

# Balanced Binaries – AVL Trees

# Balanced Binaries – AVL Trees

Height of a tree node:

1. The height of a node with no elements is 0
2. The height of a node with 1 element is 1
3. The height of a node with  $> 1$  element is  $1 +$  the height of its tallest subtree

# Balanced Binaries – AVL Trees

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AVL tree:

A binary tree in which the difference between the height of the right and left subtrees of the root is never more than one.

# Balanced Binaries – AVL Trees

Height of a tree node:

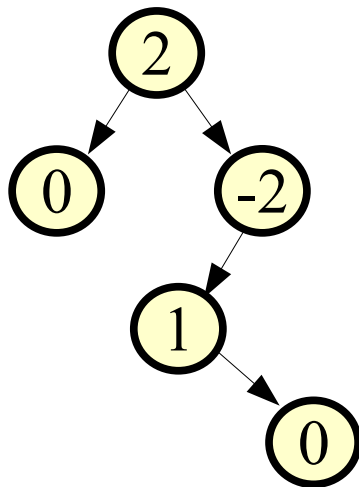
1. The height of a node with no elements is 0
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3. The height of a node with  $> 1$  element is  $1 +$  the height of its tallest subtree

AVL tree:

A binary tree in which the difference between the height of the right and left subtrees of the root is never more than one.

Each node keeps a balance number which is the difference in heights of its two subtrees.

For example,

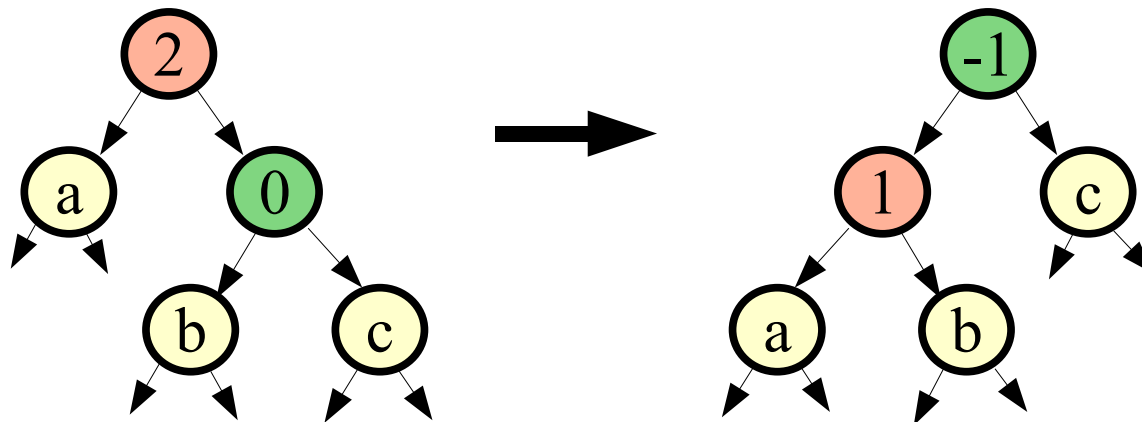
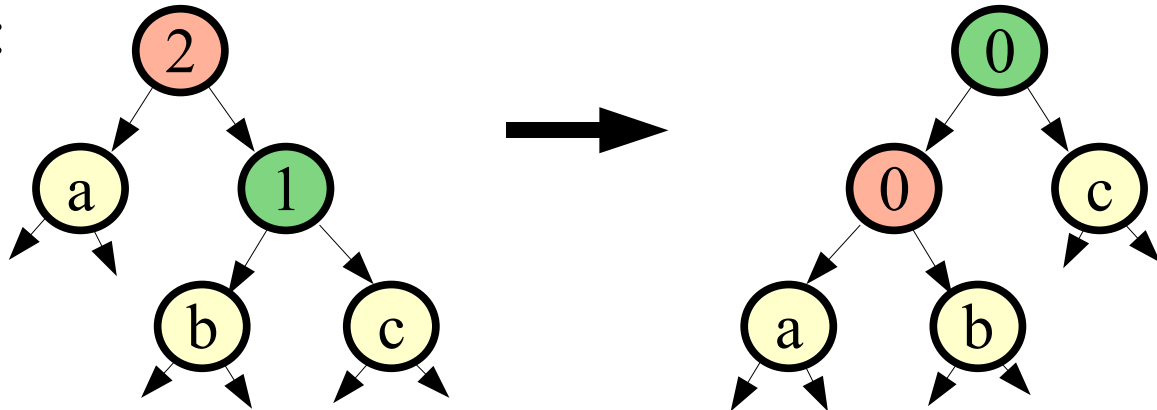


Whenever a balance number is not 0, -1, +1, perform some rotations according to some rules on following pages

# Balanced Binaries – AVL Trees

Rules for rotation:

If:

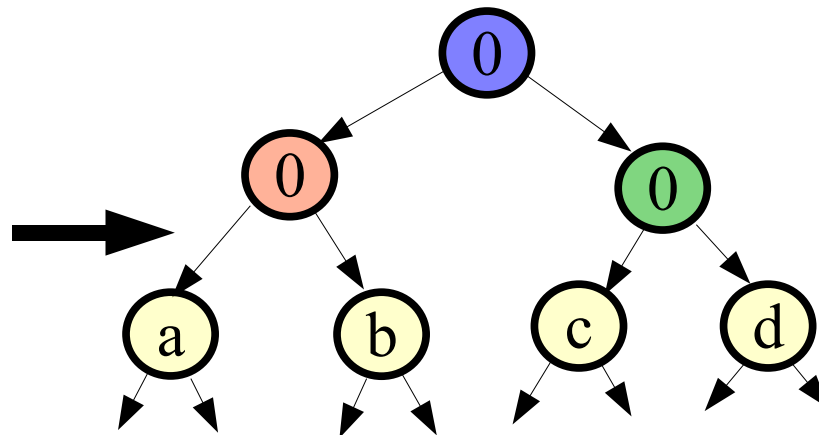
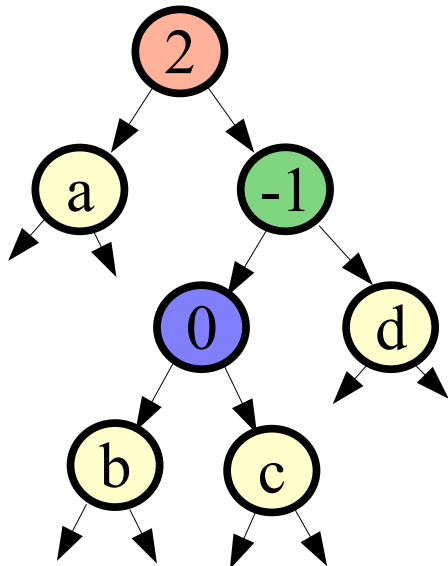
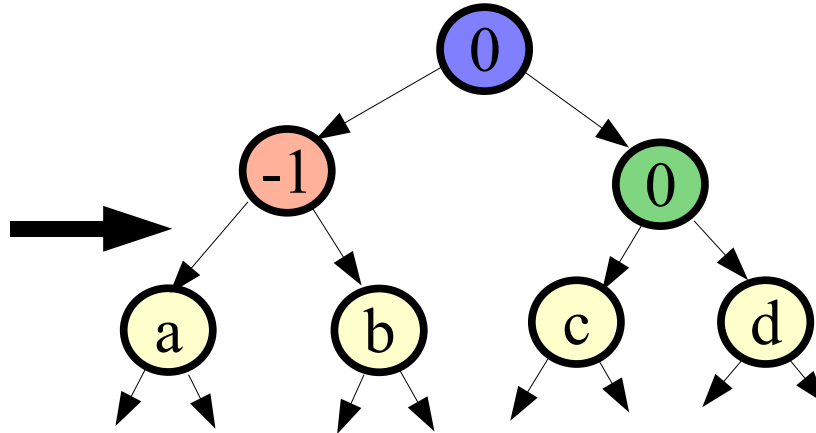
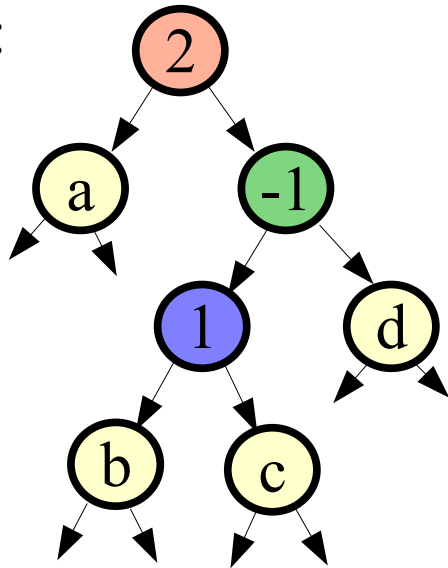


Plus mirror image of these two cases

# Balanced Binaries – AVL Trees

Rules for rotation:

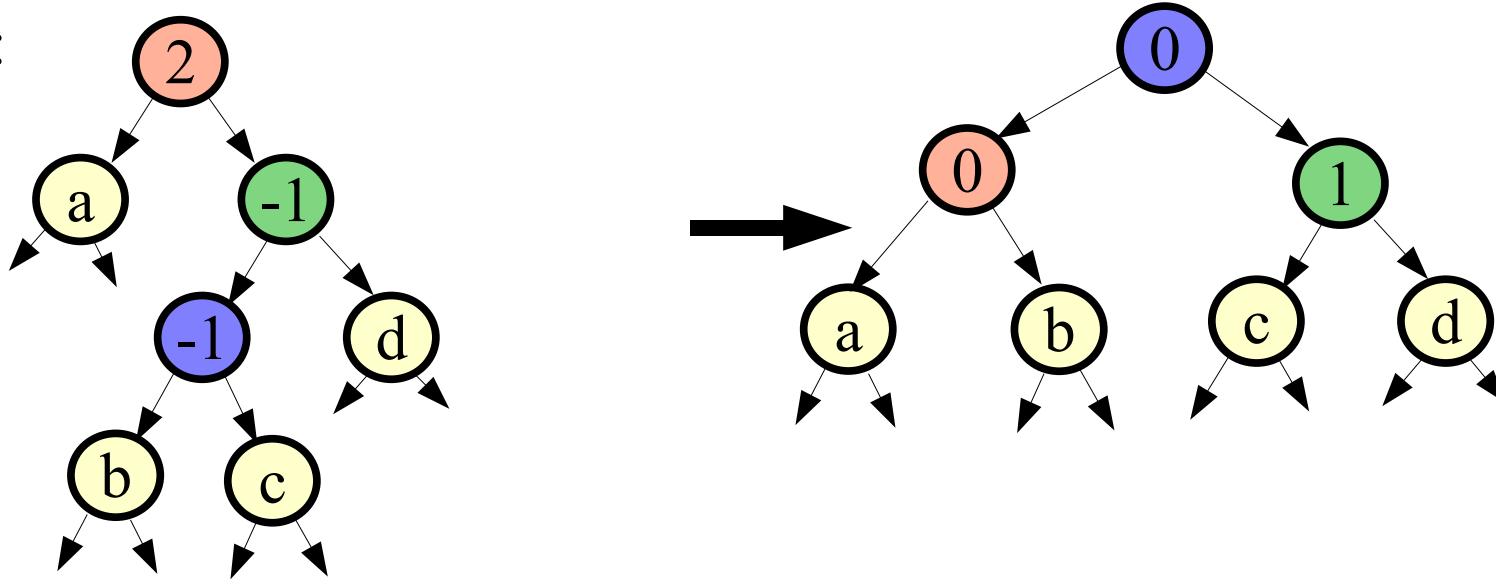
If:



# Balanced Binaries – AVL Trees

Rules for rotation:

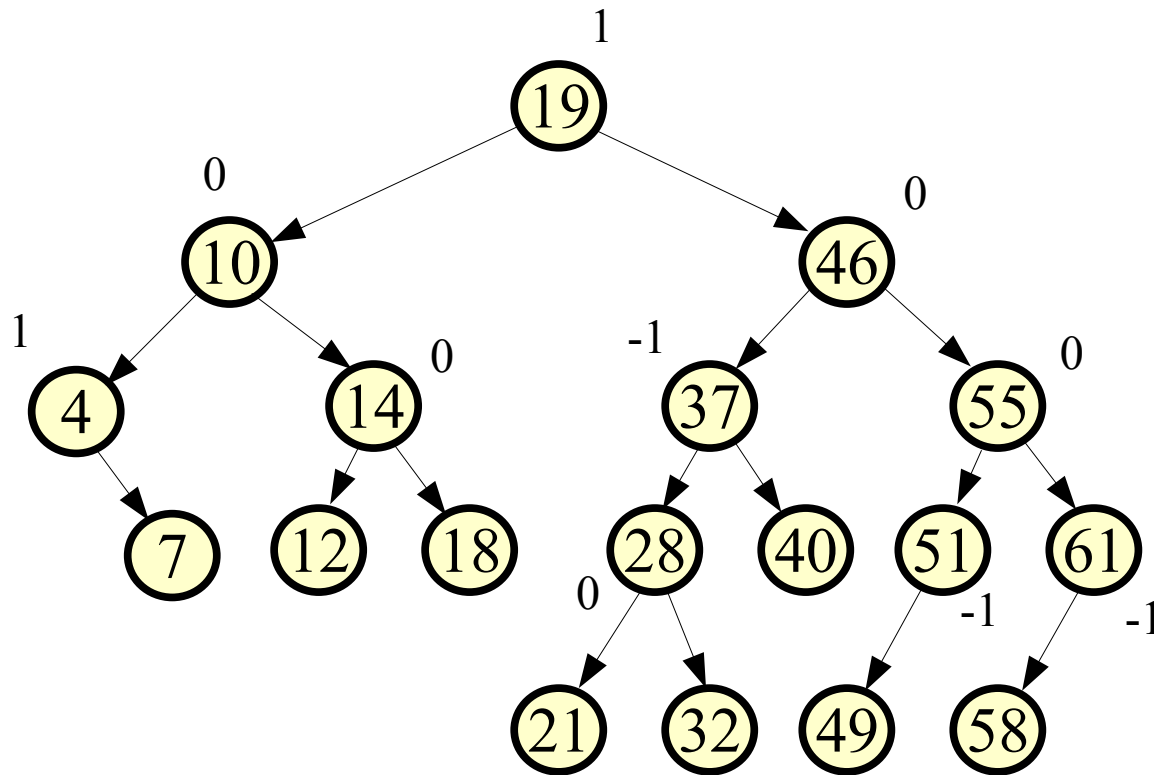
If:



Plus mirror image of these three cases

# Balanced Binaries – AVL Trees

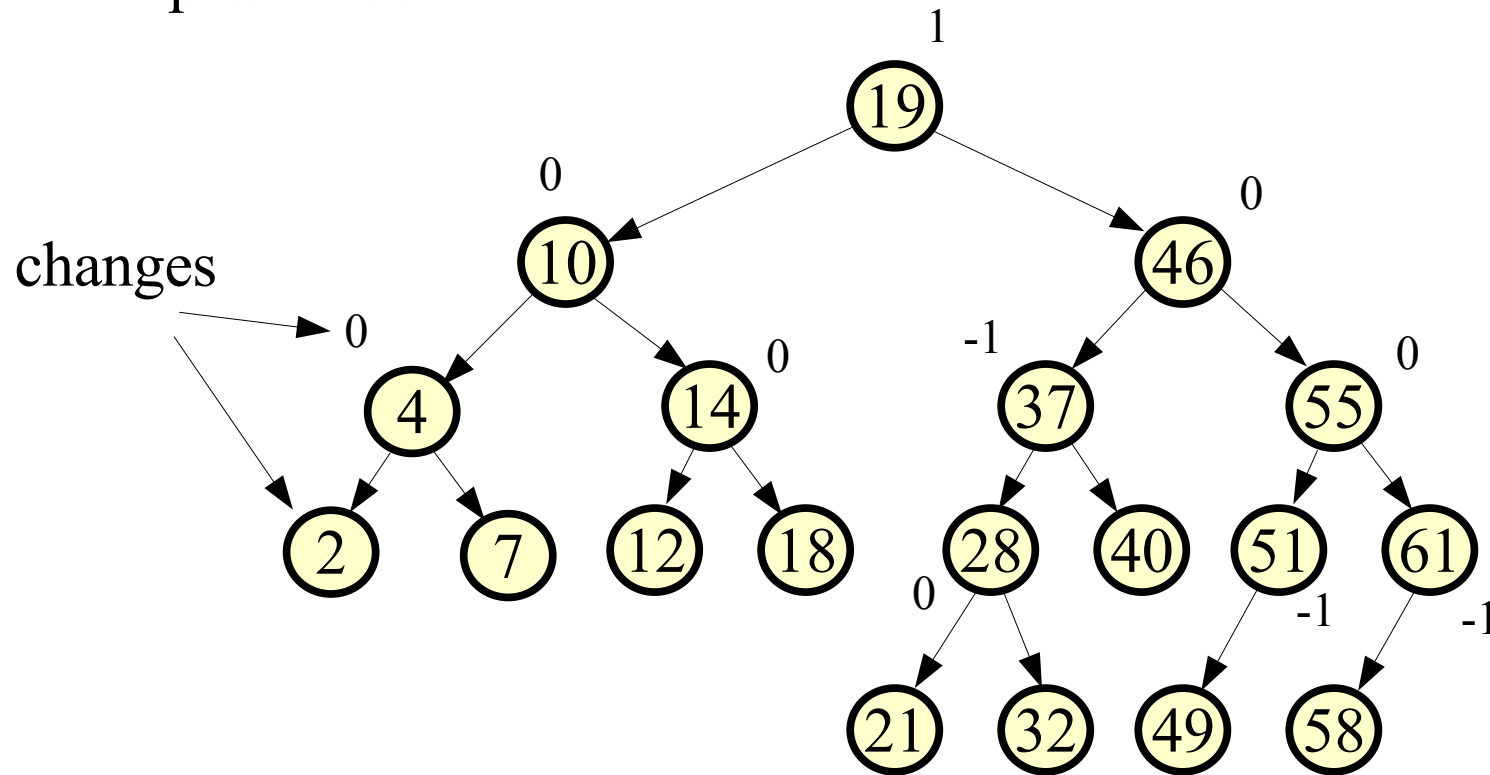
Example:





