

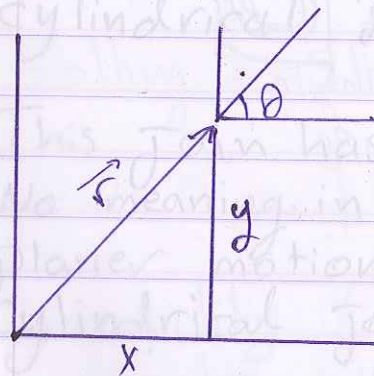
Machine Dynamic

⇓
Mechanism

↙ ↘
body Joint

Degree of freedom

number of independent parameters
need to totally control the body
machine

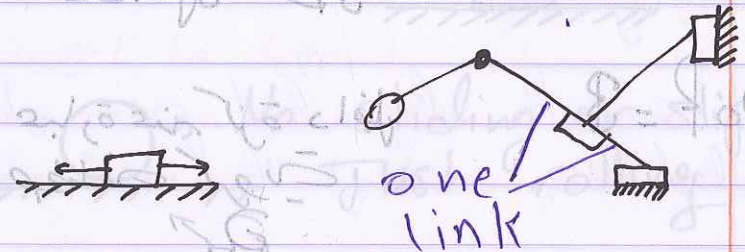


In the Plan 2D, each body
has 3 degree of freedom
① Revolute Joint = Pin

in 3D plane each body
has 6 degree

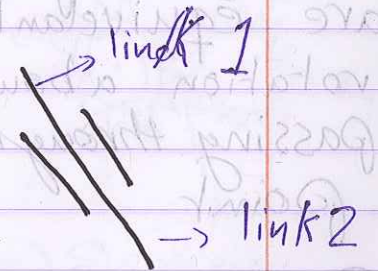
Joints

- ① Revolute Joint $Dof = 1$
- ② Prismatic Joint $Dof = 1$
≡ Slider



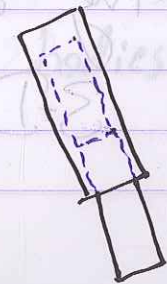
③ cylindrical Joint (3D space)

This joint has
No meaning in
planer motion
Cylindrical joint



يحتل 3 متغيرات

x, y, θ



④ Helical Screw Joint

Dof = 1 3D space

⑤ Spherical Joint 3D-space

Dof = 3

عبارة عن كرة داخل
قشانة كروية

Note: any 3 axes
passing through about
are equivalent a single
rotation about axes
passing through the same
point



⑥ Sliding on the plane has 3-Dof

⑥ Contact

- translation without rolling

Dof = 1



○ → No sliding or slipping
Just rolling

(Pure Rolling)

Rolling + Sliding

Dof = 1

mobility (m)

n = number of bodies including
the ground

$$m = 3(n-1) - 2J_1 - J_2$$

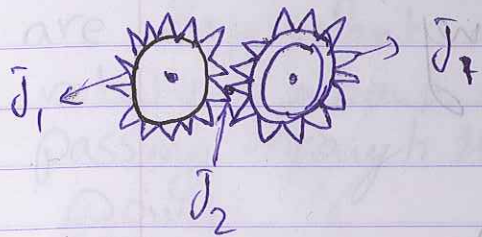
J_1 :- number of joint that have 1 DoF

J_2 :- number of joint that have 2 DoF

mobility m

Number of independent parameters needed to control the mechanism

n = number of bodies including the ground



$n=3$

$$m = 3(n-1) - 2J_1 - J_2$$

$$m = 3(3-1) - 2(2) - 1 = 1$$

* Mobility [Degree of Freedom] :-

Number of input that must be control independently to put a mechanism in to particular position "Number of independent part required to describe motion of mechanism"

"Number of independent parameters"

* Kutzbach criteria :-

$$m = 3(n-1) - 2J_1 - J_2$$

m :- mobility (DOF)

n :- No. of links including ground

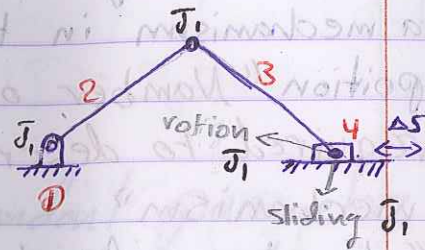
J_1 :- No. of single DOF joints.

J_2 :- No. of low DOF joints.

if $m=0 \rightarrow$ Structure (No motion)
 if $m>0 \rightarrow$ mechanism (motion)
 if $m<0 \rightarrow$ indetermined structure
 \Rightarrow "No motion"

Ex]

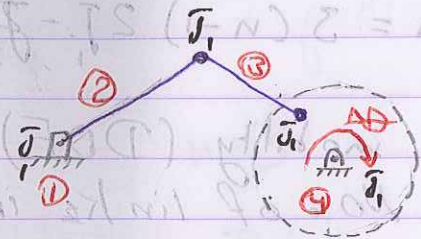
$$\begin{aligned}
 n &= 4 \\
 J_1 &= 4 \\
 J_2 &= 0
 \end{aligned}$$



$$m = 3(4-1) - 2(4) - 0 = 1$$

Ex]

$$\begin{aligned}
 n &= 4 \\
 J_1 &= 4 \\
 J_2 &= 0 \\
 m &= 1
 \end{aligned}$$



Ex]

3 link
فانہ یکتا
2 J1

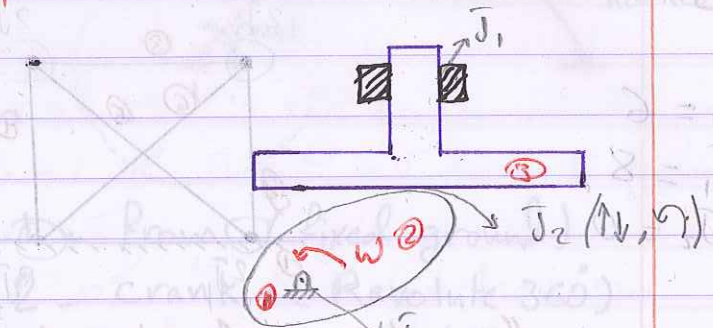
$$\begin{aligned}
 n &= 6 \\
 J_1 &= 7 \\
 J_2 &= 0
 \end{aligned}$$

$$m = 1$$

Ex]

$$\begin{aligned}
 n &= 3 \\
 J_1 &= 2 \\
 J_2 &= 1
 \end{aligned}$$

$$m = 1$$



Ex]

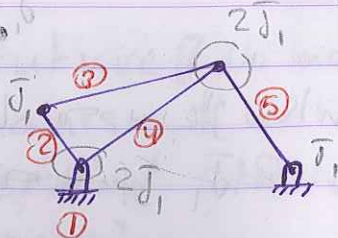
$$n = 5$$

$$J_1 = 6$$

$$J_2 = 0$$

$$m = 3(5-1) - 2(6) - 0$$

$$= 0 \text{ "structure"}$$



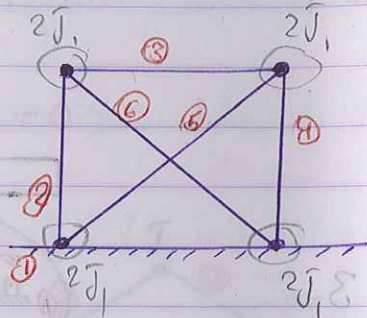
Ex]

$$n = 6$$

$$J_1 = 8$$

$$J_2 = 0$$

$$m = -1 \text{ "indetermind structur"}$$

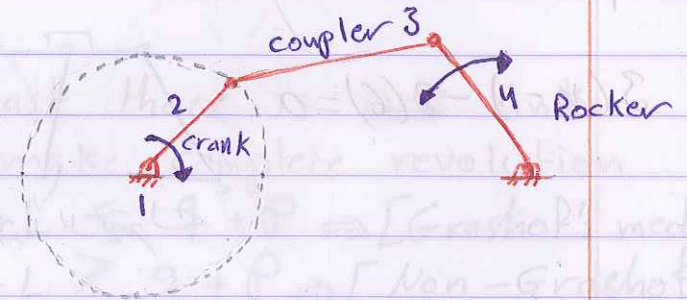


ch.2

* Four bar mechanism analysis
Simplest closed loop single
degree-of-freedom mechanism

* Crank - Rocker mechanism

link



link 1 - Frame (fixed ground)

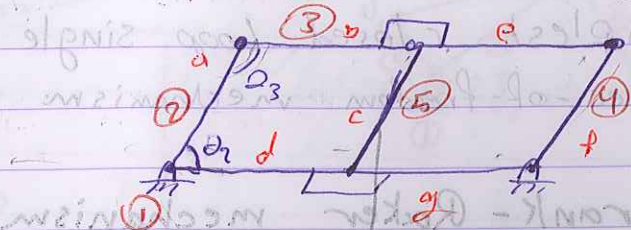
2 - crank (Revolute 360°)

3 - coupler [connected rod]

4 - Rocker [does not rotate
but oscillates]

$$m = 1$$

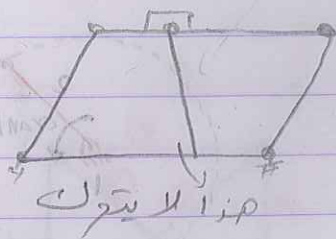
Exceptions to Kutzbach Theory



$$n = 5$$

$$J_1 = 6$$

$$3(n-1) - 2(6) = 0$$



* Four bar mechanism analysis

* Grashof's four bar mechanism

S = shortest link

L = longest link

P, q = other link

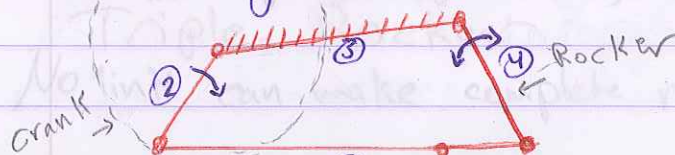
at least there is one link that make complete revolution

if $S + L \leq q + P \Rightarrow$ [Grashof's mechanism]

if $S + L > q + P \Rightarrow$ [Non-Grashof's]

Types of 4-bar mechanism

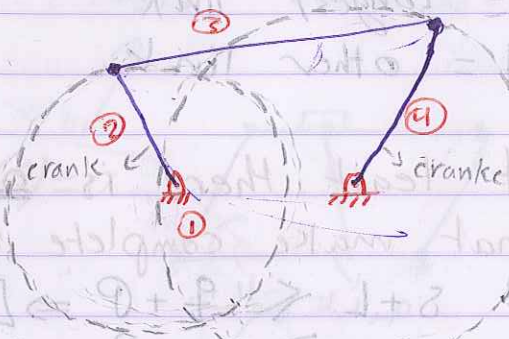
1- Crank-Rocker mechanism
 Shortest link must be one of the side link "shortest link will make complete revolution"



2- Double Crank mechanism

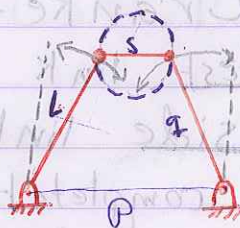
$$S + L \leq P + 7$$

shortest link is the frame (fixed)
both side link will revolute 360°



3- Double Rocker mechanism

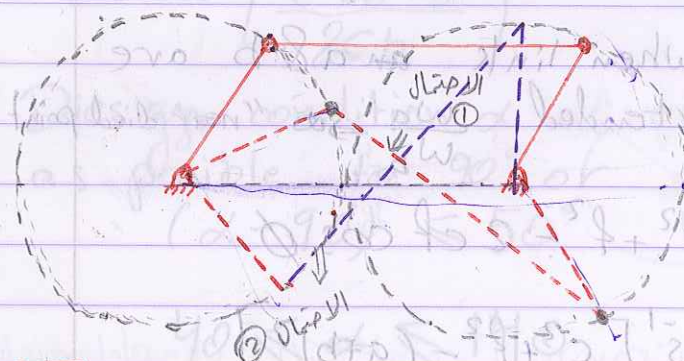
$$S + L \leq P + 7$$



4- change point mechanism

$$P + 7 = S + L$$

when all link become aligned
The Interminant motion



5- Triple Rocker mechanism

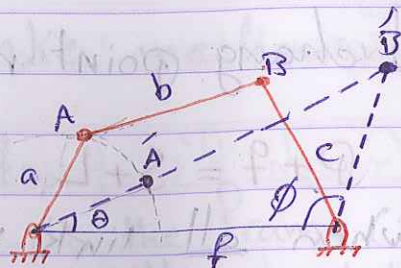
$$S + L > P + 7$$

* Dead Point Configurations

mechanism لا يتحرك في هذه الحالة

Triple Rocker

No link can make complete revolution



when link ~~an~~ a & b become aligned a torque applied on link c will not rotate the mechanism

State 1: - when link a & b are completely extended $\frac{w_a}{w_b} \rightarrow \infty$ "near dead point"

$$(a+b)^2 = c^2 + f^2 - 2cf \cos \phi$$

$$\therefore \phi = \cos^{-1} \left[\frac{c^2 + f^2 - (a+b)^2}{2cf} \right]$$

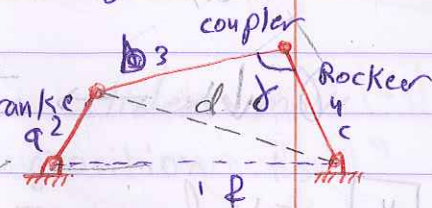
$$\theta = \cos^{-1} \left[\frac{f^2 + (b+a)^2 - c^2}{2f(b+a)} \right]$$

