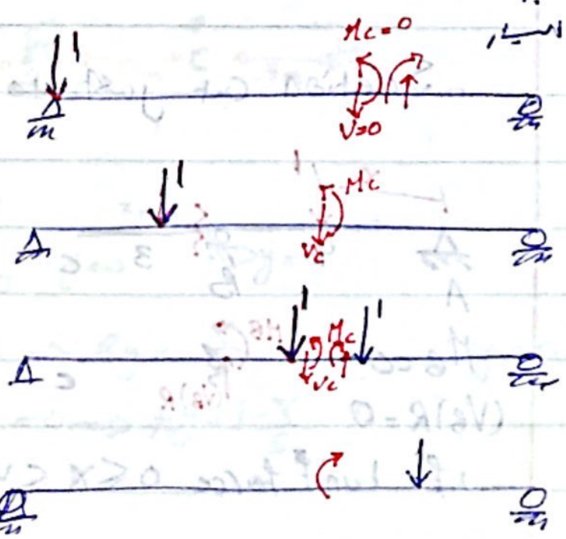
$x = 4 \Rightarrow Ay = 0.2$

$x = 5 \Rightarrow Ay = 0$

اسوأ وضعية لل ~~consentrated~~ Reaction (Ay) load عندما يكون ال consentrated force in A.

- Influence line for Vc :-

الجهة اليمين (+) الجهة اليسار (-)



$x=0 \Rightarrow V_c = 0 \ , Mc = 0$

$x=1 \Rightarrow V_c = -0.2 \ , Mc = 0.4$

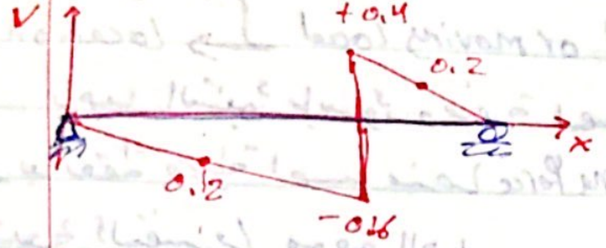
$x = 3^- \Rightarrow Vc = -0.6 \ , M = 1.2$

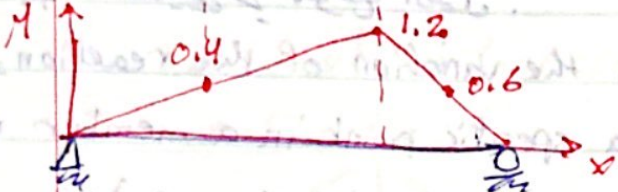
$x = 3^+ \Rightarrow Vc = +0.4 \ , M = 1.2$

$x = 4 \Rightarrow Vc = 0.2 \ , M = 0.6$

$x = 5 \Rightarrow Vc = 0 \ , M = 0$

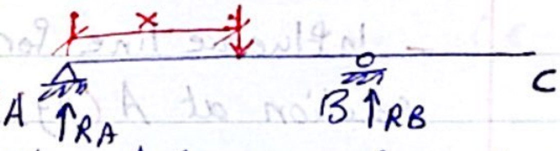
مرة قبل ومرة بعد





Example :- Construct the influence lines for $R_A, R_B, M_B, (V_B)_L$ and (V_{BR}). Using equation method. طريقة اخرى ابسط

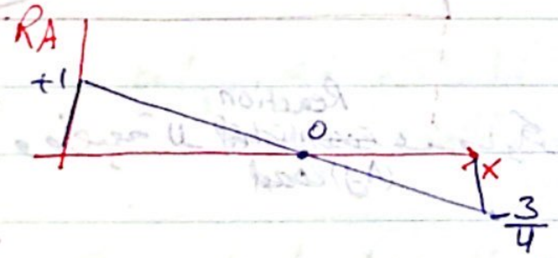
~~X(m) Vc(KN) Mc~~

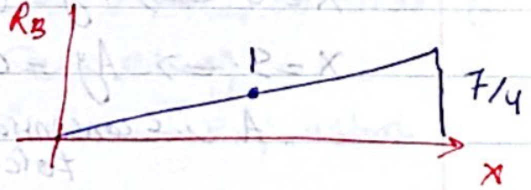


Apply a unit force at a distance x from A.

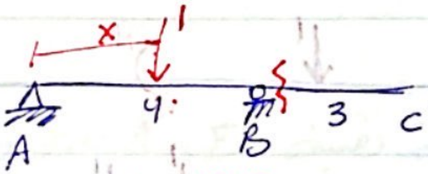
$+\circlearrowleft \Sigma M_A = 0 : -1(x) + R_B(4) = 0 \rightarrow \boxed{R_B = \frac{x}{4}}$ $0 \le x \le 7m$

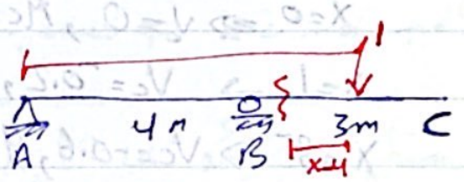
$+\uparrow \Sigma F_y = 0 : R_A - 1 + x/4 = 0 \rightarrow \boxed{R_A - 1 - x/4}$ $0 \le x \le 7m$





- Section cut just to the right of B :-





$M_B = 0$

$(V_B)R = 0$

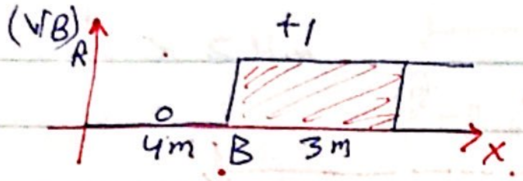
if 1 unit force $0 \le x \le 4^- m$

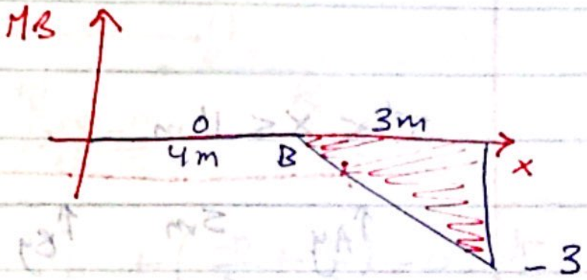
$\Sigma F_y = 0$ $4m^+ \le x \le 7m$

$(V_B)R - 1 = 0 \Rightarrow (V_B)R = {}^+1$

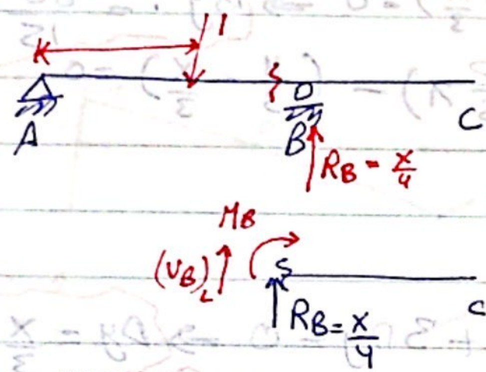
$\Sigma M = 0 - M_B - 1(x-4) = 0$

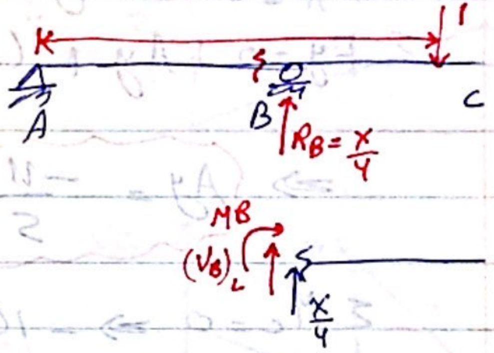
$M_B = -x + 4$





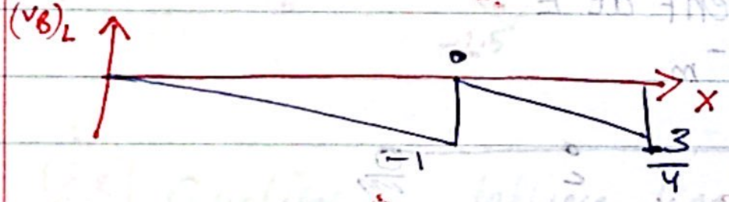
*Section cut just befor B :-





$\Sigma F_y = 0 : (V_B)_L + \frac{x}{4} = 0$ $\qquad (V_B)_L + \frac{x}{4} - 1 = 0$

