Chapter 1: probability And distributions.

1.1 : Introduction.

- Terms :

- Experement / Random experiment.
- outcome / possible outcome / sample point.
- sample space / Experimental.

exmple 1:

Experement : Toss a coin

outcomes : Tails (T) and heads (H).

sample space : {T, H}.

example 2 :

Experement : Roll | cast one red die and one white die.

outcomes : (1,1), (1,2), (1,3), ..., (6,6).

sample space : {(1,1), (1,2), ..., (6,6)}.

Notations :

$\mathcal{C}$: sample space.

C : Event.

c : sample point.

P : probability.

Terms :

- Event .
- probability / probability measure .
- classical Approach .
- Relative frequency Approach .
- subjective or personal Approach .

exp3 :

$\mathcal{E} = \{ (1,1), \ldots , (6,6) \}$.

$C = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$.

$N = 400$ times .

f: frequancy of a sum of seven $\rightarrow f = 60$

$\rightarrow$ Relative frequancy $= \frac{f}{N} = \frac{60}{400} = 0.15$.

Thus, we might associate with C a number p that is close to 0.15, and p would be called the probability of the event C .

1.2 : set Theory .

- Terms :

· set : collection .
· member : point . } undefined terms .

Def 1 :

If $A_1 \subset A_2$ and $A_2 \subset A_1 \Leftrightarrow A_1 = A_2$.

example 1 :

$A_1 = \{x : 0 \leq x < 1\} = [0,1)$.

$A_2 = \{x : -1 \leq x \leq 2\} = [-1,2]$. } ON $\mathbb{R}^1$.

↳ $A_1 \subset A_2$.

Notation : $A_1 \subset A_2$: A_1 subset A_2 .

example 2 :

$A_1 : \{(x,y) : 0 \leq x = y \leq 1\}$ → خط مستقيم

$A_2 : \{(x,y) : 0 \leq x \leq 1 , 0 \leq y \leq 1\}$ → Rejon } ON $\mathbb{R}^2$

↳ $A_1 \subset A_2$

Def 2 : If $A = \{ \ \} \Rightarrow A$ is called the null set .

This indicated by $A = \phi$.

$\rightarrow$ probability $\emptyset = 0$.

Def 3 :

$A_1 \cup A_2$: A_1 union A_2 .

$A_1 \cup A_2 = \{ x : x \in A_1 \text{ or } x \in A_2 \}$

- $A_1 \cup A_2 \cup \dots = \{ x : x \in A_j \text{ for some } j = 1, \dots \} = \{ x : x \in A_1 \text{ or } x \in A_2 \text{ or } x \in A_3, \dots \}$
- $A_1 \cup A_2 \cup \dots \cup A_K = \{ x : x \in A_j \text{ for some } j = 1, 2, \dots, K \}$.

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example 3 : let $A_1 = \{ x : x = 0, 1, \dots, 10 \}$ and

$A_2 = \{ x : x = 8, 9, 10, 11 \text{ OR } 11 < x \leq 12 \}$

Find $A_1 \cup A_2$.

$A_1 \cup A_2 = \{ 0, 1, \dots, 11 \text{ OR } 11 < x \leq 12 \}$

$= \{ 0, 1, \dots, 10 \text{ OR } 11 \leq x \leq 12 \}$.

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example 4 : let $A_1 = \{ x : 0 \leq x \leq 1 \}$ and

$A_2 = \{ x : -1 \leq x \leq 2 \}$, Find $A_1 \cup A_2$.

$A_1 \cup A_2 \Rightarrow A_1 \subset A_2$ so $A_1 \cup A_2 = A_2$

$A_1 \cup A_2 = \{ x : -1 \leq x \leq 2 \}$.

example 5: let $A_2 = \phi$ Then $A_1 \cup A_2 = A_1$ for every set A_1.

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example 6: For every set A, $A \cup A = A$.