⇒ the mechanism can be used as velocity magnifier near dead point

Transmission Angle Page 26

if link 2 = input [crank]

link 4 = output [Rocker]



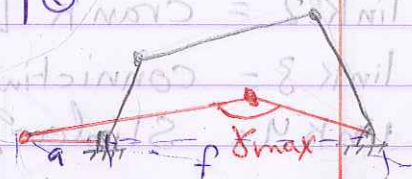
γ is Transmission angle

$$\gamma = \cos^{-1} \left[\frac{c^2 + b^2 - d^2}{2cb} \right]$$

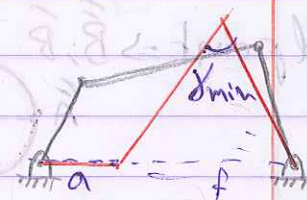
Design condition α as closed as possible to 90° or $(\alpha - 90^\circ) \leq 50^\circ$

$$40 \leq \alpha \leq 140$$

γ_{\max}
 $d = d_{\max} = f + a$



γ_{\min}
 $d = d_{\min} = f - a$



$$T_{out} = f \sin \gamma (c)$$

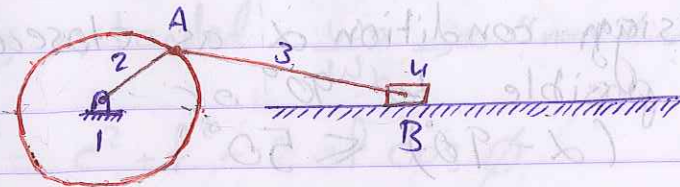
Torque
 T_{\max} when $\gamma = 90^\circ$

$\gamma \downarrow \rightarrow T \downarrow$

* Slider crank mechanism :-

[Ex]

- [1] Piston in internal combustion engine
- [2] Pump

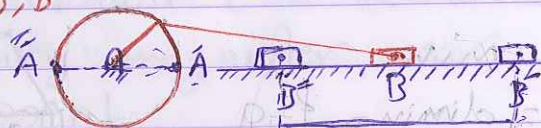


link 2 = crank [rotat 360°] complete revolution

link 3 = connecting rod

link 4 = Slider [move in reciprocating motion]

Dead point $\Rightarrow \bar{B}, \bar{B}$



stroke : distance between extreme point of slider

stroke :-

The distance that the slider move from extrem right position to extrem left

$$\text{Stroke} = X_p - X_L$$

$$= (a+b) - (b-a)$$

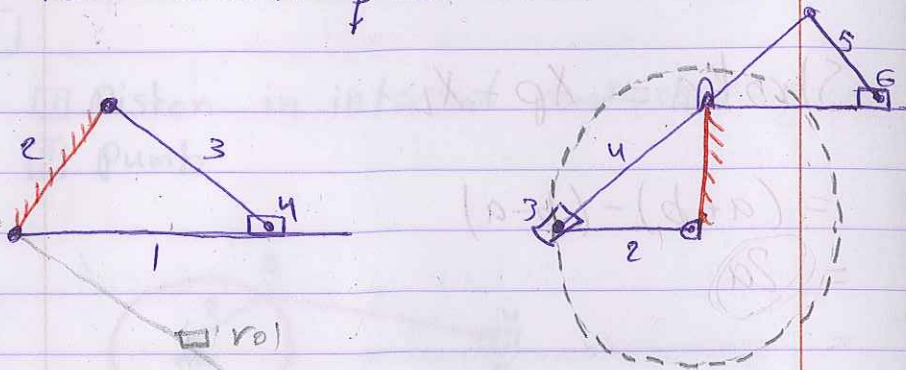
$$= 2a$$

example

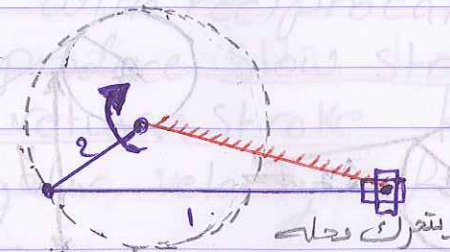
* Type of slider crank mechanism

1- Crank is fixed

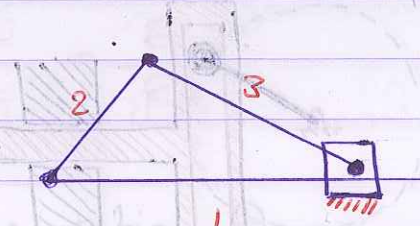
"Rotary engine"
Whitworth quick return mechanism



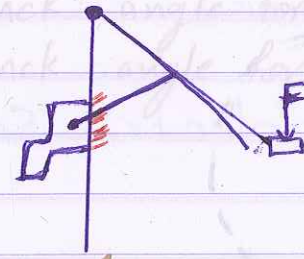
2- Connecting rod fixed



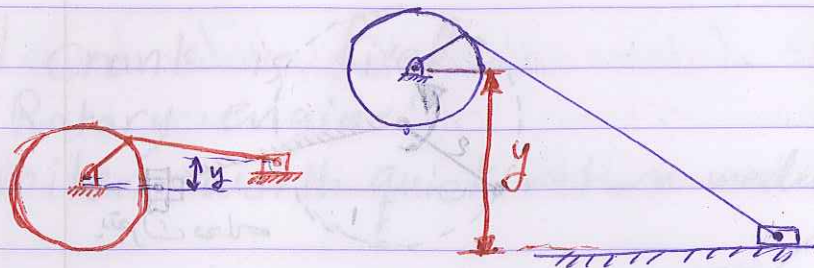
3- Slider fixed



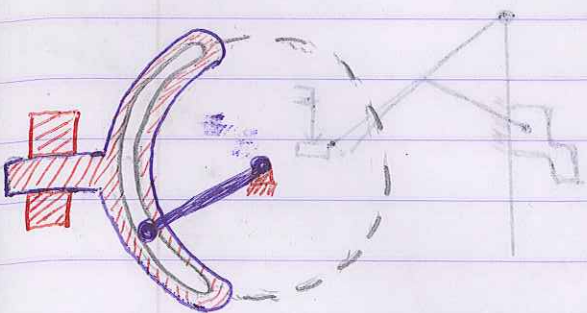
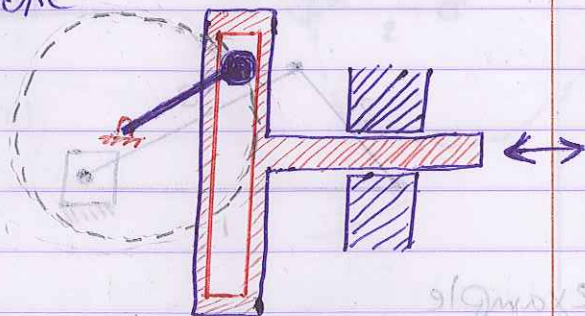
example



4 Offset slider-Crank mechanism



5 Scotch Yoke



Quick Return mechanism:-

It is used with reciprocating engines to produce slow stroke and quick return stroke for constant angular velocity of crank.

Time ratio = $\frac{\text{time of working stroke}}{\text{time of return stroke}}$

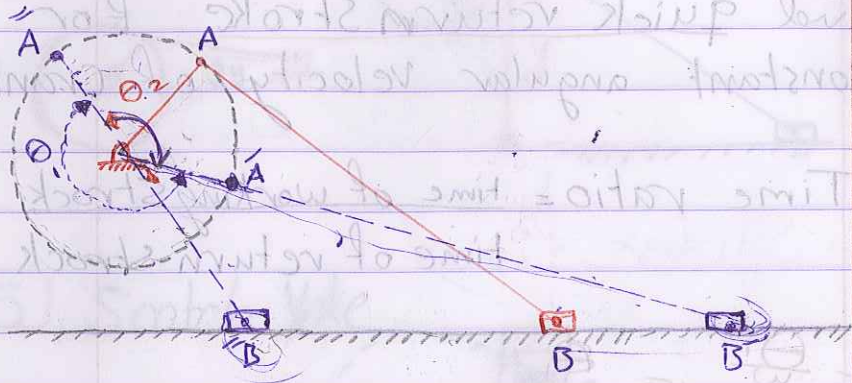
$$= \frac{\theta_1}{\theta_2} = \frac{\theta_1}{\theta_2}$$

∴ Time ratio = $\frac{\theta_1}{\theta_2} \Rightarrow \text{time ratio} > 1$

where θ_1 : Crank angle for working stroke
 θ_2 : Crank angle for return stroke

X Quick return mechanism

I Offset slider crank



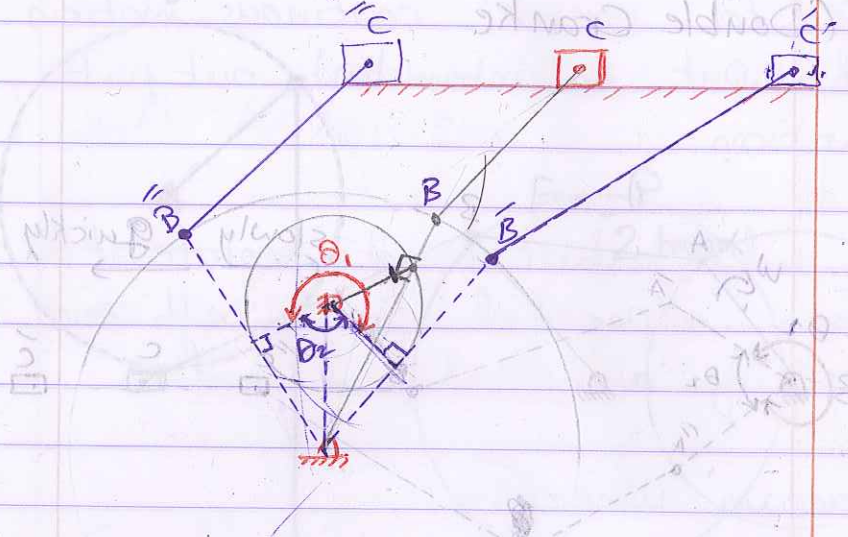
Slider move from $\bar{B} \rightarrow B$ with
Slow stroke

Time ratio ≈ 1.0

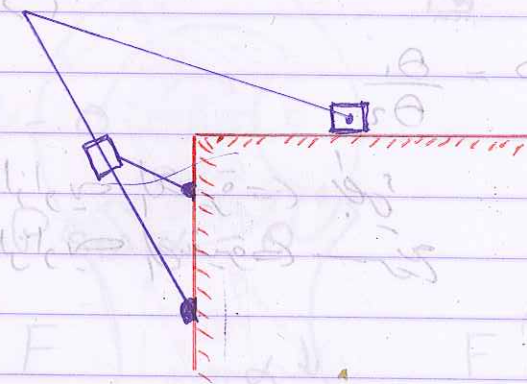
So \Rightarrow Not an effecient slider crank
Mechanism

Example:- electric saws, shapers

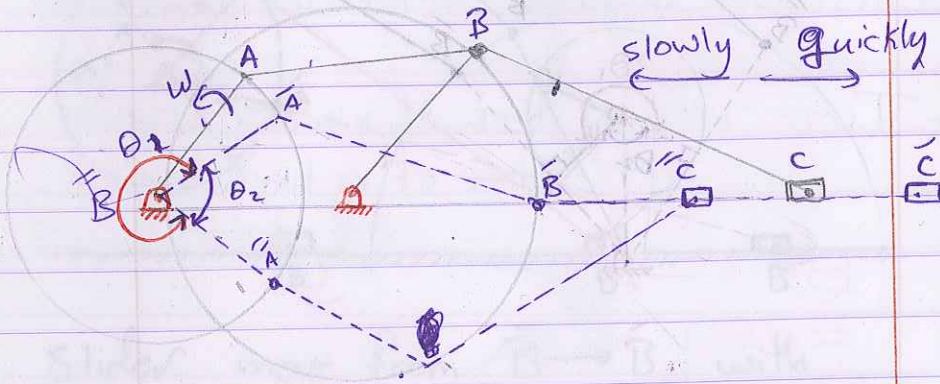
2- Crank shape mechanism



another



3 Drag link Mechanism :- (Double Crank)



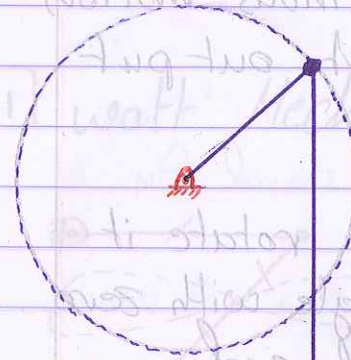
$$\theta_1 > \theta_2$$

$$t_1 > t_2$$

$$\text{time ratio} = \frac{\theta_1}{\theta_2}$$

الزاوية الكبيرة = θ_1
الزاوية الصغيرة = θ_2

Toggle



$$F = \frac{P}{2 \tan \alpha}$$



$$\sum F_y = 0$$

$$F - 2P \sin \alpha = 0$$

$$P = \frac{F}{2 \sin \alpha}$$

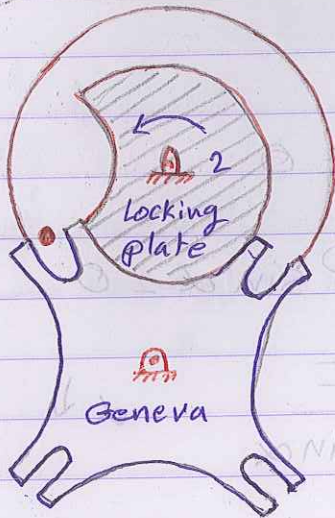
$F \uparrow$

\times ^{zero} Intermetent motion Mechanism
 Used to convert continous motion
 of input to intermetent out put
 motion

to index a shaft \circ - to rotate it
 into a spacetied angle with zero
 velocity at start and end