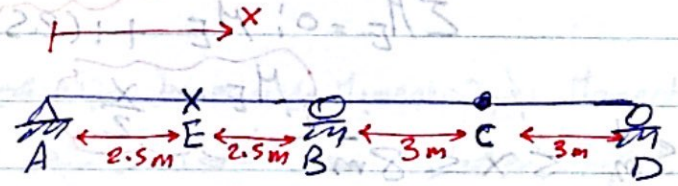
$(V_B)_L = \frac{-x}{4} \quad 0 \leq x \leq 4m$ $\qquad (V_B)_L = 1 - \frac{x}{4} \quad 4m^{+} \leq x \leq 7m$



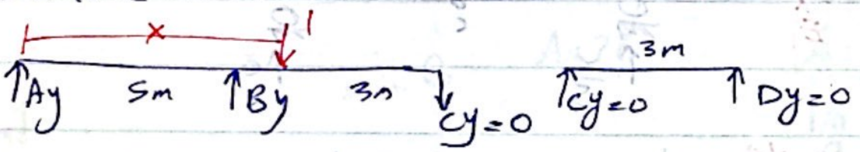
Moment → مابختلف قبل وبعد B

Shear → بختلف قبل وبعد B

Example :- Construct the influence lines for R_A, R_B, R_D, V_E and M_E . The beam shown has a pin support at A, rollers at B and D, and internal hinge at C.



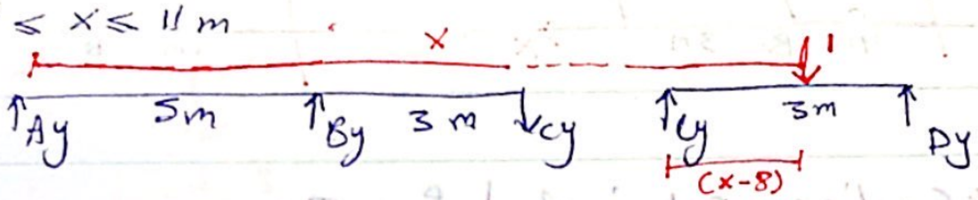
$0 \leq x \leq 8m$



$\Sigma M_B = 0 : -1(x) + 5B_y = 0 \Rightarrow B_y = \frac{x}{5}$ $\qquad D_y = 0$

$\Sigma F_y = 0 : A_y + \frac{x}{5} - 1 = 0 \Rightarrow A_y = 1 - \frac{x}{5}$

$8^{+} \leq x \leq 11\,m$



$$\Sigma M_A = 0 \; ; \; 5By - 8\left(\frac{11}{3} - \frac{x}{3}\right) = 0 \Rightarrow \boxed{By = \frac{88}{15} - \frac{8}{15}x}$$

$$\Sigma Fy = 0 \; ; \; Ay + \left(\frac{88}{15} - \frac{8}{15}x\right) - \left(\frac{11}{3} - \frac{x}{3}\right) = 0$$

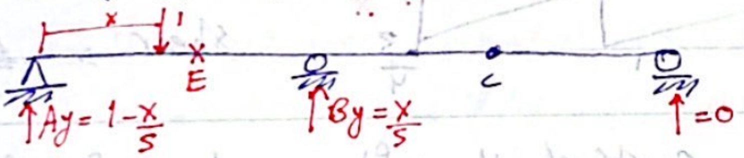
$$\Rightarrow \boxed{Ay = \frac{-11}{5} + \frac{x}{5}}$$

$$\Sigma M_c = 0 \Rightarrow -1(x-8) + 3Dy = 0 \Rightarrow \boxed{Dy = \frac{x}{3} - \frac{8}{3}}$$

$$\Sigma Fy = 0 \Rightarrow Cy - 1 + \frac{x}{3} - \frac{8}{3} = 0 \Rightarrow \boxed{Cy = \frac{11}{3} - \frac{x}{3}}$$

* Shear and moment at E :-

$0 \leq x \leq 2.5^{-}\,m.$



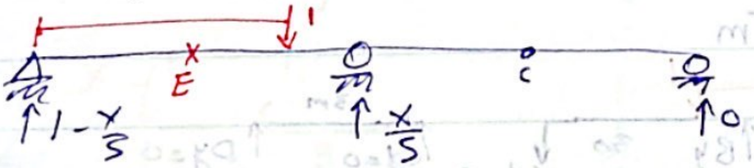
$$\Sigma Fy = 0 \; ; \; -V_E - 1 + \left(1 - \frac{x}{5}\right) = 0$$

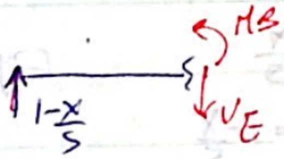
$$\boxed{V_E = \frac{-x}{5}}$$

$$\Sigma M_E = 0 \; ; \; M_E + 1(2.5 - x) - 2.5\left(1 - \frac{x}{5}\right) = 0$$

$$\boxed{M_E = \frac{x}{2}}$$

$2.5^{+}m \leq x \leq 8^{-}m$



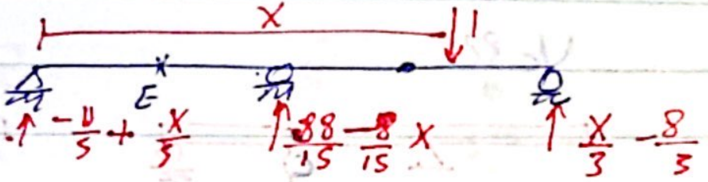


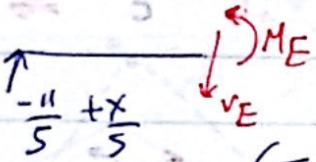
$$+\uparrow \Sigma Fy = 0 \; ; \; -V_E + 1 - \frac{x}{5} = 0 \Rightarrow \boxed{V_E = 1 - \frac{x}{5}}$$

$$+\circlearrowleft \Sigma M_E = 0 \; ; \; M_E - 2.5\left(1 - \frac{x}{5}\right) = 0$$

$$\Rightarrow \boxed{M_E = 2.5 - \frac{x}{2}}$$

$8^{+}_{m} \leq x \leq 11m$

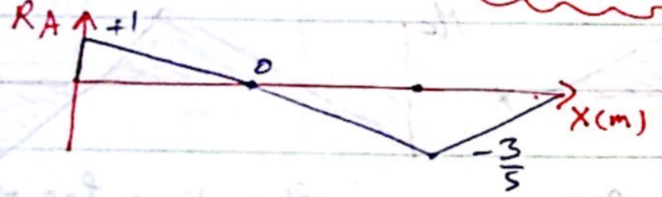


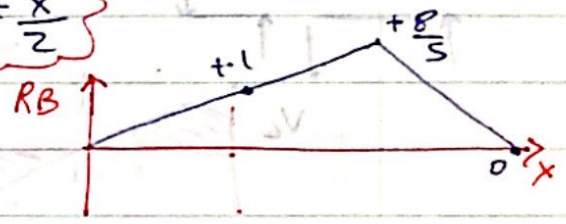


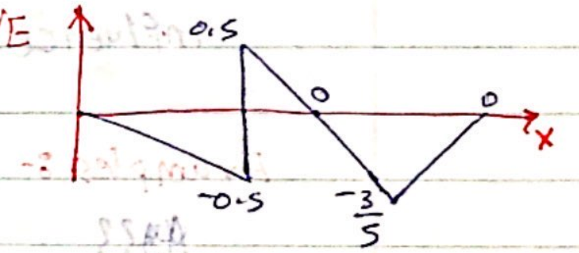
$\overset{+}{\uparrow}\Sigma F_y = 0 : -V_E + \left(\frac{-11}{5} + \frac{x}{5}\right) = 0 \rightarrow \boxed{V_E = \frac{-11}{5} + \frac{x}{5}}$

