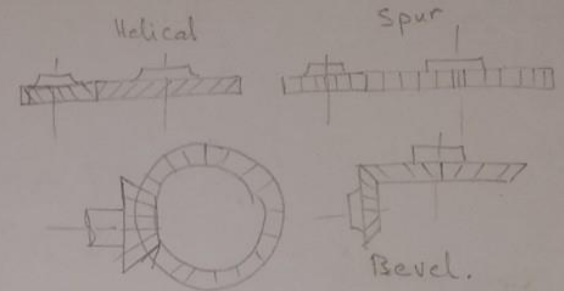
Spur Gears:

The simplest and most common types of gears
Transfere motion between parralle shafts
Teeth are parallel to shaft axis.
gear study:
1- geometry, 2- Force analysis, 3- Gear tooth bending stress, 4- Surface durability

1- Geometry and Nomenclulure:



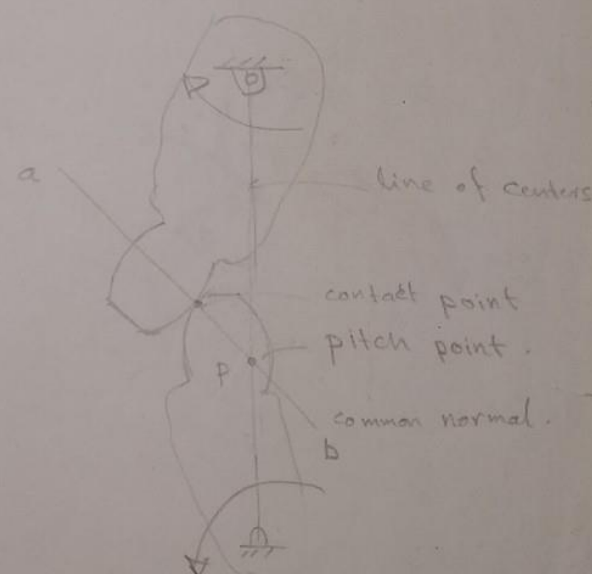
Basic requirment of gear-tooth geometry
⇒ const angular velocity ratio.
The action of gear teeth for const angular velocity – Conjugate gear-tooth action.

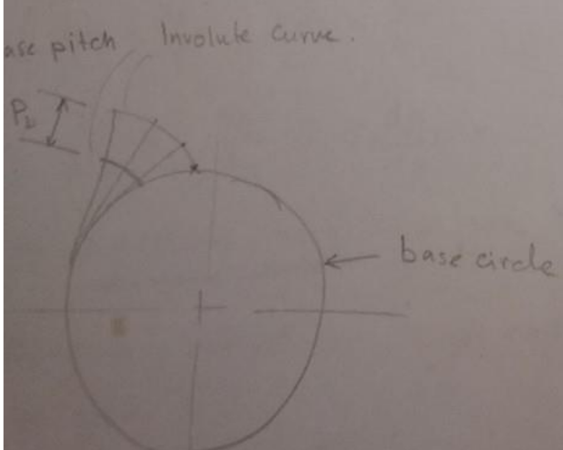
Rule for conjugate action:

The common normal of teeth surface at point of contact must Intersect the line of Centers at the same point P [Pitch point].

Involute satisfies the conjugate action.

Involute: The curve generated by any point on the tuat of thread as it unwinds from a cricle [base circle].
[Involute outside the base circle]