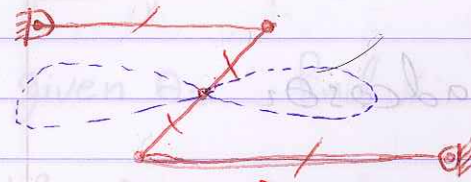
Geneva wheel \circ -

$$\frac{\omega_2}{\omega_1} = \frac{4}{1}$$

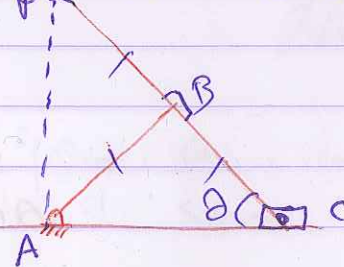


\times Straight Line Mechanism

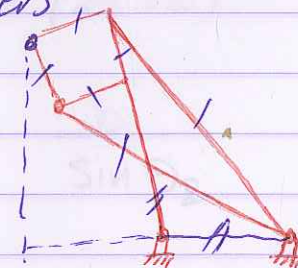
1 watt Mechanism



2 Scott Bussel



3 Peaucellier's



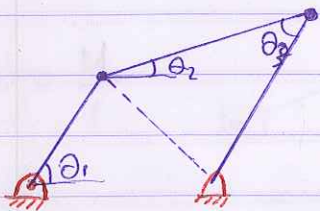
* position Analysis

Graphical Method

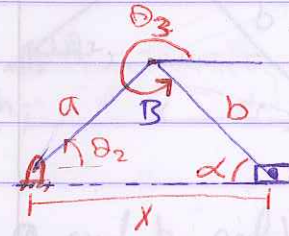
Give $\theta_2 \Rightarrow$ find θ_3 and θ_4

$$f^2 = a^2 + d^2 - 2ad \cos \theta_2$$

$$\frac{f}{\sin \theta_2} = \frac{a}{\sin B} \Rightarrow \alpha = 180 - \theta_2$$



Example 3-



given θ_2 find x

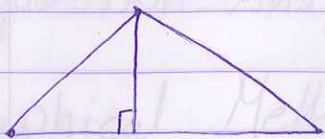
$$b^2 = a^2 + x^2 - 2ax \cos \theta_2$$

$$x^2 - 2a \cos \theta_2 x + \sqrt{(2a \cos \theta_2)^2 - 4(1)(a^2 - b^2)} = 0$$

$$\frac{b}{\sin \theta_2} = \frac{a}{\sin \alpha} \Rightarrow \alpha = 180 - \theta_2$$

$$B = 180 - (\theta_2 + \alpha)$$

$$\frac{x}{\sin B} = \frac{b}{\sin \theta_2}$$



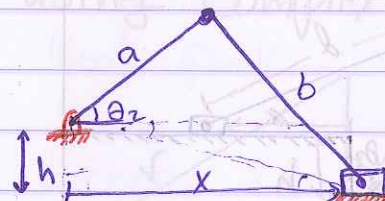
$$x = a \cos \theta_2 + b \cos \alpha$$

$$a \sin \theta_2 = b \sin \alpha$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha}$$

$$= \sqrt{1 - \left(\frac{a \sin \theta_2}{b} \right)^2}$$

$$\Rightarrow x = a \cos \theta_2 + b \sqrt{1 - \left(\frac{a \sin \theta_2}{b} \right)^2}$$



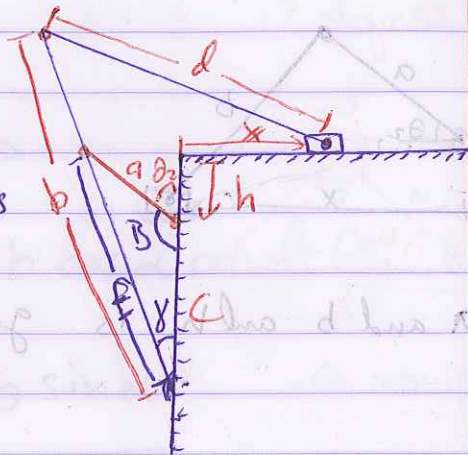
a and b and h is given $\Rightarrow x = ??$

given θ_2 and links
length fixed x ??

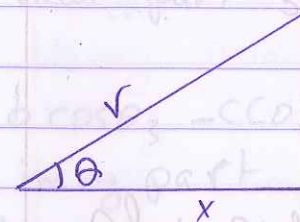
$$\beta = 180 - \theta_2$$

$$\frac{a}{\sin d} = \frac{pr}{\sin B} \quad \left(\cos d = \sin \left(\frac{a \sin B}{p} \right) \right)$$

$$p^2 = a^2 + c^2 - 2ac \cos B$$



Using Complex Number



$$\vec{r} = x + jy$$

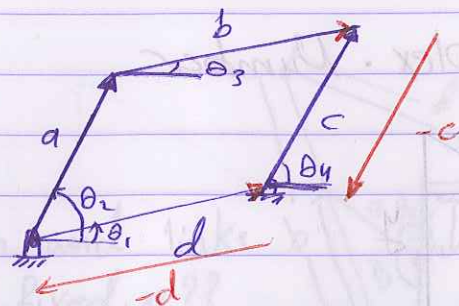
$$\vec{r} = r e^{j\theta}$$

$$r = \sqrt{x^2 + y^2}$$

$$r = r e^{j\theta}$$

$$= x + jy$$

$$r = r \cos \theta + jr \sin \theta$$



given link length and θ_1 and θ_2
find θ_3 , θ_4 ??

Loop equation

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = 0$$

$$a e^{i\theta_2} + b e^{i\theta_3} - c e^{i\theta_4} - d e^{i\theta_1} = 0$$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

$$\frac{d}{dt} e^{ax} = \frac{d}{dx} e^{ax} \frac{dx}{dt}$$

$$\frac{d f(x(t))}{dt} = \frac{d}{dx} f(x) \frac{dx}{dt}$$

$$a (\cos \theta_2 + j \sin \theta_2) + \dots$$

Real part

$$b \cos \theta_3 - c \cos \theta_4 = d \cos \theta_1 - a \cos \theta_2 \quad \text{--- (1)}$$

imag part

$$b \sin \theta_3 - c \sin \theta_4 = d \sin \theta_1 - a \sin \theta_2 \quad \text{--- (2)}$$

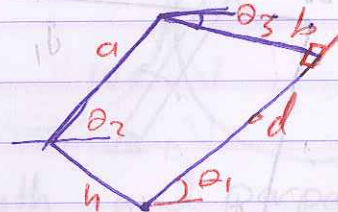
Position analysis \Rightarrow non linear equation

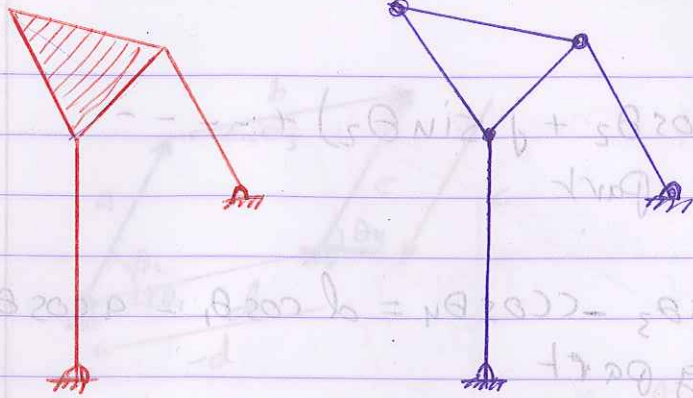
$$(\sin \theta_3)^2 + (\cos \theta_3)^2 = 1 = f_1(\theta_4)^2 + f_2(\theta_4)^2$$

Ex:- given θ_1 , θ_2 length, h
find |d|

$$\vec{h} + \vec{a} + \vec{b} - \vec{d} = 0$$

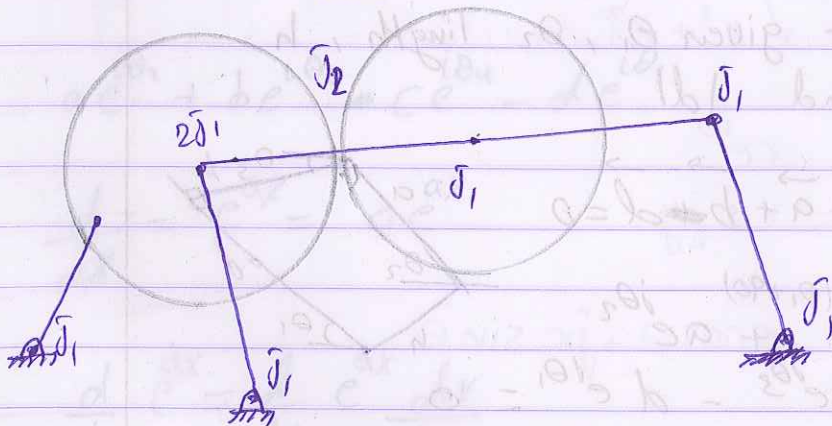
$$h e^{j(\theta_1 + 90)} + a e^{i\theta_2} + b e^{i\theta_3} - d e^{i\theta_1} = 0$$





if we consider the triangle as one body the mobility is same as the triangle (three bars)

Gears



Velocity analysis

- 1- Using velocity polygons
- 2- Using instant center
- 3- Using complex number

Translation

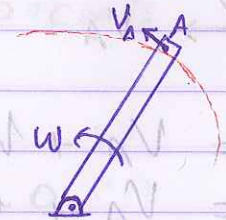
$$\omega = 0$$

$$\vec{V}_B = \vec{V}_A$$



Rotation about fixed axis (O)

$$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/O}$$



V_A tangent to path and perpendicular to \vec{r} in the direction of $\vec{\omega}$

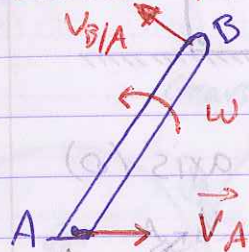
$\vec{V}_{A/O}$ = positive vector of point A with respect to O

from O to A

$+K^{\wedge}, -K^{\wedge} \vec{\omega}$

direction of $\vec{\omega}$

General motion



عودي في الخط الواحد
منه A/B و باتجاه $\vec{\omega}$

$$V_B = V_A + V_{B/A}$$

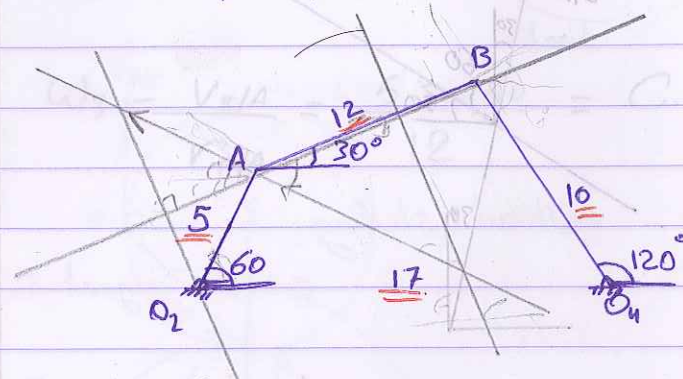
$$V_B = V_A + \vec{\omega} \times r_{B/A}$$

$V_{B/A}$ \perp to $r_{B/A}$ in the direction of $\vec{\omega}$

$$V_{B/A} = \omega r_{B/A}$$

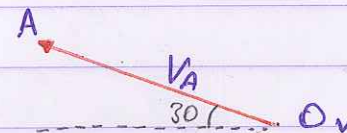
Four bar mechanism

input ω_2 (given) = 3 rad/s
find ω_3, ω_4 and \vec{V}_B ?