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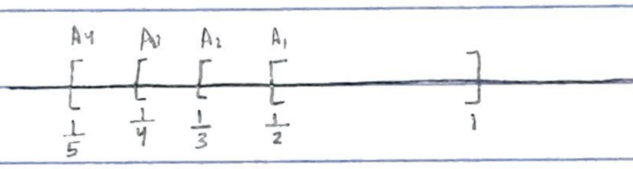
example 7: let $A_K = \{ x : \frac{1}{K+1} \leq x \leq 1 \}$, $K = 1, 2, \ldots$, Find $A_1 \cup A_2 \cup \cdots$?

فضها عملي $\rightarrow A_K = \left[\frac{1}{K+1}, 1 \right]$, $K = 1, 2, 3, \ldots$



So $A_1 \cup A_2 \cup A_3 \cup \cdots = (0, 1]$.

Def 2:

- $A_1 \cap A_2$: A_1 intersection A_2.
- $A_1 \cap A_2 = \{ x : x \in A_1 \text{ and } x \in A_2 \}$
- $A_1 \cap A_2 \cap \cdots = \{ x : x \in A_j, \ j = 1, 2, \ldots \}$.
- $A_1 \cap A_2 \cap \cdots \cap A_K = \{ x : x \in A_j, \ j = 1, 2, \ldots, K \}$.

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example 8: let $A_1 = \{ (0,0), (0,1), (1,1) \}$ and $A_2 = \{ (1,1), (1,2), (2,1) \}$ Find $A_1 \cap A_2$?

$A_1 \cap A_2 = \{ (1,1) \}$.

example 9: let $A_1 = \{ (x,y) : 0 \leq x+y \leq 1 \}$ and $A_2 = \{ (x,y) : 1 < x+y \}$ Find $A_1 \cap A_2$?

$A_1 \cap A_2 = \phi$

example 10: For every set A : $-\ A \cap A = A$

$-\ A \cap \phi = \phi$.

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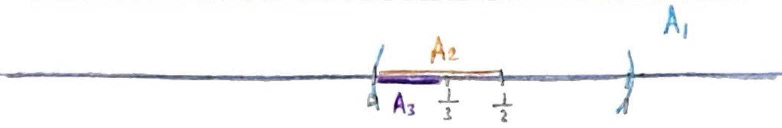
example 11: let $A_K = \{ x : 0 < x < \frac{1}{K} \}$, $K = 1, 2, 3, \ldots$.



So $A_1 \cap A_2 \cap A_3 \cap \ldots = \phi$.

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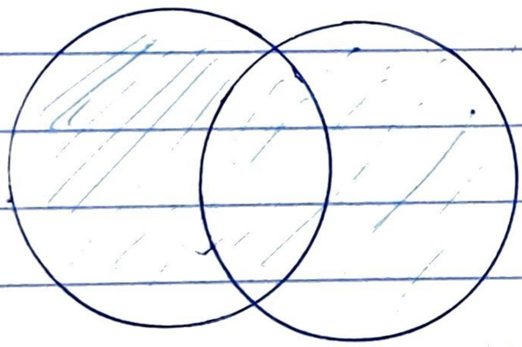
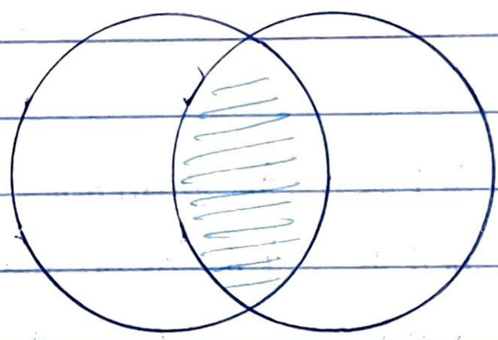
example 12: Venn diagram.

let A_1 and A_2 two sets :



$\hookrightarrow A_1 \cup A_2$



$\hookrightarrow A_1 \cap A_2$.