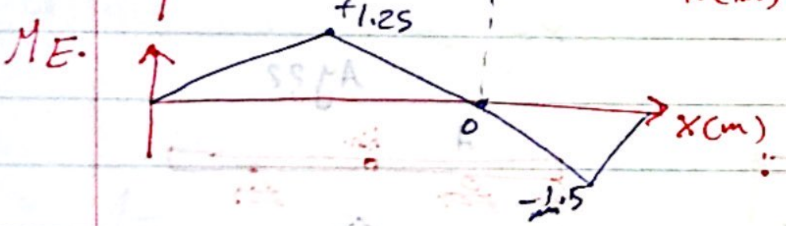
$+\circlearrowleft \Sigma M_E = 0 : M - 2.5\left(-\frac{11}{5} + \frac{x}{5}\right) = 0.$

$\boxed{M_E = \frac{-11}{2} + \frac{x}{2}}$







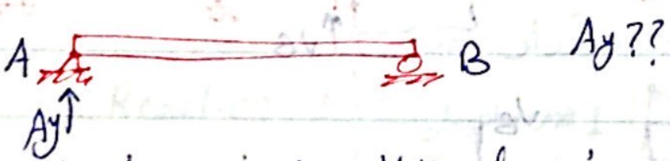


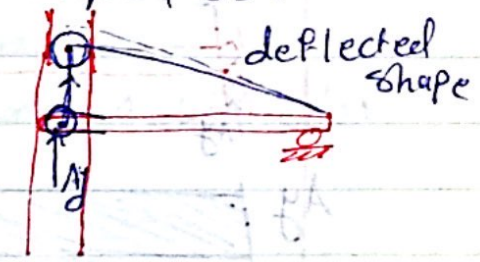
* تسليميات مثال لل Truss *

6.3 Qualitative Influnce line:-

It's states that the influnce line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.

مثلا اريد Moment influnc line بجيب Moment ع ال beam بشكل معين بعمل Vertual beam بنفس نوع deformation الـ Moment وزسم ال Deflected shape ← شكله نفس شكل الـ Influnce Line لـ beam.

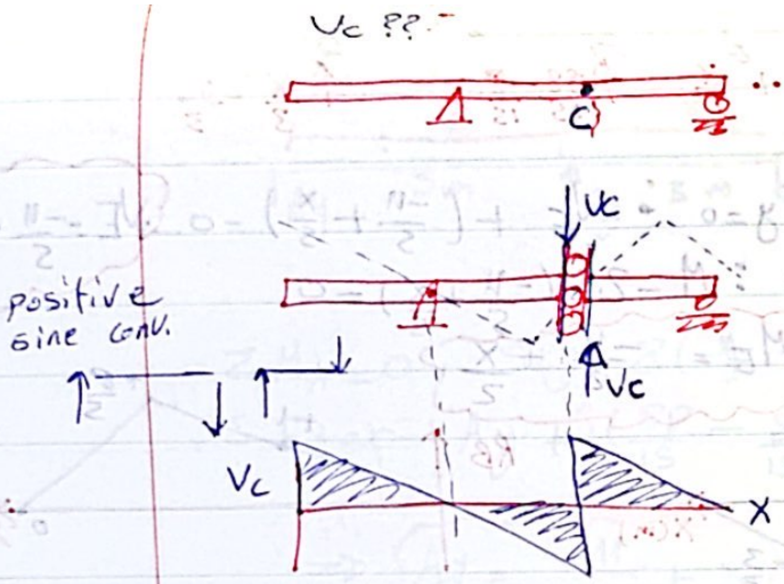




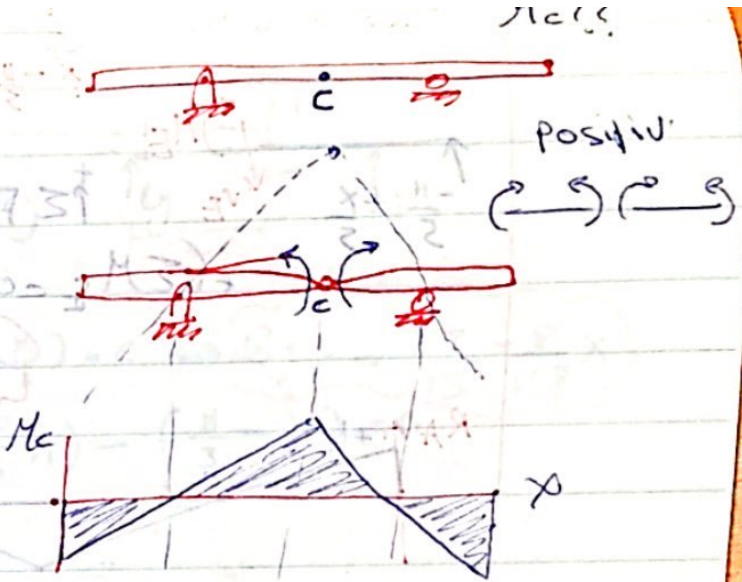
بعمل Vartual desplacemnt لل A بنفس نوع ال reaction الي عليه Force وتكون (Virtica dirction) ↑Ay

تكون ازاحة وليس زاوية desplacemnt. ← not Rot.

ال Influnce Ay Line هو نفسه شكل deflected shape تبع ال beam عند Ay



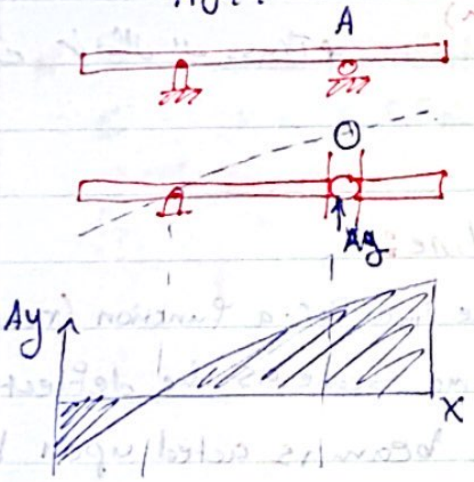
influence line for Vc.



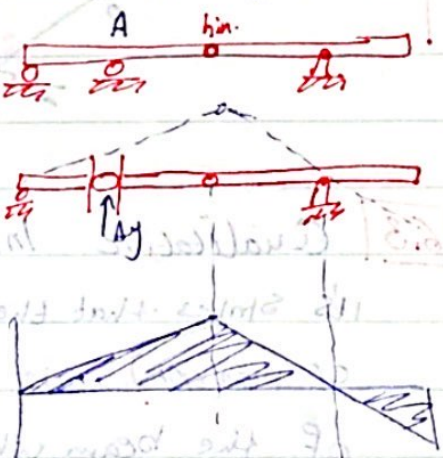
influnce line for Mc

Examples :-

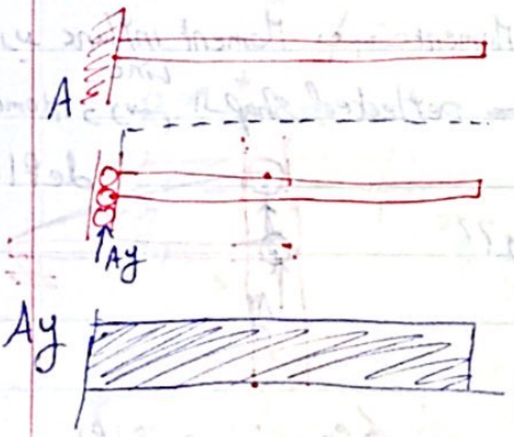
Ay??