$$\vec{V}_A = \vec{\omega}_2 \times \vec{r}_{A/O_2}$$

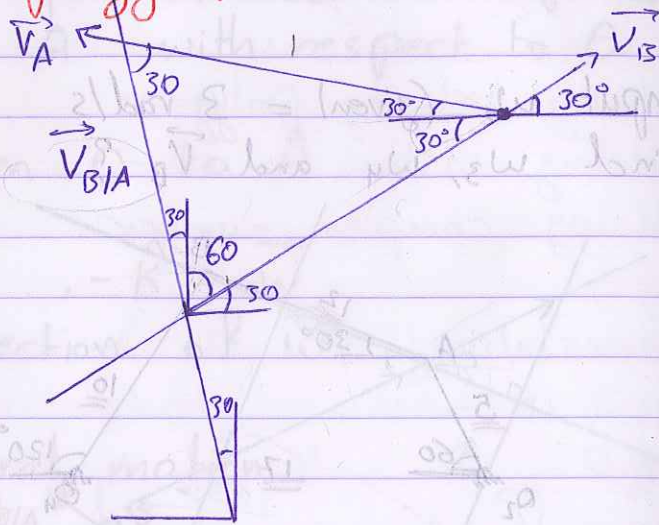
$$V_A = 3 \times 5 = 15 \text{ cm/s}$$



$$V_B = V_A + V_{B/A}$$

$$V_{B/A} = \omega_3 r_{B/A}$$

Velocity Polygon



قاعدة (sin)

$$\frac{V_A}{\sin 90} = \frac{V_B}{\sin 30}$$

$$\frac{15}{1} = \frac{V_B}{0.5}$$

$$V_B = 7.5 \text{ cm/s}$$

$$V_B = V_A + V_{B/A}$$

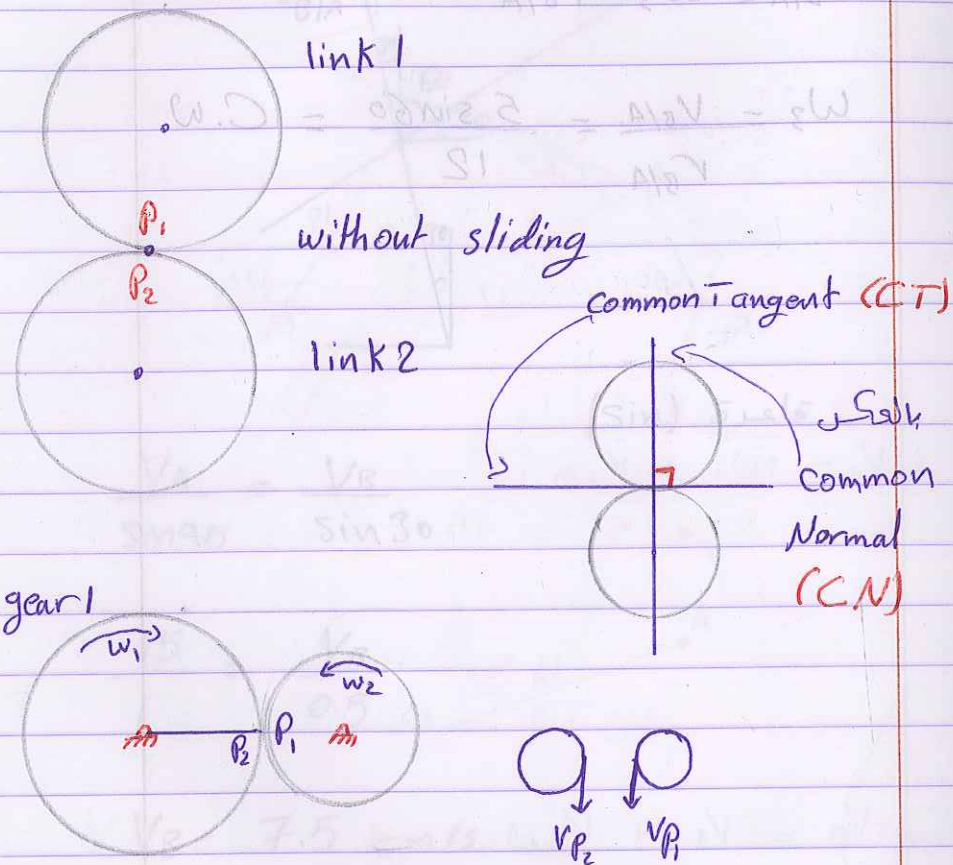
$$\omega_4 = \frac{V_B}{r_{B/O_4}} = \frac{7.5}{10} = 0.75 \text{ rad/s} \quad \curvearrowright$$

$$V_{B/A} = \omega_3 r_{O/A}$$

$$\omega_3 = \frac{V_{B/A}}{r_{B/A}} = \frac{5 \sin 60}{12} = \text{C.W.}$$

At contact point

- without sliding: velocity is equal on both bodies

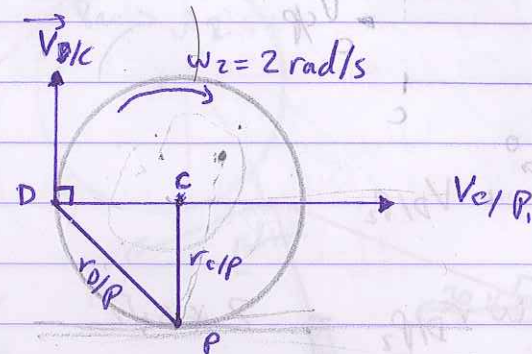
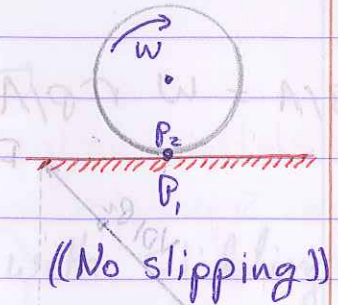


$$v_{P_1} = \omega_1 r_1 \text{ (without sliding)}$$

$$v_{P_2} = v_{P_1}$$

$$\vec{v}_{P_2/P_1} = 0$$

$$v_{P_1} - v_{P_2} = 0$$



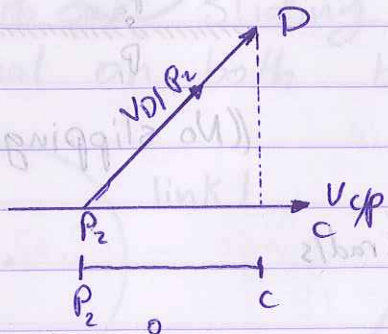
$$v_{P_1} = v_{P_2}$$

$$v_{P_1} = 0 = v_{P_2}$$

$$\vec{v}_C = \vec{v}_{P_2} + \vec{v}_{C/P_2}$$

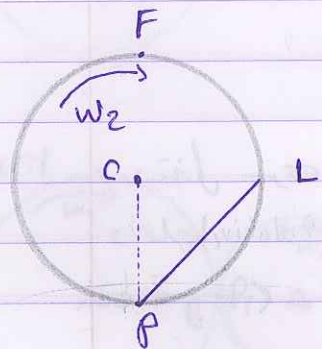
$$v_{C/P_2} = \omega_2 r_{C/P_2} = 2 \times 5 = 10$$

$$V_{B/A} = \omega r_{P/A}$$



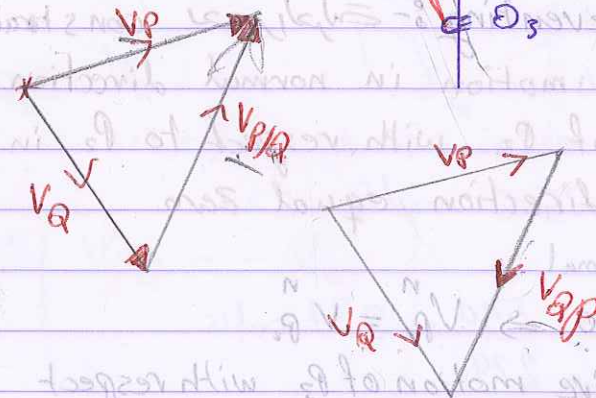
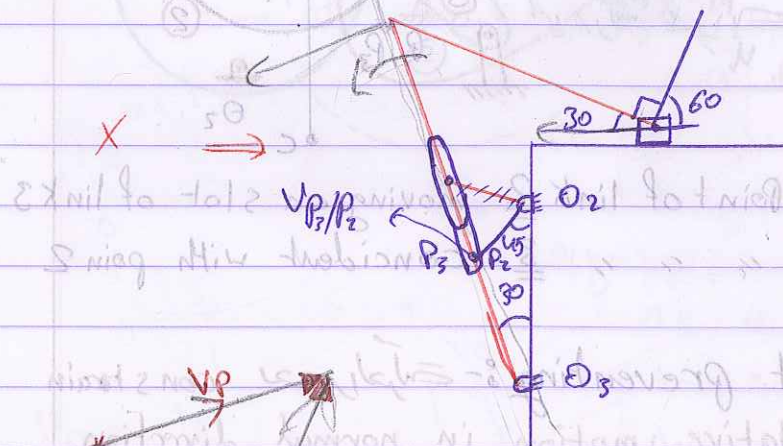
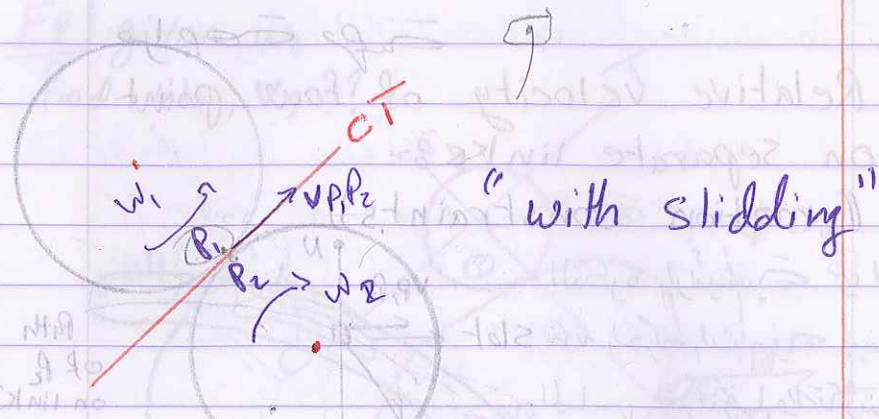
$$V_D = V_P + V_{D/P_2}$$

$$V_{D/P_2} = \omega r_{D/P_2} = 2 \times 5\sqrt{2}$$

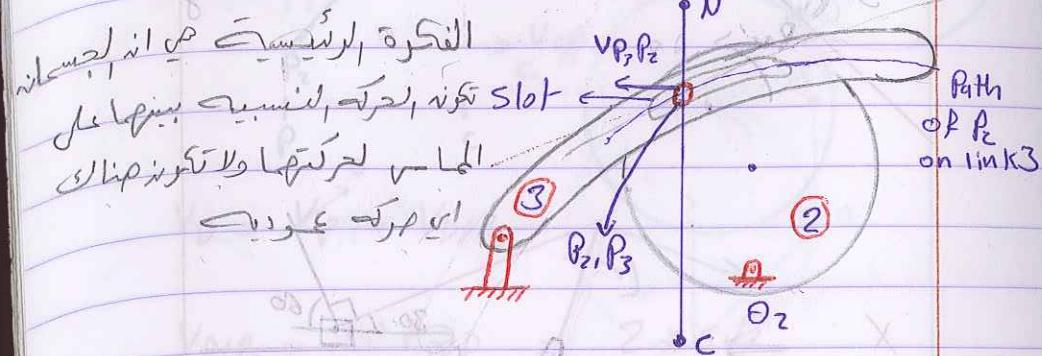


نأخذ نقطة عند المركز
مع أي نقطة خارج الدائرة

with sliding: the relative velocity is
along the common tangent



مراجعة سريعة
 [X] Relative velocity of two point on separate links :-
 (motion constraints)

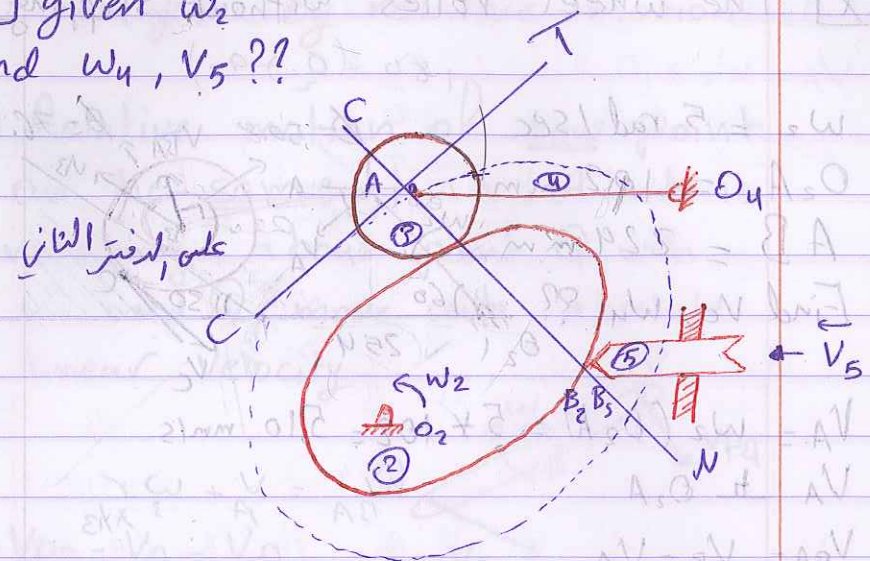


P_2 : Point of link 2 moving on slot of link 3
 P_3 : " " " 3 coincident with point 2

Slot preventing :- \Rightarrow constraint prevent relative motion in normal direction motion of P_3 with respect to P_2 in normal direction equal zero

$V_{P_2 P_3}^n = 0 \Rightarrow V_{P_3}^n = V_{P_2}^n$
 \rightarrow relative motion of P_3 with respect to P_2 is only permitted in tangential
 $\therefore V_{P_2 P_3}^t \neq 0$ tangent

Ex] given w_2
 find w_4, V_5 ??



