Problem

Let X and Y be sets, let A and B be any subsets of X, and let C and D be any subsets of Y. Determine which of the properties are true for all functions F from X to Y and which are false for at least one function F from X to Y. Justify your answers.

Exercise

For all subsets C and D of Y,

F-1$(C \cup D) = F$-1$(C) \cup F$-1(D).

Step-by-step solution

Step 1 of 3

Let F be a function from set x to set y.

And suppose $C \subseteq y$ and $D \subseteq y$ we must show that

$F^{-1}(C \cup D) = F^{-1}(C) \cup F^{-1}(D)$ in two parts

Step 2 of 3

We show $F^{-1}(C \cup D) \subseteq F^{-1}(C) \cup F^{-1}(D)$

Let $x \in F^{-1}(C \cup D)$ then we show that $x \in F^{-1}(C)$ or $x \in F^{-1}(D)$

$x \in F^{-1}(C \cup D) \Leftrightarrow F(x) \in (C \cup D)$

$\Leftrightarrow F(x) \in C \text{ or } F(x) \in D$

$\Leftrightarrow x \in F^{-1}(C) \text{ or } x \in F^{-1}(D)$

$\Leftrightarrow x \in F^{-1}(C) \bigcup F^{-1}(D)$

Therefore $F^{-1}(C \cup D) \subseteq F^{-1}(C) \cup F^{-1}(D)$ ...... (1)

Step 3 of 3

Now we show that

$F^{-1}(C) \cup F^{-1}(D) \subseteq F^{-1}(C \cup D)$

Let $x \in F^{-1}(C) \cup F^{-1}(D)$

$\Leftrightarrow \quad x \in F^{-1}(C) \text{ or } x \in F^{-1}(D)$

$\Leftrightarrow \quad F(x) \in C \text{ or } F(x) \in D$

$\Leftrightarrow \quad F(x) \in (C \cup D)$ by definition of union

$\Leftrightarrow \quad x \in F^{-1}(C \cup D)$

So $F^{-1}(C) \cup F^{-1}(D) \subseteq F^{-1}(C \cup D)$ ...... (2)

So by (1) and (2) we can write

$F^{-1}(C \cup D) = F^{-1}(C) \cup F^{-1}(D)$