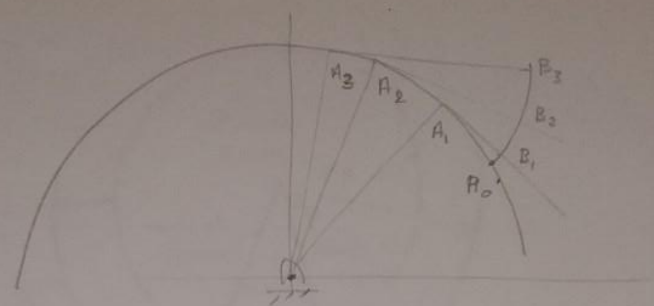




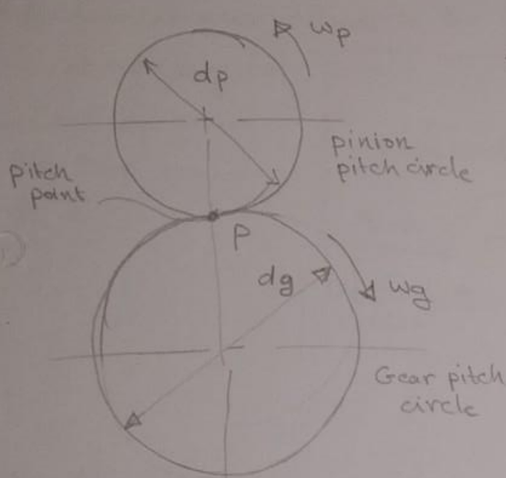
For constant angular velocity (P) must be fixed
⇒ All points of contact pass through line (a-b)
⇒ All normals coincide with line (a-b)

Conjugate action developed in three-steps.

1- friction drive
2- Belt drive
3- Involute gear-tooth drive.



1- Friction Drive [Rolling]



Smaller gear = pinion
other gear = gear

For no (slippage):

$$\frac{\omega_p}{\omega_g} = -\frac{d_g}{d_p}$$

2- Belt Drive

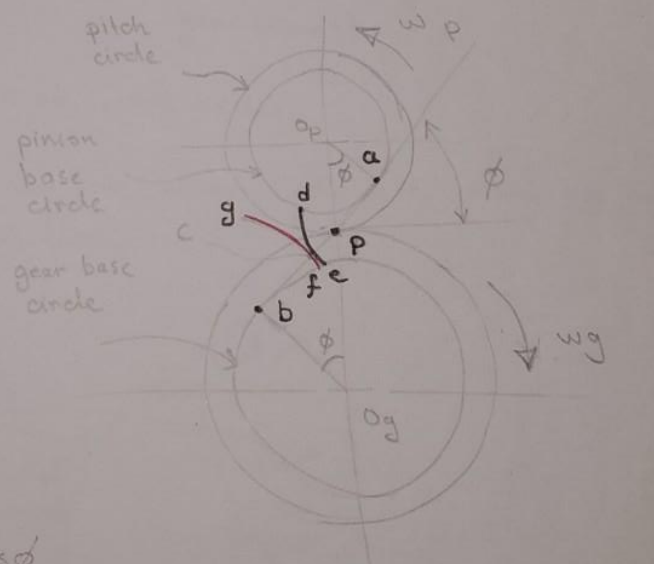
Adding belt at base circles

$$\frac{\omega_p}{\omega_g} = -\frac{d_{bg}}{d_{bp}} = -\frac{d_g}{d_p}$$

From similar triangles: $o_p P a$, $o_g P b$

$$\omega_p = \frac{V}{o_p} , \quad \omega_g = \frac{V}{o_g}$$

$$\frac{\omega_p}{\omega_g} = -\frac{o_g}{o_p} = \frac{-d_g}{d_p} \qquad r_b = r\cos\phi$$



ϕ = pressure angle = a teeth pushes the other in direction of ϕ

Cutting the belt at c, both ends generate involute (de) and (fg) for pinion and gear.

⇒ Involute satisfy the conjugate action law maintaining constant pressure angle.

