Integration

Def A function $F(x)$ is called Antiderivative of function $f(x)$ on interval I if

$$F'(x) = f(x) \quad \forall\, x \in I$$

Exp Find 3 antiderivatives for $f(x) = 2x$

$F_1(x) = x^2 + 2$

$F_2(x) = x^2 - \frac{1}{2}$

$F_3(x) = x^2 + \pi$

F_4

F_5

$\vdots$

$$\boxed{F(x) = x^2 + C}$$

التكامل غير المحدود

Indefinite Integral

The set of all anti derivatives

Exp $\int \underbrace{3x^2}_{f(x)}\, dx = \cancel{3}\frac{x^3}{\cancel{3}} + c = \underbrace{x^3 + c}_{F(x)}$

$$-\int \underbrace{\quad}_{f(x)} \qquad \cancel{(3)} \qquad \underbrace{\quad}_{F(x)}$$

$$F(x) = \int f(x)\,dx$$

(1) $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c$, $n \neq -1$

$\int x^4 \, dx = \dfrac{x^5}{5} + c$

$\int (2x^1 - 5x^2 + 4)\,dx = \int (2x^1 - 5x^2 + 4x^0)\,dx$

$= 2\dfrac{x^2}{2} - 5\dfrac{x^3}{3} + 4\dfrac{x^1}{1} + c$

$= x^2 - \dfrac{5}{3}x^3 + 4x + c$

(2) $\int \sin x \, dx = -\cos x + c$

(3) $\int \cos x \, dx = \sin x + c$

(3) $\int \sec^2 x \, dx = \tan x + c$

④ $\int \csc^2 x \, dx = -\cot x + c$

⑤ $\int \sec x \tan x \, dx = \sec x + C$

⑥ $\int \csc x \cot x \, dx = -\csc x + C$

Exp Find

① $\int (x^{-3} - 3x^2 + \sqrt{2}) dx = \frac{x^{-2}}{-2} - 3\frac{x^3}{3} + \sqrt{2}x + c$

$= \frac{-1}{2x^2} - x^3 + \sqrt{2}x + C$

② $\int \cos^2 x \, dx$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \cos^2 x - \sin^2 x$

$\cos 2x + 1 = 2\cos^2 x$

$\boxed{\frac{1+\cos 2x}{2} = \cos^2 x}$

$2\sin^2 x = 1 - \cos 2x$

$\int \frac{1+\cos 2x}{2} dx$

$\int (\frac{1}{2} + \frac{1}{2}\cos 2x) dx$

$\frac{1}{2}x + \frac{1}{2}\int \cos 2x \, dx$

$\xrightarrow{f'} \frac{1}{2} \quad \cos(2x) \to 2 \quad \cos 2x$

$\frac{x}{2} + \frac{1}{2} \frac{\sin 2x}{2} + C$

$$\frac{x}{2} + \frac{1}{2}\left(\frac{\quad}{2}\right) + C$$

$$\frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

(3) $\int \sin^2 x \, dx$

$$\int \frac{1-\cos 2x}{2}\, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \frac{1}{2}x - \frac{1}{2}\int \cos 2x \, dx$$

$$= \frac{x}{2} - \frac{1}{2}\,\frac{\sin 2x}{2} + C$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(4) $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\, d\theta$

$$\int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\, d\theta$$

$$\int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}}\, d\theta$$

$$\int \frac{\frac{1}{\cancel{\sin\theta}}}{\frac{1-\sin^2\theta}{\cancel{\sin\theta}}}\, d\theta = \int \frac{d\theta}{1-\sin^2\theta}$$

$$\int \frac{d\theta}{\cos^2\theta} = \int \sec^2\theta \, d\theta$$

$$= \tan\theta + C$$

(5) $$\int \frac{\sec t \;\; \tan t}{\sqrt{\sec t}}\, dt$$

محاولة فاشلة

$$\int \frac{\sec t \;\; \tan t}{\sqrt{\sec t}} \; \frac{\sqrt{\sec t}}{\sqrt{\sec t}} \, dt$$

$$\int \frac{\sec t \ \tan t \ \sqrt{\sec t}}{\sec t} \, dt$$

$$\int \frac{\sec t \ \tan t}{\sqrt{\sec t}} \, dt = \int \frac{\frac{1}{\cos t} \ \frac{\sin t}{\cos t}}{\sqrt{\frac{1}{\cos t}}} \, dt$$