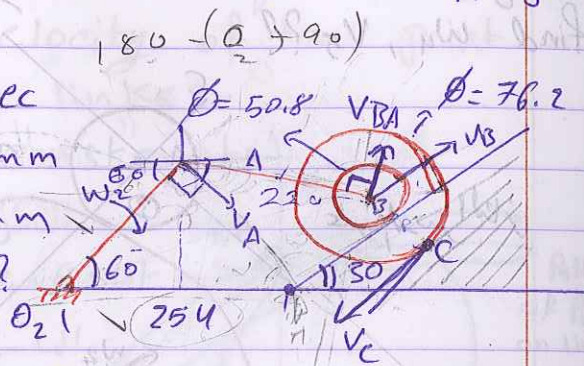
Ex] The wheel rolls without slipping

$$\omega_2 = 5 \text{ rad/sec}$$

$$O_2A = 102 \text{ mm}$$

$$AB = 229 \text{ mm}$$

Find V_C ; ω_4 ??



$$V_A = \omega_2 (O_2A) = 5 \times 102 = 510 \text{ mm/s}$$

$$V_A \perp O_2A$$

$$V_{BA} = V_B - V_A$$

V_B tangent to inclined surface (moving without slipping)

$$V_{BA} \perp AB$$

$$V_B = 650 \text{ mm/s}$$

$$V_{BA} = 590 \text{ mm/s}$$

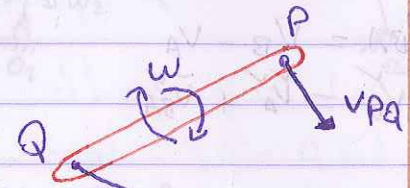
$$\omega_4 = \frac{V_B}{\frac{AB}{2}} = \frac{650}{50.8/2} = 25.6 \text{ rad/s c.w.}$$

$$\omega_3 = \frac{V_{BA}}{AB} = \frac{590}{229} = 2.57 \text{ rad/s}$$

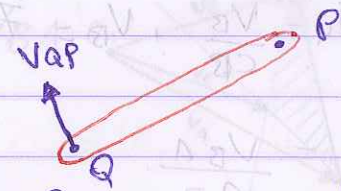
$$\omega_4 = \frac{V_B}{\frac{AB}{2}} = \frac{V_C}{\frac{PC}{2}} \Rightarrow V_C = 650 \left(\frac{25.4}{50.8} \right) = 325 \text{ mm/s}$$

2] Analysis by relative motion

a] Relative motion of two point on the same link. A link is rotating about point Q with ω and Q move with absolute linear velocity



$$V_{PQ} = V_P - V_Q = \omega \times (QP) \perp QP \text{ in sense of } \omega$$



$$V_{QP} = V_Q - V_P = \omega \times (PQ) \perp PQ \text{ in sense of } \omega$$

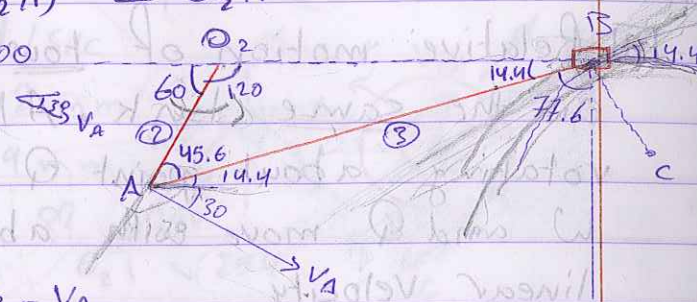
$$\therefore V_{PQ} = -V_{QP}$$

EX if $\omega_2 = 1 \text{ rad/s}$ find $V_B, \omega_4??$

$$V_A = \omega_2 \times (O_2 A) \perp O_2 A$$

$$= 1 \times 100$$

$$= 100 \text{ mm/s}$$



$$V_{BA} = V_B - V_A$$

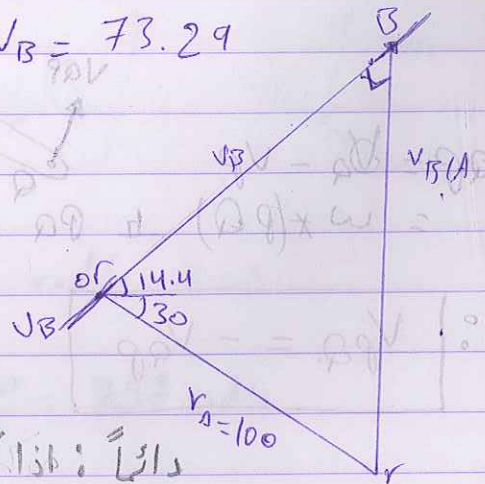
$$\therefore \vec{V}_B = \vec{V}_A + \vec{V}_{BA}$$

V_B = Known magnitude, direction known $\nearrow 14.4$

V_{BA} = magnitude is unknown direction known $\nearrow A \rightarrow B$

$$\therefore \omega_4 = \frac{V_B}{CB}, \quad V_B = 73.29$$

$$\omega_3 = \frac{V_{BA}}{AB}$$

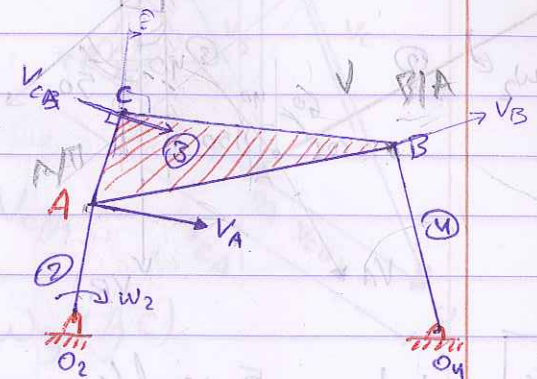


rolling = $\text{دائراً : لا كالمزلق}$

with slipping

C.T = نقطة الاتصال

EX given ω_2 find $V_B, V_C, \omega_3, \omega_4??$



Solution

$$V_A = \omega_2 (O_2 A) \perp O_2 A$$

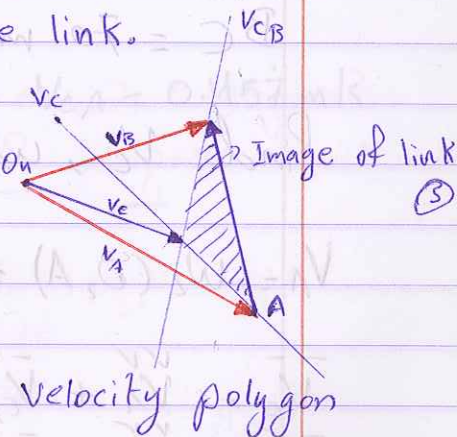
B, A lie on the same link.

$$\vec{V}_B = \vec{V}_A + \vec{V}_{BA}$$

$$V_{BA} \perp AB$$

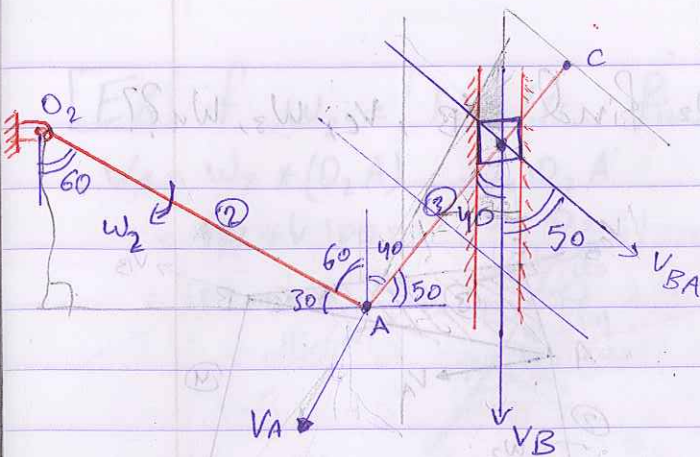
$$\vec{V}_C = \vec{V}_A + \vec{V}_{CA}$$

$$\vec{V}_C = \vec{V}_B + \vec{V}_{CB}$$



$$V_{BA} = \omega_3 (AB), \text{ or, } V_{CA} = \omega_3 (CA)$$

$$\therefore \omega_3 = \frac{V_{BA}}{AB} = \frac{V_{CA}}{AC} = \frac{V_{CB}}{BC}$$



Ex

$$w_2 = 5 \text{ rad/s}$$

$$O_2A = 76.2 \text{ mm}$$

$$AB = 152 \text{ mm}$$

$$BC = 70 \text{ mm}$$

Find V_C , w_3 ??

$$V_A = w_2 (O_2A) = 5 \times 76.2 = 0.381 \text{ m/s}$$

$$\vec{V}_C = \vec{V}_A + \vec{V}_{CA}$$

$$\vec{V}_B = \vec{V}_A + \vec{V}_{BA}$$

$$\frac{V_B}{\sin 100} = \frac{V_{BA}}{\sin 30} = \frac{V_A}{\sin 50}$$

$$\sin 100 \quad \sin 30 \quad \sin 50$$

$$w_2 = 5 \text{ rad/s}$$

$$O_2A = 76.2 \text{ mm}$$

$$\therefore V_C = V_A + V_{CA}$$

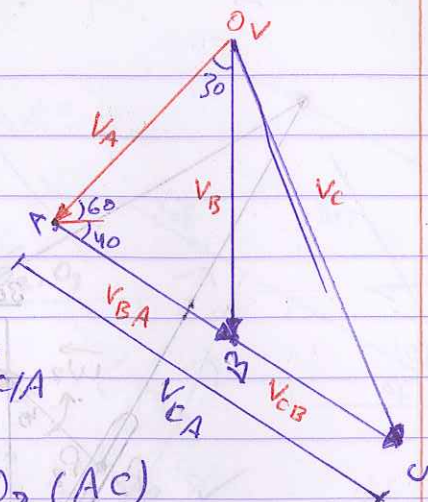
$$\text{but } V_{CA} = w_3 (AC)$$

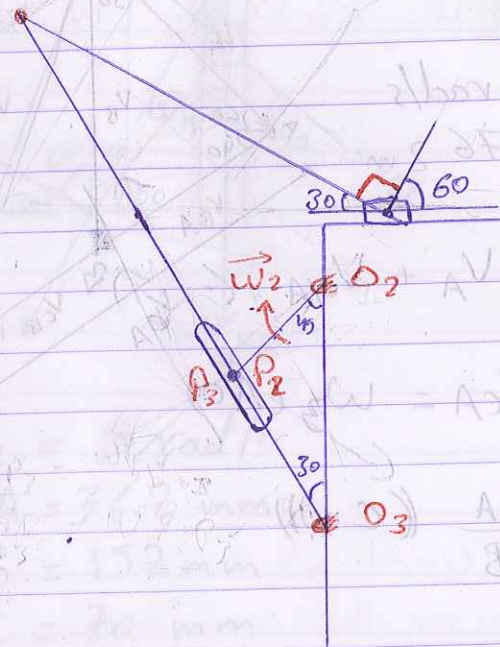
$$w_3 = \frac{V_{BA}}{AB} \quad (\text{C.W.})$$

$$w_3 = \frac{V_{BA}}{AB} = \frac{V_{CA}}{AC} \Rightarrow V_{CA} = 0.457 \text{ m/s}$$

$$\vec{V}_C = \vec{V}_A + \vec{V}_{CA}$$

$$\vec{V}_C = \vec{V}_B + \vec{V}_{CB}$$



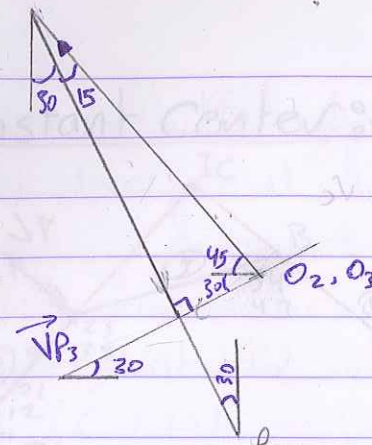


Example: Given $\omega_2 = 2 \text{ rad/s}$ and link lengths find V_5 at this instant

$$V_{P_3} = V_{P_2} + V_{P_3/P_2}$$

$$V_{P_3/P_2} \neq \omega r_{P_3/P_2}$$

$$V_{P_2} = \omega_2 \times r_{P_2/O_2} = 2 \times 5 = 10 \text{ cm/s}$$



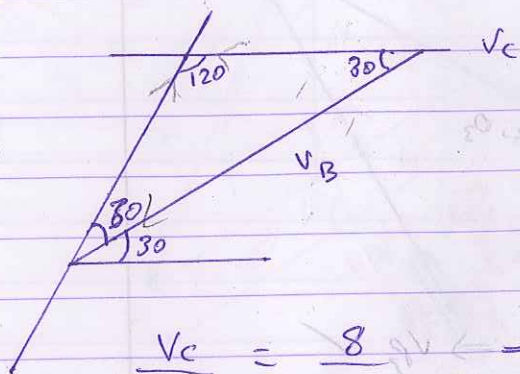
$$\frac{10}{\sin(90)} = \frac{V_{P_3}}{\sin(15)} \Rightarrow V_{P_3} = 10 \sin(15)$$

$$\omega_3 = \frac{V_{P_3}}{r_{P_3/O_3}}$$

$$\Rightarrow \omega_3 = \frac{V_{P_3}}{r_{P_3/O_3}}$$

$$V_{P_2} = \omega_2 \times r_{P_2/O_2} = 2 \times 5 = 10$$

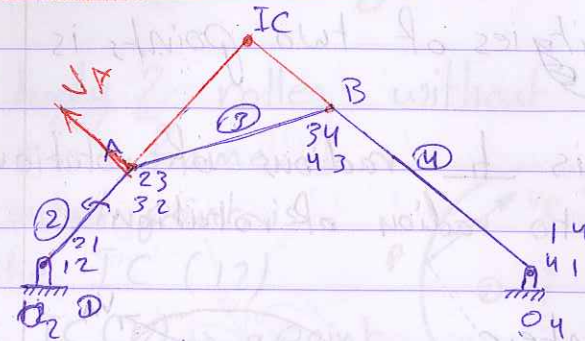
$$V_B = \omega_B \times r_{B/O} = 0.25 \times 35 = 8$$



$$\frac{V_c}{\sin 30} = \frac{8}{\sin(120)} \Rightarrow V_c \text{ to the left}$$

~~W₁ / r₁~~
~~V_{C/B}~~

Instant Center:-



a point on a body (link) about which other link is rotating either at the instant or permanently OR Point Common to two links with same velocity in magnitude and direction.