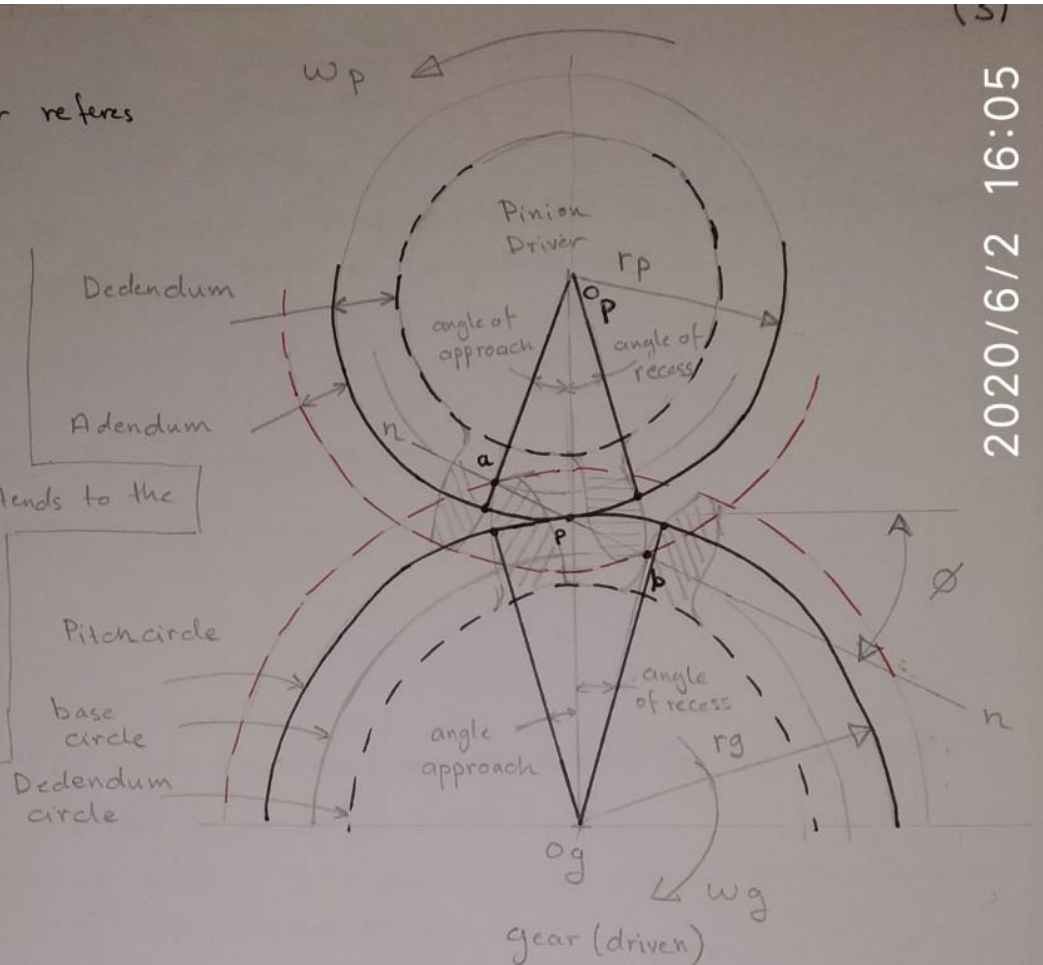
Note:
Diameter of the gear referes to pitch Dia.

d = pitch Dia.

Involute between the base and dedendum circle do not participate in the conjugate action.

Addendum of gear extends to the point of tangency (a)

