Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Previous Problems

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1. Suppose we want to estimate the average weight of an adult male in a County. We draw a random sample of 40 men from a population of 1,000,000 men and weigh them. We find that the average man in our sample weighs 180 pounds, and the standard deviation of the sample is 30 pounds. What is the 96% confidence interval?
2. For a certain commodity which you buy, you can make either a \$500 profit with probability 0.5 when you sell it, or \$200 with probability 0.3 or lose \$100 with probability 0.2.

a. Find the mean and variance of your net profit if you sell one item.
b. Suppose you sell 80 items separately and independently, find the mean and standard deviation of your total net profit.

3. Two random variables X and Y are related by Y = aX + b, where X is a random variable with zero mean and unit variance.

a. Find the mean and variance of Y
b. Find the correlation coefficient between X and Y.

4. Let X and Y be random variables with a joint pdf $f_{X,Y}(x,y) = C$ for $0 \le X + Y \le 1$, $0 \le X \le 1$, $0 \le Y \le 1$

a. Find C so that this is a valid joint pdf
b. Find the marginal density functions of X and Y.
c. Are X and Y independent?
d. Find the conditional pdf of Y given X = 0.5

5. If X and Y are independent, normal random variables with E(X) = 10, Var(X) = 4, E(Y) = 0, and Var(Y) = 9.

a. Let T = X – Y, find the mean and variance of T
b. Let Z = XY, find the mean and variance of Z.

6. The random variables X and Y are independent and uniformly distributed in the interval (0,1). Find $P(Y \le \sqrt{X})$.

7. Let X be a uniformly distributed random variable on the interval $0 \le x \le 10$ and zero elsewhere and let Y be another uniformly distributed random variable on $0 \le y \le 20$ and zero elsewhere. Assuming that X and Y are independent, find

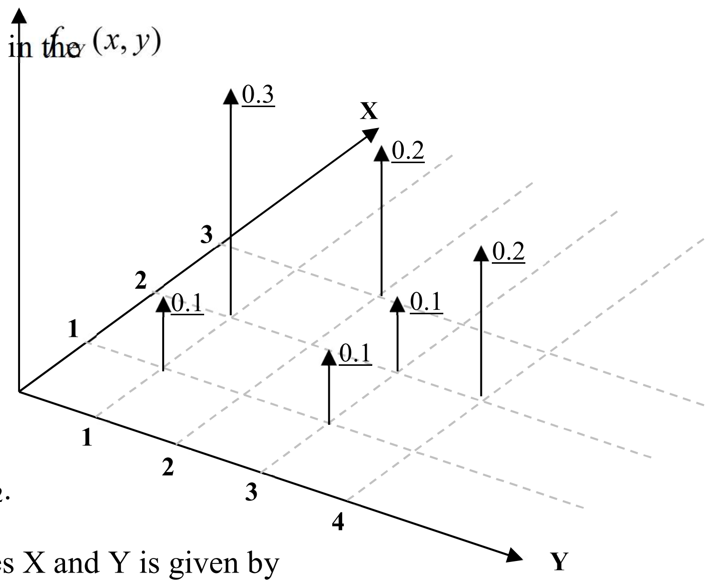
a. $P(X \le 4 \cap Y \le 8)$
b. E {X + Y}
c. E {XY}

d. Var (X + Y)

8. The lifetime of a structure **T** is a Gaussian distribution which is dependent on the strength of used concrete. B250 has μ = 35 years, σ = 10 years, whereas B300 has μ = 50 years, σ = 5 years.

a- If a structure with design period of 40 years will be designed, which concrete is better to be used?
b- For B300, find time in years at which the lifetime of the structure will exceed 95% of its design period.

9. For the joint density function shown in the figure, find the followings:



a- Marginal density functions of X and Y
b- P (X < 3)
c- P (Y $\geq$ 2)
d- P [X = x / (Y = 1)]
e- P (X $\geq$ Y)

10. 9) Let X_1 and X_2 be independent normal random variables with means 23 and 4 and variances 3 and 1, respectively. Find the probability density function of $Y = 4X_1 - X_2$.

11. The joint pdf of two random variables X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} kxy & 0 \le x \le 2 \quad 0 \le y \le 3 \\ 0 & otherwise \end{cases}$$

a. Find the constant k so that this is a valid pdf.
b. Are X and Y statistically independent?
c. Find the expected value of the function g(X,Y) = 2X + 3Y
d. Find P(X + Y < 1), P(Y – X < 1).

12. A manufacturer of semiconductor devices takes a random sample of size n of chips and tests them, classifying each chip as defective or non-defective. Let X_i = 0 if the chip is non-defective and X_i = 1 if the chip is defective.

a. Find the mean and variance of the sample average defined as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
b. Compare the sample variance for the case when n = 50 and n = 100. Comment on the effect of sample size on the variance of the sampling distribution
c. If p is the probability of a defective chip, find an unbiased estimator of p.

13. Consider a random sample of size n taken from a discrete distribution, the pmf of which is given by: $f(x) = \theta^x (1-\theta)^{1-x}$, x = 0, 1. Two estimators for θ are proposed

$$\hat{\Theta}_1 = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\hat{\Theta}_2 = \frac{n\overline{X}+1}{n+2}$$

a. Which one of these two estimators is an unbiased estimator of the parameter θ?
b. Which one has a smaller variance?

14. In a random sample of 500 persons in the city of Ramallah, it was found that 372 voted for Abu Mazen in the 2005 presidential elections for the Palestinian Authority. Determine a 95% confidence interval for p, the actual proportion of Ramallah residents supporting Abu Mazen.

15. The compressive strength of concrete is being tested by a civil engineer. He tests 12 specimens and obtains the following data (in psi)

2216 2237 2249 2204 2225 2301 2281 2283
2318 2255 2275 2295

a. Find point estimates for the mean and variance of the strength
b. Construct a 95% confidence interval on the mean strength
c. Construct a 95% confidence interval on the variance of the strength.