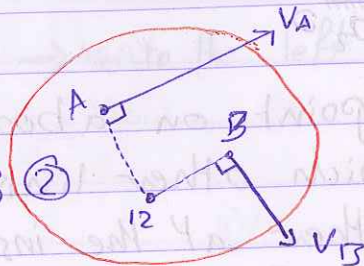
Primary instant centers:-

- ① Instant center at pin connection
Instant center (12) :- point on link 1 about which link 2 is rotating (link 2 is rotating about link 1 at (12))
12, 14 → Fixed IC 23, 34 → moving IC

⇒ To find IC when the direction of velocities of two points is known

velocity is \perp radius of rotation
 ∴ $V_A \perp$ to radius of rotation

Instant center
 Intersection of two perpendicular at A, B ②

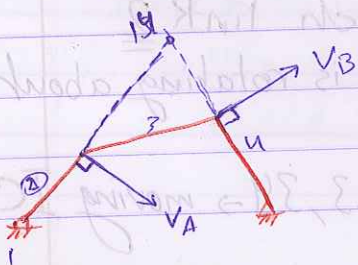


$$V_A = \omega_2 (I_2 - A)$$

$$V_B = \omega_2 (I_2 - B)$$

$$\frac{V_A}{(I_2 - A)} = \frac{V_B}{(I_2 - B)} \quad (\text{IC : } \text{مركز لحظي})$$

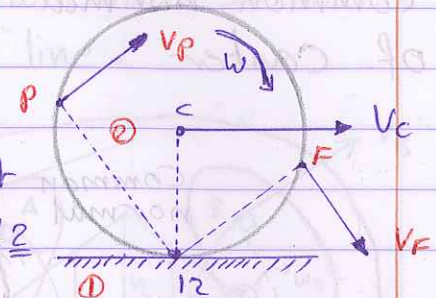
⇒ For 4 bar mechanism



② IC of Rolling contact :-

body 2 rolls without slipping on frame 1
 contact point is the IC (12)

∴ IC (12) → a point common to 1 and 2

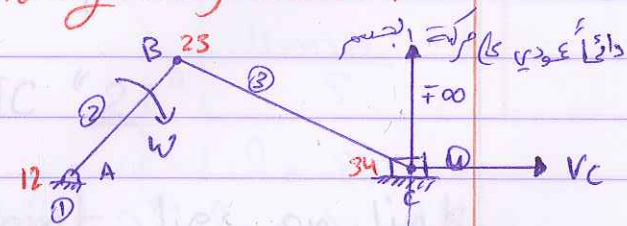


$$V_C = \omega (I_2 - C)$$

$$V_P = \omega (I_2 - P)$$

$$V_F = \omega (I_2 - F)$$

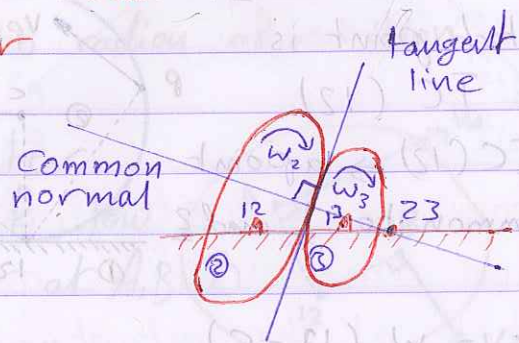
③ IC of sliding body contact



IC of sliding link \perp surface of sliding

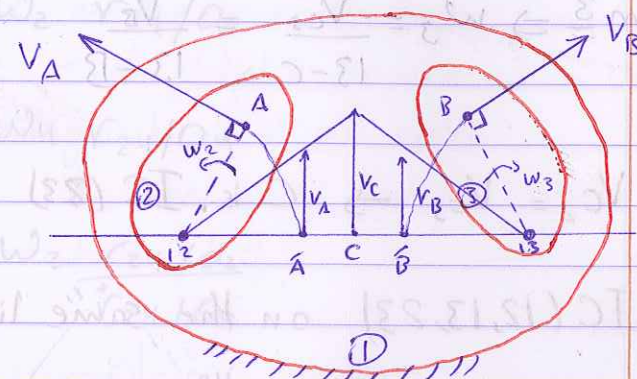
④ IC of sliding and Rolling contact

IC is the intersection of common normal with line of center



Kennedys Theorem:-

Any three body in motion relative to each other will have IC that lie on straight line.



(12, 13) is the IC of body ① and ② with respect to frame

C is the IC "23"

IC (23) : point lies on link 2 and link 3 of the same velocity

$$V_{C2} =$$

$$V_{C3} = \omega_3 (I3-C), \quad V_{C2} = V_{C3} \rightarrow \text{instant center}$$

$$\text{From } \underline{2} \Rightarrow \omega_2 = \frac{V_{C2}}{I2-C} = \frac{V_A}{I2-A}$$

$$\text{from } \underline{3} \Rightarrow \omega_3 = \frac{V_{C3}}{I3-C} = \frac{V_B}{I3-B}$$

$$V_{C2} = V_{C3} \Rightarrow C \text{ is IC (23)}$$

IC (12, 13, 23) on the same line

\Rightarrow Number of IC of a mechanism:-

$$N = \frac{n(n-1)}{2}$$

$n = \#$ of links

$(n-1) = \#$ of IC with respect to any link

$n(n-1) = \#$ of IC's

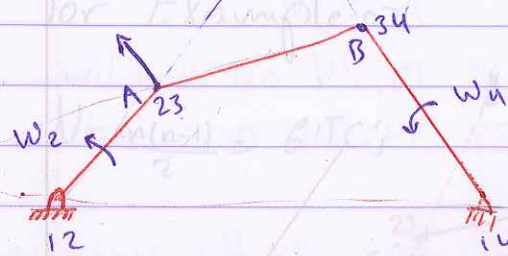
$N = \#$ of location of IC's

$$\begin{matrix} 12 & 13 & 23 & 123 \\ 12 & 14 & \boxed{24} & 124 \\ 23 & \boxed{24} & 34 & 234 \\ 13 & 14 & 34 & 134 \end{matrix}$$

$$V_{24} = \omega_2 r_{24/O_2}$$

$$V_{24} = \omega_4 r_{24/O_4}$$

$$\omega_4 = \omega_2 \frac{r_{24/O_2}}{r_{24/O_4}}$$

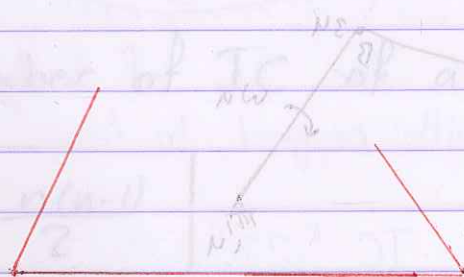
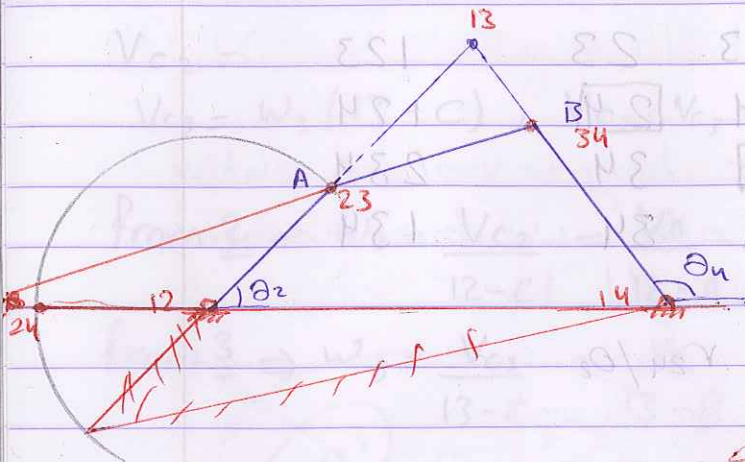


$$V_A = \omega_2 r_{A/O_2}$$

$$\omega_3 = \frac{V_A}{r_{A/I3}}$$

$$\omega_3 = \frac{V_B}{r_{B/I3}} = \frac{V_A}{r_{A/I3}}$$

$$\vec{V}_B = \vec{V}_{I3} + \vec{V}_{B/I3}$$



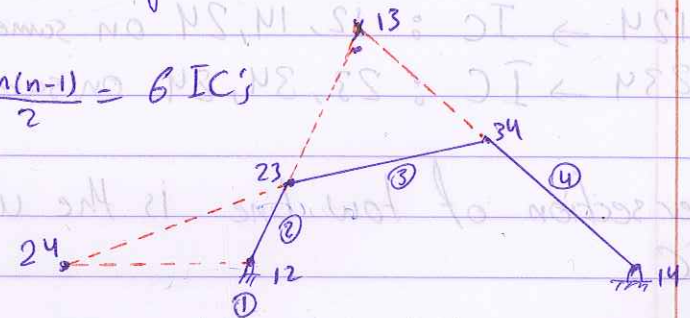
Circle diagram to find IC:

Steps:-

- ① Find # of IC's
- ② find all primary IC's
- ③ draw circle, lie point's equally spaced on the circumference of circle each point represent a link
 $\therefore \# \text{ of point} = \# \text{ of links}$
- ④ each line between these point represent an IC

For Example:-

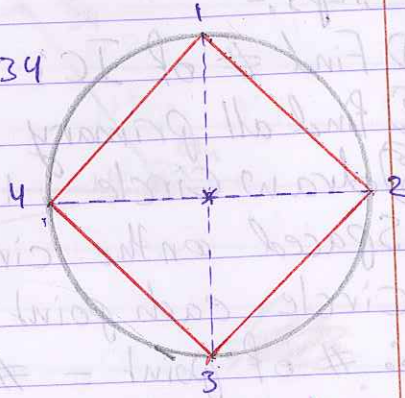
$$N = \frac{n(n-1)}{2} = 6 \text{ IC's}$$



of point \neq # of link = 4

IC (13) $\Rightarrow \Delta 123, \Delta 134$

$\Delta 123 \Rightarrow IC's$



$\Delta 123 \Rightarrow IC's : 12, 23, \underline{13}$ on the same line

$\Delta 134 \Rightarrow IC's : 14, 34, \underline{13}$ on the same line

IC (24) $\Rightarrow \Delta 124 \quad \Delta 234$

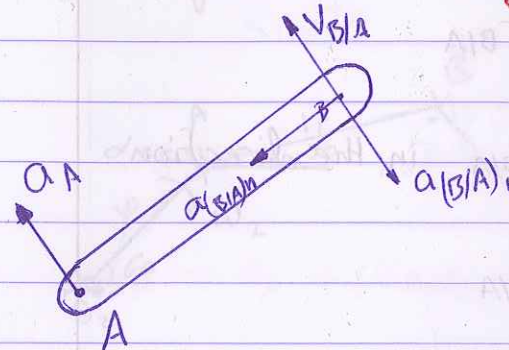
$\Delta 124 \rightarrow IC : 12, 14, 24$ on same line

$\Delta 234 \rightarrow IC : 23, 34, 24$ on same line

intersection of two line is the unknown
IC

الافتتاحية، البنية، الثاني، الفردي

Acceleration analysis



$$a_B = a_A + a_{B/A}$$

$$(a_{B/A})_n$$

$$= \omega^2 r_{B/A}$$

$$(a_{B/A})_t$$

$$\alpha \times r_{B/A}$$

$(\vec{a}_{B/A})_n$ = Centripital acceleration of point B relative to A in the oppisit direction of $\vec{r}_{B/A}$

حل مسئله (A → B = $v_{B/A}$) سکتاں انتہا $(a_{B/A})_n$
B → A

(In the direction towards the center of rotation A)