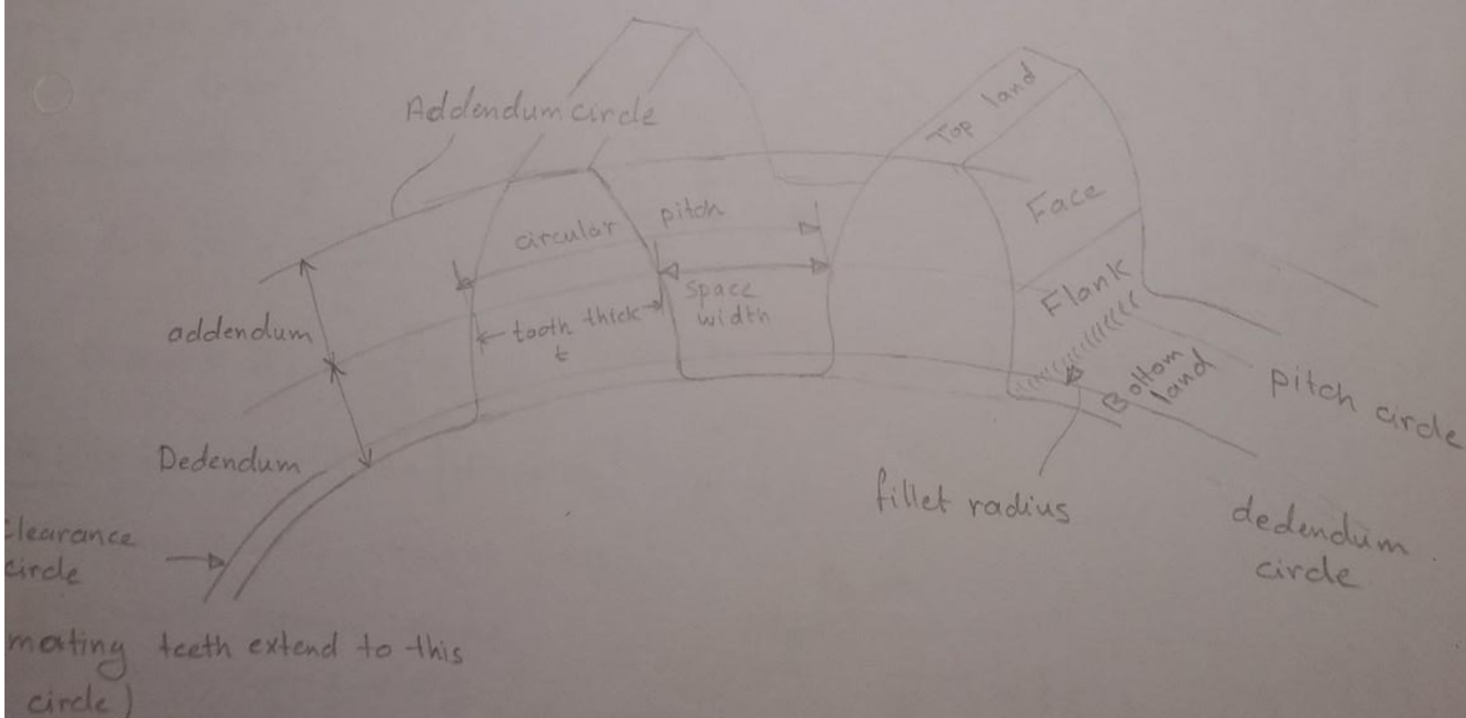
Addendum of pinion shorter than tangency point (b).



Angle of approach } measured to points on pitch circles.
Angle of Recess }

line n-n = line of action = pressure line → force between teeth acts along line n-n

Angle of action = Angle of approach + Angle of recess.



Diametral pitch (P) = $\frac{N}{d}$ = $\frac{\text{No. of teeth}}{\text{Pitch dia.}}$ [English units]

Module (m) = $\frac{d}{N}$ [SI units]

Circular pitch = Distance, measured on pitch circle from a point on one teeth to the corresponding point on adjacent teeth.

$$p = \frac{\pi d}{N}$$

$\Rightarrow$ $pP = \pi$ [p in in., P = teeth per in]

$\frac{p}{m} = \pi$ [p in mm, m = mm per tooth]

$$m = \frac{25.4}{P}$$

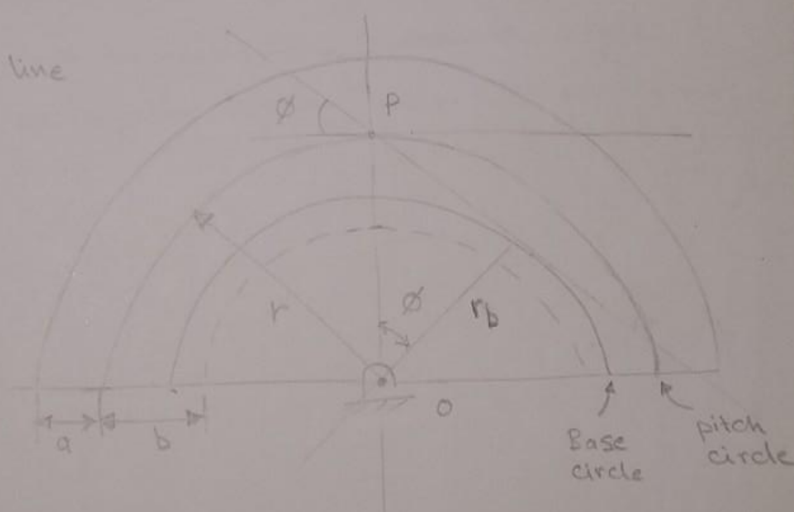
English units: 12- pitch gear $\Rightarrow$ gear with 12 teeth / in of pitch dia.
pitch $\to$ diametral pitch