16. A random sample of n = 36 observations has been drawn from a normal distribution with mean 50 and standard deviation 12. Find the probability that the sample mean is in the interval 47 to 53.

17. Given the following pair of measurements, which are suspected to be linearly related. Do a regression analysis to find the linear relationship $y = \alpha x + \beta$

X_i	0.77	4.39	4.11	2.91	0.56	0.89	4.09	2.38	0.78	2.52
Y_i	14.62	22.21	20.12	19.42	14.69	15.23	24.48	16.88	8.56	16.24

18. A machine produces metal rods used in an automobile suspension system. A random sample of 9 rods is selected and the diameter is measured. The resulting data (in mm) are:

8.24 8.23 8.20 8.21 8.22 8.28 8.17 8.26 8.19

If the sampling comes from a normal population with a mean rod diameter μ and a variance σ^2, find

a. point estimates for the mean and the variance
b. a 95% confidence interval on the mean
c. a 95% confidence interval on the variance

19. A random sample of n = 10 structural elements is tested for compressive strength. We know that the true mean compressive strength is μ = 5000 psi and the standard deviation is σ = 100 psi. Find the probability that the sample mean compressive strength exceeds 4985 psi.

20. Let X_1 and X_2 be a sample of size two drawn from a population with mean μ and variance σ^2. Two estimators for μ are proposed:

$$\hat{\mu}_1 = \frac{X_1 + X_2}{2}$$

$$\hat{\mu}_2 = \frac{X_1 + 2X_2}{3}$$

Which is the better estimator and in what sense?

21. Suppose that X has the following discrete distribution

$$P(X = x) = \begin{cases} 1/3 & x = 1,2,3 \\ 0 & otherwise \end{cases}$$

A random sample of n = 200 is selected from this population. Approximate the probability that the sample mean is greater than 2.1 but less than 2.5.

22. The amount of waiting time that a customer spends waiting at a bank is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of n = 50 customers is observed. Find the probability that the average waiting time for these customers is less than 8 minutes.

23. A computer, in adding numbers, round each number to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5). If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

24. Suppose that X has a normal distribution with mean μ and variance σ^2, where μ and σ^2 are unknown. A sample of size 15 yielded the values $\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$. Obtain a 95% confidence interval on the variance.

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1. Suppose we want to estimate the average weight of an adult male in a County. We draw a random sample of 40 men from a population of 1,000,000 men and weigh them. We find that the average man in our sample weighs 180 pounds, and the standard deviation of the sample is 30 pounds. What is the 96% confidence interval?

$$P\left\{\hat{\mu}_x - \mathfrak{z}_{\frac{\alpha}{2}} \frac{\hat{\sigma}_x}{\sqrt{n}} \le \mu_x \le \hat{\mu}_x + \mathfrak{z}_{\frac{\alpha}{2}} \frac{\hat{\sigma}_x}{\sqrt{n}}\right\} = 1 - \alpha$$

$$P\left\{180 - 2.05 * \frac{30}{\sqrt{1000}} \le \mu_x \le 180 + 2.05 * \frac{30}{\sqrt{1000}}\right\} = 0.96$$

$$P\{180 - 1.94 \le \mu_x \le 180 + 1.94\} = 0.96$$

2. For a certain commodity which you buy, you can make either a \$500 profit with probability 0.5 when you sell it, or \$200 with probability 0.3 or lose \$100 with probability 0.2.

a. Find the mean and variance of your net profit if you sell one item.

b. Suppose you sell 80 items separately and independently, find the mean and standard deviation of your total net profit.

Profit	500	200	-100
probability	0.5	0.3	0.2

$$\mu_x = \sum_{i=1}^{n} x_i \, P(X = x_i) = 500 * .5 + 200 * .3 - 100 * .2 = 300\$$$

$$\sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu_x)^2 \, P(X = x_i) = 48000$$

$$\hat{\mu}_x = \mu_x = 300$$

$$\sigma_x^2 = 80 * 48000 = 3840000$$

$$\sqrt{\sigma_x^2} = \sqrt{3840000} = 1960\$$$

your total net profit 80 items $f_x(x) = \sum_{i=1}^{80} x_i$

$$\hat{\mu}_x = n\mu_x = 80 * 300 = 24000\$$$

$$\sqrt{\sigma_x^2} = \mathrm{n}\sqrt{3840000} = 1960 * 80 = 156800\$$$

3. Two random variables X and Y are related by Y = aX + b, where X is a random variable with zero mean and unit variance.

a. Find the mean and variance of Y

$$E[y] = aE[x] + b = b$$

$$Var(y) = a^2E[x] = a^2$$

b. Find the correlation coefficient between X and Y.

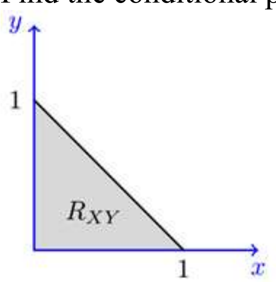
$$cov(X,Y) = E[XY] - \mu_x\mu_y = E[X(\mathrm{a}X + \mathrm{b})] - 0 * b = \mathrm{a}E[X^2] + bE[X] = \mathrm{a}E[X^2]$$

$$\mathrm{a}E[X^2] = a(\sigma_x^2 + E[X]^2) = a(1+0)$$

$$\rho_{xy} = \frac{cov(X,Y)}{\sigma_x\sigma_y} = \frac{a}{1 * a^2} = \frac{1}{a}$$

4. Let X and Y be random variables with a joint pdf $f_{X,Y}(x,y) = C$ for $0 \le X + Y \le 1$, $0 \le X \le 1$, $0 \le Y \le 1$

a. Find C so that this is a valid joint pdf
b. Find the marginal density functions of X and Y.
c. Are X and Y independent?
d. Find the conditional pdf of Y given X = 0.5



$$\int_0^{\infty}\int_0^{\infty} f(x,y)dydx = 1 = \int_0^1\int_0^{1-x} cdydx = c\int_0^1 (1-x)dx = c\left(x - \frac{x^2}{2}\right)_0^1 = \frac{c}{2}$$