طريقة صحيحة

$$\int \frac{\sec t \ \tan t}{\sqrt{\sec t}} \, dt = \int \sec^{1} t \, (\sec t)^{-\frac{1}{2}} \tan t \, dt$$

$$= \int \sqrt{\sec t} \ \tan t \ dt$$

طريقة صحيحة

$$\int \frac{\sec t \ \tan t}{\sqrt{\sec t}} \, dt$$

$$(\sec t)' = \sec t \ \tan t$$

$$u = \sec t$$

$$du = \sec t \ \tan t \ dt$$

$$\int \frac{du}{\sqrt{u}}$$

$$\int u^{-\frac{1}{2}} \, du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C$$

$$\int u^{-\frac{1}{2}}\,du = \frac{u}{\frac{1}{2}} + c = 2\sqrt{u} + c$$

$$= 2\sqrt{\sec t} + c$$

$\int f(x)\,dx$ indefinite integral

$\int_a^b f(x)\,dx$ definite integral

جواب رقم

Exp $\int_0^1 (6x^2 - 4x + 2)\,dx$

$$= 6\frac{x^3}{3} - 4\frac{x^2}{2} + 2x \Big|_0^1$$

$$= 2x^3 - 2x^2 + 2x \Big|_0^1$$

$$= 2(1)^3 - (2)(1)^2 + (2)(1) - 0$$

$$= 2 - 2 + 2$$

$$= \boxed{2}$$

Exp Find y' if $y = \int_0^3 \frac{\tan^3 x}{1 - \sin^4 x}\, dx$

$y =$ رقم

$y' = 0$

Exp $\int_0^{\pi} \left(\cos x + |\cos x|\right) dx$ رقم

$$\boxed{\int_0^{\pi} \cos x \, dx} + \int_0^{\pi} |\cos x| \, dx$$

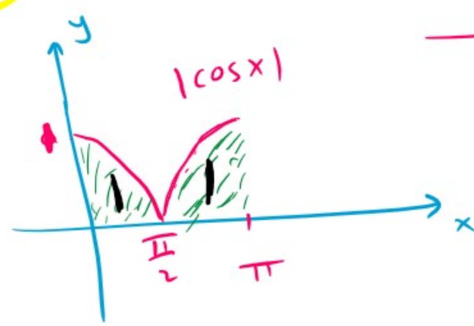
موجب

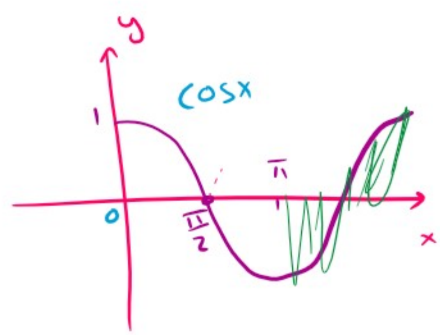
$\sin x \Big|_0^{\pi}$

$\sin \pi - \sin 0$

$0 - 0$

0





$$|\cos x| = \begin{cases} \cos x & \text{if } 0 \le x \le \frac{\pi}{2} \\ -\cos x & \text{if } \frac{\pi}{2} \le x \le \pi \end{cases}$$