SI Units: Gear of pitch = 3.14 (mm) $\Rightarrow$ gear circular pitch = 3.14 mm
pitch $\to$ circular pitch.

Base circle - tangent to pressure line

$r_b = r \cos \phi$

r_b = base radius.
r = pitch radius



For $\phi = 20^\circ$

Addendum (a) = $\frac{1}{P}$

dedendum (b) = $\frac{1.25}{P}$

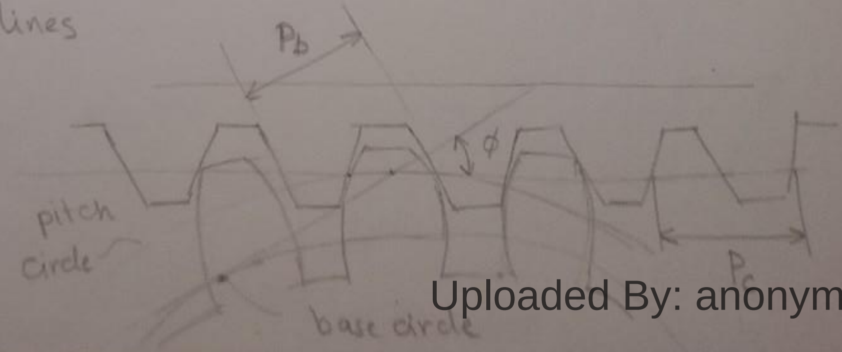
Tooth thickness (t) = $\frac{p}{2}$

Table [13-2] a, b for different ϕ

Rack: spur gear with [$d \to \infty$], $N \to \infty$

Involute sides : straight lines

Base pitch = const. distance between parallel involute curves along a common normal.

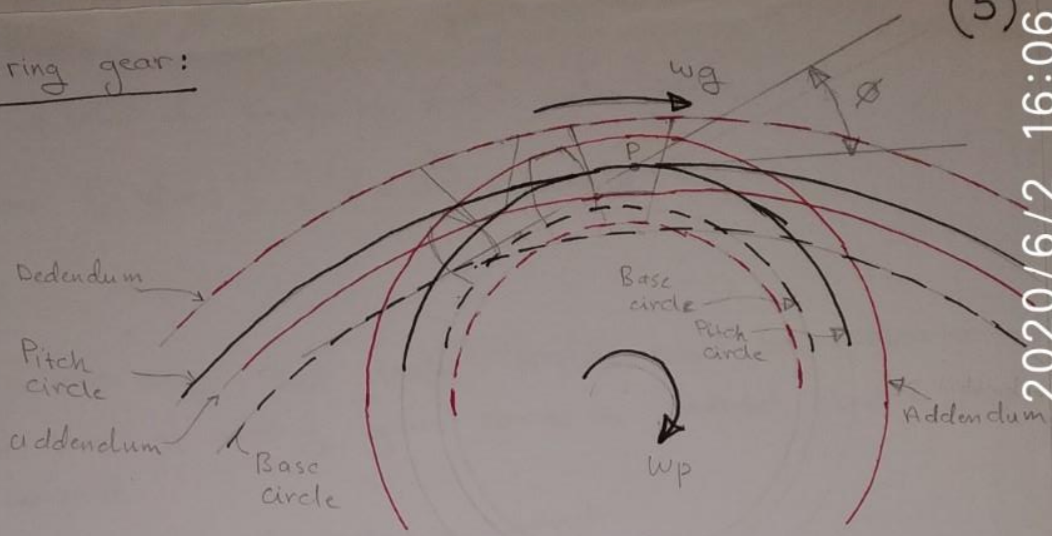


$p_b = p_c \cos \phi$

Internal - annular - ring gear:

d — negative

$$\frac{w_p}{w_g} = +\frac{d_g}{d_p}$$



Note:
that position of addendum and dedendum circles are reversed with respect to pitch point.