$c = 2$

the marginal density functions of X and Y

$$f(x) = \int_0^{1-x} 2dy = 2 - 2x$$

$$f(y) = \int_0^{1-y} 2dy = 2 - 2y$$

Are X and Y independent? No $f(x)f(y) \ne f(x,y)$

$(2-2x)(2-2y) \ne 2$

Find the conditional pdf of Y given X = 0.5

$$f(y \mid x = 0.5) = \frac{f(x,y)}{f(x = 0.5)} = \frac{2}{2 - 0.5 * 2} = 2$$

1. variables with E(X) = 10, Var(X) = 4, E(Y) = 0, and Var(Y) = 9.
a. Let T = X – Y, find the mean and variance of T
b. Let Z = XY, find the mean and variance of Z.

See **EXAMPLE (5-6)**

T = X – Y

$$E(T) = E(X - Y) = E(X) - E(Y) = 10 - 0 = 10$$
$$Var(T) = Var(X - Y) = \sigma_x^2 + (-1)^2\sigma_y^2 + 0 = 4 + 9 = 13$$

Z = XY

$$E(Z) = E(XY) = E(X)E(Y) = 10 * 0 = 0$$
$$Var(Z) = Var(XY) = E((XY)^2) - E(XY)^2 =$$
$$E(X^2)E(Y^2) - E(XY)^2 = (\sigma_x^2 + E(X)^2)\left(\sigma_y^2 + E(y)^2\right) - 0 =$$
$$(4 + 100)\left(9 + 0^2\right) = 936$$

2. The random variables X and Y are independent and uniformly distributed in the interval (0,1). Find $P(Y \le \sqrt{X})$.

$$f_{x,y}(x,y) = f_x(x)\, f_y(y) = \frac{1}{1-0} * \frac{1}{1-0} = 1$$
$$P\left(Y \le \sqrt{X}\right) = \int_0^1 \int_0^{\sqrt{X}} f_{x,y}(x,y)dydx = \int_0^1 \int_0^{\sqrt{X}} 1dydx =$$
$$\int_0^1 \sqrt{X}dx = \frac{2}{3}$$

3. Let X be a uniformly distributed random variable on the interval $0 \le x \le 10$ and zero elsewhere and let Y be another uniformly distributed random variable on $0 \le y \le 20$ and zero elsewhere. Assuming that X and Y are independent, find

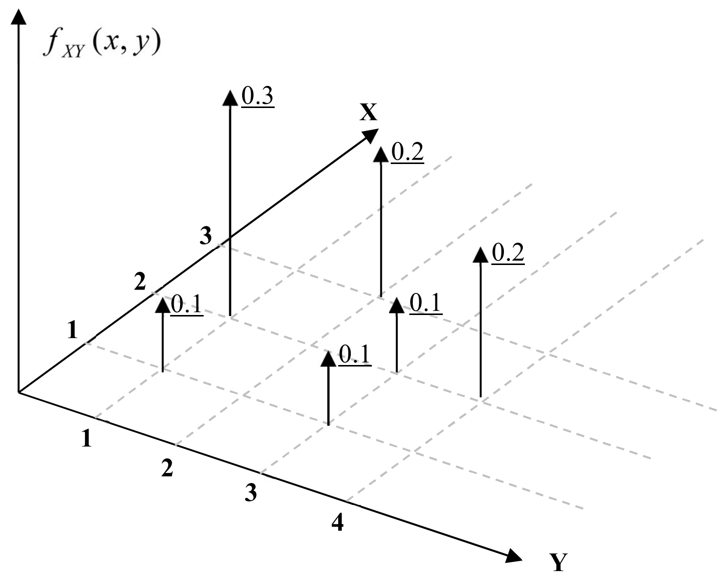
a. $P(X \le 4 \cap Y \le 8)$
b. E {X + Y}
c. E {XY}
d. Var (X + Y)

$$f_{x,y}(x,y) = f_x(x)\, f_y(y) = \frac{1}{10-0} * \frac{1}{20+0} = \frac{1}{200}$$
$$P(X \le 4 \cap Y \le 8) = \int_0^4 \int_0^8 f_{x,y}(x,y)dydx = \int_0^4 \int_0^8 \frac{1}{200}dydx =$$
$$\int_0^4 \frac{1}{25}dx = \frac{4}{25}$$
$$E(X+Y) = E(X) + E(Y) = \frac{10+0}{2} + \frac{20+0}{2} = 15$$
$$E(XY) = E(X)E(Y) = \frac{10+0}{2} * \frac{20+0}{2} = 50$$
$$Var(X+Y) = \sigma_x^2 + \sigma_y^2 + 0 = \frac{(10-0)^2}{12} * \frac{(20-0)^2}{12} = 277.78$$

4. For the joint density function shown in the figure, find the followings:

a- Marginal density functions of X and Y

b- P (X < 3)
c- P (Y ≥ 2)
d- P [X = x / (Y = 1)]
e- P (X ≥ Y)



x\y	1	2	3	4		f(x)
1	0.1		0.1			0.2
2	0.3		0.1	0.2		0.6
3		0.2				0.2
f(y)	0.4	0.2	0.2	0.2		1

P (X < 3)=0.2+0.6=0.8
P (Y ≥ 2)=0.2+0.2+0.2=0.6
P [X = x / (Y = 1)]=0.4
P (X ≥ Y)=0.1+0.3+0.2=0.6

5. Let X1 and X2 be independent normal random variables with means 23 and 4 and variances 3 and 1, respectively. Find the probability density function of Y = 4X1 – X2.