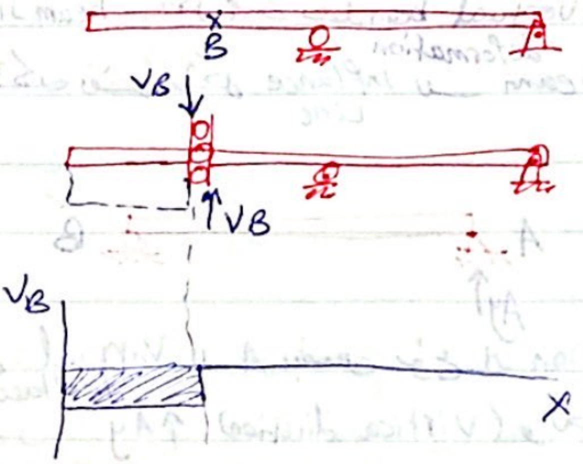
Ay??



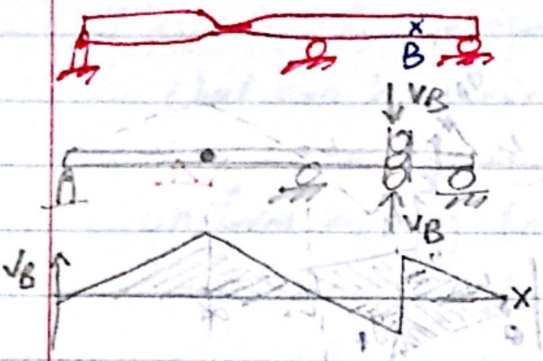
Ay??



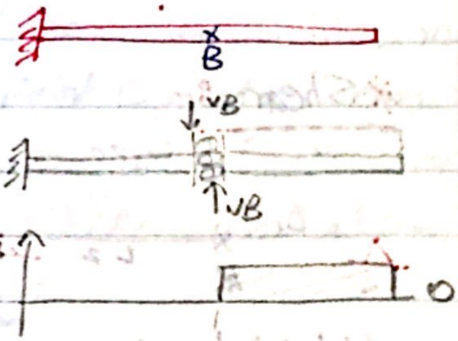
VB.??



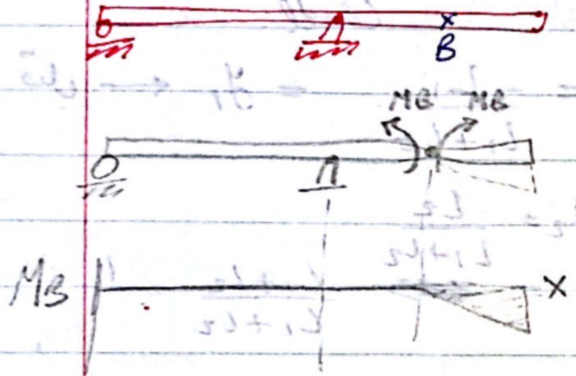
VB?



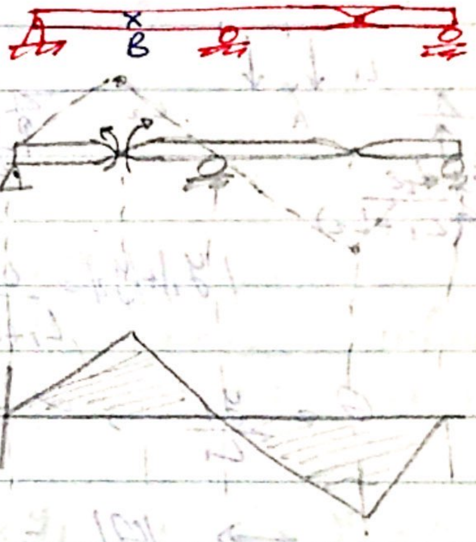
VB??

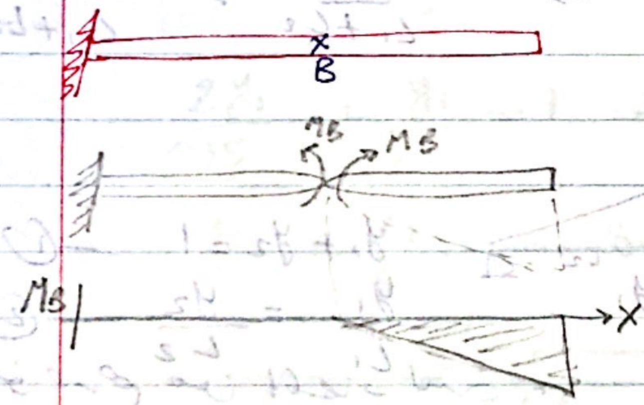


MB?



MB?





M1

* The value of the influnce line :-

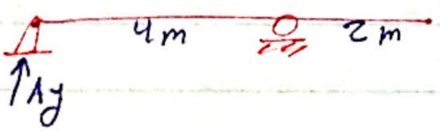
Virtual displeacement / Rotation = 1 unit.

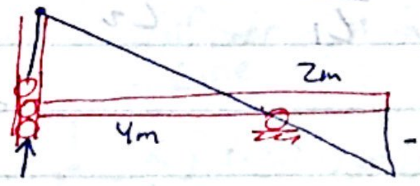
→ 1mm displeacement , 1 rad rotation.

اثبت بمقدار هذا ال deformation بمقدار 1unit

— Reaction :- desplacement 1

Ay??



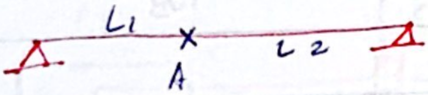


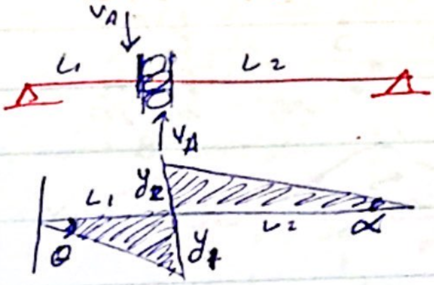
$-\frac{1}{2}$ من تشابه المثلثات

$\frac{1}{4} = \frac{x}{2} = x = \frac{2}{4} = \frac{1}{2}$

*Shear :-

VA??

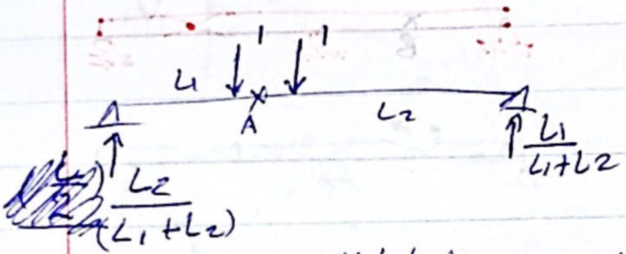




$|y_1|+|y_2| = 1$

بفرض مرة قبل النقطة A بشوي ومرة بعد النقطة A مباشرة

Moving Load



$VA = \frac{-L_1}{L_1+L_2} = y_1 \leftarrow$ قبل

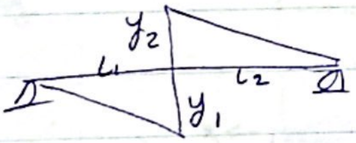
$y_2 = \frac{L_2}{L_1+L_2}$

$|y_1|+|y_2| = \frac{L_1}{L_1+L_2} + \frac{L_2}{L_1+L_2} = \frac{L_1+L_2}{L_1+L_2} = 1$

$\theta = \frac{y_1}{L_1}$, $\alpha = \frac{y_2}{L_2} \Rightarrow \theta = \frac{-1}{L_1+L_2}$, $\alpha = \frac{1}{L_1+L_2}$

$\Rightarrow |\theta| = |\alpha|$

Summary :-



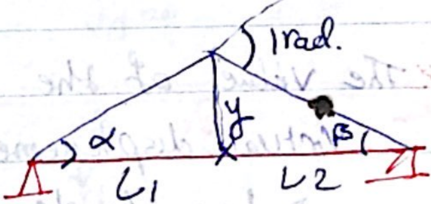
$y_1 + y_2 = 1$ — ①

$\frac{y_1}{L_1} = \frac{y_2}{L_2}$ — ②

نوجدهم من المعادلتين .