اوپر انتہا مرکز گردان

$$(a_{B/A})_n = \omega^2 r_{B/A}$$

$$(a_{B/A})_t = \alpha r_{B/A} \text{ in the direction of } \alpha$$

$$(a_{B/A})_t = \alpha r_{B/A}$$

$$a_{B/A} = a_B - a_A$$

$$a_{B/A} = a_{B/A}^n + a_{B/A}^t$$

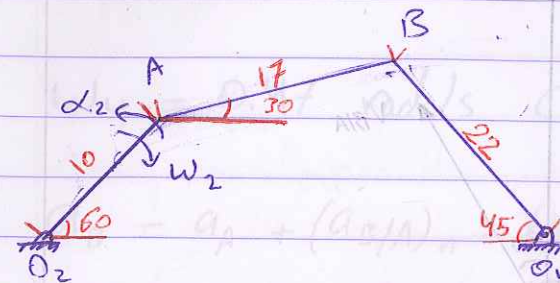
$$\text{but } a_{B/A}^n = \omega^2 r_{B/A} = \frac{V^2}{BA} \text{ (Rotation about fixed axis)}$$

$$a_{B/A}^t = \alpha_2 (BA) \perp AP \text{ in the direction of } \alpha$$

$$a_{AB}^t = -a_{BA}^t$$

$$a_{AB}^n = -a_{BA}^n$$

Example: for the given Mechanism



$$\omega_2 = 2 \text{ rad/s C.W}$$

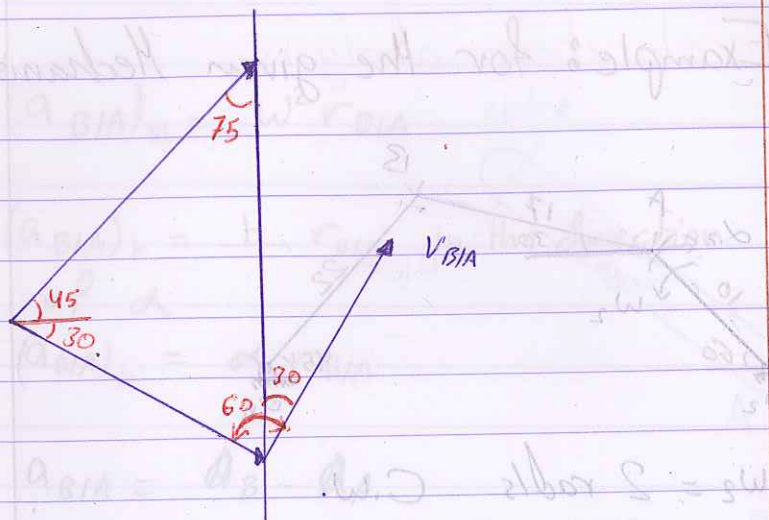
$$\alpha_2 = 3 \text{ rad/s C.C.W}$$

Determine a_B and α_1, α_3

$$V_A = \omega_2 r_{A/O_2}$$

$$= 2 \times 10 = 20 \text{ cm/s C.C}$$

$$V_B = V_A + V_{B/A}$$



$$V_{B/A} = V_A \\ = 20 \text{ cm/s}$$

$$\omega_3 = V_{B/A} / r_{B/A} = \frac{20}{17} = 1.18 \text{ rad/s}$$

C.C.W

$$\frac{V_B}{\sin 30} = \frac{V_A}{\sin 75}$$

$$V_B = \frac{20 \times \sin 30}{\sin 75} = 10.35 \text{ cm/s}$$

$$V_B = \frac{20 \sin 30}{\sin 75} = 10.35 \text{ cm/s}$$

$$\omega_4 = 0.47 \text{ rad/s C.W}$$

$$a_B = a_A + (a_{B/A})_n + (a_{B/A})_t$$

$$(a_B)_n + (a_B)_t = (a_A)_n + (a_A)_t + (a_{B/A})_n + (a_{B/A})_t$$

$$\omega_2 = 2$$

$$\omega_3 = 1.18$$

$$\omega_4 = 0.47$$

$$a_B = \omega_4^2 r_{B/O} = (0.47)^2 (22) = 4.85 \text{ cm/s}$$

$$(a_A)_n = \omega_2^2 r_{A/O_2} = 40 \text{ cm/s}$$

$$(a_A)_t = \alpha_2 r_{A/O_2} = 3 (10) = 30 \text{ cm/s}^2$$

$$\vec{a}_{B/A} = \omega^2 r_{B/A} = 23.7 \text{ cm/s}$$

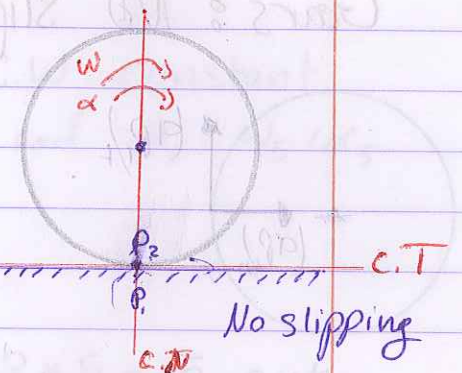
$$\alpha_A = (a_B)_t / r_{B/O_A}$$

C.C.W

At the contact point without Slipping

Remark

Component of the acceleration along the common tangent direction is equal on both bodies

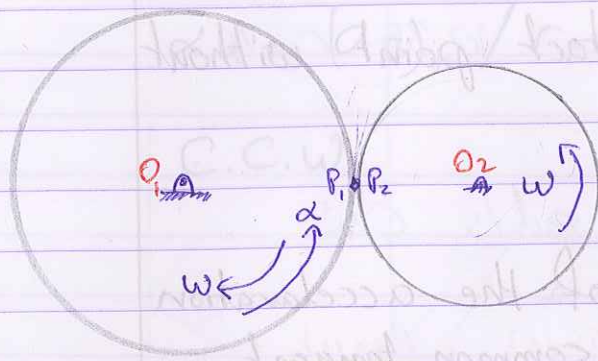


$$\vec{a}_{P_1} = 0$$

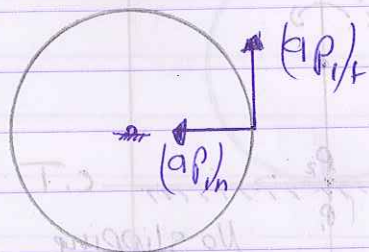
$$\vec{a}_{P_1} = 0 = (a_{P_1})_T + (a_{P_1})_N$$

$$\vec{a}_{P_2} = (a_{P_2})_T + (a_{P_2})_N$$

slipping



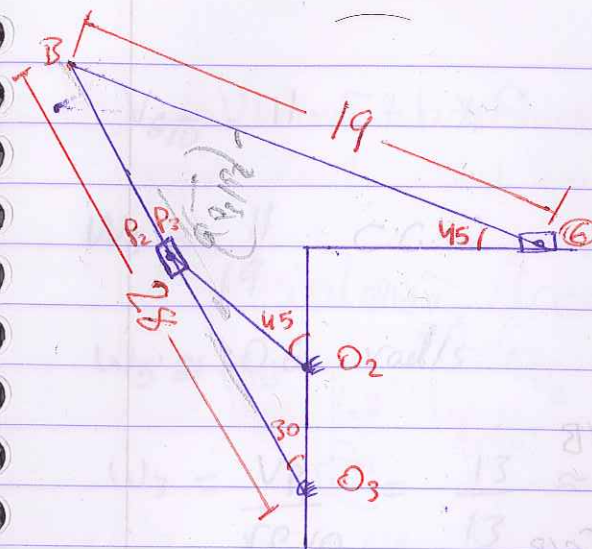
Gears: No slipping



$$(a_P)_n = \omega^2 \cdot r_{P/O}$$

$$(a_P)_t = \alpha \cdot r_{P/O}$$

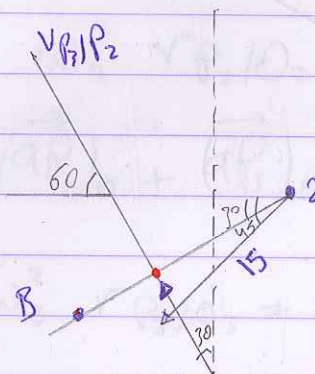
$$\alpha_2 r_2 = \alpha_1 r_1$$



$\omega_2 = 3 \text{ rad/s}$ C.C.W constant
determine V_G and a_G, ω_5, α_5

$$V_{P_3} = V_{P_2} + V_{P_3/P_2}$$

$$V_{P_2} = \omega_2 r_{P_2/O_2} = 3 \times 5 = 15 \text{ cm/s}$$

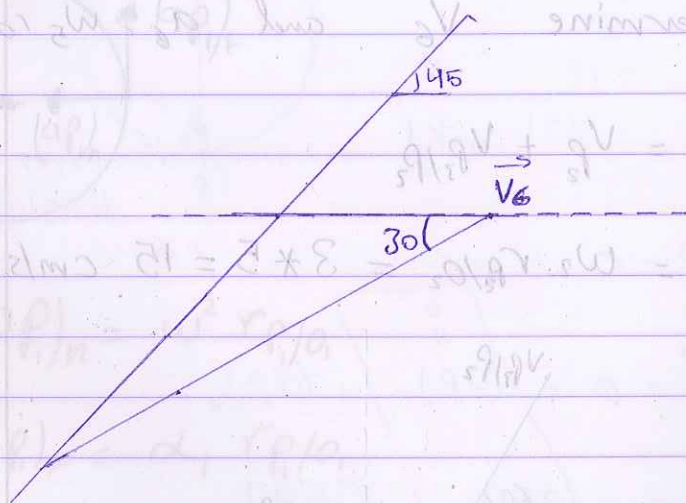


$$V_{P_3} = 13 \text{ cm/s}$$

$$\frac{28}{13} = \frac{V_B}{V_{P_3}}$$

$$\vec{V}_G = \vec{V}_B + \vec{V}_{G/B}$$

$$V_{G/B} = \omega_5 r_{G/B}$$



$$V_{G/B} = 11$$

$$\omega_5 = \frac{11}{19} \text{ C.G.W}$$

$$\omega_5 \approx 0.6 \text{ rad/s}$$

$$\omega_3 = \frac{V_{P_3}}{r_{P_3/O_3}} = \frac{13}{13} \approx 1 \text{ rad/s C.G.W}$$

$$V_{P_3/P_2} = 7$$

$$\vec{a}_{P_3} = \vec{a}_{P_2} + (a_{P_3/P_2})_{\text{rel}} + 2\omega_3 \times (\vec{V}_{P_3/P_2})$$

$$a_{P_2} = (a_{P_2})_n + (a_{P_2})_t = \alpha_2 \times r_{P_2/O_2}$$

$$(a_{P_2})_n = \omega_2^2 r_{P_2/O_2} = (3)^2 (5) = 45 \text{ cm/s}^2$$

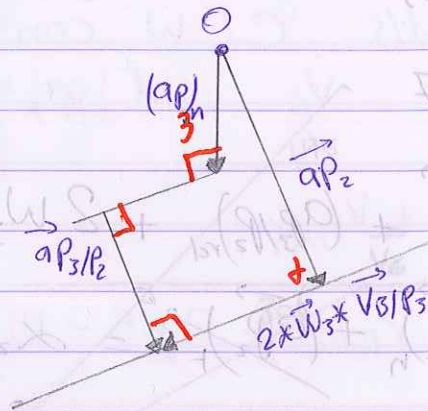
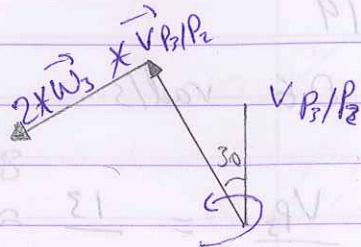
$$(\vec{a}_{P_3}) = (\vec{a}_{P_3})_n + (\vec{a}_{P_3})_t$$

$$a_{P_3} = \omega_3^2 r_{P_3/O_3} = (1)^2 (13) = 13 \text{ cm/s}^2$$

$$|2 \vec{\omega}_3 \times \vec{V}_{P_3/P_2}| = 2 \times 1 \times 7 = 14 \text{ cm/s}^2$$

$$\omega_3 = 1 \text{ rad/s}$$

↻ C.C.W



$$\frac{a_B}{28} = \frac{a_{P_3}}{13} \Rightarrow a_B = \frac{28}{13} a_{P_3}$$

$$a_B = a_B + a_{B/P}$$

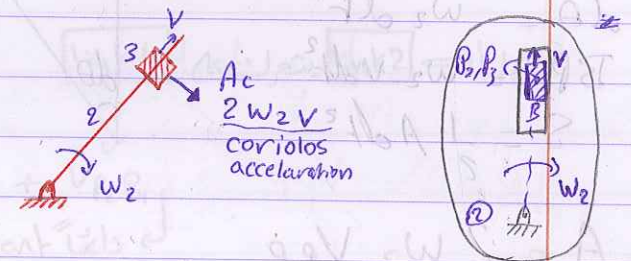
2 Relative acceleration of coincident point on separate links-

Coriolis Acceleration

$$2 \omega_2 \times V_{P_3/P_2}$$

② يتحرك على ③

Coriolis acceleration دائياً عندما يتحرك جسم على آخر يدور مع يمينه



P_3 = Point on moving link 3 along slot of link 2 with linear velocity

P_2 = Coincident point on link 2 which is rotating with ω_2

after time dt

displacement of P_3

$$P_3 \rightarrow \bar{P}_3 = P_3 \bar{P}_2 + \bar{P}_2 B + B \bar{P}_3$$

arc $P_2 \bar{P}_2$ with $w_2 = \text{constant}$

$\bar{P}_2 B =$ with $w_2 = \text{const}$

and constant $V = v dt$

$$\text{arc } B \bar{P}_3 = P_2 B d\theta$$

$$d\theta = w_2 dt$$

$$B \bar{P}_3 = w_2 v dt^2$$

$$S = \frac{1}{2} A dt^2$$

$$A = 2 w_2 V_{P_2}$$

$A \perp V$ in sense of w_2

$$A_c = 2 \times w_2 \times V_{P_2}$$

Ex

$$O_2 P = 152, O_2 O_4 = 457$$

$$O_4 P = 514$$

$$w_2 = 9.5 \text{ rpm}$$

find α_4 where $\alpha_2 = 0$??