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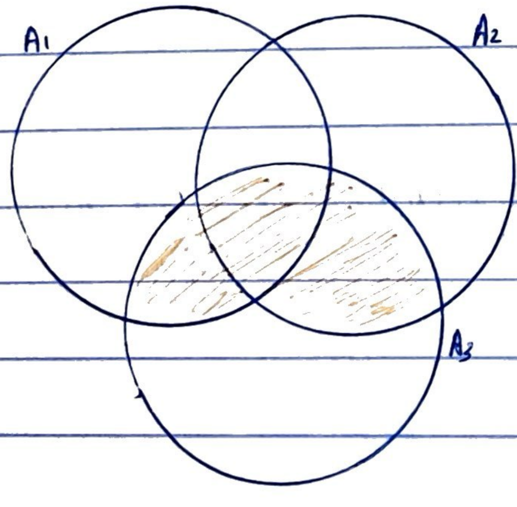
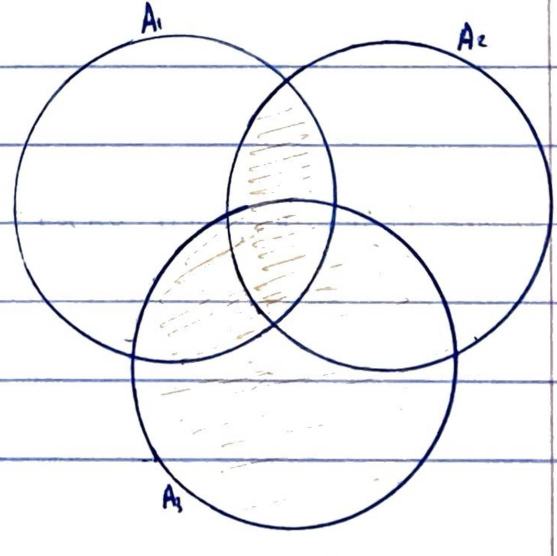
example 13 : let A_1 and A_2 and A_3 three sets :



$(A_1 \cup A_2) \cap A_3$



$(A_1 \cap A_2) \cup A_3$

Def 5: space is the set of all elements.

we denote spaces by capital script letters: $\mathcal{A}, \mathcal{B}, \mathcal{C}$.

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example 14:

# of heads = 4 times.

The number of heads will be one of the numbers: 0, 1, 2, 3, 4.

so the space:

$\mathcal{A} = \{0, 1, 2, 3, 4\}$.

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example 15: consider all nondegenerate rectangles of base x and height y.

both x and y must be positive.

so The space $\mathcal{A} = \{(x,y): x > 0, y > 0\}$.

Def 6: complement

A^*: the complement of A.

$A^* = \{x : x \notin A\}$

complement of A w.r.t $\mathcal{A}$.

Inparticular, $\mathcal{A}^* = \emptyset$.

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example 16: $\mathcal{A} = \{0, 1, 2, 3, 4\}$

If $A = \{0, 1\}$ find A^*.

$A^* = \{2, 3, 4\}$ ✓

example 17: Given $A \subset \mathcal{A}$, Then
- $A \cup A^* = \mathcal{A}$
- $A \cap A^* = \emptyset$
- $A \cup \mathcal{A} = \mathcal{A}$
- $A \cap \mathcal{A} = A$
- $(A^*)^* = A$

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* Set Functions.

example 18: $A \subset \mathbb{R}$, $Q(A)$ = number of points in a which correspond to a positive integer.

- $A = \{x : 0 < x < 5\}$

  $\emptyset(A) = 4$.

- $A = \{-2, -1\}$: $\emptyset(A) = 0$.

- $A = \{x : -\infty < x < 6\}$: $\emptyset(A) = 5$.