6. The joint pdf of two random variables X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} kxy & 0 \le x \le 2 \quad 0 \le y \le 3 \\ 0 & otherwise \end{cases}$$

a. Find the constant k so that this is a valid pdf.
b. Are X and Y statistically independent?
c. Find the expected value of the function g(X,Y) = 2X + 3Y
d. Find P(X + Y < 1), P(Y – X < 1).

$$a)\ f_{x,y}(x, y) = \int_0^2 \int_0^3 kxydyd \quad = 1$$

$$\int_0^2 kx\frac{3^2}{2}dx = 1$$

$$k\frac{2^2 3^2}{4} = 1$$
$$k = \frac{1}{9}$$

$$b)\ \ f_x(x) = \frac{1}{9}\int_0^3 xydy = \frac{x}{2}$$
$$f_y(y) = \frac{1}{9}\int_0^2 xydy = \frac{2y}{9}$$
$$f_{x,y}(x,y) \stackrel{?}{=} f_x(x)f_x(y)$$
$$\frac{xy}{9} = \frac{2y}{9}\frac{x}{2}$$

X and Y statistically independent

$$\text{d)}\ \ \mathrm{P(X + Y < 1) = P(Y < 1 - X)} = \frac{1}{9}\int_0^2\int_0^{1-x} xydydx =$$
$$\frac{1}{9}\int_0^2 x\frac{(1-x)^2}{2}dx = \frac{1}{9}\int_0^2 \frac{x^3 - 2x^2 + x}{2}dx = \frac{1}{9}[\frac{x^4}{8} - \frac{x^3}{3} + \frac{x^2}{4}|_0^2] = \frac{1}{27}$$
$$\mathrm{P(Y - X < 1) = P(Y < 1 + X)} = \frac{1}{9}\int_0^2\int_0^{1+x} xydydx =$$
$$\frac{1}{9}\int_0^2 x\frac{(1+x)^2}{2}dx = \frac{1}{9}\int_0^2 \frac{x^3 + 2x^2 + x}{2}dx = \frac{1}{9}[\frac{x^4}{8} + \frac{x^3}{3} + \frac{x^2}{4}|_0^2] = \frac{17}{27}$$

7. A manufacturer of semiconductor devices takes a random sample of size n of chips and tests them, classifying each chip as defective or non-defective. Let X_i = 0 if the chip is non-defective and X_i = 1 if the chip is defective.

a. Find the mean and variance of the sample average defined as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.
b. Compare the sample variance for the case when n = 50 and n = 100. Comment on the effect of sample size on the variance of the sampling distribution
c. If p is the probability of a defective chip, find an unbiased estimator of p.

See **EXAMPLE (5-8)**

$$E(\bar{X}) = \frac{1}{n}(\mu_1 + \mu_2 + \cdots + \mu_n) = E(X)$$
$$Var(\bar{X}) = \frac{\sigma_x^2}{n}$$

when n = 50 and n = 100.

$$E(\bar{X}_{n=50}) = E(\bar{X}_{n=100}) = E(X)$$
$$Var(\bar{X}_{n=50}) = \frac{\sigma_{x,n=50}^2}{50} < Var(\bar{X}_{n=100}) = \frac{\sigma_{x,n=50}^2}{100}$$

$$Var(\bar{X}_{n=50}) = 50 * \sigma_x^2 < Var(\bar{X}_{n=100}) = 100 * \sigma_x^2$$

If p is the probability of a defective chip, find an unbiased estimator of p

$$E(p) = \frac{1}{n}(\mu_1 + \mu_2 + \cdots + \mu_n) = \frac{1}{n}(p + p + \cdots + p) = p$$

8. Consider a random sample of size n taken from a discrete distribution, the pmf of which is given by: $f(x) = \theta^x (1-\theta)^{1-x}$, x = 0, 1. Two estimators for θ are proposed

$$\hat{\Theta}_1 = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\hat{\Theta}_2 = \frac{n\overline{X} + 1}{n + 2}$$

a. Which one of these two estimators is an unbiased estimator of the parameter θ?
b. Which one has a smaller variance?

$$E[\widehat{\Theta_1}] = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{1}{n}\sum_{i=1}^{n} \theta^{x_i} (1-\theta)^{1-x_i}$$

9. In a random sample of 500 persons in the city of Ramallah, it was found that 372 voted for Abu Mazen in the 2005 presidential elections for the Palestinian Authority. Determine a 95% confidence interval for p, the actual proportion of Ramallah residents supporting Abu Mazen.

10. The compressive strength of concrete is being tested by a civil engineer. He tests 12 specimens and obtains the following data (in psi)

2216	2237	2249	2204	2225	2301	2281	2283
2318	2255	2275	2295				

a. Find point estimates for the mean and variance of the strength

$$\hat{\mu}_x = \frac{1}{n}\sum_{i=1}^{n} x_i = 2261$$

$$\hat{\sigma}_x^2 = \frac{n\sum_{i=1}^{n} {x_i}^2 - (\sum_{i=1}^{n} x_i)^2}{\mathrm{n}(n-1)} = 426339$$

b. Construct a 95% confidence interval on the mean strength

c. Construct a 95% confidence interval on the variance of the strength.

11. A random sample of n = 36 observations has been drawn from a normal distribution with mean 50 and standard deviation 12. Find the probability that the sample mean is in the interval 47 to 53.