Addendum circle lies inside the pitch circle for internal gear.

Effect of increasing gears centers distance:

- Increasing center distance → Increases the pitch circles
- Base circle does not change. [used to generate tooth profile]
- Increase center distance → Increases the pressure angle.
- The teeth still conjugate → angular velocity ratio will not change.

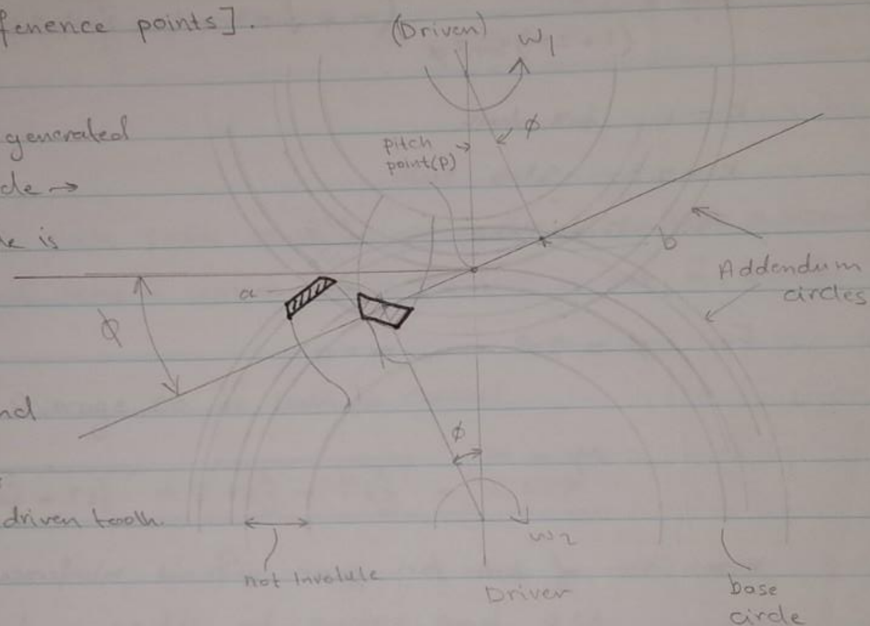
Interference:

prevents rotation of mating gears

occurs: if either Addendum circles extends beyond tangent points (a) and (b) [Interference points].

Since the involute is generated outside the base circle → tooth below base circle is not on involute.

⇒ The portion of the tooth extend beyond tangent point will dig in an non involute of driven tooth.



Interference Prevention:

1- Interfering tooth tips are normaly removed.

2- Both flanks of mating gears are under cut → weaken the teeth.

Maximum possible addendum radius without interference:

$$r_{a(max)} = \sqrt{r_b^2 + C^2 \sin^2\phi}$$

$r_{a(max)}$ = max. noninterfering addendum circle radius of pinion or gear.

r_b = base circle radius of the same member

C = Center distance $= (r_p + r_g)$

ϕ = Pressure angle

3- using more teeth, by increasing pitch diameter → larger the gear and increase the pitch-line velocity → noisier gears [not desirable].

4- larger pressure angles → Smaller base circle → more of tooth profile min. no. of teeth on the pinion without interference

$$N_p = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$k = 1$, full depth

$k = 0.8$, Stub teeth

m = Gear ratio $= \frac{N_G}{N_p}$

For $\phi = 20^\circ$, $m = 4$

For ϕ $N_p = 16$

max. no. of teeth for gear without interference

$$N_G = \frac{N_p^2 \sin^2\phi - 4k^2}{4k - 2N_p \sin^2\phi}$$

Table [13-6] max. no. of Tooth numbers to avoid interference.