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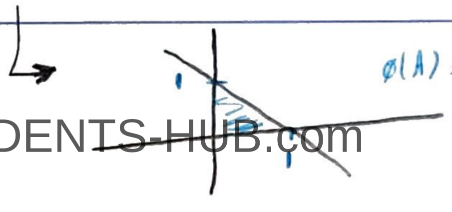
example 19: $A \subset \mathbb{R}^2$

$$\emptyset(A) = \begin{cases} \text{area of } A, & A \text{ has finite area} \\ \text{undefind}, & \text{otherwise.} \end{cases}$$

① $A = \{(x,y) : x^2 + y^2 \leq 1\} \rightarrow \emptyset(A) = \pi$

② $A = \{(0,0), (1,1), (0,1)\} \rightarrow \emptyset(A) = 0$

③ $A = \{(x,y) : 0 \leq x, 0 \leq y, x + y \leq 1\} \rightarrow \emptyset(A) = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$



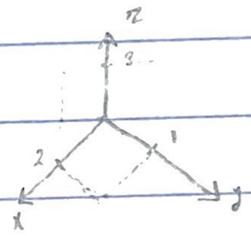
$\emptyset(A)$ = area of triangle.

example 20 : $A \subset \mathbb{R}^3$

$$\phi(A) = \begin{cases} \text{Volume of } A, & A \text{ has finite volume} \\ \text{undefined}, & \text{otherwise}. \end{cases}$$

① $A = \{(x,y,z) : 0 \leq x \leq 2,\ 0 \leq y \leq 1,\ 0 \leq z \leq 3\}$

$\hookrightarrow \phi(A) = 3 \times 2 \times 1 = 6$



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② $A = \{(x,y,z) : x^2 + y^2 + z^2 \geq 1\}$

$\hookrightarrow \phi(A)$: undefind.

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example 21: $A \subset \mathbb{R}$

$\phi(A) = \sum_{x \in A} f(x)$ and $f(x) = \begin{cases} \left(\frac{1}{2}\right)^x, & x = 1, 2, \dots \\ 0, & \text{else where} \end{cases}$

- $A = \{x : 0 \leq x \leq 3\}$

$\phi(A) = \sum_{x \in A} f(x) = f(1) + f(2) + f(3) + f(0)$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + 0 = \frac{7}{8}$

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example 22: $\phi(A) = \sum_{x \in A} f(x)$, $f(x) = \begin{cases} p^x (1-p)^{1-x}, & x = 0, 1 \\ 0, & \text{else where} \end{cases}$

$0 < p < 1$

- $A = \{0\}$

$\phi(A) = \sum_{x \in A} f(x) = f(0) = p^0 (1-p)^{1-0}$

$= 1 - p$ ✓

exp22. ② $A = \{x : 1 \le x \le 2\}$.

$$\phi(A) = f(1) = p^1 (1-p)^{1-1} = p.$$

example 23: $A \subset R$, $Q(A) = \int_A e^{-x}\, dx$.

- $A = \{x : 0 \le x < \infty\}$

$$Q(A) = \int_0^\infty e^{-x}\, dx = 1$$ improper integral

- $A = \{x : 1 \le x \le 2\}$

$$Q(A) = \int_1^2 e^{-x}\, dx = e^{-1} - e^{-2}$$

- $A_1 = \{x : 0 \le x \le 1\}$ and $A_2 = \{x : 1 < x \le 3\}$

$$Q(A_1 \cup A_2) = \int_0^3 e^{-x}\, dx = \int_0^1 e^{-x}\, dx + \int_1^3 e^{-x}\, dx$$

$$= Q(A_1) + Q(A_2)$$

- $A = A_1 \cup A_2$ where $A_1 = \{x : 0 \le x \le 2\}$ and $A_2 = \{x : 1 \le x \le 3\}$

$$Q(A) = Q(A_1 \cup A_2) = \int_0^3 e^{-x}\, dx$$

$$= \int_0^2 e^{-x} dx + \int_1^3 e^{-x} dx - \int_1^2 e^{-x} dx$$

$$= Q(A_1) + Q(A_2) - Q(A_1 \cap A_2).$$