Fundamentals Physics

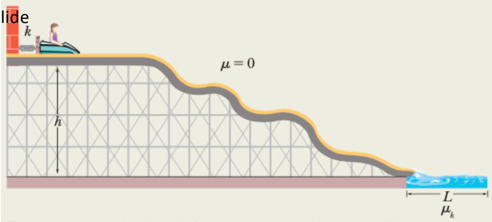
Tenth Edition

Halliday

Chapter 8_3

Potential Energy and Conservation of Energy

Sample Problem 8.06 Lots of energies at an amusement park water slide



- A glider is shot by a spring, (frictionless) track, ground-level track gradually brought to rest by friction, m = 200 kg, the initial compression of the spring is d = 5.00 m, the spring constant is k = 3.20 x 10^3 N/m, the initial height is h = 35.0 m, and the μ_k along the ground-level track = 0.800.
- Through what distance L does the glider slide along the ground-level track until it stops?

The system is *isolated* and thus its total energy cannot change, no external force doing work on the system:

Eq. 8-37: $E_{mec,2} = E_{mec,1} - \Delta E_{th}$.

To find distance L?

In the initial state : $E_{\text{mec},1} = K_1 + U_{e1} + U_{g1} = 0 + ½\, kd^2 + mgh$.
In the final state : $E_{\text{mec},2} = K_2 + U_{e2} + U_{g2} = 0 + 0 + 0$
$\Delta E_{\text{th}} = f_k\, L = \mu_k F_N\, L = \mu_k\, mg\, L$

Substituting we have: $0 = \frac{1}{2}kd^2 + mgh - \mu_k mgL,$

$$L = \frac{kd^2}{2\mu_k mg} + \frac{h}{\mu_k}$$
$$= \frac{(3.20 \times 10^3\ \text{N/m})(5.00\ \text{m})^2}{2(0.800)(200\ \text{kg})(9.8\ \text{m/s}^2)} + \frac{35\ \text{m}}{0.800}$$
$$= 69.3\ \text{m.} \qquad \text{(Answer)}$$

*13 A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring.

(a) What is the change ΔU_g of the marble–Earth system during the 20 m ascent?
(b) What is the change ΔU_s elastic potential energy of the spring during its launch of the marble?
(c) What is k of the spring?

Take the reference point for gravitational potential energy to be at the position of the marble when the spring is compressed. The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$

The energy stored in the spring is $U_s = kx^2/2$

(a) The height of the highest point is h = 20 m. With initial gravitational potential energy set to zero, we find

$$\Delta U_g = mgh = (5.0\times10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(20\ \text{m}) = 0.98\ \text{J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is U_s = 0.98 J. This must be ½ k x^2, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{0.98\ \text{J}}{(0.080\ \text{m})^2} = 3.1\times10^2\ \text{N/m} = 3.1\ \text{N/cm}.$$

In general, the marble has both kinetic and potential energies: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgy$

At the maximum height y_{max}=h, v = 0 and mgh=kx^2/2, or $h = \frac{kx^2}{2mg}$

••25 At t = 0 a 1.0 kg ball is thrown from a tall tower with $\vec{v} = (18\text{ m/s})\hat{\text{i}} + (24\text{ m/s})\hat{\text{j}}$

What is Δ*U* of the ball–Earth system between t = 0 and t = 6.0 s (still free fall)?

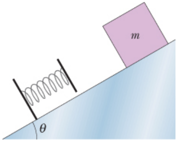
Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0_y} t - \frac{1}{2} g t^2$$ = 24x6 – ½ x 9.8 x 36 = 144-176 = -32

This leads to $\Delta y = -32\text{ m}$. Therefore

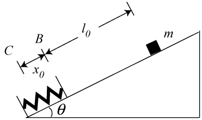
$$\Delta U = mg\Delta y = -318\text{ J} \approx -3.2\times10^{-2}\text{ J}$$

** 29 In the Figure, a block of mass m = 12 kg is released from rest on a frictionless incline of angle θ = 30°. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm.
(a) How far does the block move down the incline from its rest position to this stopping point?
(b) What is the speed of the block just as it touches the spring?



The block stops momentarily before sliding back up again. Refer to its starting point as *A*, into contact with the spring as *B*, and the point where the spring is compressed by x_0=0.055 m as C.

Point *C* is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed.



Information in the question allows us to compute the spring constant. From Hooke's law, we find $k = \frac{F}{x} = \frac{270\ \text{N}}{0.02\ \text{m}} = 1.35\times10^4\ \text{N/m}$.

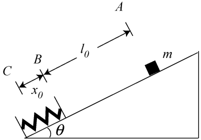
The distance between points *A* and *B* is l_0, and we note that the total sliding distance l_0+x_0 is related to the initial height h_A of the block (measured relative to *C*) by $\sin\theta = \frac{h_A}{l_0+x_0}$, where the incline angle θ is 30°.