12. Given the following pair of measurements, which are suspected to be linearly related. Do a regression analysis to find the linear relationship $y = \alpha x + \beta$

X_i	0.77	4.39	4.11	2.91	0.56	0.89	4.09	2.38	0.78	2.52
Y_i	14.62	22.21	20.12	19.42	14.69	15.23	24.48	16.88	8.56	16.24

$$\alpha = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i{}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2} = \frac{457.6 - \frac{1}{10} * 23.4 * 172.5}{75.7 - \frac{1}{10} * (23.4)^2}$$

$$\beta = \hat{\mu}_y + \alpha\hat{\mu}_x$$

13. A machine produces metal rods used in an automobile suspension system. A random sample of 9 rods is selected and the diameter is measured. The resulting data (in mm) are:

8.24 8.23 8.20 8.21 8.22 8.28 8.17 8.26 8.19

If the sampling comes from a normal population with a mean rod diameter μ and a variance σ^2, find

a. point estimates for the mean and the variance
b. a 95% confidence interval on the mean
c. a 95% confidence interval on the variance

14. A random sample of n = 10 structural elements is tested for compressive strength. We know that the true mean compressive strength is $\mu = 5000$ psi and the standard deviation is $\sigma = 100$ psi. Find the probability that the sample mean compressive strength exceeds 4985 psi.

15. Let X_1 and X_2 be a sample of size two drawn from a population with mean μ and variance σ^2. Two estimators for μ are proposed:

$$\hat{\mu}_1 = \frac{X_1 + X_2}{2}$$

$$\hat{\mu}_2 = \frac{X_1 + 2X_2}{3}$$

Which is the better estimator and in what sense?

See **EXAMPLE (5-16): the same**

16. Suppose that X has the following discrete distribution

$$P(X = x) = \begin{cases} 1/3 & x = 1,2,3 \\ 0 & otherwise \end{cases}$$

A random sample of n = 200 is selected from this population. Approximate the probability that the sample mean is greater than 2.1 but less than 2.5.

17. The amount of waiting time that a customer spends waiting at a bank is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of n = 50 customers is observed. Find the probability that the average waiting time for these customers is less than 8 minutes.

18. A computer, in adding numbers, round each number to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5, 0.5). If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?

The exact answer.

$$P(\mathrm{X}>15)=1-\frac{1}{n!}\sum_{0}^{n}\binom{15}{\mathrm{x_i}}P^{x}(1-P)^{n-x}=1-\sum_{0}^{15}\binom{\mathrm{n}}{\mathrm{x_i}}P^{x}(1-P)^{n-x}$$

We need to compute:

$$(1-P(X<765))\cdot 2\approx 0.1797$$

Using the normal approximation:

$$\hat{\mu}_x=\frac{0.5+-0.5}{2}=0$$

$$\hat{\sigma}_x^2=\frac{(-0.5-0.5)^2}{12}=\frac{1}{12}$$

$$P\{z>15\}=1-\phi\left(\frac{15}{\sqrt{\frac{1*1500}{12}}}\right)=0.1797$$

19. Suppose that X has a normal distribution with mean μ and variance σ^2, where μ and σ^2 are unknown. A sample of size 15 yielded the values $\sum_{i=1}^{15}X_i=8.7$ and $\sum_{i=1}^{15}X_i^2=27.3$. Obtain a 95% confidence interval on the variance.

$$\hat{\mu}_x=\frac{1}{n}\sum_{i=1}^{n}x_i=\frac{1}{15}8.7=0.58$$

$$\hat{\sigma}_x^2=\frac{n\sum_{i=1}^{n}{x_i}^2-(\sum_{i=1}^{n}x_i)^2}{\mathrm{n}(n-1)}$$

$$\hat{\sigma}_x^2=\frac{15*27.3-8.7^2}{15(15-1)}=1.59$$

$$t_{0.025,14}=0.6938$$

$$P\left\{\hat{\mu}_x-t_{\frac{\alpha}{2},14}\frac{\hat{\sigma}_x}{\sqrt{n}}\le\mu_x\le\hat{\mu}_x+t_{\frac{\alpha}{2},14}\frac{\hat{\sigma}_x}{\sqrt{n}}\right\}=1-\alpha$$

$$P\left\{0.58 - 0.6938\frac{\sqrt{1.59}}{\sqrt{15}} \le \mu_x \le 0.58 + 0.6938\frac{\sqrt{1.59}}{\sqrt{15}}\right\} = 0.95$$

$$P\{0.354 \le \mu_x \le 0.58 + 0.805\} = 0.95$$

$$\chi^2_{0.025,14} = 26.119$$

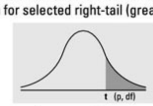
$$\chi^2_{0.975,14} = 5.629$$

$$P\left\{\frac{(n-1)\hat{\sigma}_x^2}{\chi^2_{\frac{\alpha}{2},14}} \le \sigma_x^2 \le \frac{(n-1)\hat{\sigma}_x^2}{\chi^2_{1-\frac{\alpha}{2},14}}\right\} = 1-\alpha$$

$$P\left\{\frac{14 * 1.59}{26.119} \le \sigma_x^2 \le \frac{14 * 1.59}{5.629}\right\} = 0.95$$

$$P\{0.852 \le \sigma_x^2 \le 4.2\} = 0.95$$

Numbers in each row of the table are values on a *t*-distribution with (*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	**0.95**	**0.90**	**0.75**	**0.50**	**0.25**	**0.10**	**0.05**	**0.01**
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9958	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

Chi-Square Table

Table 5-2

Critical Values of the χ^2 Distribution

df \ p	0.995	0.975	0.9	0.5	0.1	0.05	0.025	0.01	0.005	df
1	.000	.000	0.016	0.455	2.706	3.841	5.024	6.635	7.879	1
2	0.010	0.051	0.211	1.386	4.605	5.991	7.378	9.210	10.597	2
3	0.072	0.216	0.584	2.366	6.251	7.815	9.348	11.345	12.838	3
4	0.207	0.484	1.064	3.357	7.779	9.488	11.143	13.277	14.860	4
5	0.412	0.831	1.610	4.351	9.236	11.070	12.832	15.086	16.750	5
6	0.676	1.237	2.204	5.348	10.645	12.592	14.449	16.812	18.548	6
7	0.989	1.690	2.833	6.346	12.017	14.067	16.013	18.475	20.278	7
8	1.344	2.180	3.490	7.344	13.362	15.507	17.535	20.090	21.955	8
9	1.735	2.700	4.168	8.343	14.684	16.919	19.023	21.666	23.589	9
10	2.156	3.247	4.865	9.342	15.987	18.307	20.483	23.209	25.188	10
11	2.603	3.816	5.578	10.341	17.275	19.675	21.920	24.725	26.757	11
12	3.074	4.404	6.304	11.340	18.549	21.026	23.337	26.217	28.300	12
13	3.565	5.009	7.042	12.340	19.812	22.362	24.736	27.688	29.819	13
14	4.075	5.629	7.790	13.339	21.064	23.685	26.119	29.141	31.319	14
15	4.601	6.262	8.547	14.339	22.307	24.996	27.488	30.578	32.801	15