*Bending Moment :-

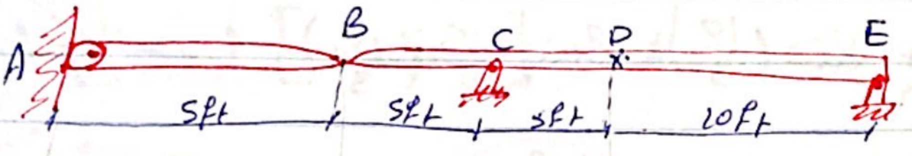
M??

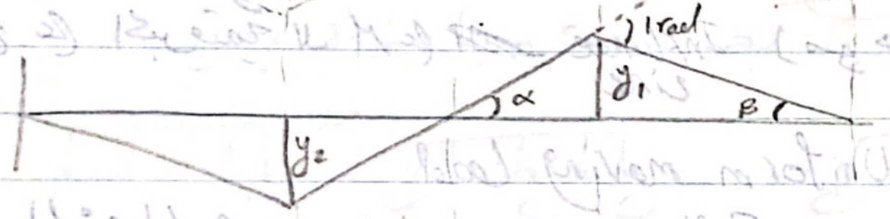


$\alpha + \beta = 1 \text{ rad}$

$\frac{y}{L_1} + \frac{y}{L_2} = 1 \Rightarrow y = ?$

Example :- Determine the maximum positive moment that can be developed at point D in the beam shown. due to a concentrated moving load of 4000 lb. a uniform moving load of 300 lb/ft, and a beam weight of 200 lb/ft : $M_{D_{max}} = ?$

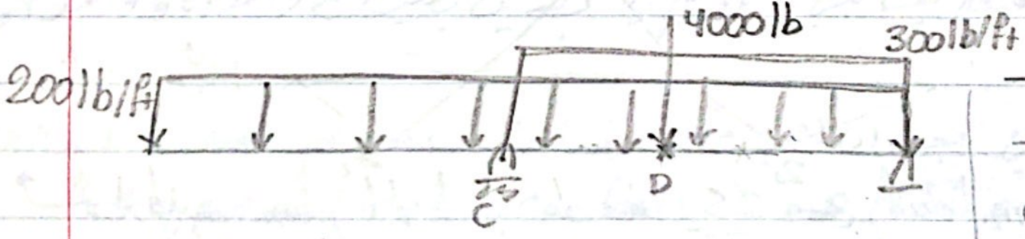




influnce line at D

$$\frac{2y_1}{(2)5} + \frac{y_1}{10} = 1 \Rightarrow \frac{3}{10}y_1 = 1 \Rightarrow y_1 = \frac{10}{3}$$

$$\frac{y_1}{5} = \frac{y_2}{5} \longrightarrow y_1 = y_2 = \frac{10}{3}$$ من تشابه المثلثات



- dead load :-
→ weight
(ع جسم ال beam.)
- Moving load :-

① Cons. Load 1 ⇒ $y_1 = \frac{10}{3}$
Cons. Load 4000 ⇒ $y_1 = ??$
$M_D = y_1 \times 4000\left(\frac{10}{3}\right)$

② Uniform load moving
W (Area under Influnce Line)

① concentrated load
لتعطي الجر M عند D بنحط الـ اعلى قيمة موجبة للـ Influnc Line (عند D).

② uniform load :-

Load ←

• لتعطي M_{max} (Positive) نغطي الجزء الذي فيه influnce line يكون موجب

$M_{DU} = 3000 \left[\frac{1}{2} \times 15 \times \frac{10}{3} \right]$.

⑤ dead load & (weight)

$M_{Dw} = w$ (area under influnce line)
مقارن

$= 200 \left(\frac{1}{2} \times 15 \times \frac{10}{3} + \frac{1}{2} (10 \times \frac{10}{3} \right)$

$M_{D\,max.} = 400 \left(\frac{10}{3} \right) + 300 \left[\frac{15}{2} \left(\frac{10}{3} \right) \right] + 200 \left[\frac{15}{2} \left(\frac{10}{3} \right) - \frac{10}{2} \left(\frac{10}{3} \right) \right]$

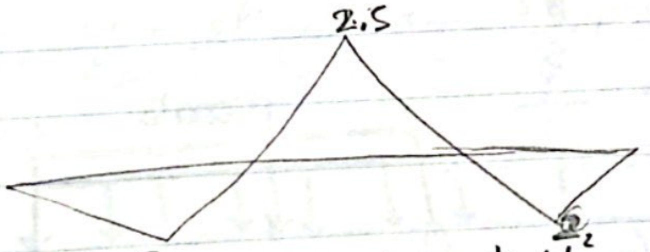
$= 2250$ lb.ft.

Summary :- ① Concentrated moving load
يوضع ع اكبر قيمة لل M ع Influnce line (موجبة)

② Uniform moving load
يوضع ع الجزء الموجب من ال Influnce line.

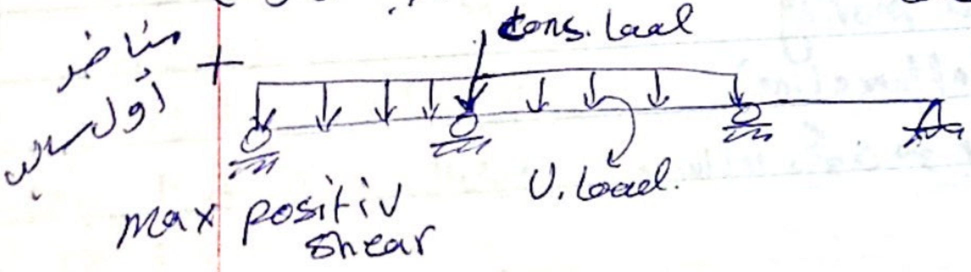
③ dead load يوضع بالمكان الذي يكون فيه.

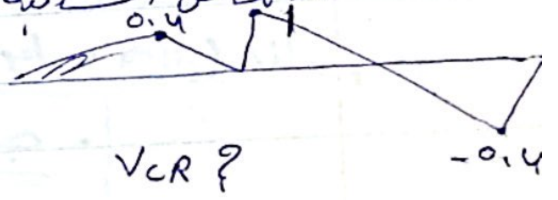
note:-

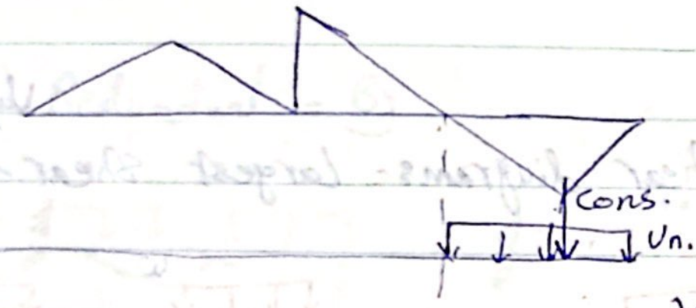


اذا طلب $+M_{Max}$ بنحط عند 2.5 ⇐ cons. m. load.
اما اذا طلب $-M_{max}$ بنحطها اما عند 2₁ أو عند 2₂.