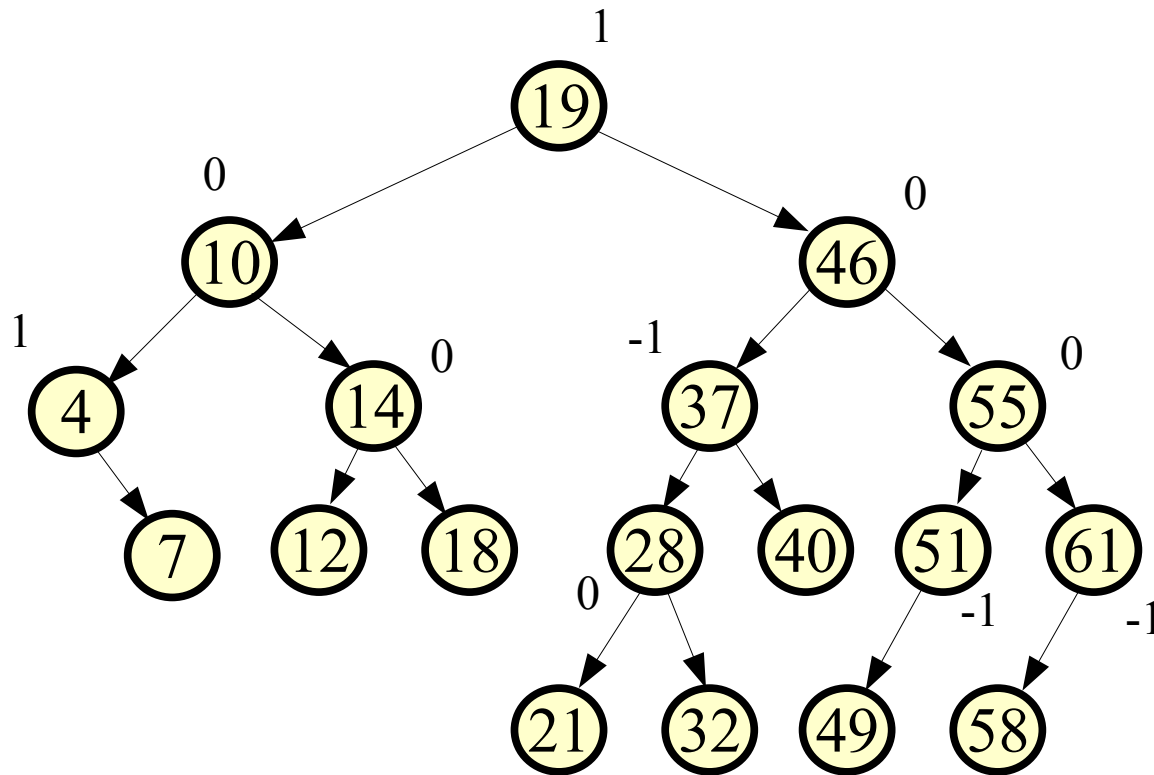
# Balanced Binaries – AVL Trees

Example: Insert 2



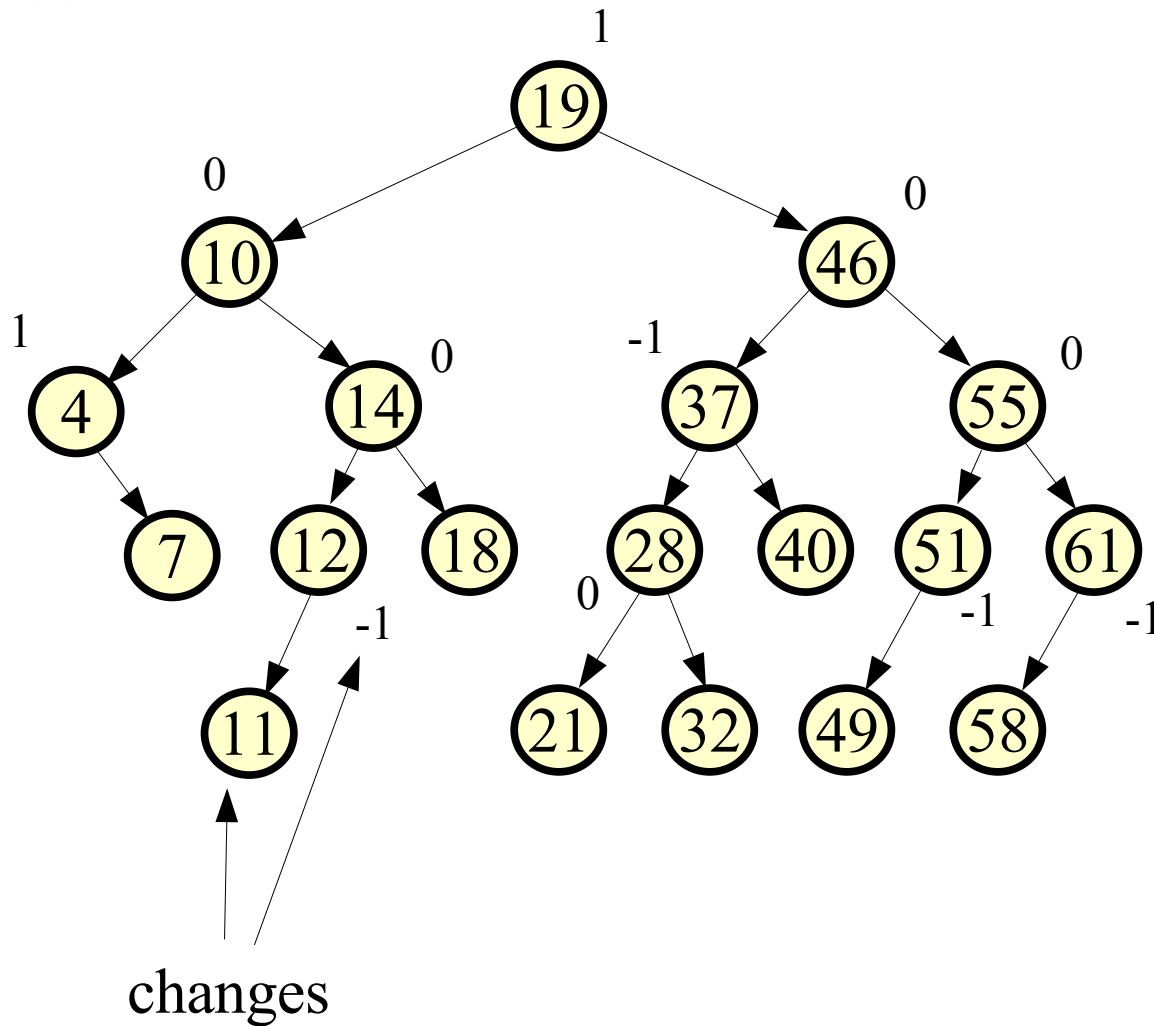
# Balanced Binaries – AVL Trees

Example:



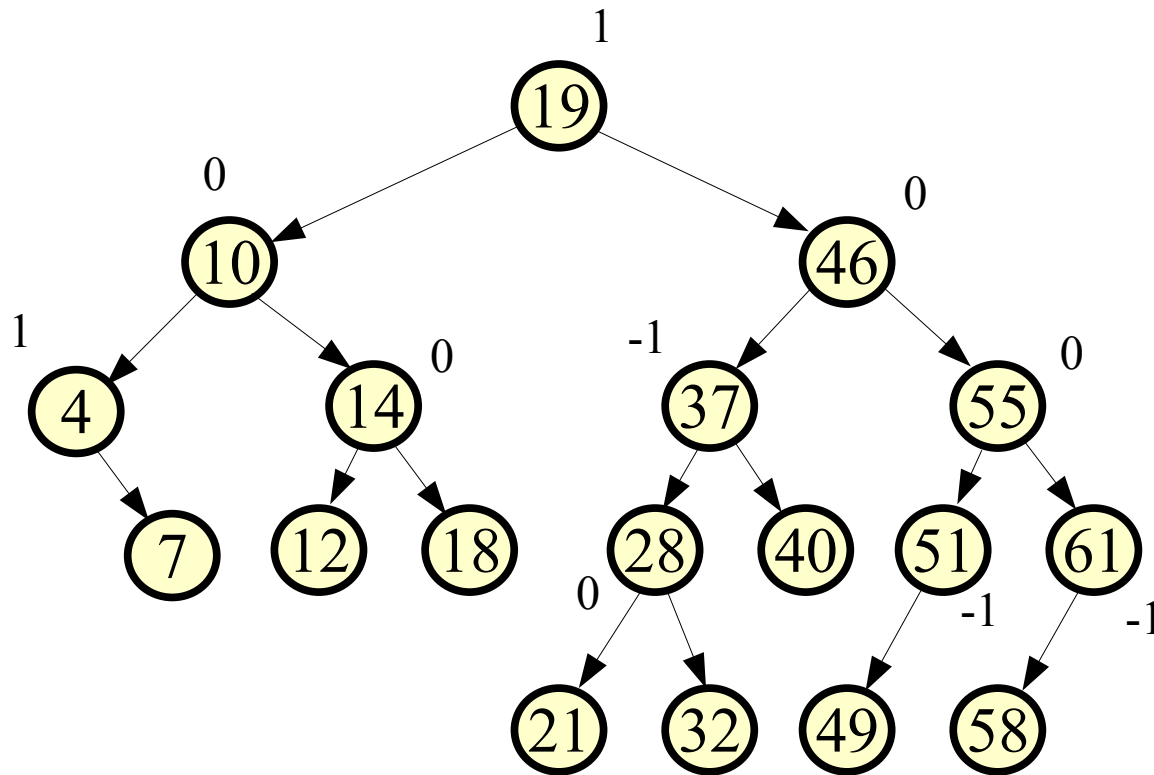
# Balanced Binaries – AVL Trees

Example: Insert 11



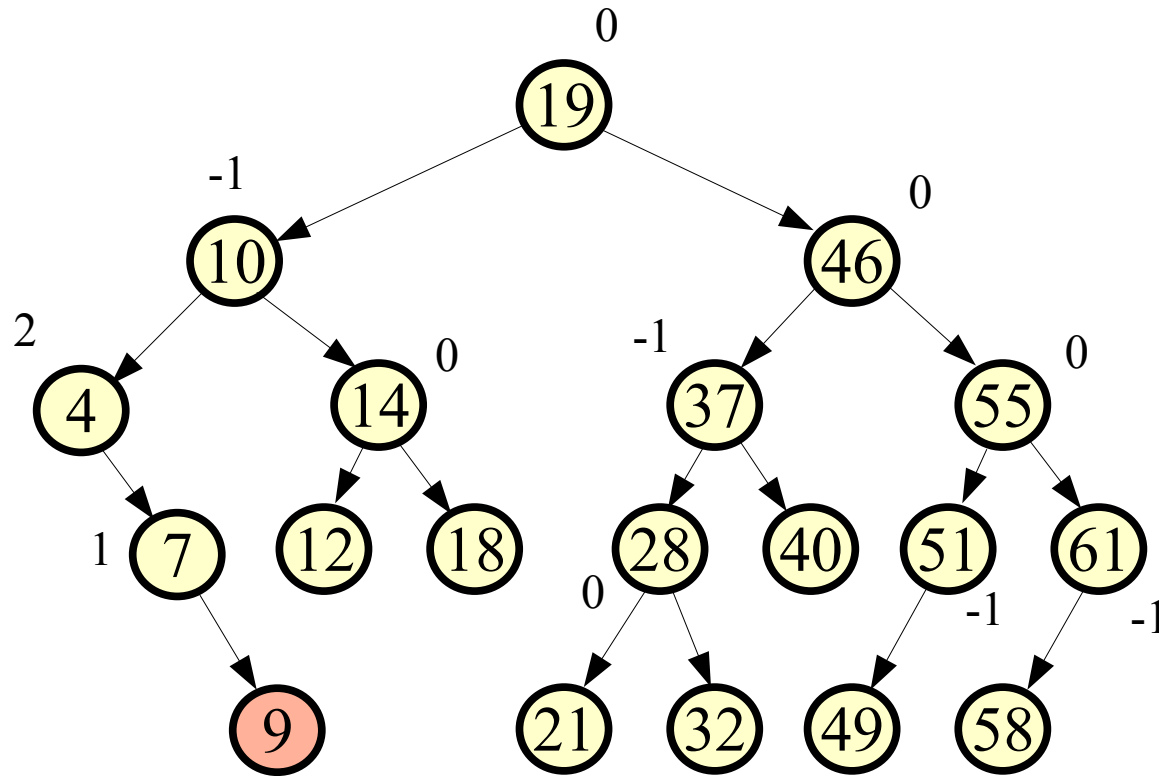
# Balanced Binaries – AVL Trees

Example:



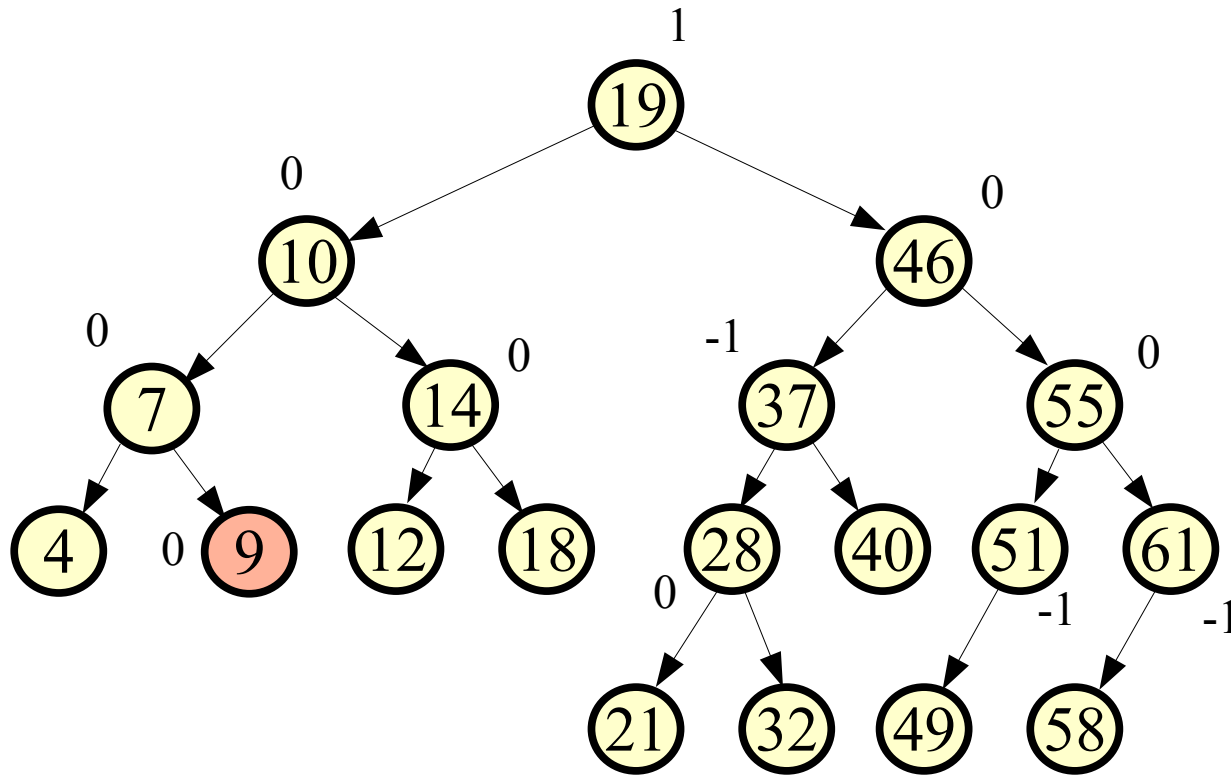
# Balanced Binaries – AVL Trees

Example: Insert 9



# Balanced Binaries – AVL Trees

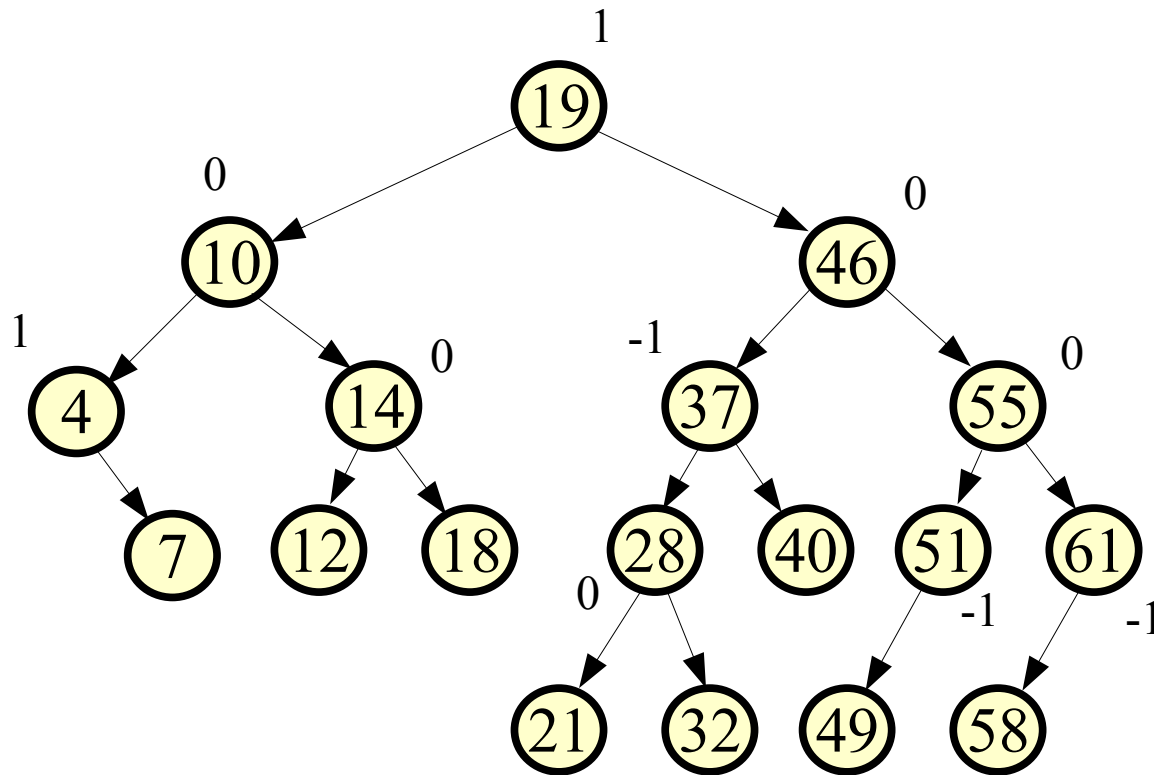
Example: Insert 9



Rotation around 7

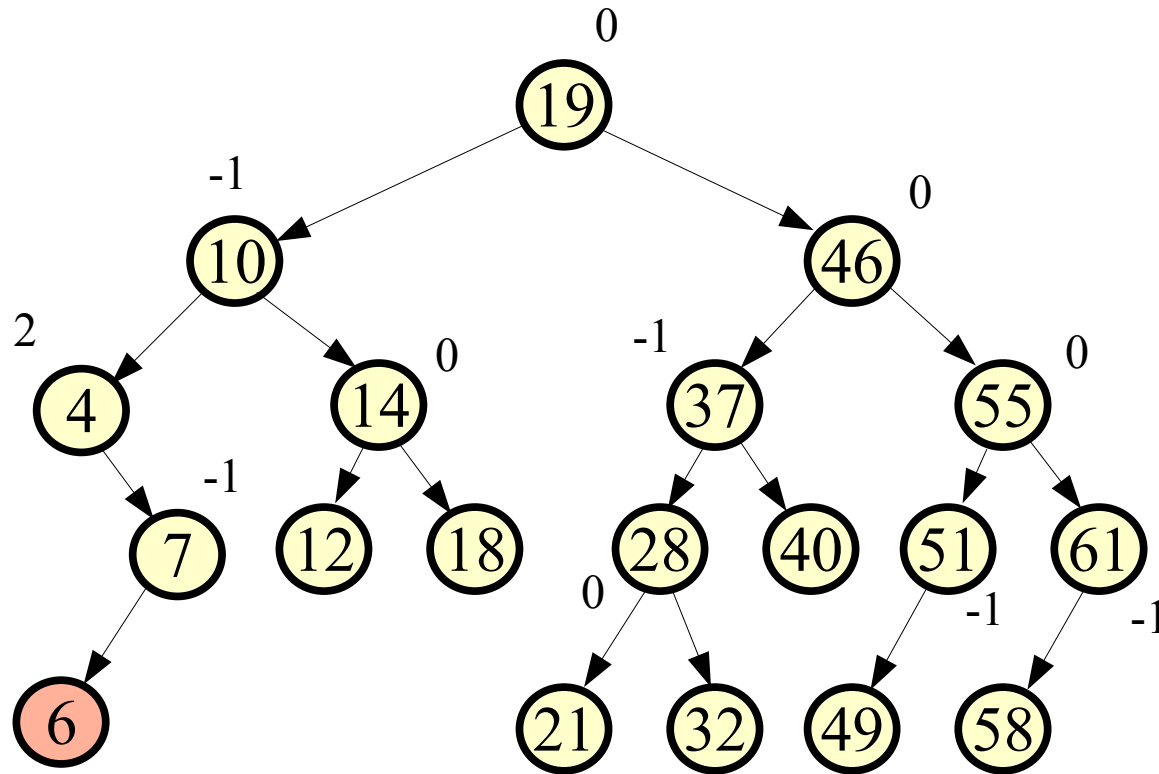
# Balanced Binaries – AVL Trees

Example:



# Balanced Binaries – AVL Trees

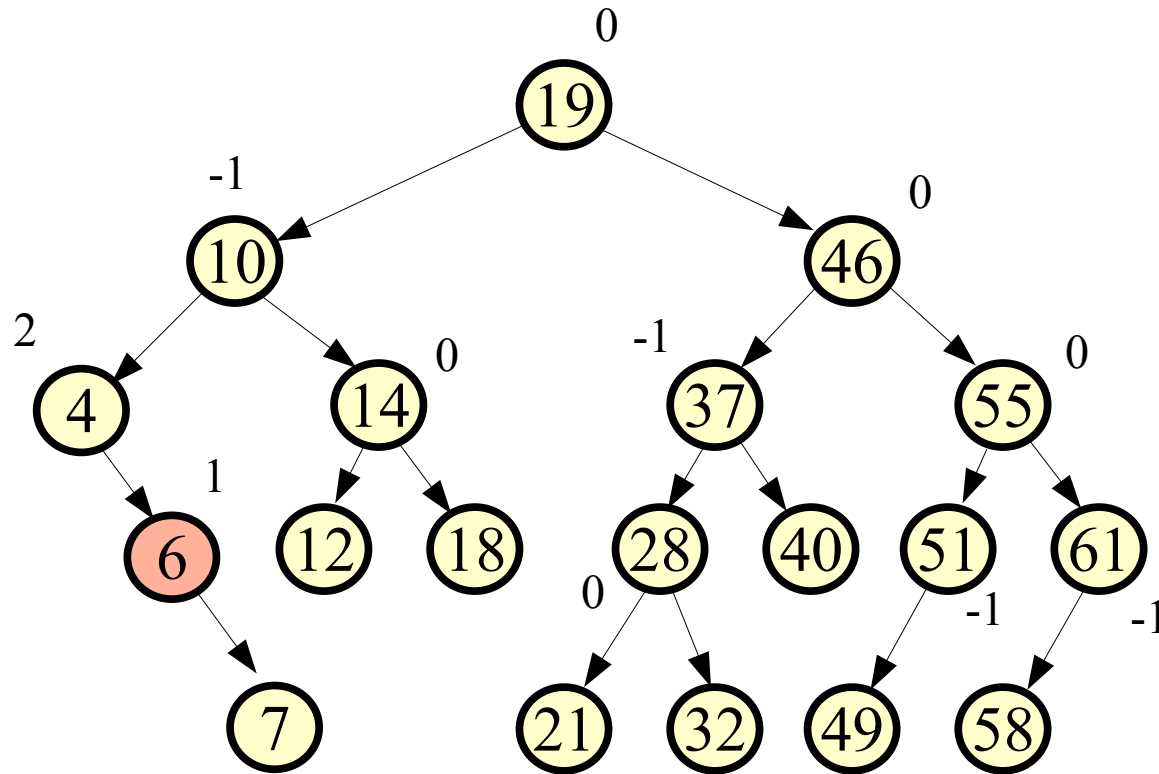
Example: Insert 6





# Balanced Binaries – AVL Trees

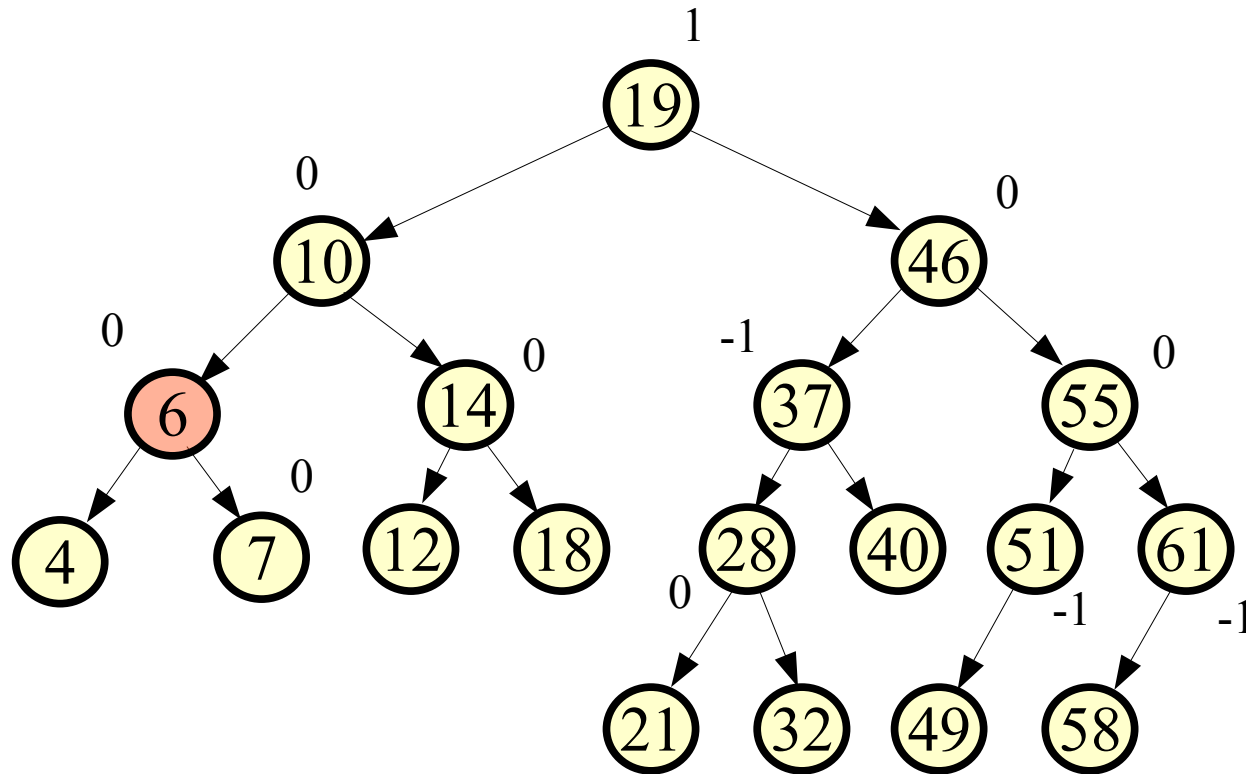
Example: Insert 6



Double rotation

# Balanced Binaries – AVL Trees

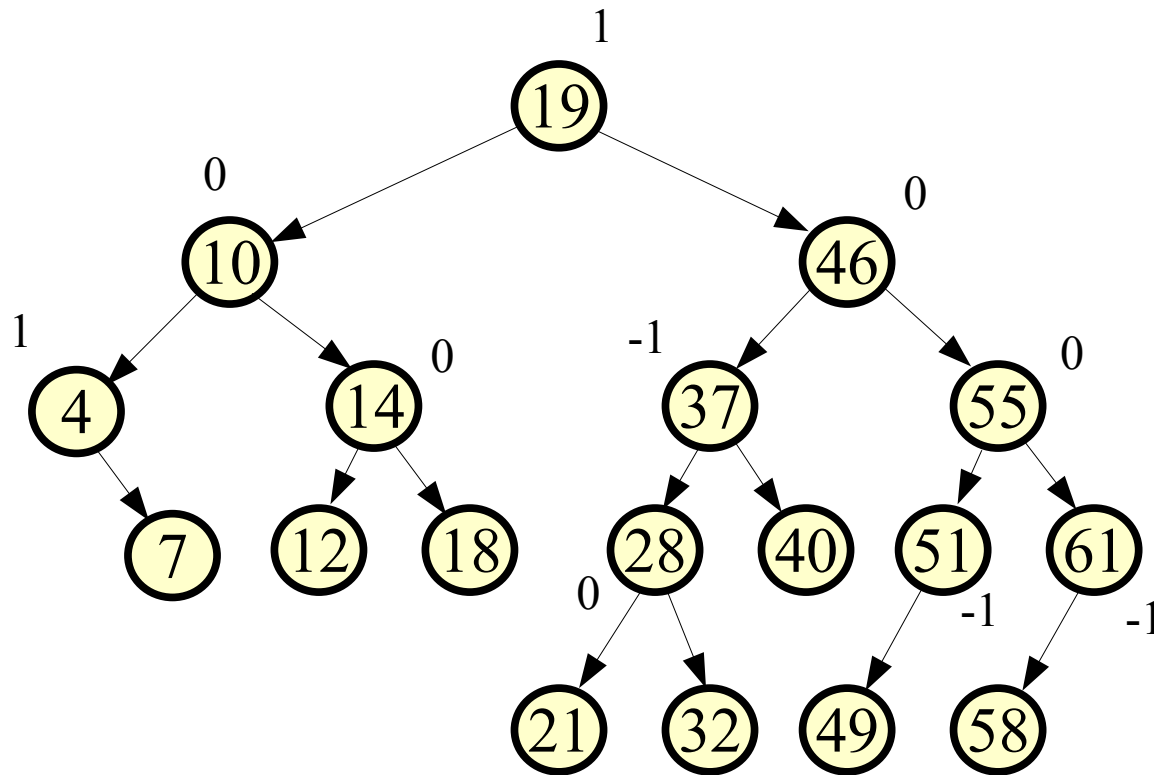
Example: Insert 6



Double rotation

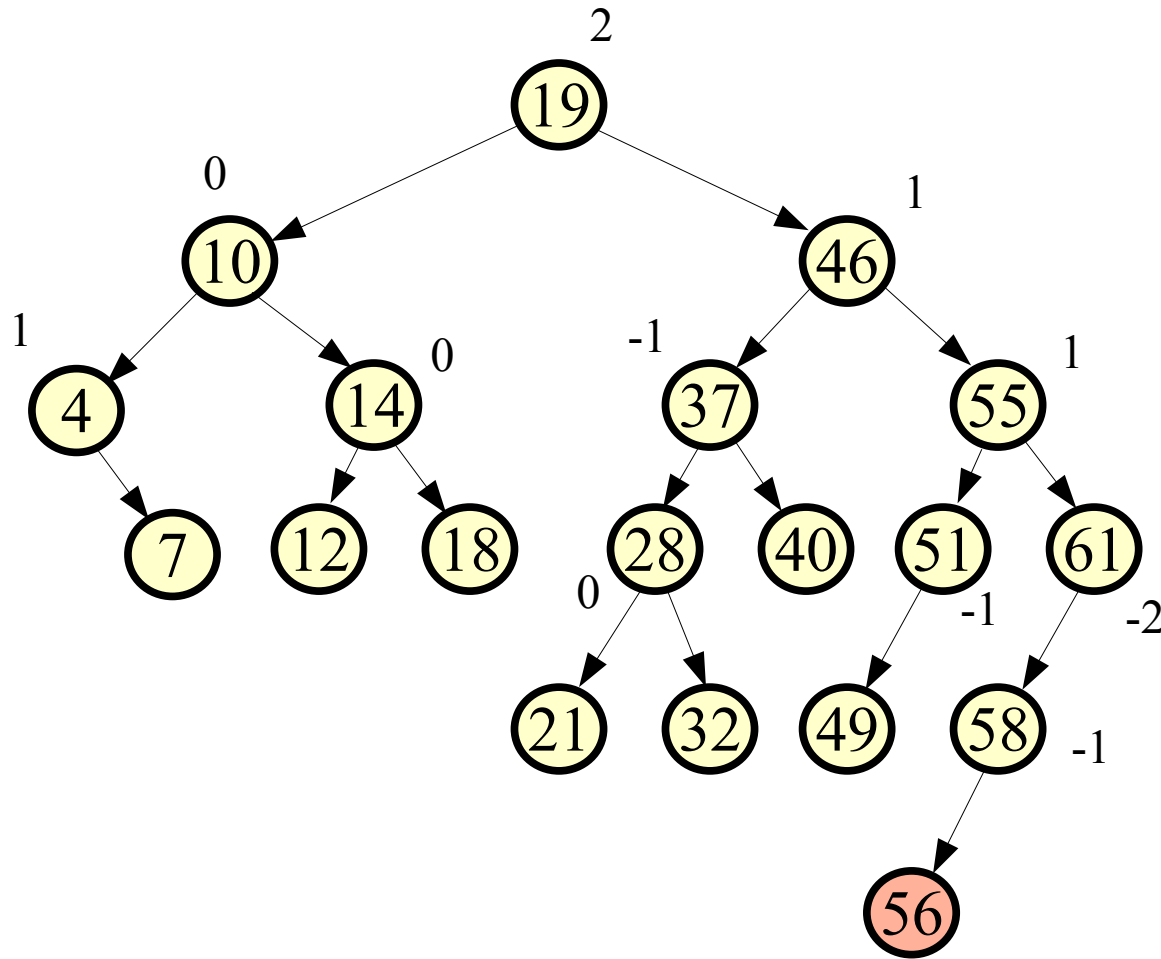
# Balanced Binaries – AVL Trees

Example:



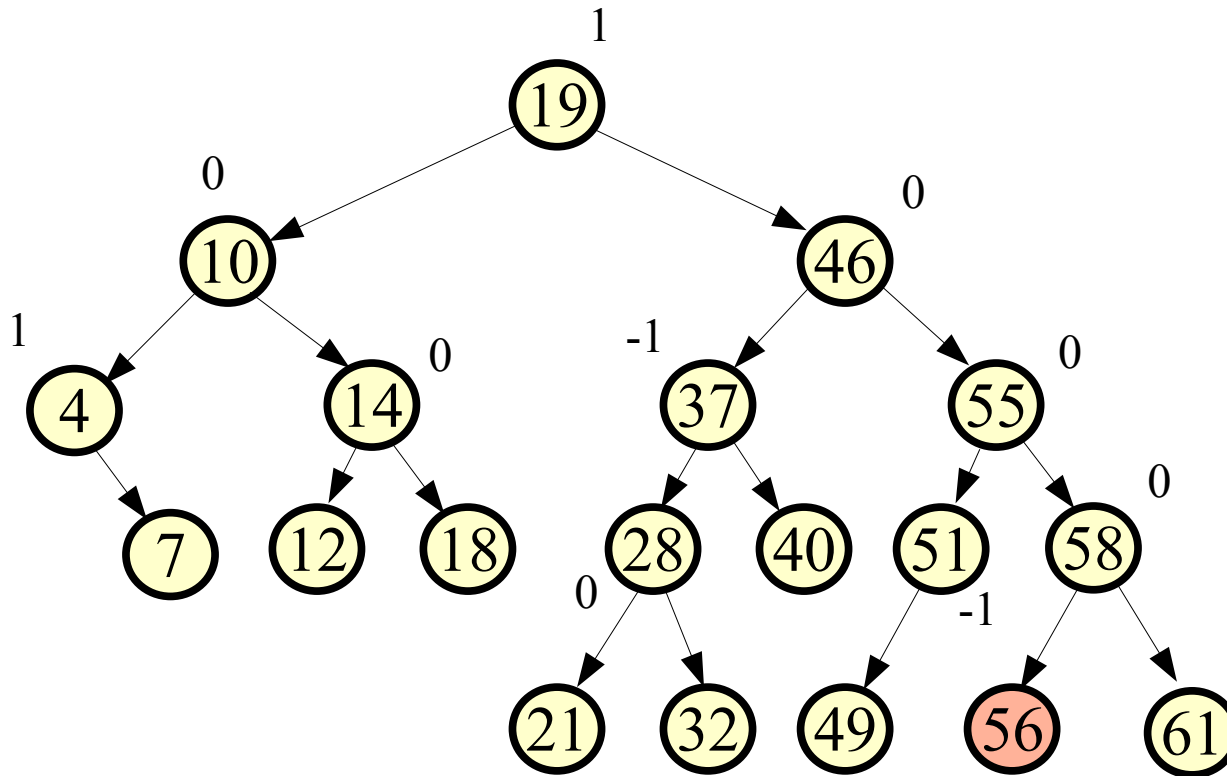
# Balanced Binaries – AVL Trees

Example: Insert 56



# Balanced Binaries – AVL Trees

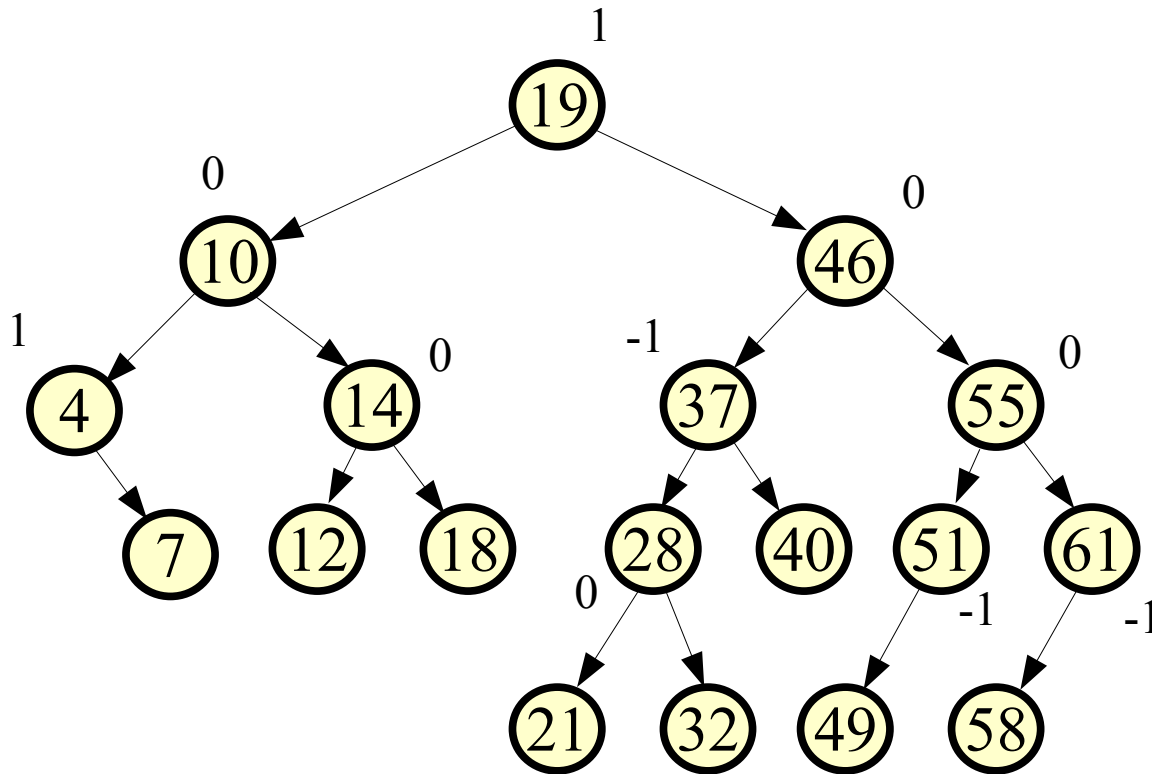
Example: Insert 56



Single rotation around 58

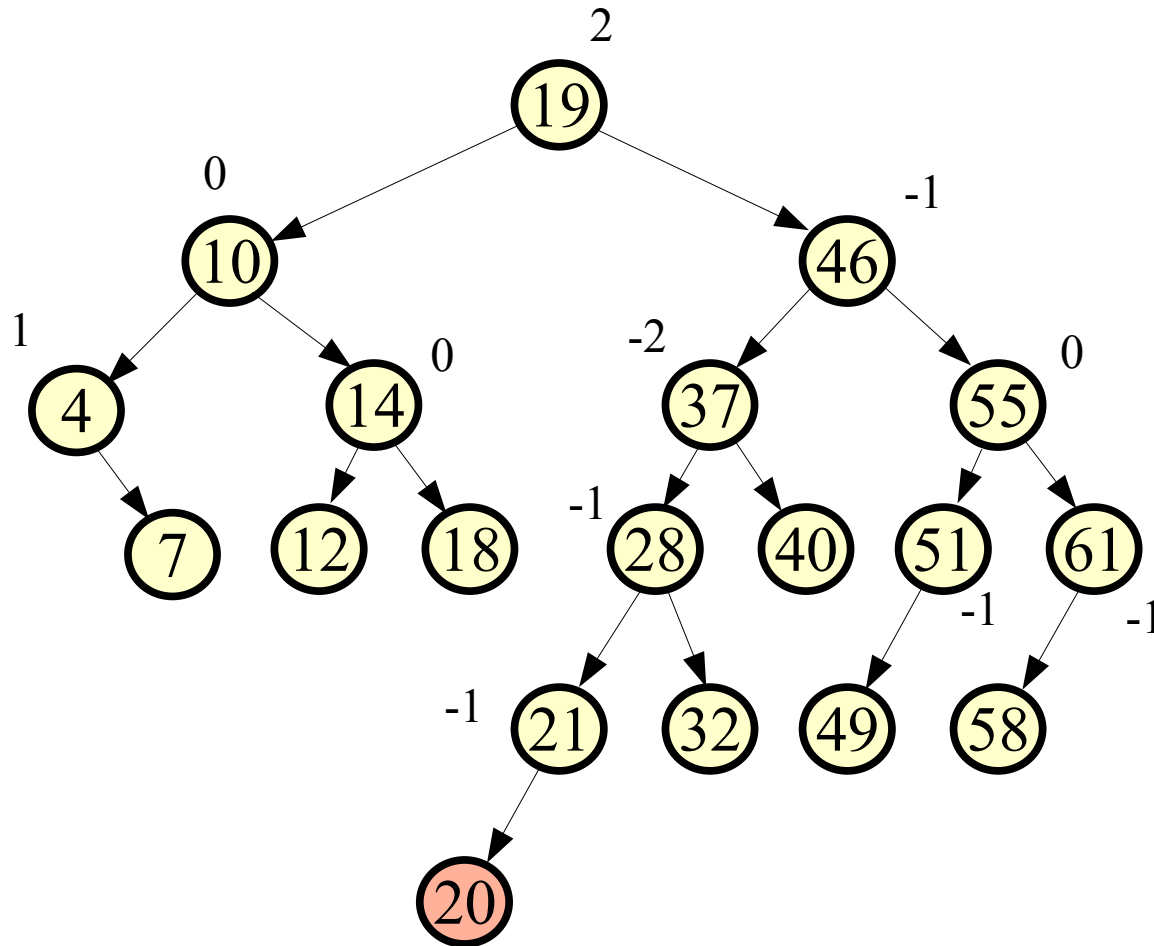
# Balanced Binaries – AVL Trees

Example:



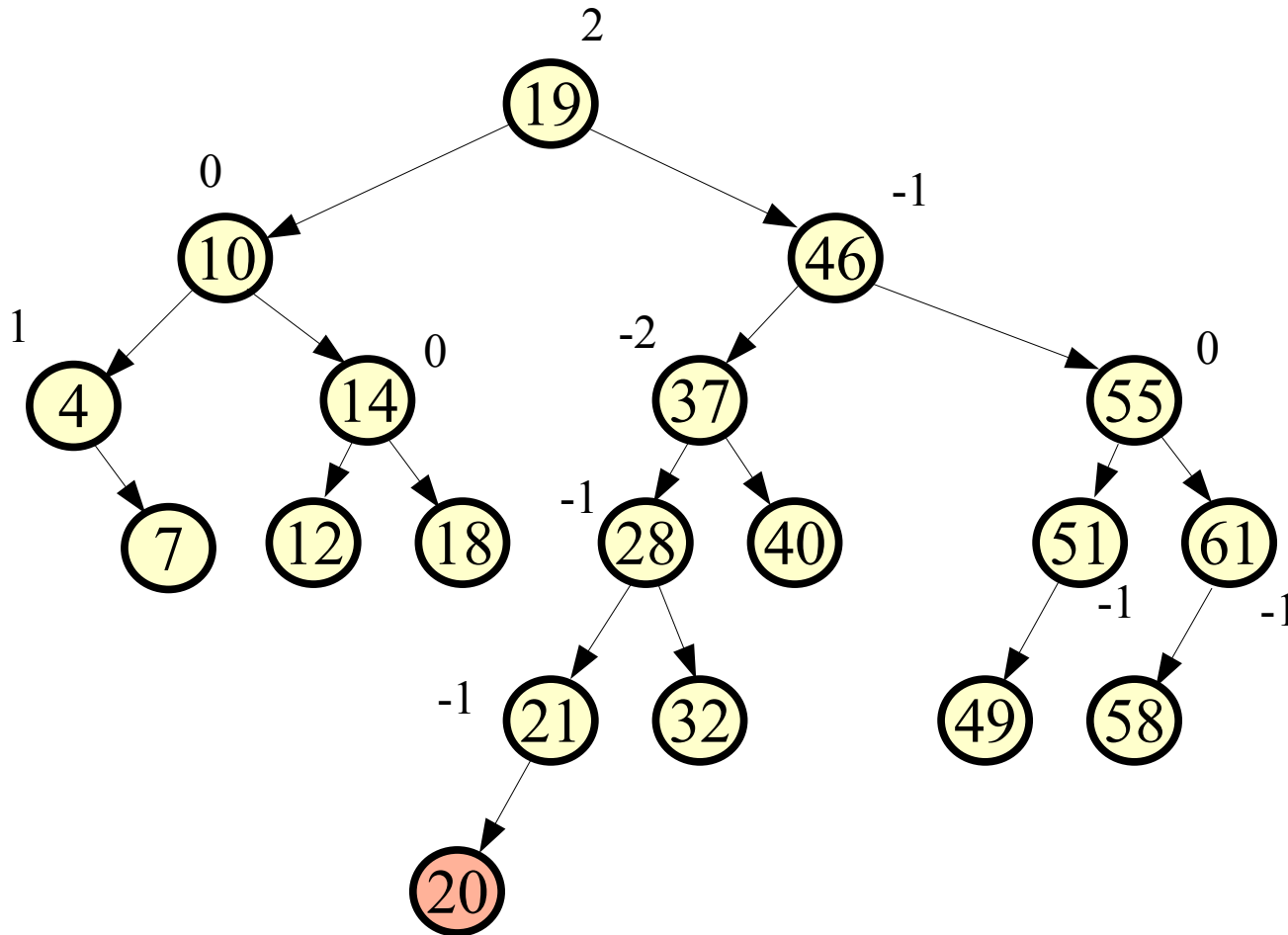
# Balanced Binaries – AVL Trees

Example: Insert 20



# Balanced Binaries – AVL Trees

Example: Insert 20

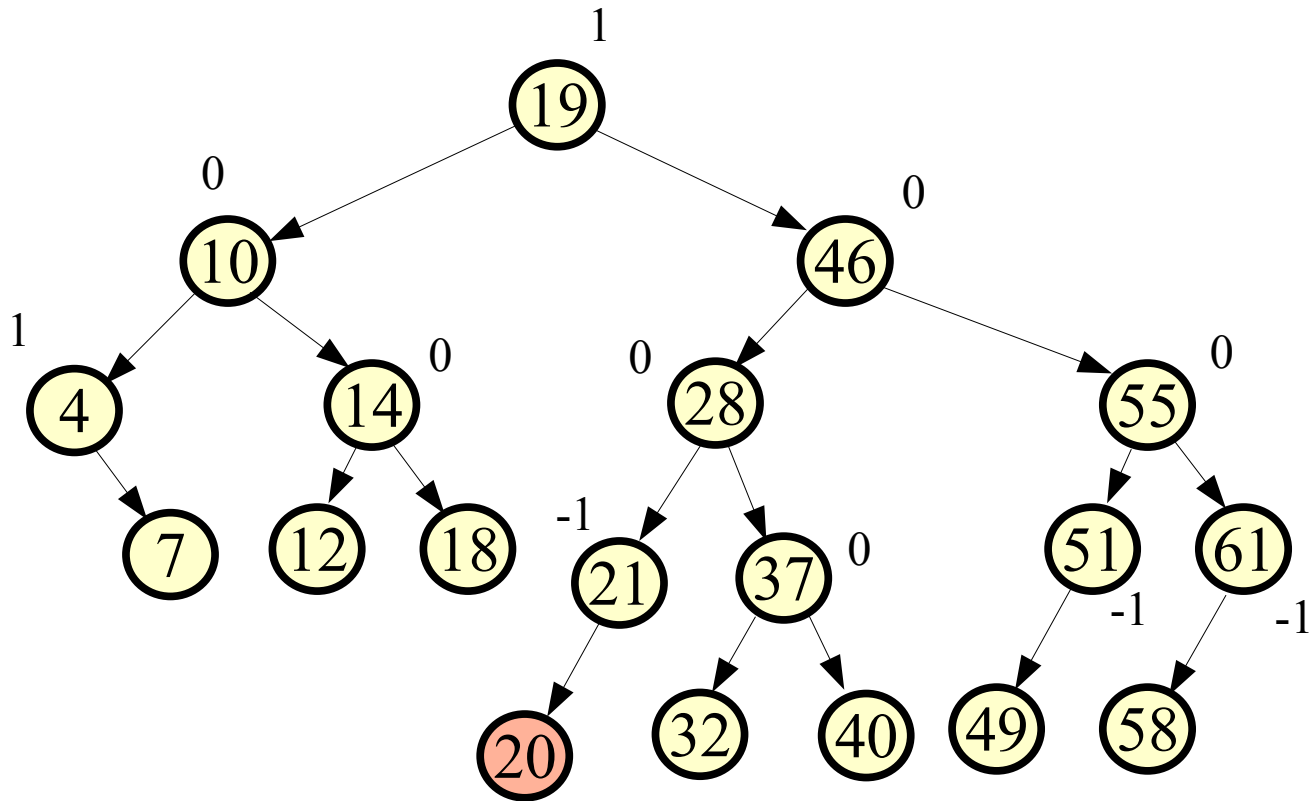


Rotate around 28



# Balanced Binaries – AVL Trees

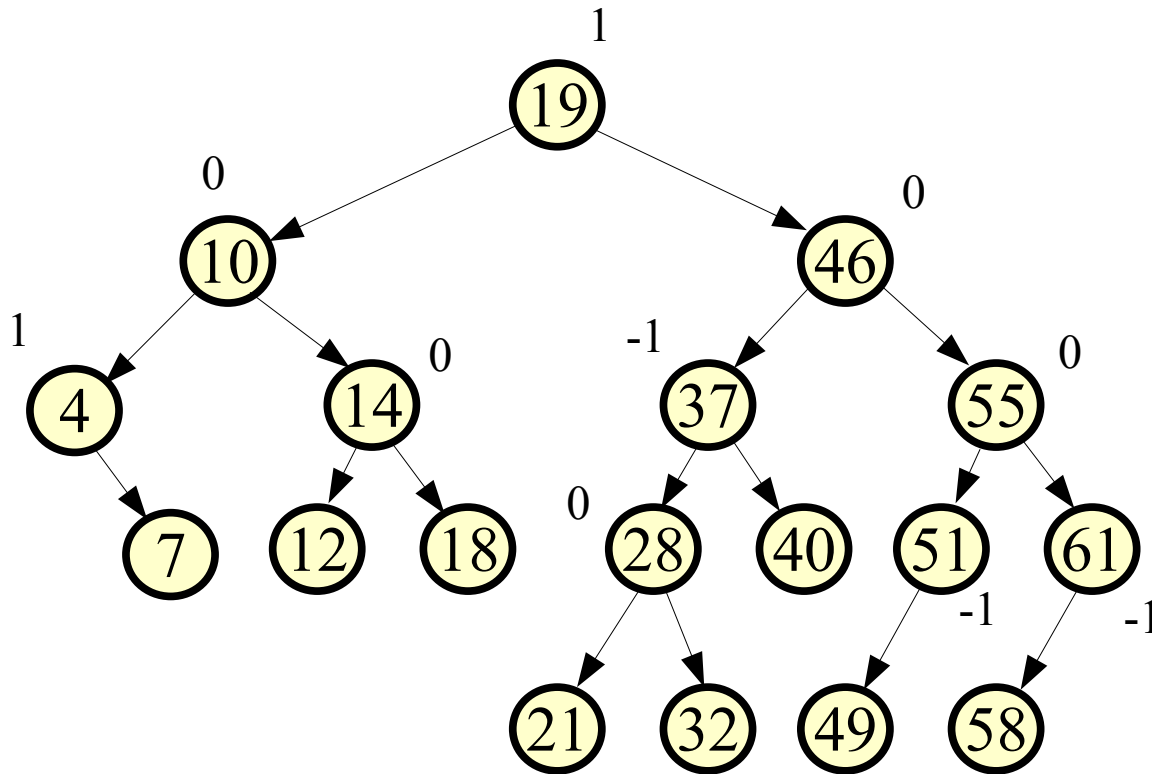
Example: Insert 20



Rotate around 28

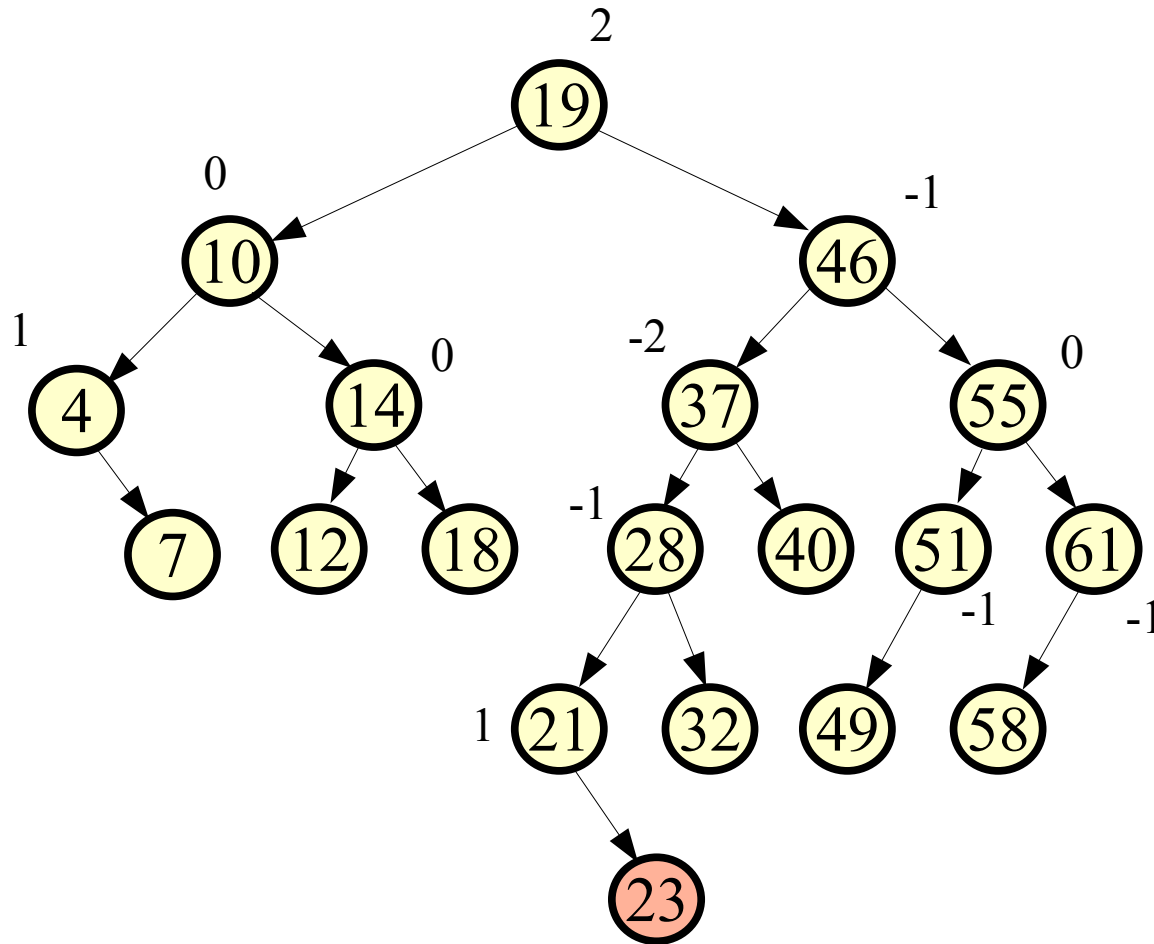
# Balanced Binaries – AVL Trees

Example:



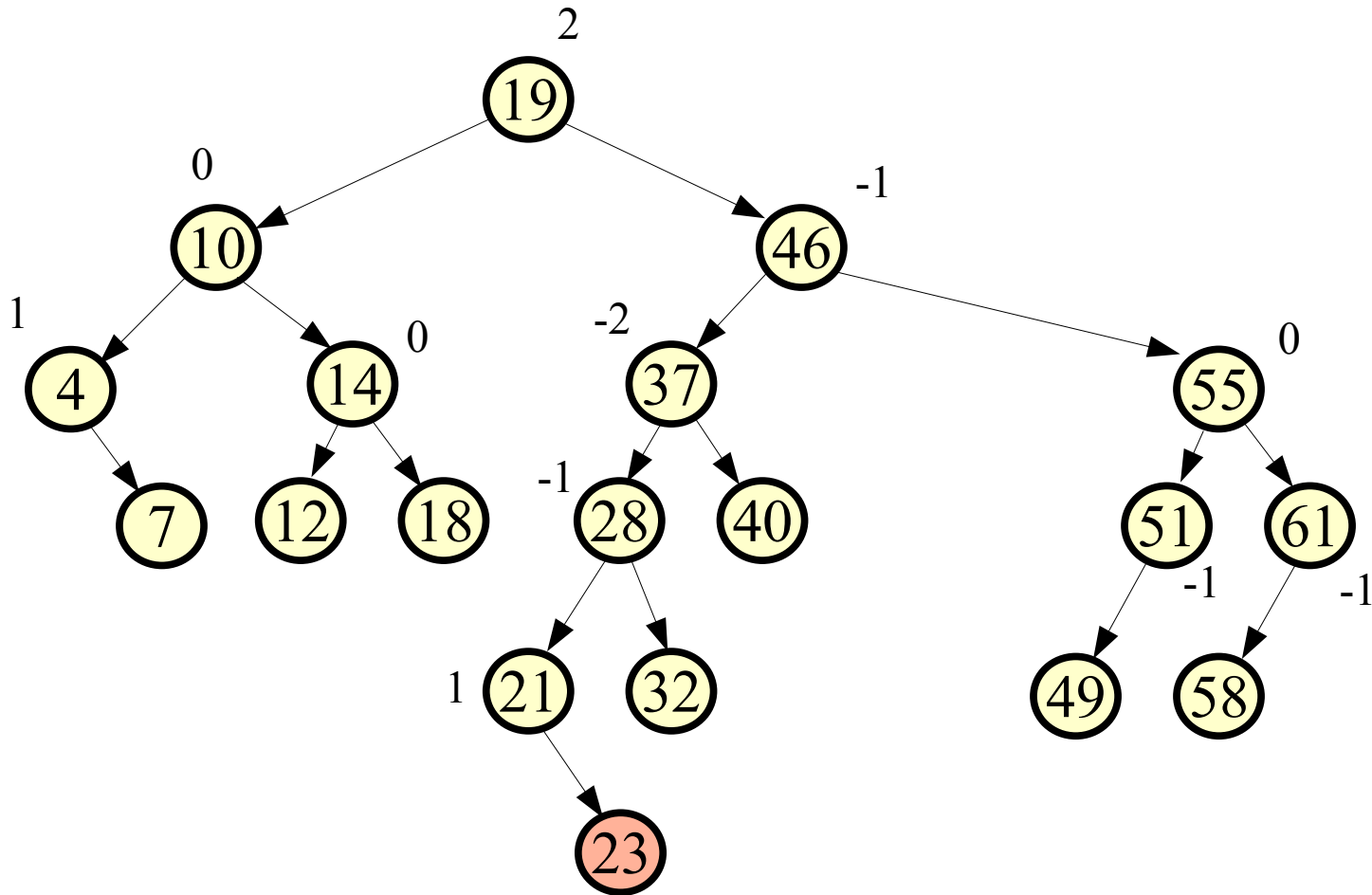
# Balanced Binaries – AVL Trees

Example: Insert 23



# Balanced Binaries – AVL Trees

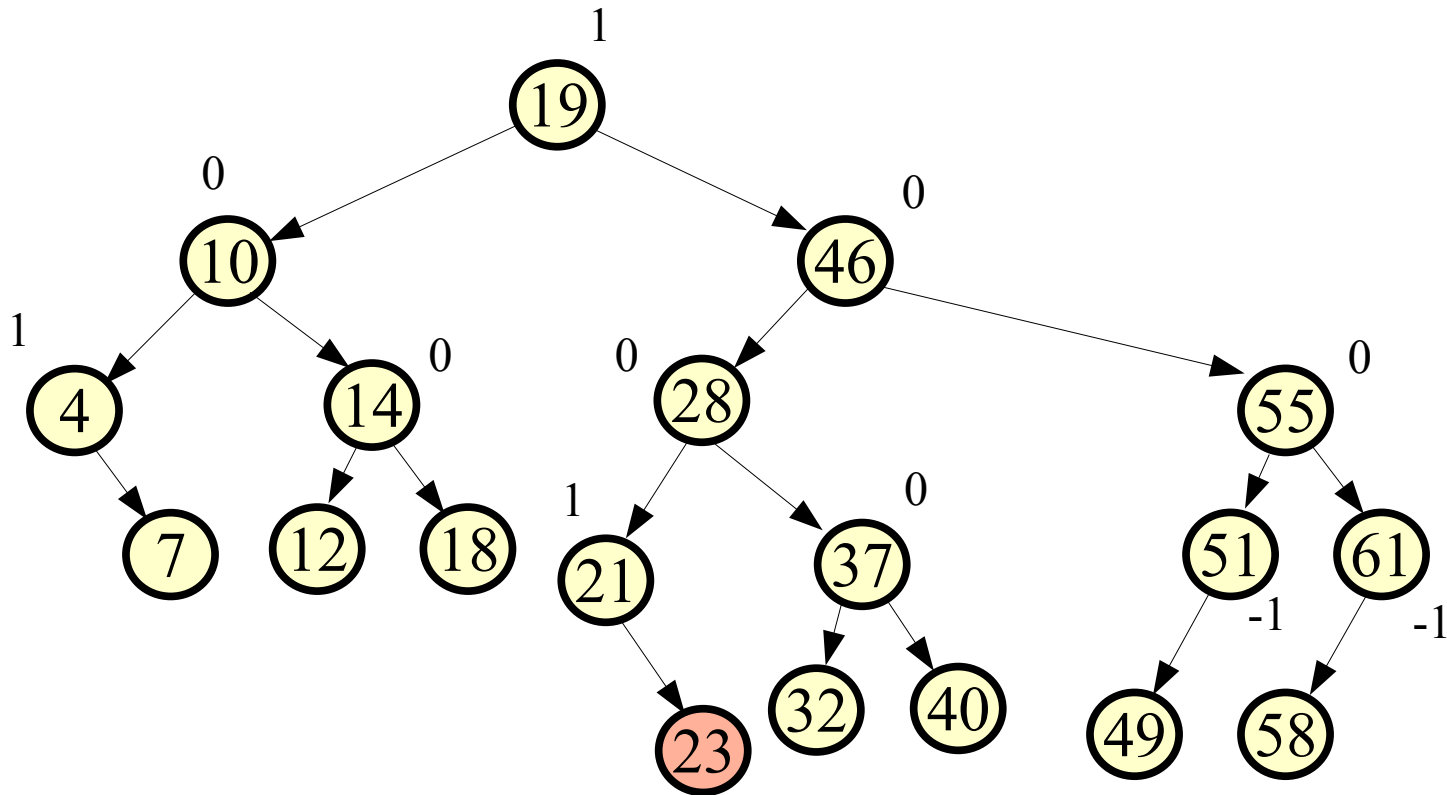
Example: Insert 23



Rotation around 28

# Balanced Binaries – AVL Trees

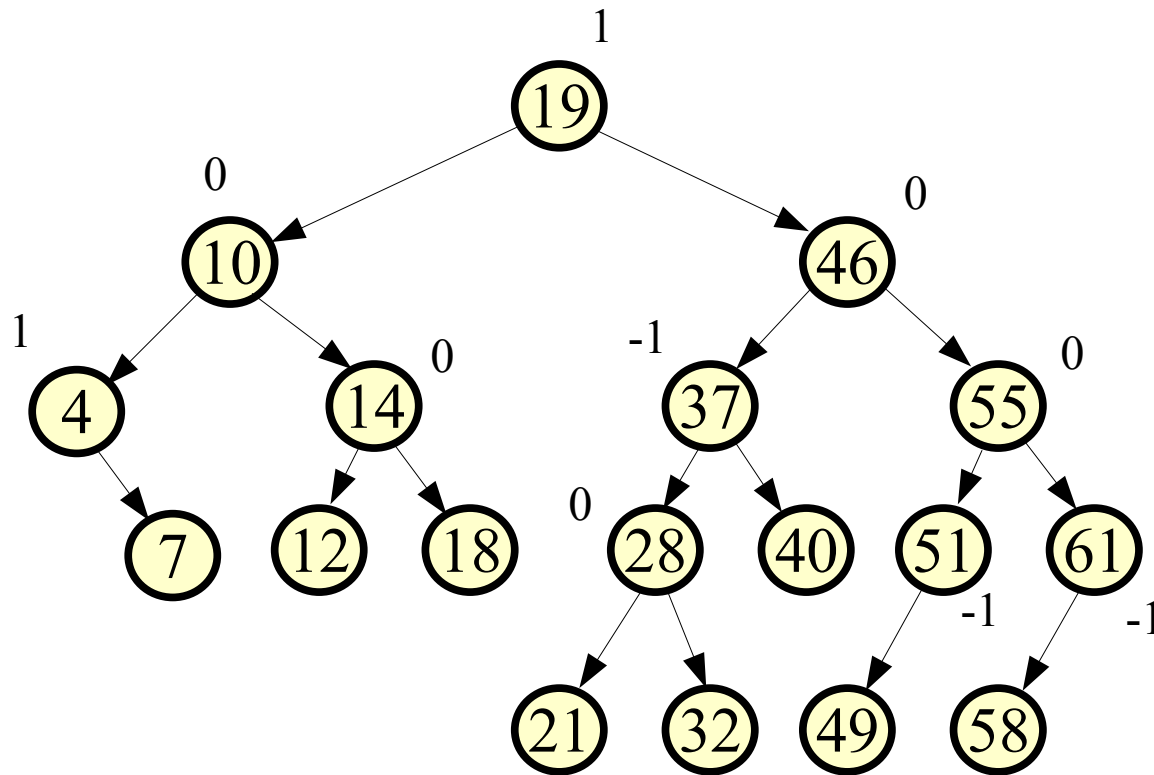
Example: Insert 23



Rotation around 28

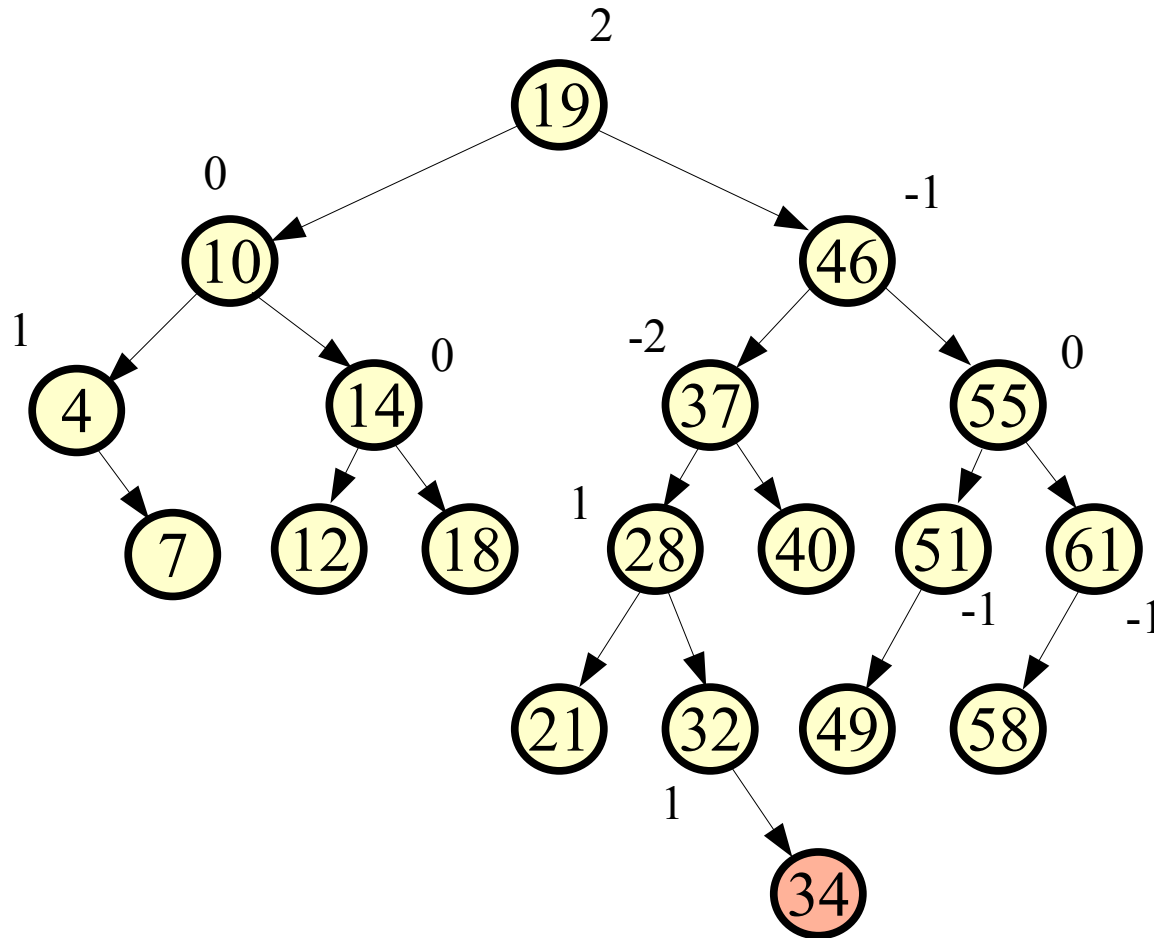
# Balanced Binaries – AVL Trees

Example:



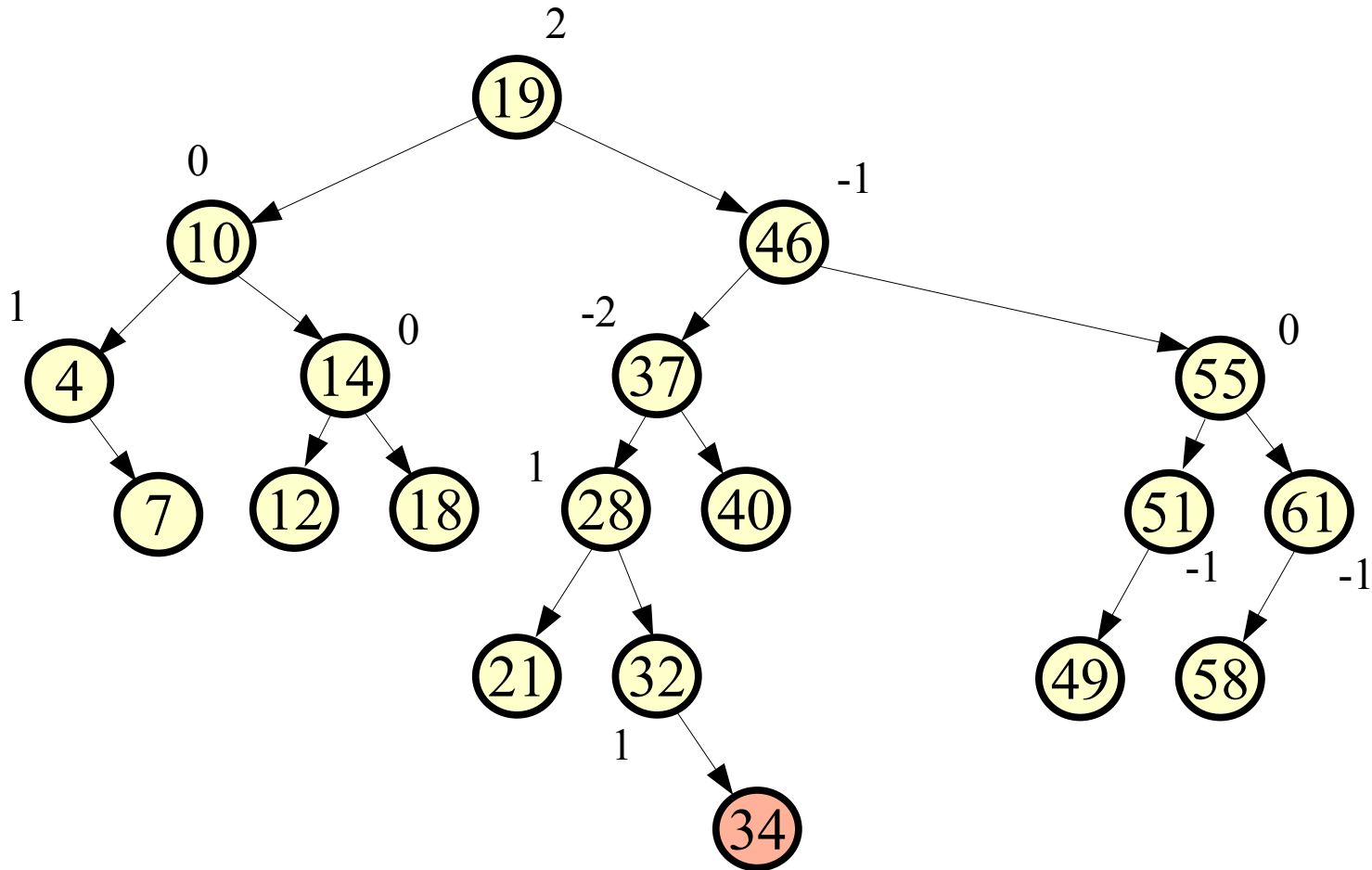
# Balanced Binaries – AVL Trees

Example: Insert 34



# Balanced Binaries – AVL Trees

Example: Insert 34

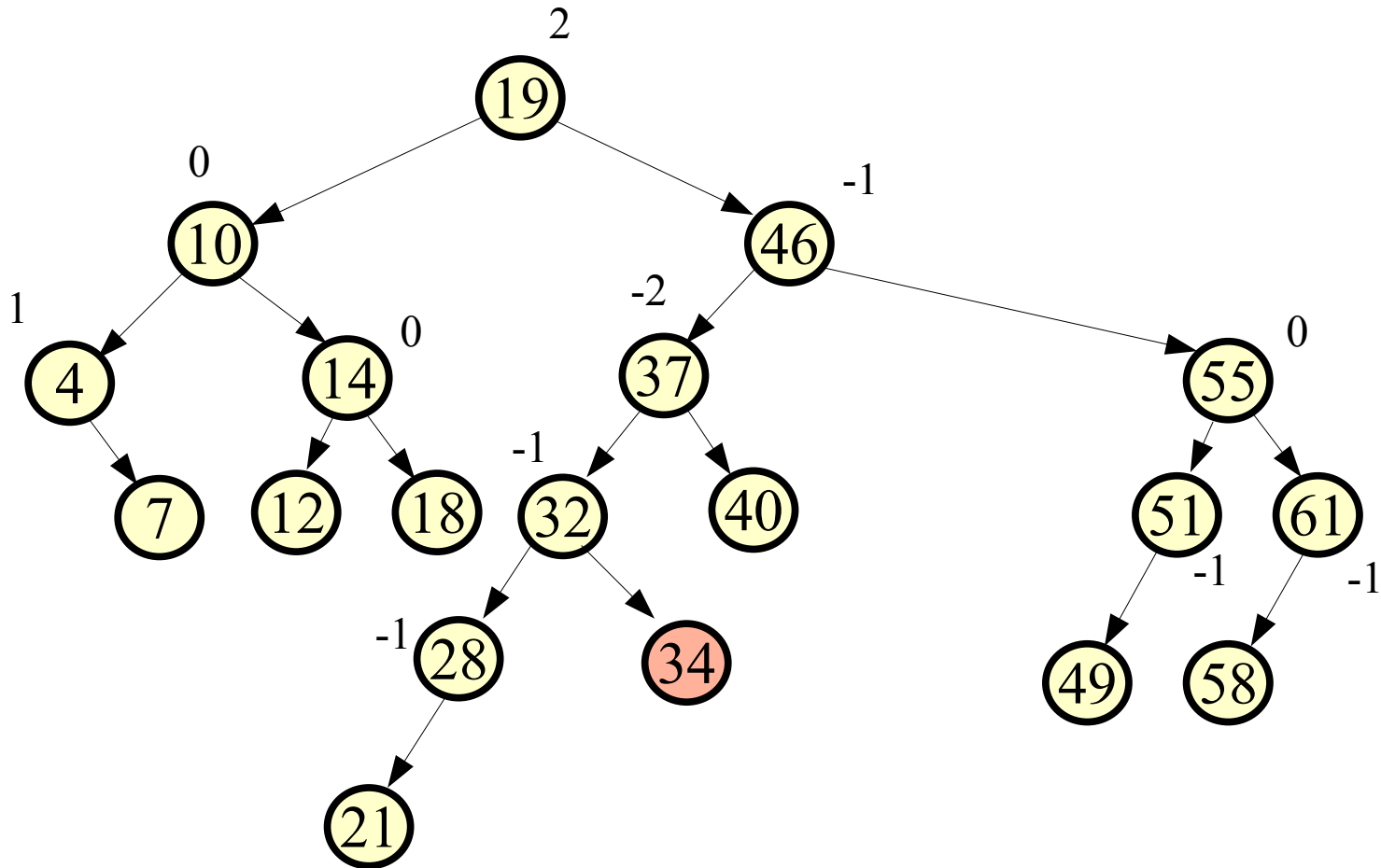


Double rotation around 32



# Balanced Binaries – AVL Trees

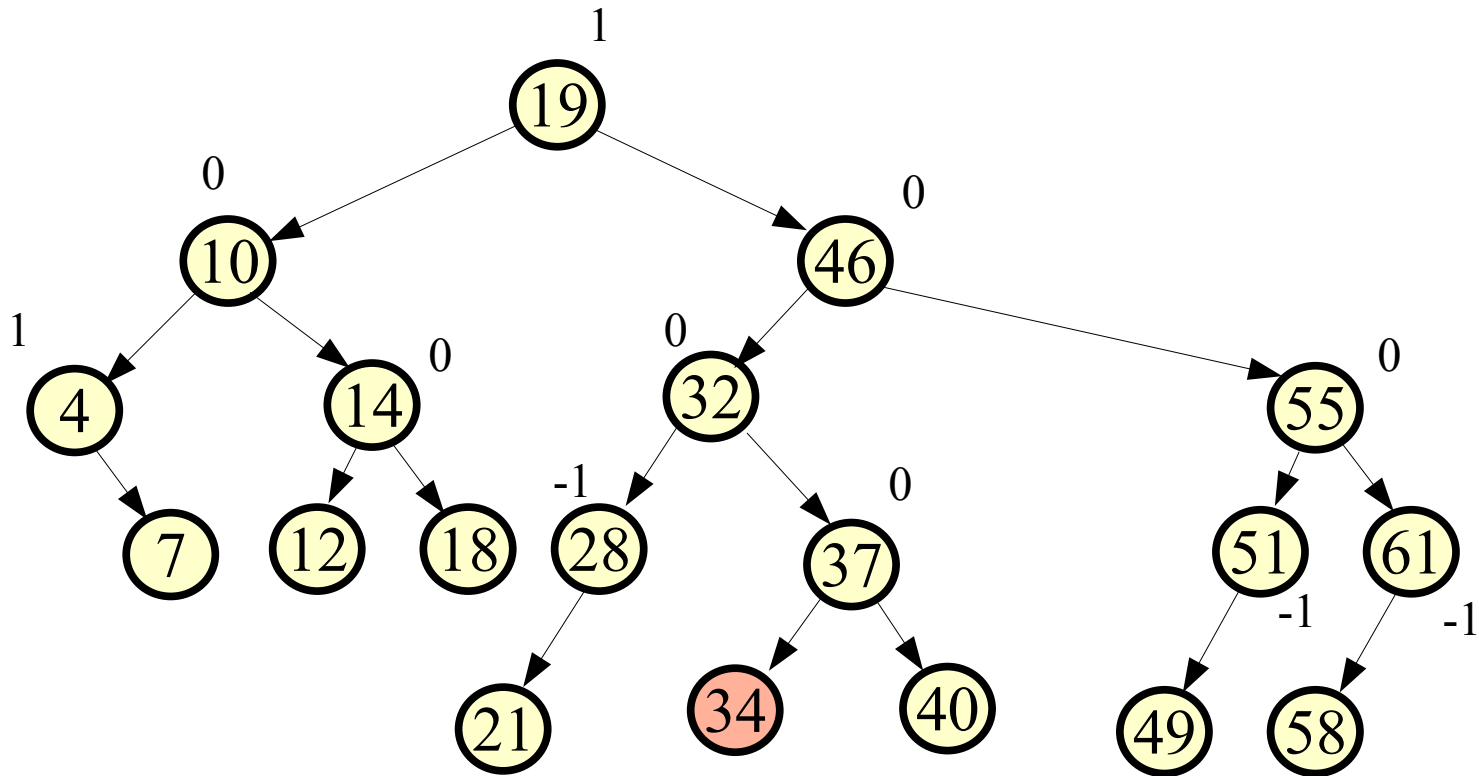
Example: Insert 34



Double rotation around 32

# Balanced Binaries – AVL Trees

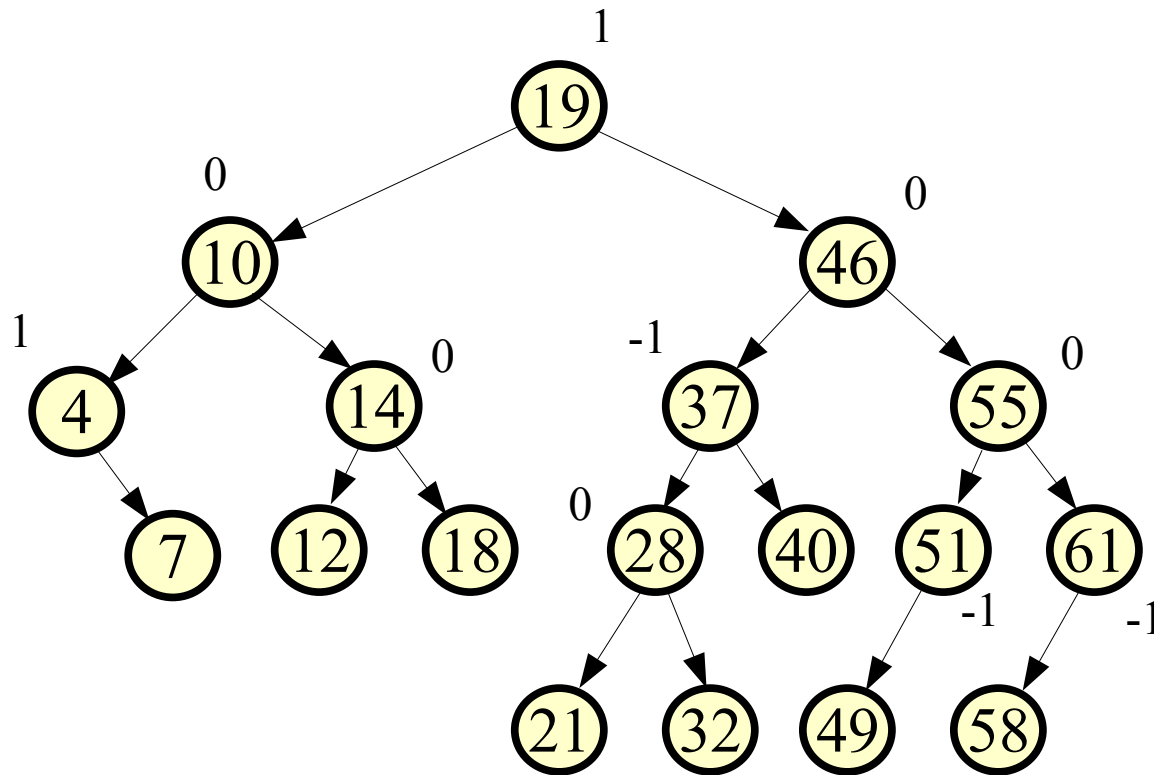
Example: Insert 34



Double rotation around 32

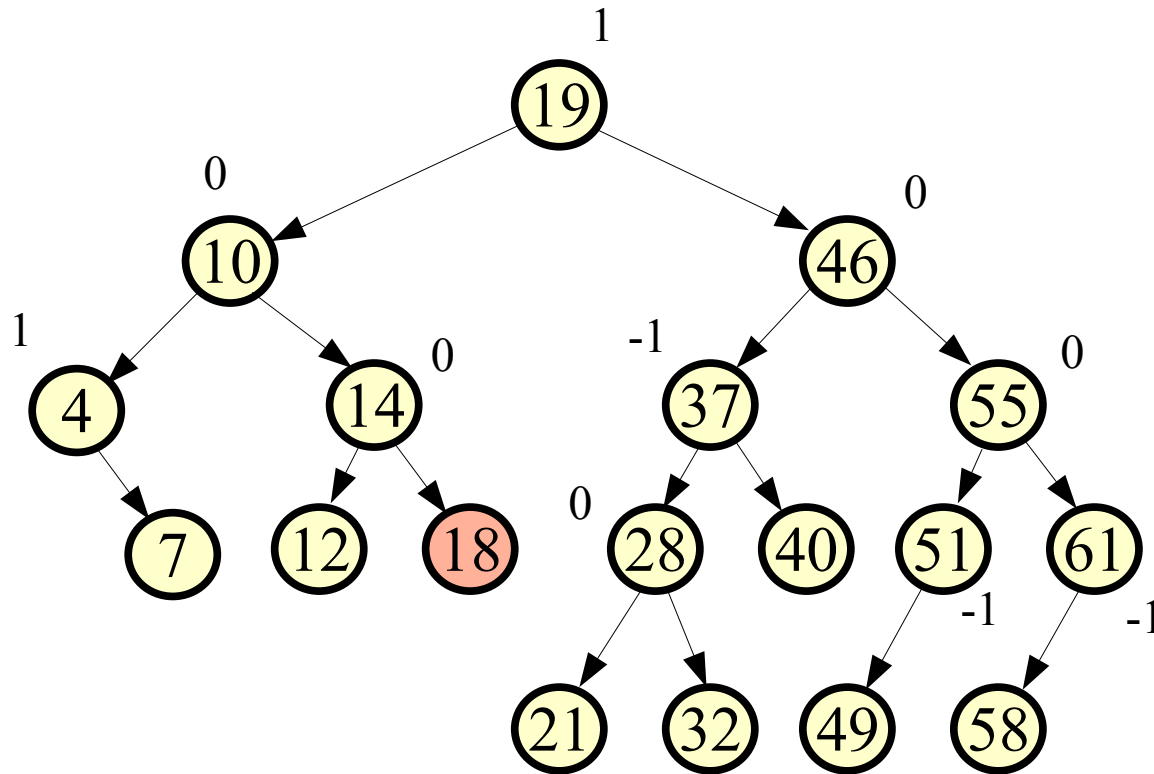
# Balanced Binaries – AVL Trees

Example:



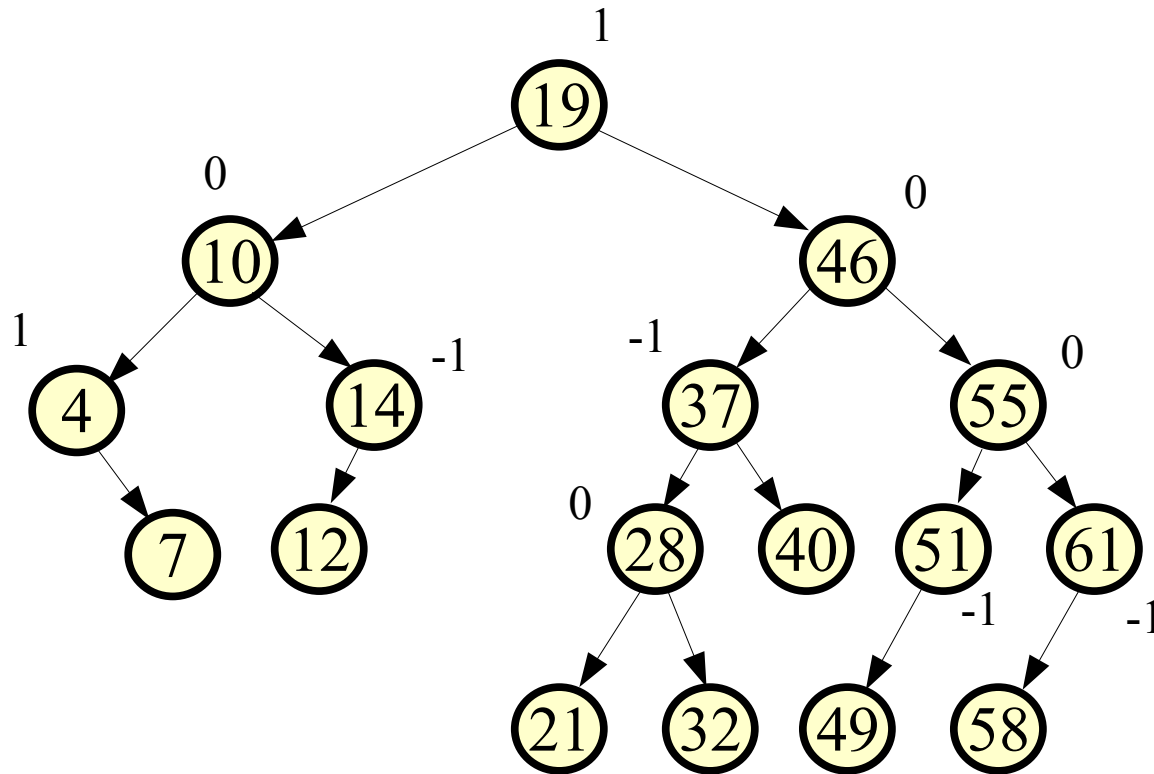
# Balanced Binaries – AVL Trees

Example: Delete 18



# Balanced Binaries – AVL Trees

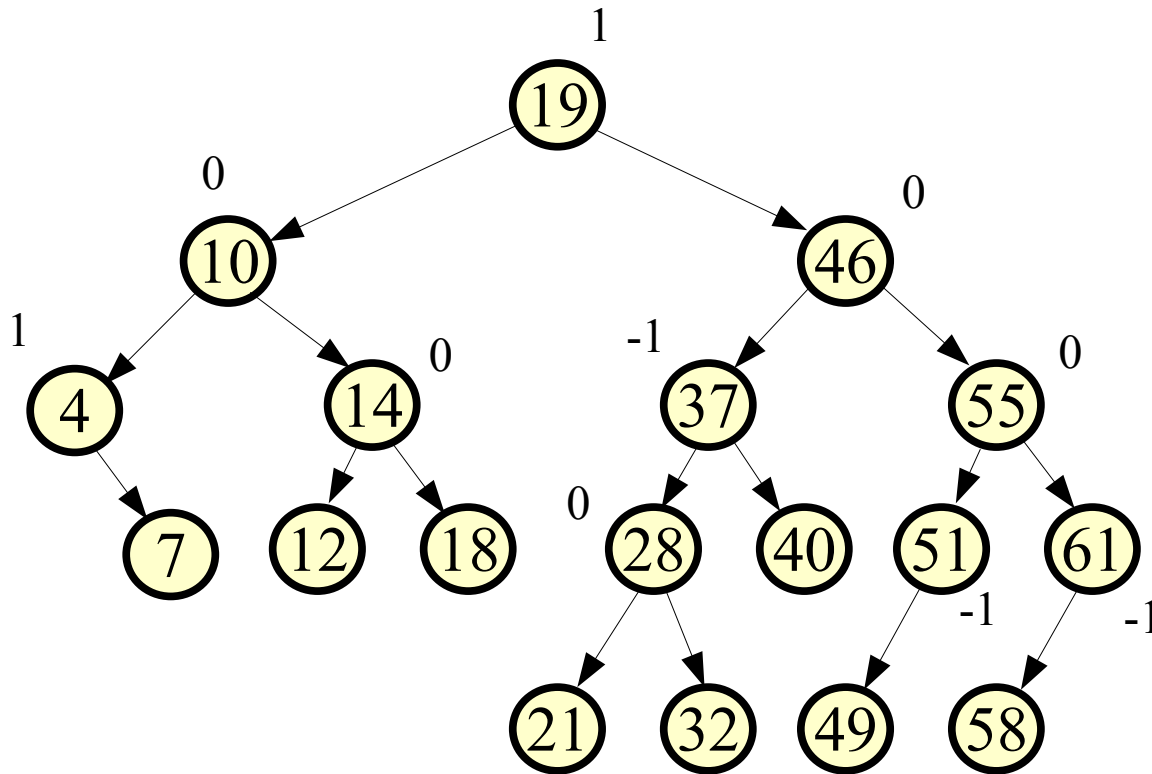
Example: Delete 18



No change

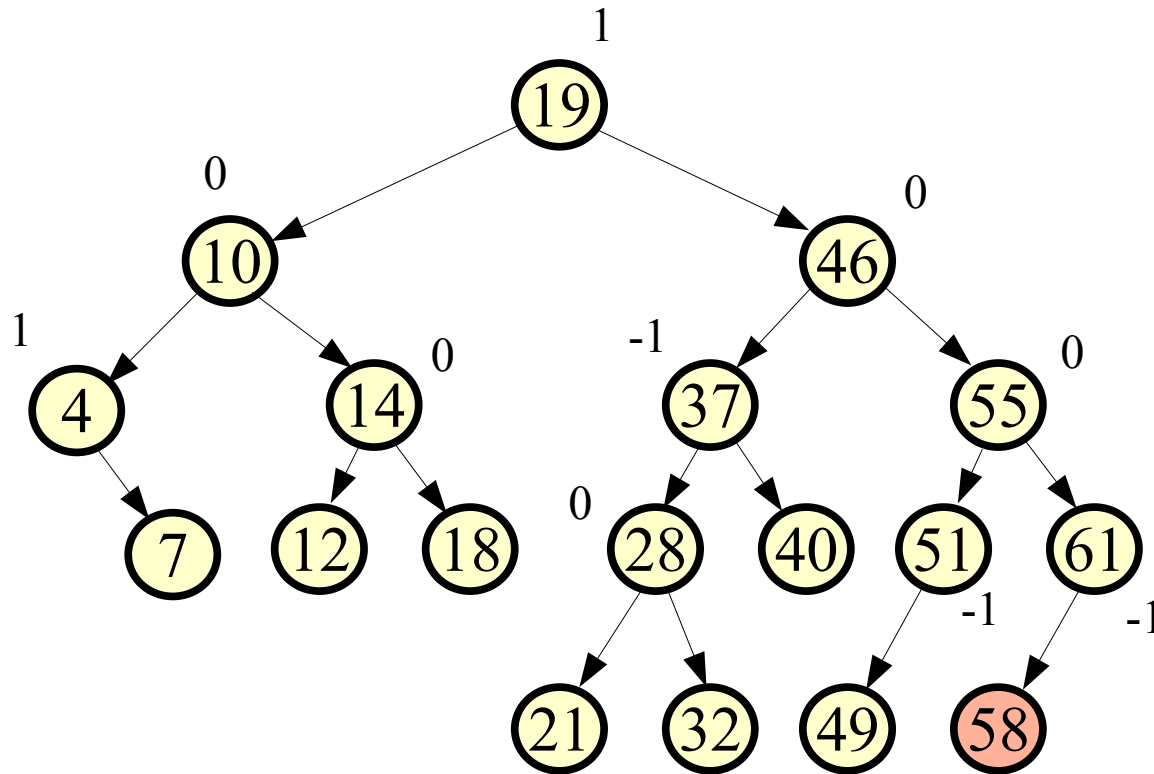
# Balanced Binaries – AVL Trees

Example:



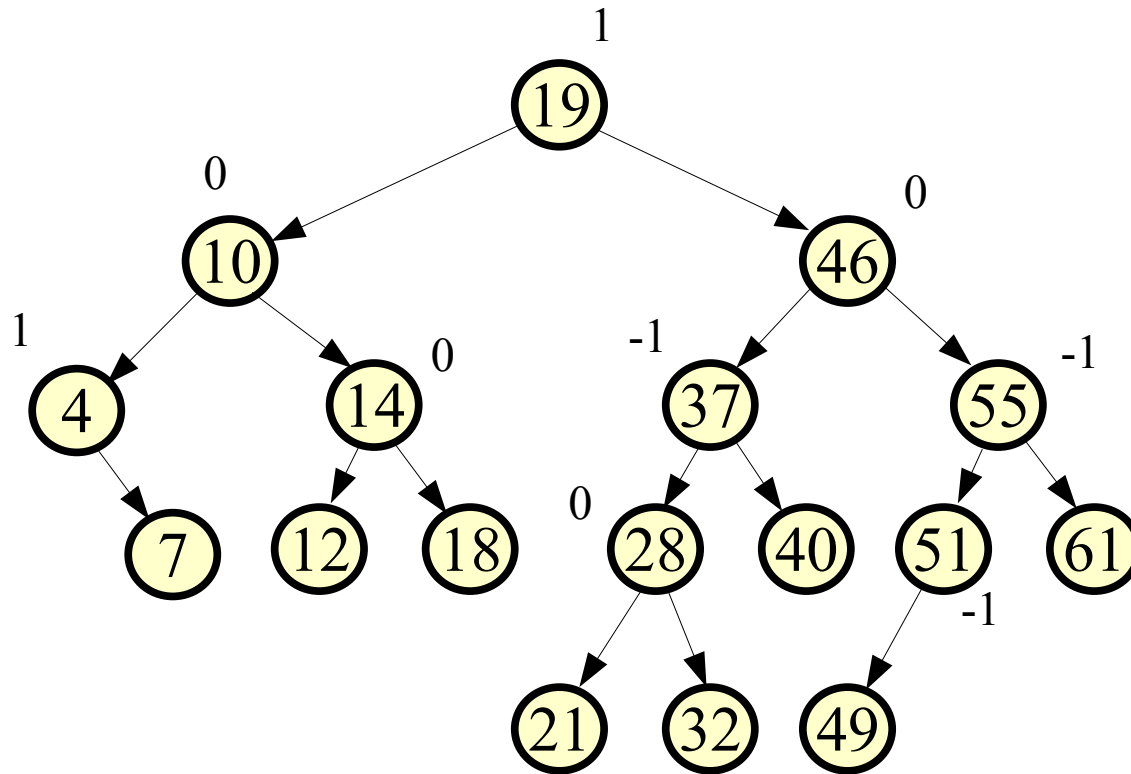
# Balanced Binaries – AVL Trees

Example: Delete 58



# Balanced Binaries – AVL Trees

Example: Delete 58

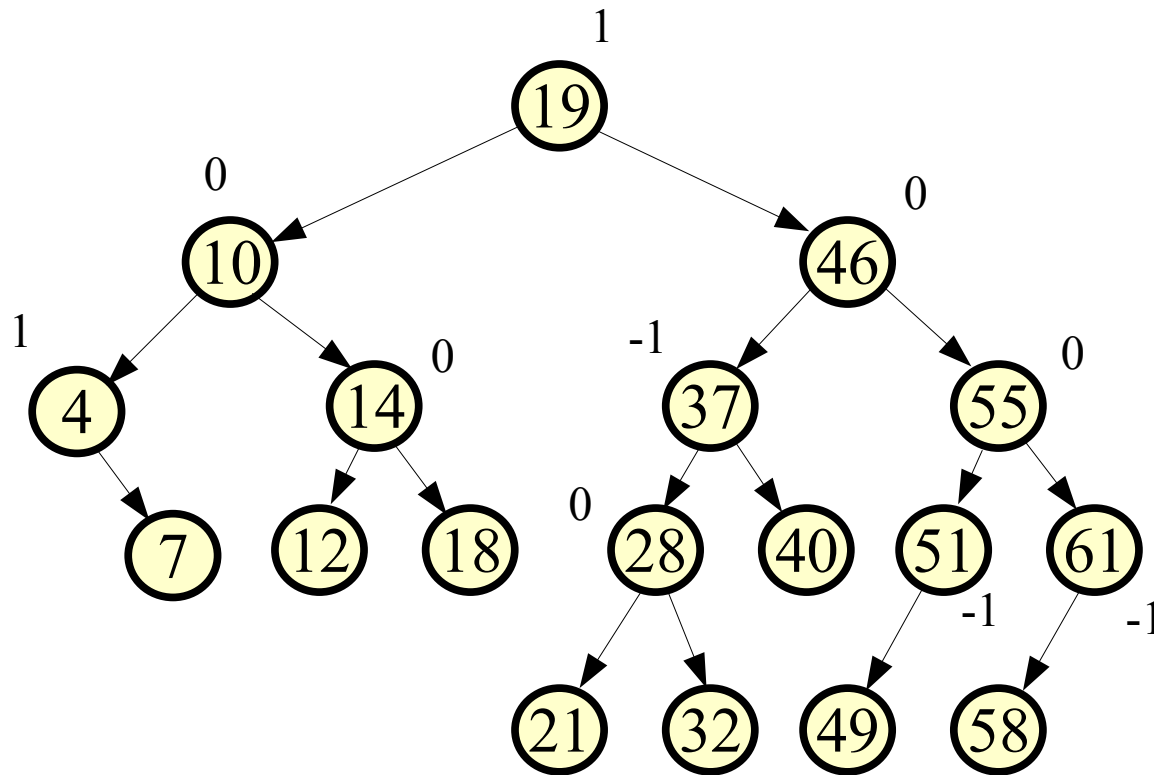


No change



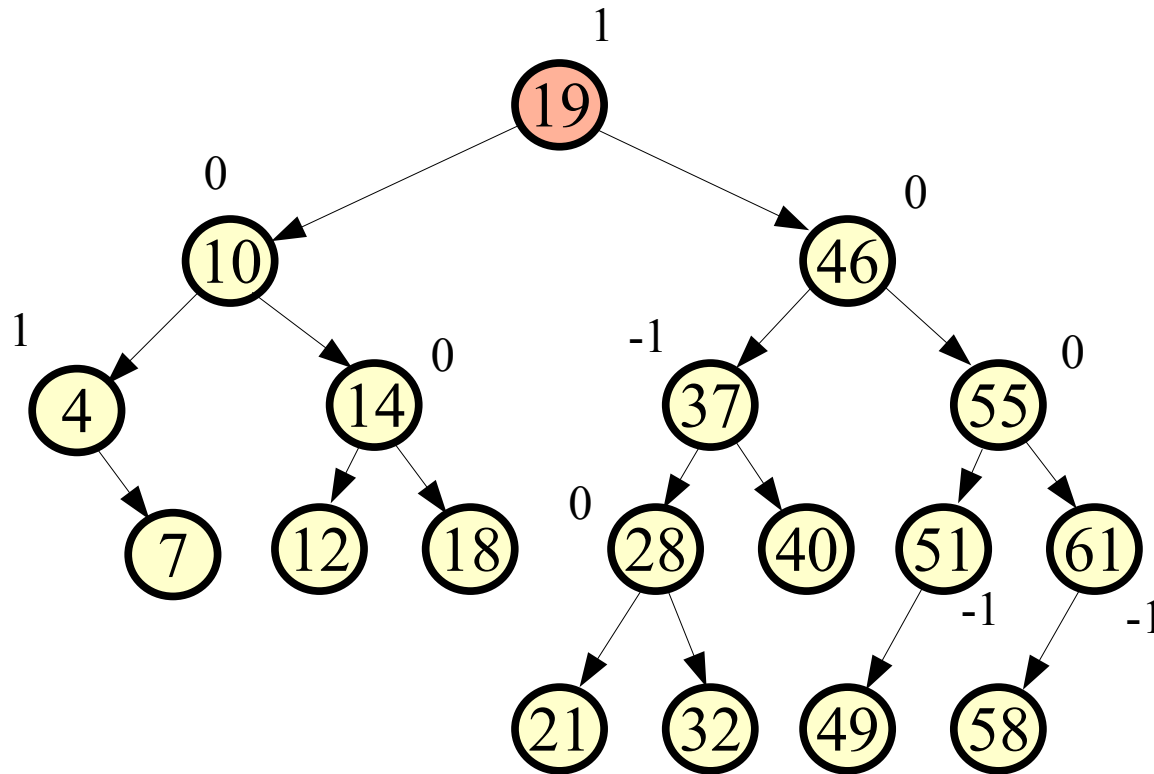
# Balanced Binaries – AVL Trees

Example:



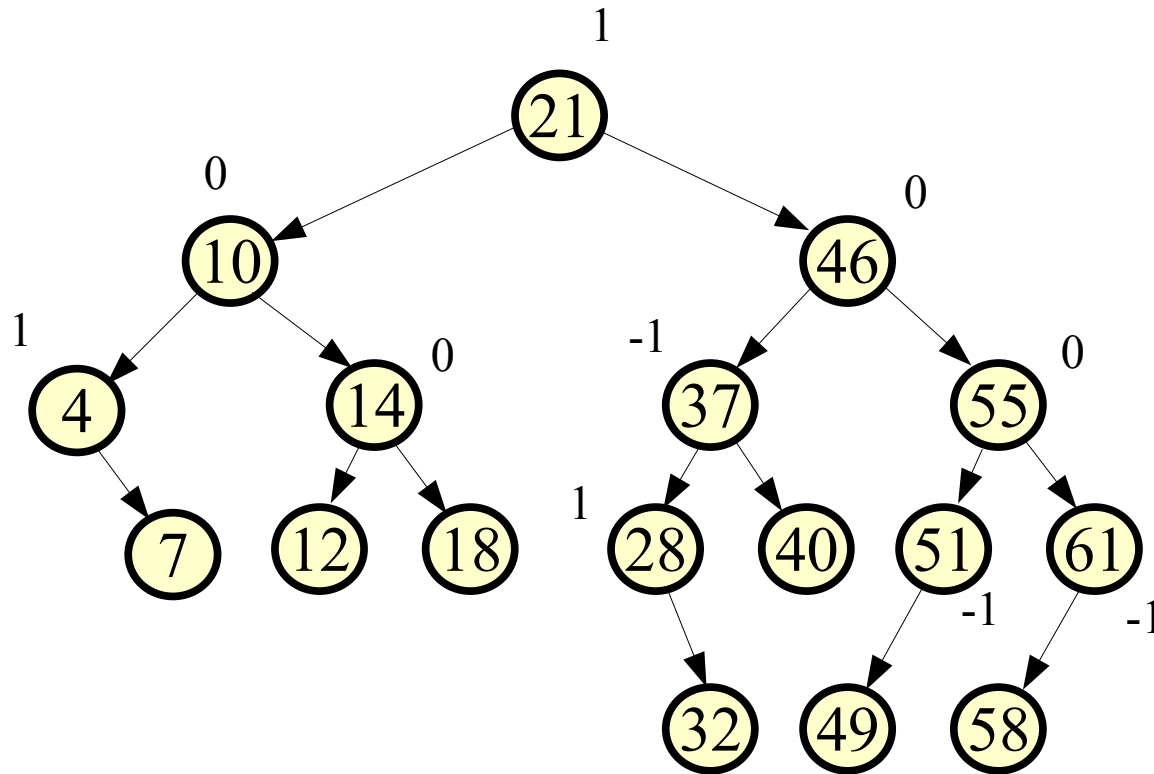
# Balanced Binaries – AVL Trees

Example: Delete 19



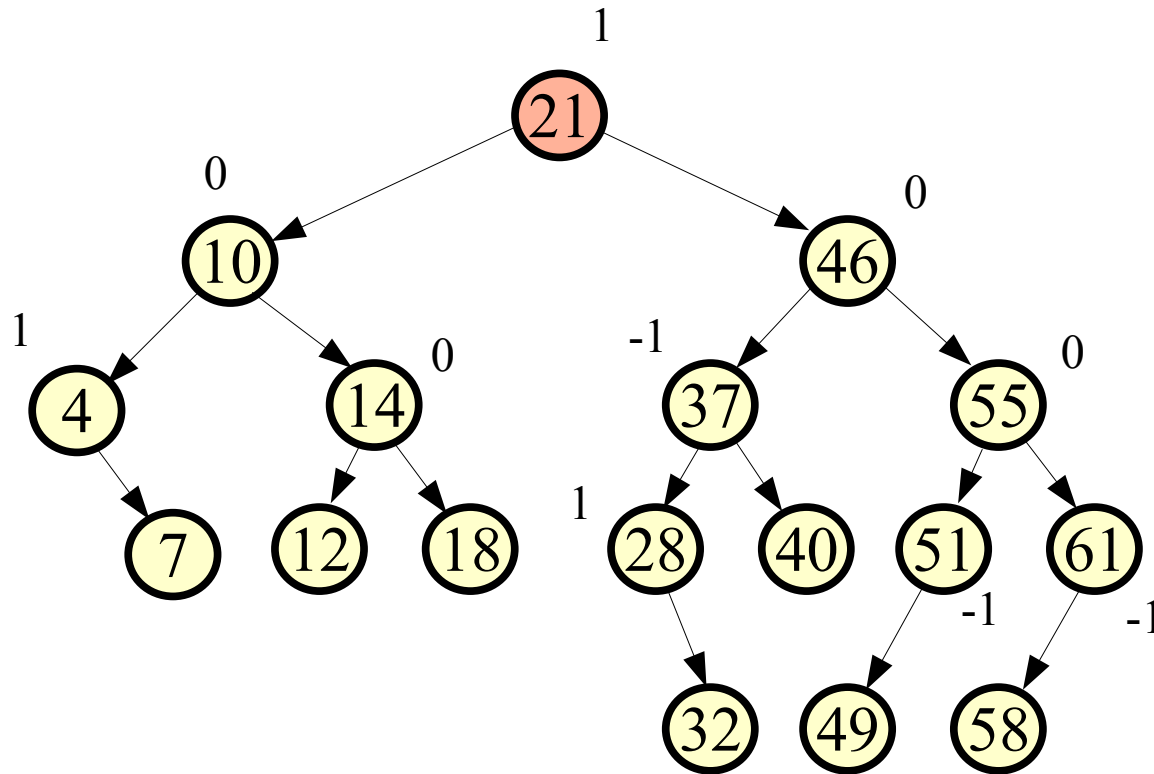
# Balanced Binaries – AVL Trees

Example: Delete 19



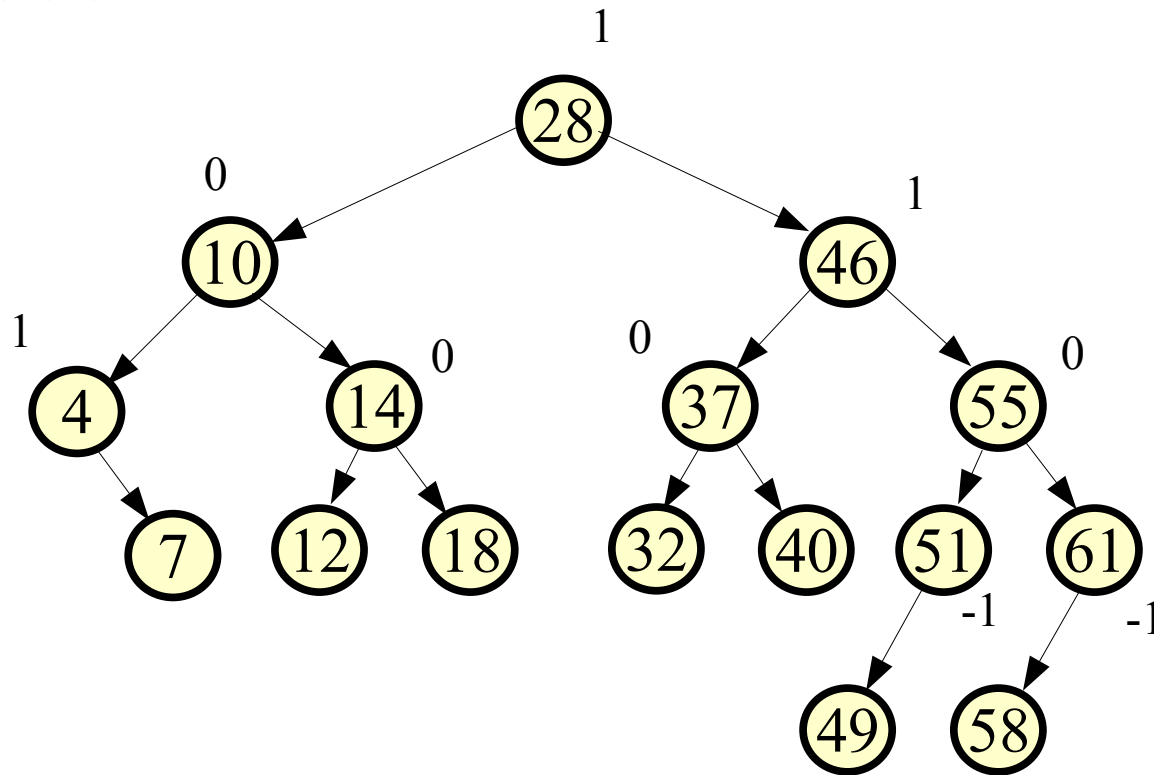
# Balanced Binaries – AVL Trees

Example: Delete 21



# Balanced Binaries – AVL Trees

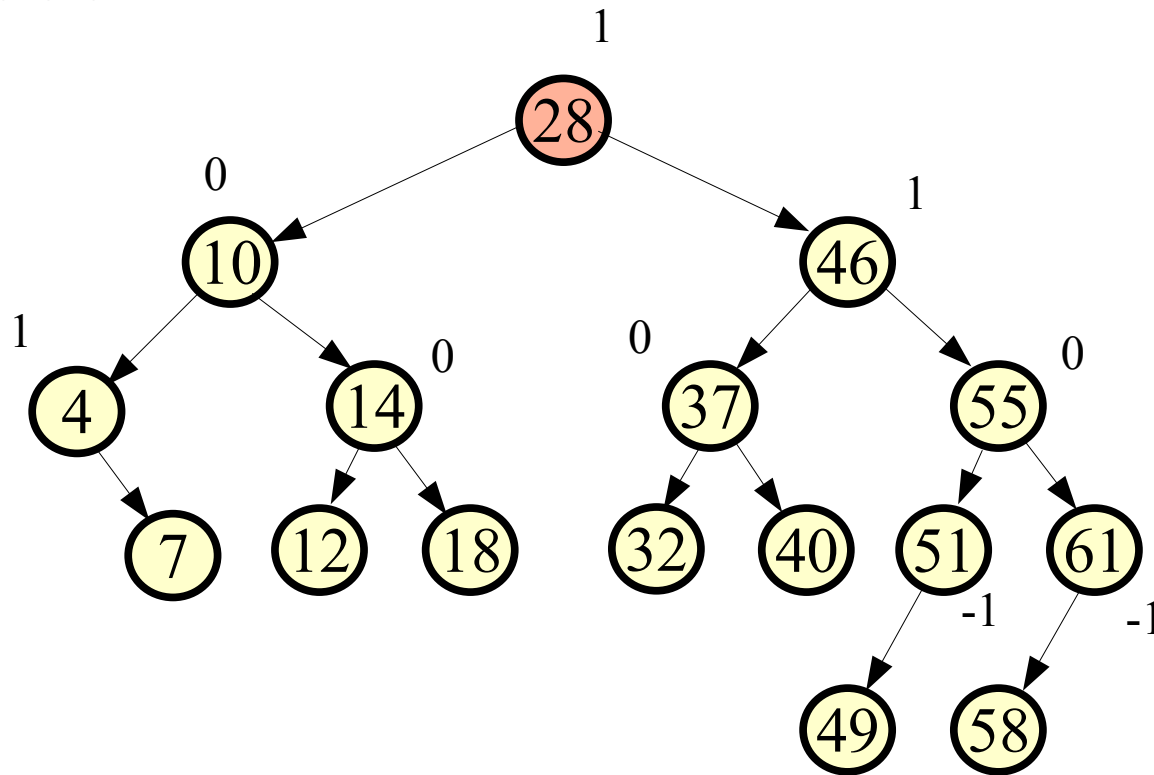
Example: Delete 21



No change

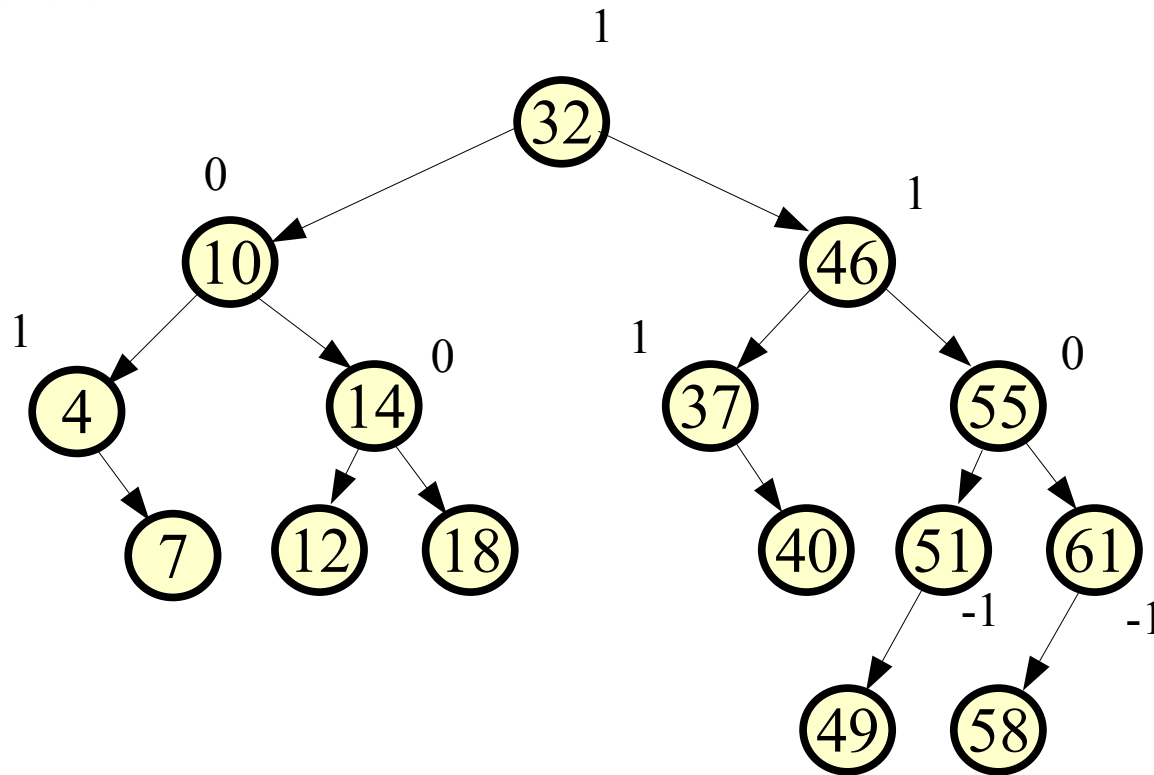
# Balanced Binaries – AVL Trees

Example: Delete 28



# Balanced Binaries – AVL Trees

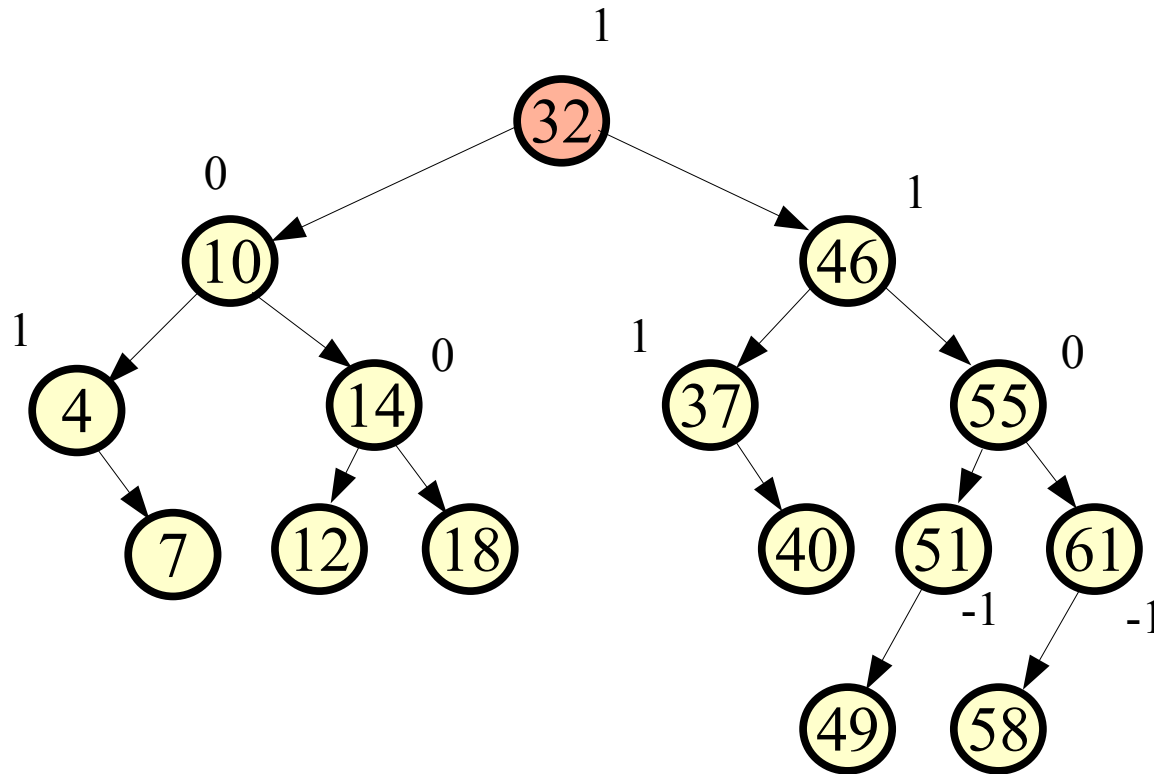
Example: Delete 28



No change

# Balanced Binaries – AVL Trees

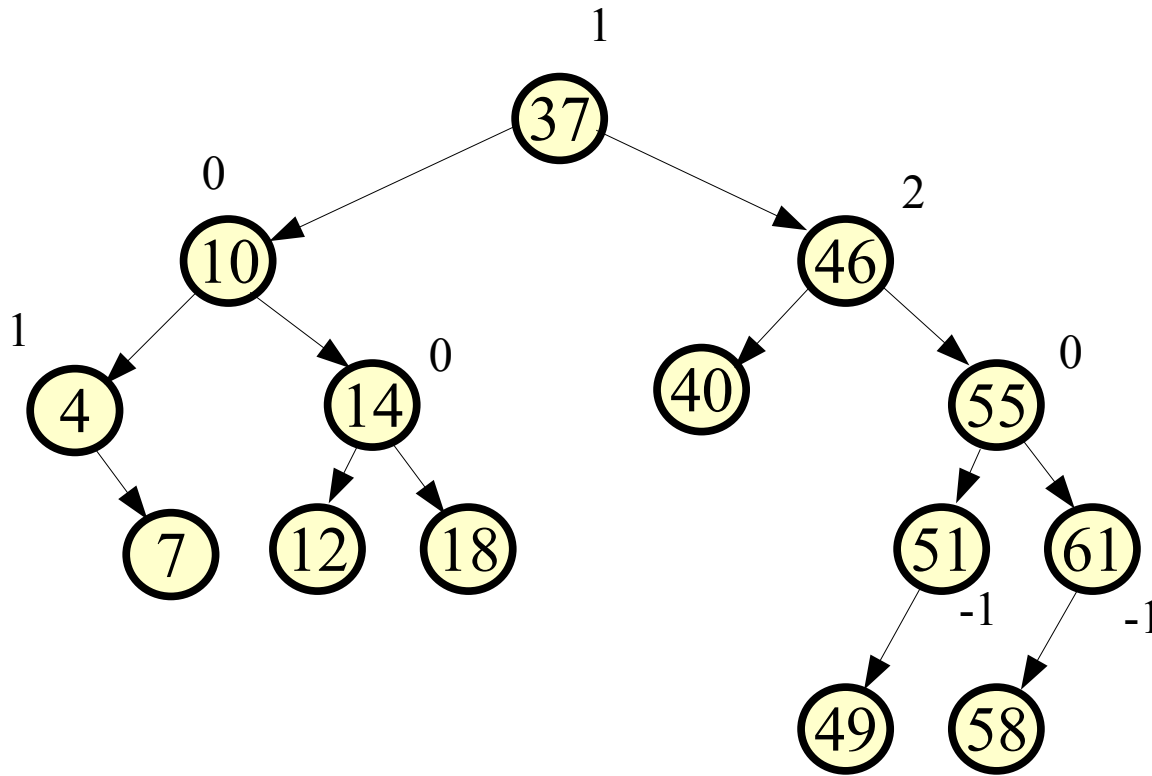
Example: Delete 32





# Balanced Binaries – AVL Trees

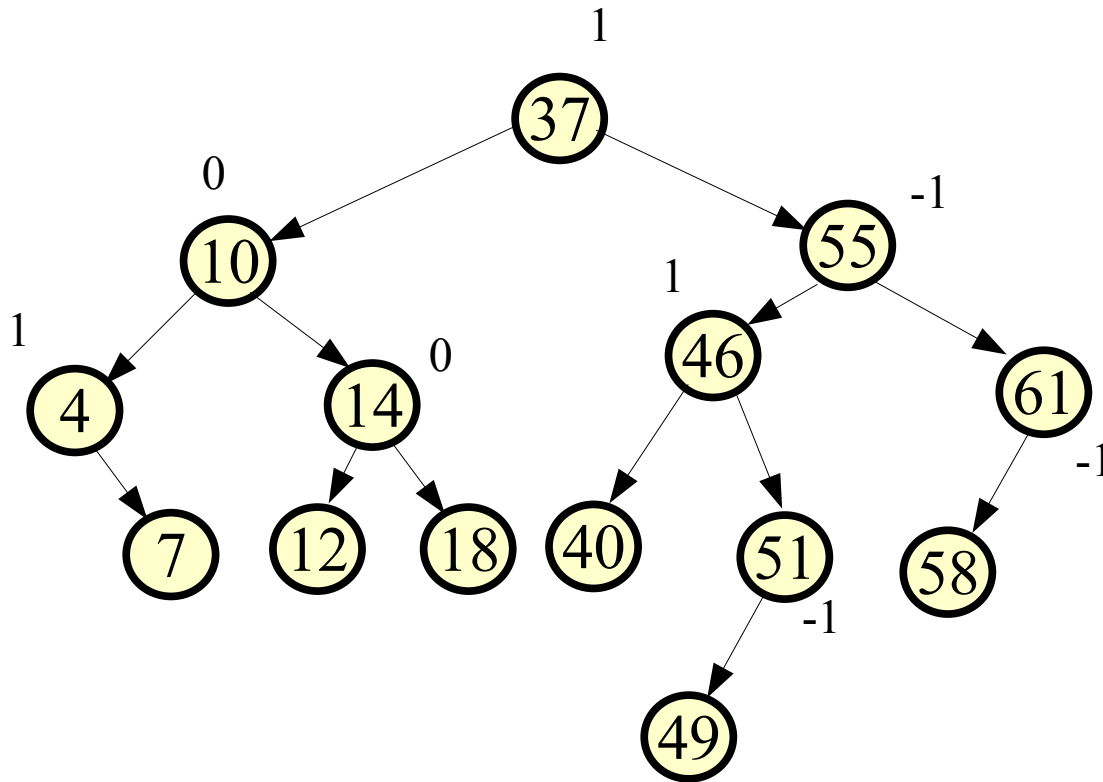
Example: Delete 32



Rotation around 55

# Balanced Binaries – AVL Trees

Example: Delete 32



Rotation around 55