Examples Ch(4,5)

The weight of an item is normally distributed with a mean of 0.1 Kg and a standard deviation of 0.01 ounce. Suppose that 16 items are placed in a package and that the weights are independent,

a. Find the mean and the variance of the net weight.
b. Find the probability that the net weight exceeds 1.65 Kg.

$X_T = X_1 + X_2 + \ldots + X_n$

$\hat{\mu}_x = n\mu_x = 1.6$ $\quad\quad$ $\hat{\sigma}_x^2 = \mathrm{n}\sigma_x^2 = 16 * (0.01)^2 = 0.0016$

$$P\{z > 1.65\} = 1 - \phi\left(\frac{1.65 - 1.6}{\sqrt{0.0016}}\right) = 1 - \phi(1.25)$$

A bank teller serves customers standing in the queue one by one. Suppose that the service time x_i for customer i has mean $E\{x_i\} = 2$ (minutes) and $Var\{x_i\} = 1$. We assume that service times for different bank customers are independent. Let Y be the total time the bank teller spends serving 50 customers. Find $P(90 < Y < 110)$.

$$X_T = X_1 + X_2 + \ldots + X_n$$

$\hat{\mu}_x = n\mu_x = 50 * 2 = 100min$ $\hat{\sigma}_x^2 = \mathrm{n}\sigma_x^2 = 50 * (1)^2 = 50$

$$P\{90 < \mathrm{Y} < 110\} = \phi\left(\frac{110 - 100}{\sqrt{50}}\right) - \phi\left(\frac{90 - 100}{\sqrt{50}}\right) = 1 - 2\phi\left(\frac{10}{\sqrt{50}}\right)$$

Suppose we want to estimate the average weight of an adult male in a County. We draw a random sample of 40 men from a population of 1,000,000 men and weigh them. We find that the average man in our sample weighs 180 pounds, and the standard deviation of the sample is 30 pounds. What is the 96% confidence interval?

$$P\left\{\hat{\mu}_x - \mathfrak{z}_{\frac{\alpha}{2}} \frac{\hat{\sigma}_x}{\sqrt{n}} \leq \mu_x \leq \hat{\mu}_x + \mathfrak{z}_{\frac{\alpha}{2}} \frac{\hat{\sigma}_x}{\sqrt{n}}\right\} = 1 - \alpha$$

$$P\left\{180 - 2.05 * \frac{30}{\sqrt{1000}} \leq \mu_x \leq 180 + 2.05 * \frac{30}{\sqrt{1000}}\right\} = 0.96$$

$$P\{180 - 1.94 \leq \mu_x \leq 180 + 1.94\} = 0.96$$

Let X be a random variable with the following probability density function

$$f_X(x) = \begin{cases} (\theta+1)x^{\theta}, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

a. Use the maximum likelihood estimator to estimate the parameter θ

See Example 6-4 $\hat{\theta} = \frac{1}{-\sum_{i=1}^{n} \ln x_i / n}$-1

A manufacturer of semiconductor devices takes a random sample of size n of chips and tests them, classifying each chip as defective or non-defective. Let $X_i = 0$ if the chip is non-defective and $X_i = 1$ if the chip is defective.

a. Find the mean and variance of the sample average defined as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

b. Compare the sample variance for the case when n = 50 and n = 100. Comment on the effect of sample size on the variance of the sampling distribution

c. If p is the probability of a defective chip, find an unbiased estimator of p.

$$E(\bar{X}) = \frac{1}{n}(\mu_1 + \mu_2 + \cdots + \mu_n) = E(X)$$

$$Var(\bar{X}) = \frac{\sigma_x^2}{n}$$

when n = 50 and n = 100.

$$E(\bar{X}_{n=50}) = E(\bar{X}_{n=100}) = E(X)$$

$$Var(\bar{X}_{n=50}) = \frac{\sigma_{x,n=50}^2}{50} < Var(\bar{X}_{n=100}) = \frac{\sigma_{x,n=50}^2}{100}$$

$$Var(\bar{X}_{n=50}) = 50 * \sigma_x^2 < Var(\bar{X}_{n=100}) = 100 * \sigma_x^2$$

If p is the probability of a defective chip, find an unbiased estimator of p

$$E(p) = \frac{1}{n}(\mu_1 + \mu_2 + \cdots + \mu_n) = \frac{1}{n}(p + p + \cdots + p) = p$$

Consider a random sample of size n taken from a discrete distribution, the pmf of which is given by:

$f(x) = \theta^x (1-\theta)^{1-x}$, x = 0, 1. Two estimators for θ are proposed

$$\hat{\Theta}_1 = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\hat{\Theta}_2 = \frac{n\overline{X}+1}{n+2}$$

a. Which one of these two estimators is an unbiased estimator of the parameter θ?
b. Which one has a smaller variance?

