7.6 Inverse Trigonometric functions

2(c) + 5(a) + 6(b) + 12 + 16 + 18 + 22 + 26 + 34 + 35 + 40 + 46

50 + 59 + 64 + 70 + 74 + 79 + 86 + 90

	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$[\frac{-\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$(\frac{-\pi}{2}, \frac{\pi}{2})$
$\sec^{-1} x$	$x \le -1$ or $x \ge 1$	$[0, \pi]$
$\csc^{-1} x$	$x \le -1$ or $x \ge 1$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cot^{-1} x$	$(-\infty, \infty)$	$(0, \pi)$

2(c) $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \boxed{\frac{-\pi}{6}}$

5(a) $\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$

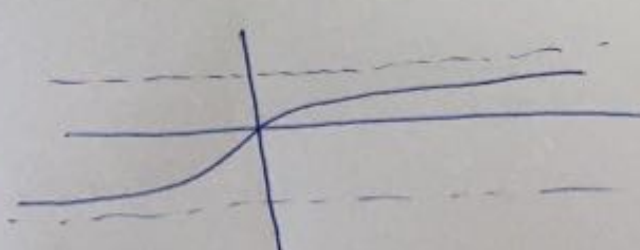
6(b) $\csc^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \boxed{\frac{-\pi}{3}}$

12 $\cot\left(\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)$

$\cot\left(-\frac{\pi}{3}\right)$

$= \dfrac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{-1}{\sqrt{3}}}$

16 $\lim_{x \to -\infty} \tan^{-1} x = \boxed{\frac{-\pi}{2}}$



18 $\lim_{x \to -\infty} \sec^{-1} x$

$= \lim_{x \to -\infty} \cos^{-1} \frac{1}{x}$

$= \boxed{\frac{\pi}{2}}$

22 $y = \cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$

$\frac{dy}{dx} = \boxed{\frac{1}{|x|\sqrt{x^2-1}}}$

26 $y = \sec^{-1} 5x$

$\frac{dy}{dx} = \frac{5}{|5x|\sqrt{25x^2-1}}$

$\frac{dy}{dx} = \boxed{\frac{1}{|x|\sqrt{25x^2-1}}}$

34 $y = \tan^{-1}(\ln x)$

$\frac{dy}{dx} = \frac{\frac{1}{x}}{\ln^2 x + 1} = \boxed{\frac{1}{x(\ln^2 x + 1)}}$

35. $y = \csc^{-1}(e^t)$

$$\frac{dy}{dt} = -\frac{e^t}{|e^t|\sqrt{(e^t)^2 - 1}}$$

$$\boxed{\frac{dy}{dt} = -\frac{1}{\sqrt{e^{2t} - 1}}}$$

45. $y = \cot^{-1}\frac{1}{x} - \tan^{-1}x$

$$= \frac{\pi}{2} - \tan^{-1}x^{-1} - \tan^{-1}x$$

$$\frac{dy}{dx} = 0 - \frac{\frac{-1}{x^2}}{x^{-2}+1} - \frac{1}{1+x^2}$$

$$= \frac{\frac{1}{x^2}}{\frac{1}{x^2}+1} - \frac{1}{1+x^2} = \frac{\frac{1}{x^2}}{\frac{1+x^2}{x^2}} - \frac{1}{1+x^2} = \boxed{0}$$

46 $\int \frac{1}{9+3x^2}\, dx$

$$\int \frac{1}{9\left[1+\frac{x^2}{3}\right]}\, dx$$

$$\frac{1}{9}\int \frac{dx}{1+\frac{x^2}{3}}$$

$$\frac{1}{9}\int \frac{dx}{1+\left(\frac{x}{\sqrt{3}}\right)^2}$$

$$u = \frac{x}{\sqrt{3}} \to du = \frac{dx}{\sqrt{3}}$$

$$= \frac{1}{9}\int \frac{\sqrt{3}\, du}{1+u^2} = \frac{\sqrt{3}}{9} \tan^{-1} u + C$$

$$= \cancel{\sqrt{3}} \boxed{\frac{1}{3\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C}$$

4

58 $\int_0^{\frac{3\sqrt{2}}{4}} \frac{dx}{\sqrt{9-4x^2}}$

$= \cancel{u = 4x^2}$ $\cancel{du = 8x}$

$4x^2 = u^2 = (2x)^2$

$u = 2x$

$du = 2dx$

$\frac{1}{2}\int \frac{du}{\sqrt{9-u^2}} = \frac{1}{2}\sin^{-1}\frac{u}{3}$

$= \frac{1}{2}\sin^{-1}\frac{2x}{3}\Big|_0^{\frac{3\sqrt{2}}{4}} = \boxed{\frac{\pi}{8}}$

59 $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}}$

$u = 2x-1 \rightarrow du = 2dx$

$\int \frac{\frac{du}{2}}{u\sqrt{u^2-4}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-4}} = \frac{1}{2}\frac{1}{2}\sec^{-1}\left|\frac{u}{2}\right| + c$

$= \boxed{\frac{1}{4}\sec^{-1}\left|\frac{2x-2}{2}\right| + C}$

5

(54) $\int_{1}^{e^{\frac{\pi}{4}}} \frac{4\,dt}{t(1+\ln^2 t)}$

$u = \ln t$

$du = \frac{dt}{t}$

$\int \frac{4\,t\,du}{t(1+u^2)} = 4\int \frac{du}{1+u^2}$

$= 4\tan^{-1}u = 4\tan^{-1}(\ln t) = 4\left(\tan^{-1}\ln e^{\frac{\pi}{4}} - \tan^{-1}\ln 1\right)$

$= 4\left(\tan^{-1}\frac{\pi}{4} - \tan^{-1}0\right) = \boxed{4\tan^{-1}0}$

(70) $\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}}$

$3+4t-4t^2 = -4\left[t^2-t-\frac{3}{4}\right] = -4\left[t^2-t+\frac{1}{4}-\frac{1}{4}-\frac{3}{4}\right]$

$= -4\left[t^2-t+\frac{1}{4}-1\right] = -4\left[\left(t-\frac{1}{2}\right)\left(t-\frac{1}{2}\right)\right]+4$

$= -4\left(t-\frac{1}{2}\right)^2+4 = 4\left[1-\left(\frac{2t-1}{2}\right)\right]$

$= \frac{4\left[2-(2t-1)\right]}{2}$

$= 2\left[2-(2t-1)\right]$

6

$3+4t-4t^2 = -4(t^2-t)+3$

$= -4\left(t^2-t+\frac{1}{4}-\frac{1}{4}\right)+3$

$= -4\left(\left(t-\frac{1}{2}\right)^2-\frac{1}{4}\right)+3$

$= -4\left(\left(t-\frac{1}{2}\right)^2\right)+1+3$

$= -4\left(\frac{2t-1}{2}\right)^2+4$

$= -\frac{4}{4}(2t-1)+4$

$= -(2t-1)+4$

$$\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{4-(2t-1)^2}}$$

$u = 2t-1$

$du = 2dt$

$$\int_0^1 \frac{6\,dt}{2\sqrt{4-u^2}} = 3\sin^{-1}\frac{u}{2}\Big|_0^1$$

$= 3\sin^{-1}\frac{1}{2} - 3\sin^{-1} 0$

$= 3\cdot\frac{\pi}{6} - 3\cdot 0 = \boxed{\frac{\pi}{2}}$