$$0 + \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$$

$$0 + \int_{0} \cos x \, dx + \int_{\frac{\pi}{2}} -\cos x \, dx$$

$$\sin x \Big|_{0}^{\frac{\pi}{2}} + -\sin x \Big|_{\frac{\pi}{2}}^{\pi}$$

$$\sin\frac{\pi}{2} - \sin 0 + \left(-\sin\pi - -\sin\frac{\pi}{2}\right)$$

$$1 - 0 + (0 + 1)$$

$$1 + 1 = 2$$

$$\int_a^b f(x)\,dx \longrightarrow$$ رقم

مساحة فقط اذا كان $f(x) \geq 0$

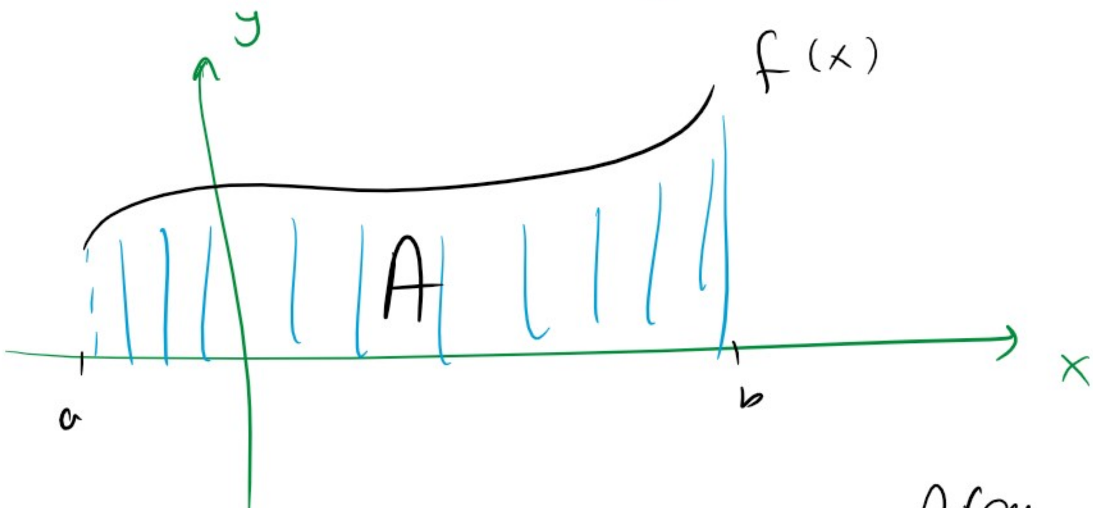
Area

If $f(x) \geq 0$ on $I = [a,b]$ then

$$\int_a^b f(x)\,dx = \text{area}$$

المساحة المحصورة بين $f(x)$ ومحور

$= \text{area bounded by } f \text{ and } x\text{-axis}$



$$\int_a^b f(x)\,dx = A \longrightarrow \text{Area}$$

Exp Find ① $\int_0^4 x \, dx$

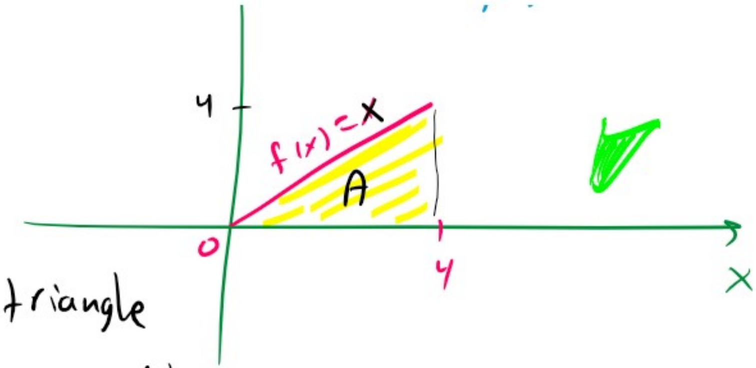
$$= \frac{x^2}{2}\Big|_0^4 = \frac{4^2}{2} - \frac{0^2}{2} = \frac{16}{2} = 8$$

$f(x) = x$ on $[0,4]$ $\qquad$ $f > 0$ on $[0,4]$

$y = x$

$f(x) = x$ on $[0, 4]$

$y = x$



$$A = \int_0^4 x \, dx = \text{area of triangle}$$

$$= \frac{1}{2}(\text{base})(\text{hieght})$$

$$= \frac{1}{2}(4)(4)$$

$$= 8$$