$$E[\widehat{\Theta}_1] = E\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{1}{n}\sum_{i=1}^{n} x_i = \widehat{\Theta}_1 \quad \text{unbiased}$$

$$E[\widehat{\Theta}_2] = E\left[\frac{nE[X]+1}{n+2}\right] = \frac{nE[X]}{n+2} + \frac{n}{n+2} = \frac{nE[X]+1}{n+2} = \widehat{\Theta}_2 \quad \text{unbiased}$$

$$Var[\widehat{\Theta}_1] = Var[E[X]]$$

$$Var[\widehat{\Theta}_2] = Var\left[\frac{nE[X]+1}{n+2}\right] = \frac{n^2 Var[E[X]]}{(n+2)^2} + 0 < Var[\widehat{\Theta}_1] = Var[E[X]]$$

The compressive strength of concrete is being tested by a civil engineer. He tests 12 specimens and obtains the following data (in psi)

2216 2237 2249 2204 2225 2301 2281 2283 2318 2255 2275 2295

a. Find point estimates for the mean and variance of the strength
b. Construct a 95% confidence interval on the mean strength
c. Construct a 95% confidence interval on the variance of the strength.

$$\hat{\mu}_x = \frac{1}{n}\sum_{i=1}^{n} x_i = 2261$$

$$\hat{\sigma}_x^2 = \frac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{\mathrm{n}(n-1)} = 426339$$

$$P\left\{2261 - 2.05\frac{\sqrt{426339}}{\sqrt{12}} \le \mu_x \le 2261 + 2.05\frac{\sqrt{426339}}{\sqrt{12}}\right\} = 0.95$$

$$P\left\{\frac{(12-1)426339}{\chi^2_{0.025,12}} \le \sigma_x^2 \le \frac{(12-1)426339}{\chi^2_{0.975,12}}\right\} = 1 - 0.05$$

$$P\left\{\frac{11 * 426339}{23.34} \le \sigma_x^2 \le \frac{11 * 426339}{4.4}\right\} = 0.95$$

A random sample of n = 36 observations has been drawn from a normal distribution with mean 50 and standard deviation 12. Find the probability that the sample mean is in the interval 47 to 53.

$$X_T = \frac{1}{n}(X_1 + X_2 + ... + X_n)$$

$$\hat{\mu}_x = \mu_x = 50 \qquad \hat{\sigma}_x^2 = \frac{\sigma_x^2}{n} = \frac{12^2}{50}$$

$$P\{47 < \hat{\mu}_x < 53\} = \phi\left(\frac{53-50}{\sqrt{\frac{12^2}{50}}}\right) - \phi\left(\frac{47-50}{\sqrt{\frac{12^2}{50}}}\right)$$

Given the following pair of measurements, which are suspected to be linearly related. Do a regression analysis to find the linear relationship

$$y = \alpha x + \beta$$

X_i	0.77	4.39	4.11	2.91	0.56	0.89	4.09	2.38	0.78	2.52
Y_i	14.62	22.21	20.12	19.42	14.69	15.23	24.48	16.88	8.56	16.24

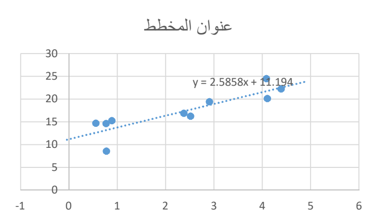
$$\hat{\sigma}_x^2 = \frac{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2}{n(n-1)} = \frac{10*76-(23)^2}{10(9)} = 2.33$$

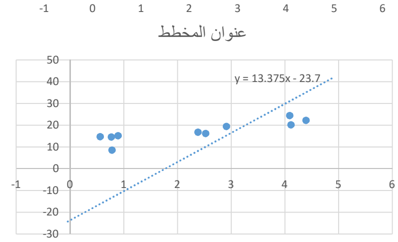
$$\alpha = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n(n-1)\hat{\sigma}_x^2}$$

$$= \frac{10*458-23*76}{10(9)2.33} = 13.37$$

$$\beta = \hat{\mu}_y - \alpha\hat{\mu}_x = 7.6 - 13.37*2.34 = -23.7$$

$$y = 13.37x - 23.7$$





Suppose that X has a normal distribution with mean μ and variance σ², where μ and σ² are unknown. A sample of size 15 yielded the values

$\sum_{i=1}^{15} X_i = 8.7$ and $\sum_{i=1}^{15} X_i^2 = 27.3$ Obtain a 95% confidence interval on the variance.

$$\hat{\mu}_x = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{15}8.7 = 0.58$$

$$\hat{\sigma}_x^2 = \frac{n\sum_{i=1}^{n} {x_i}^2 - (\sum_{i=1}^{n} x_i)^2}{\mathrm{n}(n-1)}$$

$$\hat{\sigma}_x^2 = \frac{15 * 27.3 - 8.7^2}{15(15-1)} = 1.59$$

$$t_{0.025,14} = 2.15$$

$$P\left\{\hat{\mu}_x - t_{\frac{\alpha}{2},14}\frac{\hat{\sigma}_x}{\sqrt{n}} \le \mu_x \le \hat{\mu}_x + t_{\frac{\alpha}{2},14}\frac{\hat{\sigma}_x}{\sqrt{n}}\right\} = 1-\alpha$$

$$P\left\{0.58 - 2.15\,\frac{\sqrt{1.59}}{\sqrt{15}} \le \mu_x \le 0.58 + 2.15\,\frac{\sqrt{1.59}}{\sqrt{15}}\right\} = 0.95$$

$$\chi^2_{0.025,15} = 27.5$$

$$\chi^2_{0.975,15} = 6.3$$

$$P\left\{\frac{(n-1)\hat{\sigma}_x^2}{\chi^2_{\frac{\alpha}{2},15}} \le \sigma_x^2 \le \frac{(n-1)\hat{\sigma}_x^2}{\chi^2_{1-\frac{\alpha}{2},15}}\right\} = 1-\alpha$$

$$P\left\{\frac{14 * 1.59}{27.5} \le \sigma_x^2 \le \frac{14 * 1.59}{6.3}\right\} = 0.95$$

2. For a certain commodity which you buy, you can make either a \$500 profit with probability 0.5 when you sell it, or \$200 with probability 0.3 or lose \$100 with probability 0.2.

a. Find the mean and variance of your net profit if you sell one item.

b. Suppose you sell 80 items separately and independently, find the mean and standard deviation of your total net profit.