ما Shear لا يهم موجب أو سالب نجد اكبر قيمة من موسالبة
واجرب حالتين لـ destributed load مرة ع الاماكن الموجبة ومرة ع الاماكن السالبة

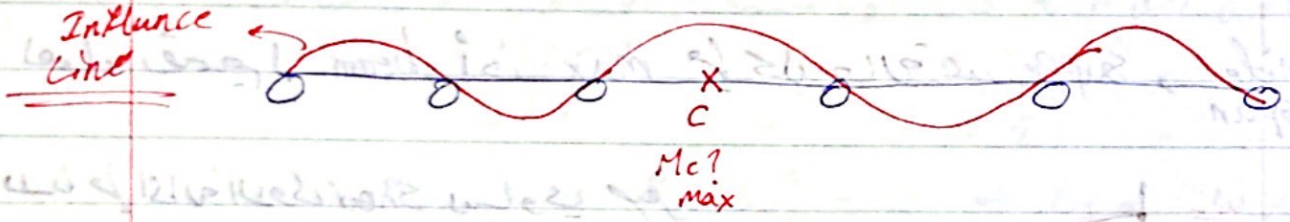




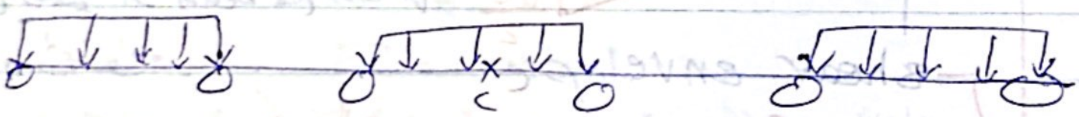


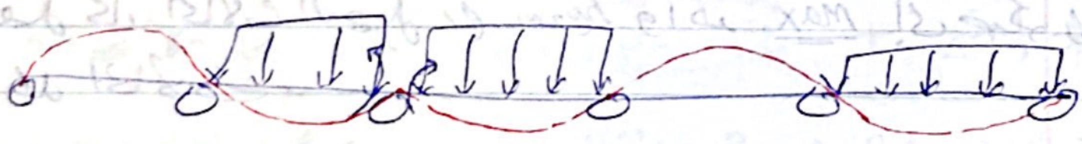
ومرة اخرى مناخذها عند
الـــــسالبة ㊀
ومنشوف الأكبر فيهم
يكون هو V max.

influnce line in determinal structure ⇒ ~~the~~ linear
. " " in Indeterminat " ⇒ ~~not always~~.
non Linear



للوصول لـ M_c Max بحمل span واي جنبه لا, يعني -:





اذا بدنا ايا ع support

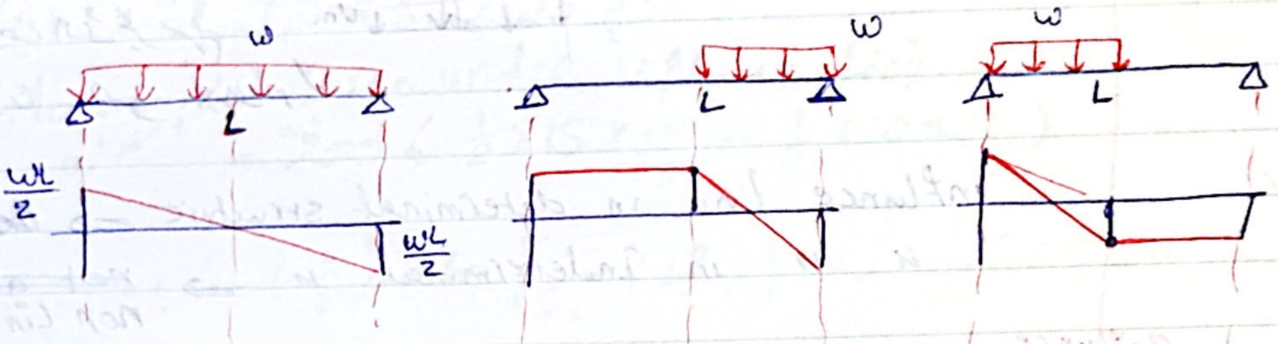
منحط اللود على جانبي الـ support بصير يصير اخر واحد او واحد كذا.

نحشين جو جريشين يجب ان اهتم فيهم عند ايجاد max. هم mide span و Supports
ويهم المنشئ بناءً على هذه القيم (worst value load) عند key points
فيمكن رسم للـ beam Largest Bending mom. عند هذه النقاط على الـ beam (اعلى قيم)
احصل عليهن من رسم mide span و support جميعا لـ (influnce line) End points.
واقارنها كم اكبر Mom. تحدث عند كل نقطة واعمل تمثيل بياني بحيث Max. Mom.
مع X-axis للـ beam فينتج رسمة للـ beanding mom. على كامل البيم تسمى moment Envelope.

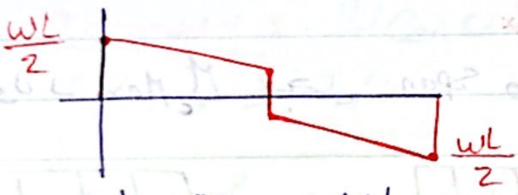
The larges beanding moment at catch point on the beam.

ونفس الفكرة للـ Shear on the beam

Envelope shear diagrams - Largest shear on the beam



لعمل تصميم لـ beam أخذ max من كل حالة عند supp. , mide span

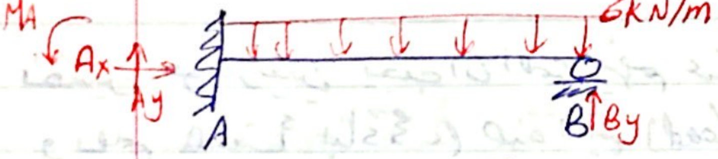


لأنه في الحالة الأولى shear يساوي صفر في mide span نأخذ الحالة الثانية لا ينفع ان يصمم الـ beam على أساس Zero shear في منتصفه .

shear envelope
يكون التصميم على حسبه وليس على حسب الثانية

⇐ باختصار نعمل كل الحالات التحميل على beam وأخذ max. إلي حيكون على كل نقطة من كل الحالات .

Indetermenet beams :-



منه نعرف ان هناك سبورت 1 احتياطي
→ 4 Reac. > 3equ
⇒ Indetermenet.
To the first degree. 4-3=1

زيادة الـ support في الـ beam. يعزز من قوته (احتياطاً) (أماناً)

support B ⇒ redundant (إذا أزيل لا يبقى الـ beam stable)

MA , By → redundant support.

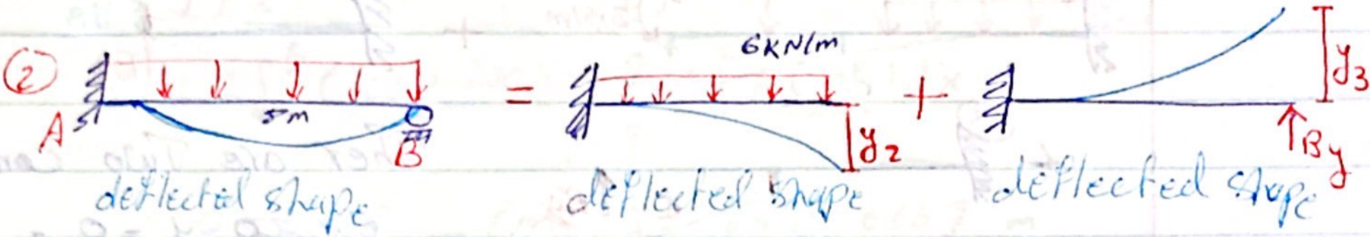
لأنه لو أزلنا أي واحد منهم دون الآخر يبقى الـ beam. ⇐ Stable

ليتم تحليل هذا الـ beam يجب استخدام contability equation تبنى من خلال (Force method) load أو desplacement desplacement