(a) Mechanical energy conservation leads to $K_A+U_A = K_C+U_C \;\Rightarrow\; 0+mgh_A = \frac{1}{2}kx_0^2$

which yields $h_A = \frac{kx_0^2}{2mg} = \frac{(1.35\times10^4\ \text{N/m})(0.055\ \text{m})^2}{2(12\ \text{kg})(9.8\ \text{m/s}^2)} = 0.174\ \text{m}.$

Therefore, the total distance traveled by the block before coming to a stop is $l_0+x_0 = \frac{h_A}{\sin30°} = \frac{0.174\ \text{m}}{\sin30°} = 0.347\ \text{m} \approx 0.35\ \text{m}.$

(b) What is the speed of the block just as it touches the spring?



From result in (a), we find $l_0 = x_0 = 0.347\text{ m} - 0.055\text{ m} = 0.292\text{ m}$,

which means that the block has descended a vertical distance:

$|\Delta y| = h_A - h_B = l_0 \sin\theta = (0.292\text{ m})\sin 30° = 0.146\text{ m}$ in sliding from point *A* to point *B*.

Thus, we have: $0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B \;\Rightarrow\; \frac{1}{2}mv_B^2 = mg|\Delta y|$

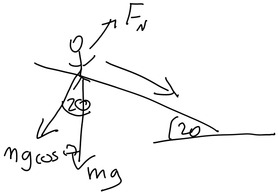
which yields $v_B = \sqrt{2g|\Delta y|} = \sqrt{2(9.8\text{ m/s}^2)(0.146\text{ m})} = 1.69\text{ m/s} \approx 1.7\text{ m/s}$

Note that energy is conserved in the process. The total energy of the block at position *B* is

$$E_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}(12\text{ kg})(1.69\text{ m/s})^2 + (12\text{ kg})(9.8\text{ m/s}^2)(0.028\text{ m}) = 20.4\text{ J},$$

which is equal to the elastic potential energy in the spring: $\frac{1}{2}kx_0^2 = \frac{1}{2}(1.35\times10^4\text{ N/m})(0.055\text{ m})^2 = 20.4\text{ J}$

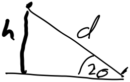
••54 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10.
(a) How much energy is transferred to thermal energy?
(b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?



(a) $\Delta E_{th} = f_k d$, $f_k = \mu_k F_N$
Using the force analysis (Chapter 6), we find the normal force

$$F_N = mg\cos\theta \quad (mg = 267\ \text{N}),\ f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Eq. 8-31: $$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20^\circ = 1.5\times10_2\ \text{J}.$$



(b) The potential energy change is $\Delta U = mg(-d\sin\theta) = (267\ \text{N})(-6.1\ \text{m})\sin 20^\circ = -5.6\times10^2$ J.
The initial kinetic energy is $$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\ \text{N}}{9.8\ \text{m/s}^2}\right)(0.457\ \text{m/s}^2) = 2.8\ \text{J}.$$

$$\sin\theta = \frac{h}{d}$$

$$h = d\sin\theta$$

Therefore, using Eq. 8-33 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - (-5.6\times10^2) - 1.5\times10^2 = 4.1\times10^2\ \text{J}.$$

Consequently, the final speed is

$$v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$$

••56 You push a 2.0 kg block against a horizontal spring, compressing the spring by 15 cm. Then you release the block, and the spring sends it sliding across a tabletop. It stops 75 cm from where you released it. The spring constant is 200 N/m. What is the block–table coefficient of kinetic friction?

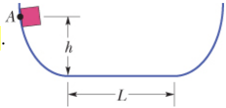
Energy conservation, as expressed by Eq. 8-33 (with W = 0) leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$

$$\Rightarrow\; \mu_k mgd = \frac{1}{2}(200\,\text{N/m})(0.15\,\text{m})^2 \;\Rightarrow\; \mu_k(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.75\,\text{m}) = 2.25\ \text{J}$$

which yields μ_k = 0.15 as the coefficient of kinetic friction.

***65 A particle can slide along a track with elevated ends and a flat central part, as shown in figure. The flat part has length L = 40 cm. Curved portions are frictionless, but the flat part μ_k = 0.20. The particle is released from rest at point A, which is at height $h = L/2$.



How far from the left edge of the flat part does the particle finally stop?

- The initial and final kinetic energies are zero.
- Energy conservation in the form of Eq. 8-33 ($W=\Delta E_{mec}+\Delta E_{th}$, with W = 0)
- Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on.
- If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$.
- If during its second pass through, then the total thermal energy is $\Delta E_{th} = \mu_k mg\,(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$).
- Generalizing to the n^{th} pass through, we see that $\Delta E_{th} = \mu_k mg\,[(n-1)L + d]$

In this way, we have $mgh = \mu_k mg\left((n-1)L + d\right)$

which simplifies (when $h = L/2$ is inserted, given in the question) to $\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$

At n=0 The first two terms give $1+1/(2\mu_k)=1+1/(2(0.2))=3.5$, so that the requirement $0 \le d/L \le 1$ demands that n = 3.

We arrive at the conclusion that d/L= ½ , or

$$d = \frac{1}{2}L = \frac{1}{2}(40\text{ cm}) = 20\text{ cm}$$ This occurs on its third pass through the flat region.