Profit	500	200	-100
probability	0.5	0.3	0.2

$$\mu_x = \sum_{i=1}^{n} x_i\, P(X = x_i) = 500 * .5 + 200 * .3 - 100 * .2 = 300\$$$

$$\sigma_x^2 = \sum_{i=1}^{n}\left(x_i - \mu_x\right)^2 P(X = x_i) = 48000$$

$$\hat{\mu}_x = \mu_x = 300$$

$$\sigma_x^2 = 80 * 48000 = 3840000$$

$$\sqrt{\sigma_x^2} = \sqrt{3840000} = 1960\$$$

your total net profit 80 items $f_x(x) = \sum_{i=1}^{80} x_i$

$$\hat{\mu}_x = n\mu_x = 80 * 300 = 24000\$$$

$$\sqrt{\sigma_x^2} = \mathrm{n}\sqrt{3840000} = 1960 * 80 = 156800\$$$

Two random variables X and Y are related by Y = aX + b, where X is a random variable with zero mean and .unit variance

a. Find the mean and variance of Y

$$E[y] = aE[x] + b = b$$

$$Var(y) = a^2 E[x] = a^2$$

b. Find the correlation coefficient between X and Y.

$$cov(X,Y) = E[XY] - \mu_x \mu_y = E[X(\mathrm{a}X + \mathrm{b})] - 0 * b = \mathrm{a}E[X^2] + bE[X] = \mathrm{a}E[X^2]$$

$$\mathrm{a}E[X^2] = a(\sigma_x^2 + E[X]^2) = a(1 + 0)$$

$$\rho_{xy} = \frac{cov(X,Y)}{\sigma_x \sigma_y} = \frac{a}{1 * a^2} = \frac{1}{a}$$

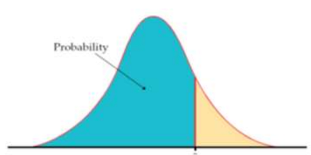


Table entry for z is the area under the standard normal curve to the left of z.

TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABLE of CRITICAL VALUES for STUDENT'S t DISTRIBUTIONS

Column headings denote probabilities (α) ***above*** tabulated values.

d.f.	0.40	0.25	0.10	0.05	0.04	0.025	0.02	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	7.916	12.706	15.894	31.821	63.656	127.321	318.289	636.578
2	0.289	0.816	1.886	2.920	3.320	4.303	4.849	6.965	9.925	14.089	22.328	31.600
3	0.277	0.765	1.638	2.353	2.605	3.182	3.482	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.333	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.191	2.571	2.757	3.365	4.032	4.773	5.894	6.869
6	0.265	0.718	1.440	1.943	2.104	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.046	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.004	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	1.973	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	1.948	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	1.928	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	1.912	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	1.899	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	1.887	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	1.878	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	1.869	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	1.862	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	1.855	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	1.850	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	1.844	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	1.840	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	1.835	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	1.832	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.256	0.685	1.318	1.711	1.828	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	1.825	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	1.822	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	1.819	2.052	2.158	2.473	2.771	3.057	3.421	3.689
28	0.256	0.683	1.313	1.701	1.817	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	1.814	2.045	2.150	2.462	2.756	3.038	3.396	3.660
30	0.256	0.683	1.310	1.697	1.812	2.042	2.147	2.457	2.750	3.030	3.385	3.646
31	0.256	0.682	1.309	1.696	1.810	2.040	2.144	2.453	2.744	3.022	3.375	3.633
32	0.255	0.682	1.309	1.694	1.808	2.037	2.141	2.449	2.738	3.015	3.365	3.622
33	0.255	0.682	1.308	1.692	1.806	2.035	2.138	2.445	2.733	3.008	3.356	3.611
34	0.255	0.682	1.307	1.691	1.805	2.032	2.136	2.441	2.728	3.002	3.348	3.601
35	0.255	0.682	1.306	1.690	1.803	2.030	2.133	2.438	2.724	2.996	3.340	3.591
36	0.255	0.681	1.306	1.688	1.802	2.028	2.131	2.434	2.719	2.990	3.333	3.582
37	0.255	0.681	1.305	1.687	1.800	2.026	2.129	2.431	2.715	2.985	3.326	3.574
38	0.255	0.681	1.304	1.686	1.799	2.024	2.127	2.429	2.712	2.980	3.319	3.566
39	0.255	0.681	1.304	1.685	1.798	2.023	2.125	2.426	2.708	2.976	3.313	3.558
40	0.255	0.681	1.303	1.684	1.796	2.021	2.123	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	1.781	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.254	0.678	1.292	1.664	1.773	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.254	0.677	1.290	1.660	1.769	1.984	2.081	2.364	2.626	2.871	3.174	3.390
120	0.254	0.677	1.289	1.658	1.766	1.980	2.076	2.358	2.617	2.860	3.160	3.373
140	0.254	0.676	1.288	1.656	1.763	1.977	2.073	2.353	2.611	2.852	3.149	3.361
160	0.254	0.676	1.287	1.654	1.762	1.975	2.071	2.350	2.607	2.847	3.142	3.352
180	0.254	0.676	1.286	1.653	1.761	1.973	2.069	2.347	2.603	2.842	3.136	3.345
200	0.254	0.676	1.286	1.653	1.760	1.972	2.067	2.345	2.601	2.838	3.131	3.340
250	0.254	0.675	1.285	1.651	1.758	1.969	2.065	2.341	2.596	2.832	3.123	3.330
inf	0.253	0.674	1.282	1.645	1.751	1.960	2.054	2.326	2.576	2.807	3.090	3.290

Chi-Square (χ^2) Distribution

Degrees of Freedom	Area to the Right of Critical Value							
	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01
1	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892