Super position

① Redundunt = By

②



③ $y_B = 0$ (deflection distance for B. zero)

$y_2 + y_3 = 0$

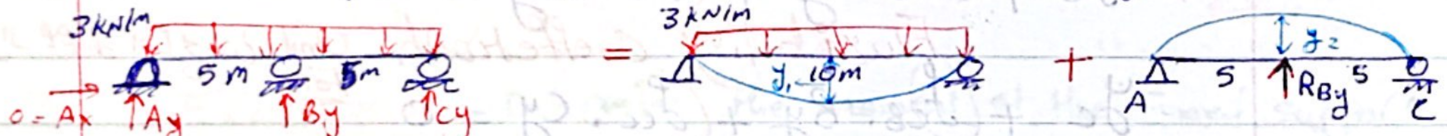
بحسب y لكل حالة كال أو باستخدام الجداول آخر الكتاب.

$$y_2 = -\frac{(6)(5)^4}{8EI} \quad , \quad y_3 = \frac{(By)(5)^3}{3EI}$$

$$-\frac{6(5)^4}{\cancel{8EI}} + \frac{By(5)^3}{\cancel{3EI}} = 0 \Rightarrow By = \frac{90}{8} \text{ KN.}$$

Example ② :- EI = constant.

R = 4 > Eq = 3 → Indet. beam to the First degree.



اعتبرنا ان B هو Redun. فنشتغل على ⟸ B و C ازالة اي support منهم لا يتغير حالة

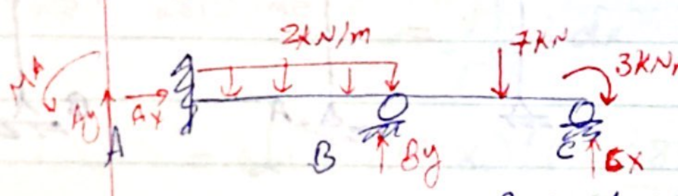
هنا ال ~~حله~~ لبناء المعادلة. ال beam يبقى (stable).

$$y_1 + y_2 = 0 \rightarrow \frac{-5wL^4}{384\cancel{EI}} + \frac{By(L)^3}{48\cancel{EI}} = 0$$ أو باستخدام Virtual

$$By = \frac{5(3)(48)(10)}{384} \text{ KN.}$$ ثم نوجد Ay و Cy

Example ③ :-

5 Reactions > 3 equations.

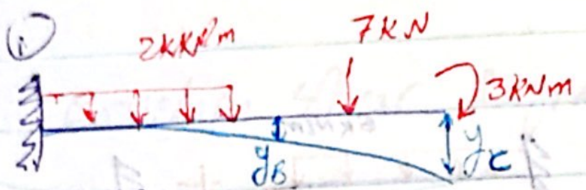


⟹ Indetermenat on 2^{nd} degree.

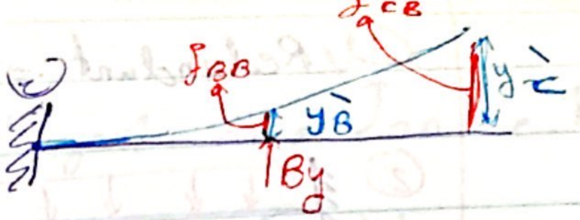
⟹ Tow comp. equations.

MA, Cy / MA, By / By, Cy

⟸ ممكن استخدام اي واحدة كـ Redundant



+



Ther are Two contab. eq.

$y_c = 0$, $y_B = 0$

$$y_B + y_B' + y_B'' = 0 \quad , \quad y_c + y_c' + y_c'' = 0$$

لو افترضنا ان مقدار By = 1 Unit ل beam 2 وايجاد f_{BB} و f_{CB}

f_{BB} → هو مقدار ال deflect الناتج عن 1 unit Force عند B ل beam ②

f_{CB} → عند C ل beam ②

وافعل نفس الخطوة عند beam 3 واوجد f_{CB} و f_{CC}

ل 1 unit عند C

$$y_B' = f_{BB} \cdot B_y \quad , \quad y_c' = f_{CB} \cdot B_y \quad , \quad y_B'' = f_{BC} \cdot C_y$$

$$y_c'' = f_{CC} \cdot C_y$$

$$\Rightarrow y_B + f_{BB} \cdot B_y + f_{BC} \cdot C_y = 0$$

Fluxability Coeffetiont. مقدار ال deff الناتج عن Unit Force Δ

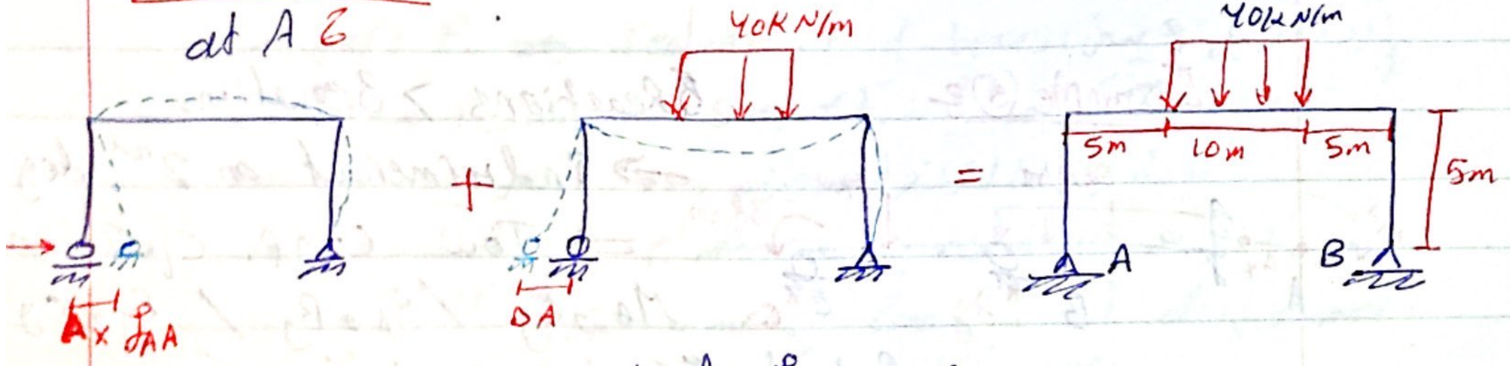
$$y_c + f_{CB} \cdot B_y + f_{CC} \cdot C_y = 0$$

يتم ايجا

$$\begin{bmatrix} y_B \\ y_c \end{bmatrix} + \begin{bmatrix} f_{BB} & f_{BC} \\ f_{CB} & f_{CC} \end{bmatrix} \begin{bmatrix} B_y \\ C_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Flexibility matrix

Example :- Determine the horizantal support reaction at A ?



$$D_A + A_x \cdot f_{AA} = 0$$

$$\Delta_A = \int_0^L \frac{Mm}{EI}\,dx = 2\int_0^5 \frac{(0)(1x_1)}{EI}\,dx_1 + 2\int_0^5 \frac{(200x)(-5)}{EI}\,dx_2$$

$$+ 2\int_0^5 \frac{(100+200x_3-20x_3^2)(-5)}{EI}\,dx_3$$

$$= 0 - \frac{25000}{EI} - \frac{66\,666.7}{EI} = \frac{-91\,666.7}{EI}\ m$$

نستخدم $\underline{m}$ التي اوجدناها للأول مطلوب Δ_A فنكون اوجدنا m لـ 1unit افقي عند A فنستخدمها نفسها.

$$f_{AA} = \int_0^L \frac{mm}{EI}\,dx = 2\int_0^5 \frac{(1x_1)^2\,dx_1}{EI} + 2\int_0^5 (5)^2\,dx_2$$