Example:

Parallel shaft with center distance $C = 4''$, connected to 6-pitch 20-deg. Spur gears → velocity Ratio = −3.0

a) d_p, d_g, N_p, N_g b) check for Interference.

c) C.R.

a) 1- $C = r_p + r_g = 4 \Rightarrow r_p + r_g = 4$ — ①

$-\frac{r_g}{r_p} = \left(\frac{w_p}{w_g}\right) = 3.0 \Rightarrow \frac{r_g}{r_p} = 3.0$ — ②

- from ① and ② $\Rightarrow r_p = 1''$ and $r_g = 3''$

$d_p = 2''$, $d_g = 6''$

2- 6-pitch gear → diametral pitch $\underline{P} = 6$ (teeth/in)

$\underline{P} = \frac{N}{d}$, $\underline{P} = \frac{N_p}{d_p} \Rightarrow N_p = 12$ $N_p = \frac{2k}{(1+2m)\sin^2\phi}\left[m + \sqrt{(2m+1)\sin^2 ...}\right.$ $+ m^2$

$\underline{P} = \frac{N_g}{d_g} \Rightarrow N_g = 36$ $m = 3$

N_1 $N_p = 15$

(b) To check for Interference.

$\underline{P} = 7.5$

$\underline{P} = 8$

$r_b = r\cos\phi$, $r_{bp} = 1 \times \cos 20 =$, $r_{bg} = 3\cos 20$

Pinion:

$r_{a(max)} = \sqrt{r_b^2 + c^2 \sin^2\phi} = 1.66$ (in) $\underline{P} = 6$

C.R = 1.6

Gear: $r_{a(max)} = 3.133$ (in)

Standard full Depth teeth : $a = \frac{1}{P} = 0.167$

Pinion:

$r_{ap} = r_p + a = 1 + 0.167 = 1.167$

gear:

$r_{ag} = r_g + a = 3 + 0.167 = 3.167 > 3.133 \Rightarrow$ Interference occur.

c) $C.R. = \dfrac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c \sin\phi}{p_b}$

$p_b = p \cos\phi = \frac{\pi}{P} \cos\phi = 0.492$

$r_{ap} = 1.167$, $r_{ag} = 3.167$

$r_{bp} = r_p \cos\phi = 1.0966$, $r_{bg} = r_g \cos\phi = 2.976$

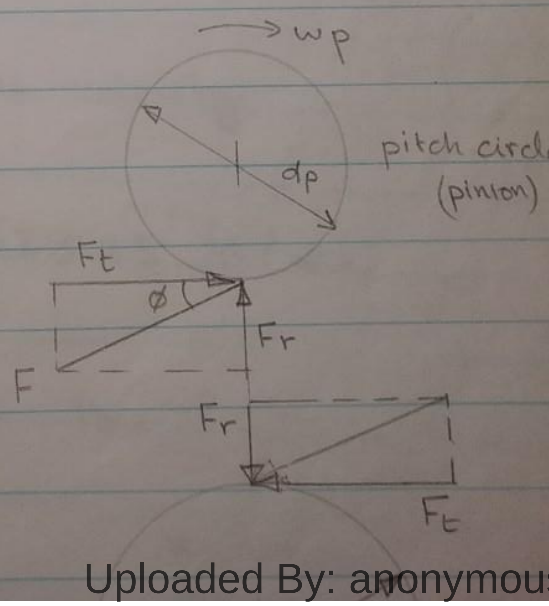
$C.R. = 0.23$ — is not a suitable value.

From the equation: $r_{a(max)} = \sqrt{r_b^2 + c^2 \sin^2\phi}$

1- Interference involve the tip of gear teeth than pinion teeth
2- Interference increases — for small no. of pinion teeth
larger no. gear teeth, smaller pressure angle.

Gear Force Analysis:

Line of action = pressure line = line of action of the forces between mating
Force are resolved at the pitch point!



- Tangential component, [Ft]

  $T = F_t \frac{d}{2}$, $P = T\omega$

- Radial component, [F_r]

  Does not work — push the gears apart

  $F_r = F_t \tan\phi$

Pitch line velocity.

$V = \pi d n / 12$