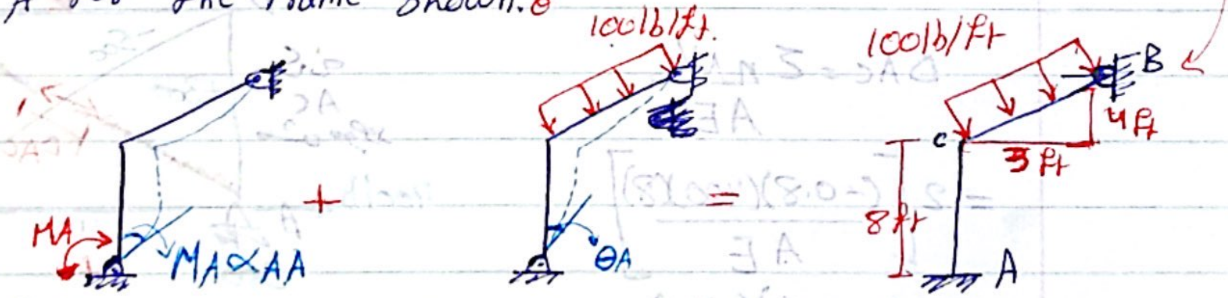
$$+ 2\int_0^5 (5)^2\,dx_3 = \frac{583.33}{EI}\ m$$

$$0 = \frac{-91666.7}{EI} + A_x\left(\frac{583.33}{EI}\right)$$

$$\Rightarrow A_x = 157\,KN$$

عندما يكون عل pin support الحائط يكون له 2Reaction

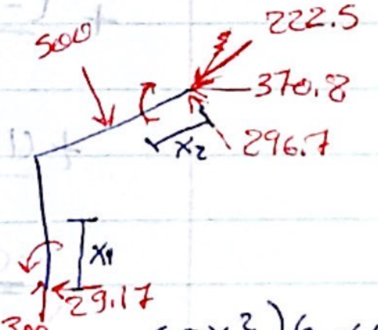
Example 8- Determine the moment at the Fixed support A for the frame shown.8



$$\theta_A + M_A \cdot \alpha_{AA} = 0$$

$$M_1 = 29.17x_1$$

$$M_2 = 296.7x_2 - 50x_2^2$$



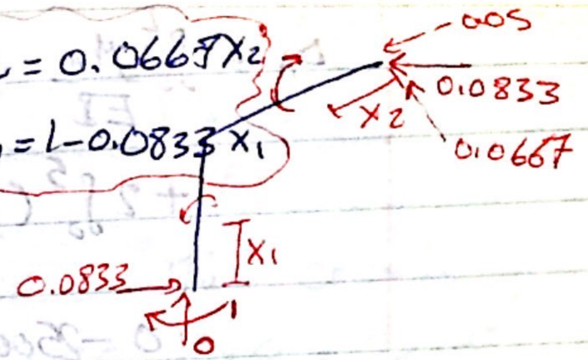
$$\theta_A = \Sigma \int_0^L \frac{Mm_\theta}{EI}\,dx$$

$$= \int_0^8 \frac{(29.17x_1)(1-0.0833x_1)}{EI}\,dx + \int_0^5 (296.7x_2 - 50x_2^2)(0.066 x_2)\,dx$$

$$= \frac{518.5}{EI} + \frac{303.2}{EI} = 821.8/EI$$

$m_2 = 0.0667x_2$

$m_1 = 1 - 0.0833x_1$



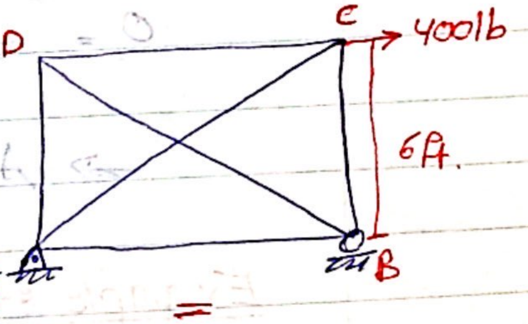
$$\alpha_{AA} = \Sigma \int_0^L \frac{m_\theta m_\theta}{EI} dx$$

$$= \int_0^8 \frac{(1-0.0833x_1)^2}{EI} dx + \int_0^5 \frac{(0.0667x_2)^2}{EI} dx_2 = \frac{3.85}{EI} + \frac{0.185}{EI} = \frac{4.04}{EI}$$

$$0 = \frac{821.8}{EI} + M_A \left(\frac{4.04}{EI}\right) \Rightarrow M_A = \ominus 804 \text{ lb.ft}$$

بعكس الاتجاه المفروض

Example 8- Determine the force in member AC of the truss shown. ?

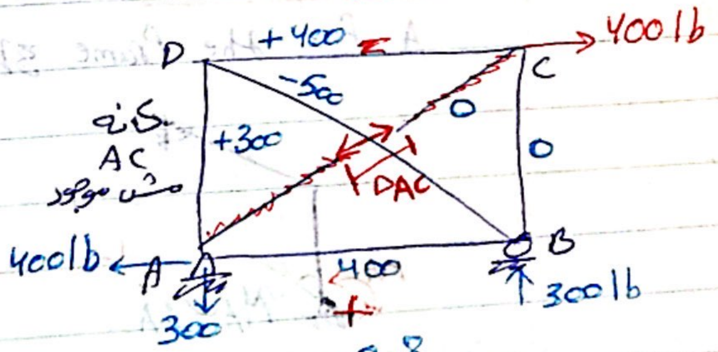


$R + m \overset{?}{=} 2j \Rightarrow 3 + 6 = 4 \times 2$

$9 > 8$ indet. to 1st degree

بنلغي member بحيث يبقى ال truss ← Stable

اصلا، عادة "ما يطلب" force عنده ال السؤال.

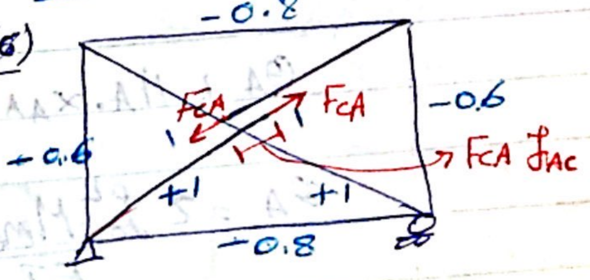


$$D_{AC} = \Sigma \frac{nNL}{AE}$$

$$= 2\left[\frac{(-0.8)(400\times8)}{AE}\right] + \frac{(-0.6)(0)(6)}{EA} + \frac{(-0.6)(300)(6)}{EA} + \frac{(1)(-500)(10)}{EA} + \frac{(1)(0)(10)}{EA}$$

$$= \frac{\ominus 11200}{AE}$$

[illegible]



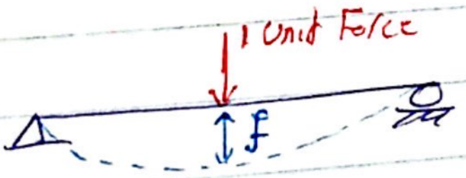
اصلا كأنه unit (1) = FcA

$$f_{AC} = \sum \frac{n^2 L}{AE} = 2\left[\frac{-0.8^2(8)}{AE}\right] + 2\left[\frac{(-0.6)^2(6)}{EA}\right]$$

$$+\ 2\left[\frac{(1)^2(10)}{EA}\right] = \frac{34.56}{AE}$$

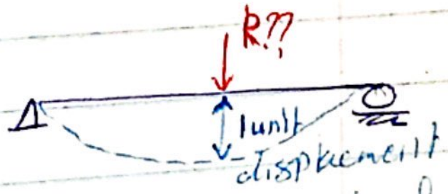
$$\frac{-11200}{AE} + \frac{34.56}{AE} F_{AC} \Rightarrow F_{AC} = 324\ lb\ (T)$$

① Force method (f).
(Flexibility).



f ?? displacement.

② Displacement method (k)
(stiffness method).



R?? Force Applyed.