④ با ② متحرك كـ

link 4 متحرك على link 2

$$\vec{V}_{P_2} = \vec{V}_{P_4} + \vec{V}_{P_2 P_4}$$

tangent لـ ④

$V_{P_2 P_4}$ in direction of link 4

$$V_{P_4} = 0.074 \text{ m/s}$$

$$V_{P_2 P_4} = 0.131 \text{ m/s}$$

$$V_{P_2} = w_2 (O_2 P)$$

$$= 151 \text{ mm/s} \perp O_2 P$$

$$(a_{P_2})_n + (a_{P_2})_t = (a_{P_4})_n + (a_{P_4})_t + (a_{P_4 P_2})_n + (a_{P_4 P_2})_t + 2\omega_4 V_{P_2 P_4}$$

$$a_{P_2}^n = \omega_2^2 (O_2 P) = 0.15 \text{ m/s}^2$$

$$a_{P_2}^t = 0 \quad \text{because } \alpha_2 = 0$$

$$a_{P_4}^n = \omega_4^2 (O_4 P) = \frac{V_{P_4}^2}{O_4 P} = 0.0007 \text{ m/s}^2 \quad P \rightarrow O_4$$

$$A_{P_4}^t \perp O_4 P$$

$$A_{P_4 P_2}^t \text{ along link 4}$$

$$A_{P_2 P_4}^n = \frac{V_{P_2 P_4}}{R \rightarrow \infty} = 0.038 \text{ m/s}^2$$

$$2\omega_4 V_{P_2 P_4} = 0.038 \text{ m/s}^2$$

$$\omega_4 = \frac{V_{P_4}}{O_4 P} = 0.144 \text{ rad/s ccw}$$

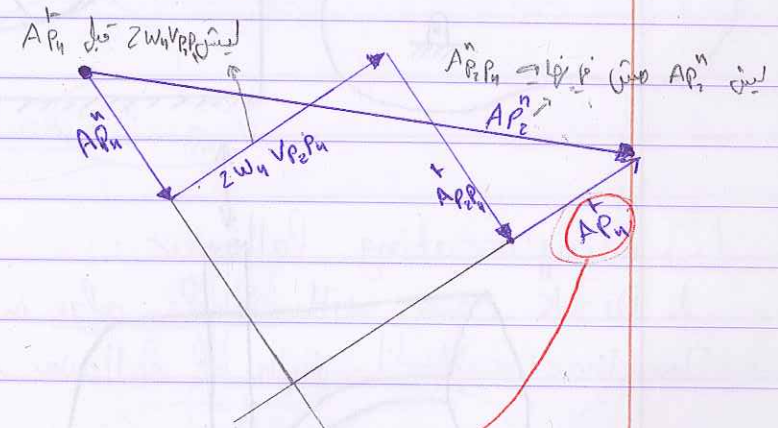
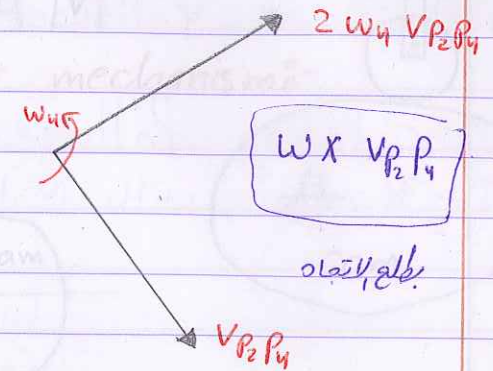
$$\alpha_4 = \frac{A_{P_4}^t}{O_4 P} = C.W$$

$$(9.0) \cdot \omega = 9V$$

$$12.0 \cdot 0.0 = 0.9V$$

$$9.0 + 12.0 = 21.0$$

$$12.0 \cdot 0.0 = 0.9V$$

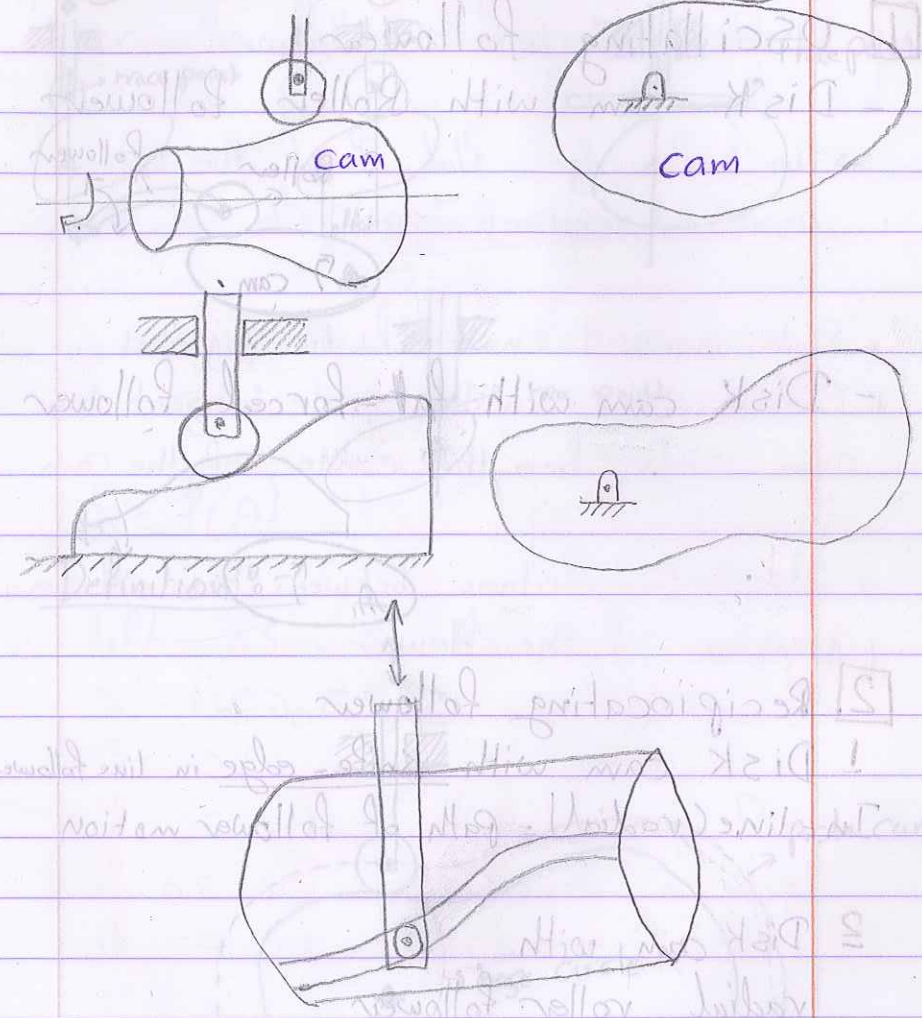


$$\alpha_4 = \frac{A_{P_4}^t}{O_4 P}$$

$$C.W$$

"CAM"

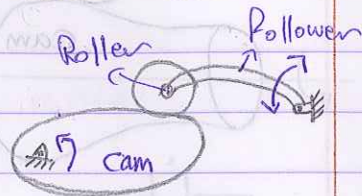
Cam follower mechanism:-



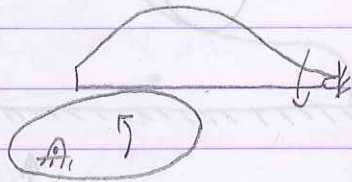
* Type of follower:-

1 Oscillating follower

- Disk cam with Roller follower



- Disk cam with flat-faced follower



2 Reciprocating follower

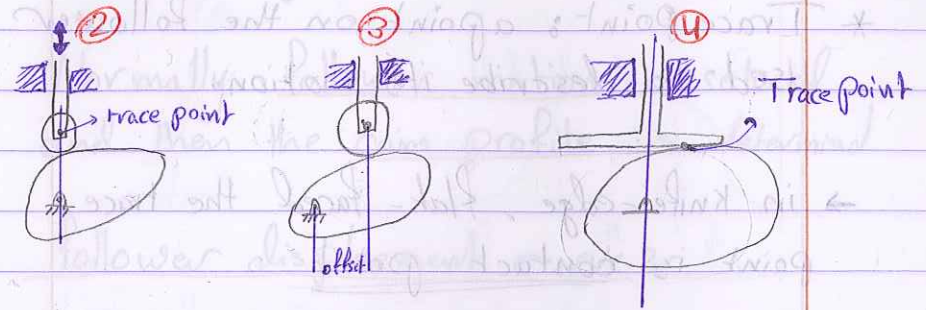
1 Disk cam with Knife-edge in line follower

In-line (radial) = path of follower motion

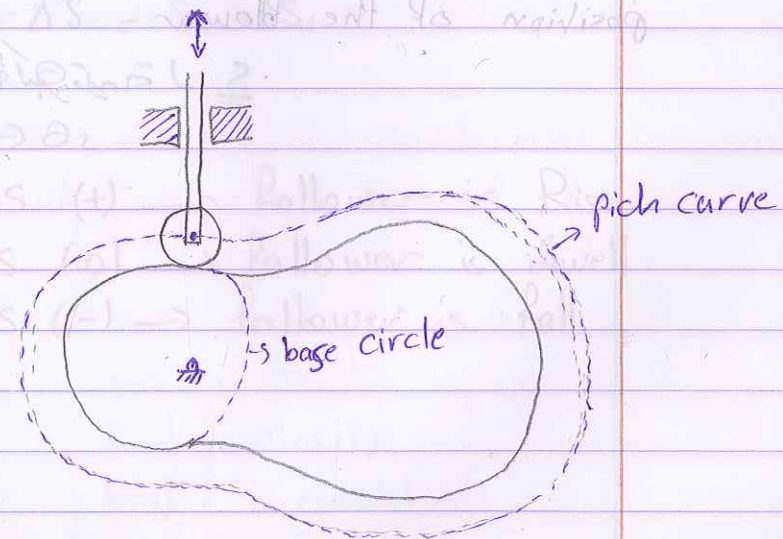
2 Disk cam with radial roller follower

3 Disk cam with radial offset roller follower

4 Disk cam with flat-faced follower



Definition:-



Definition:-

* Trace point: a point on the follower used to describe its motion.

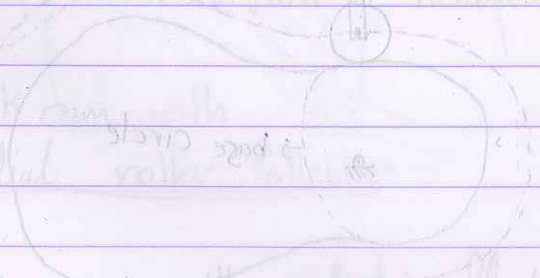
→ in knife-edge, flat-faced the trace point is contact point

* Pitch curve: curve of trace point

* Base circle: - small circle tangent to cam circle from the center of the cam

* Stroke: - distance between two extreme position of the follower

الارتفاع S



Cam Design:-

Normally follower motion is selected and then the cam profile is determined

follower displacement curve:-

S = displacement of follower

θ = cam rotation

$$S = f(\theta)$$

$$\text{lift} = \Delta S = S_2 - S_1$$

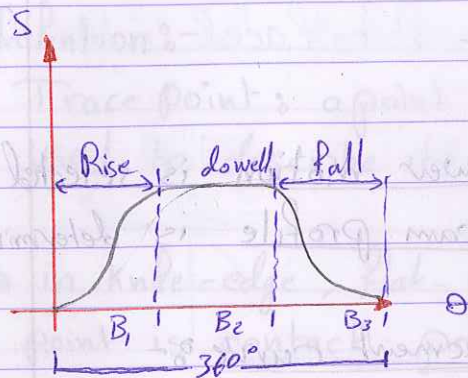
$$S_1 \rightarrow \theta_1$$

$$S_2 \rightarrow \theta_2$$

if $\Delta S (+)$ → follower is Rise

if $\Delta S (0)$ → follower is dwell

if $\Delta S (-)$ → follower is fall



B_1, B_2, B_3
is cam interval angle



$$v = \frac{ds}{d\theta}$$

$$a = \frac{d^2s}{d\theta^2}$$

Dwell $\Rightarrow S = \text{constant}$

$$\frac{ds}{d\theta} = 0 = v ; a = 0 = \frac{d^2s}{d\theta^2}$$

Design condition

\Rightarrow continuous S, v, a

$F = ma \rightarrow F$ is continuous
other wise \rightarrow high internal force
 \rightarrow vibration
 \rightarrow noise
 \rightarrow cracks

$$v = \frac{ds}{d\theta}$$

finite velocity

$$a = \frac{dv}{dt}$$

finite acceleration

$$\frac{da}{d\theta} = \text{Jerk} \quad \text{'vibration'}$$

Displacement Curve

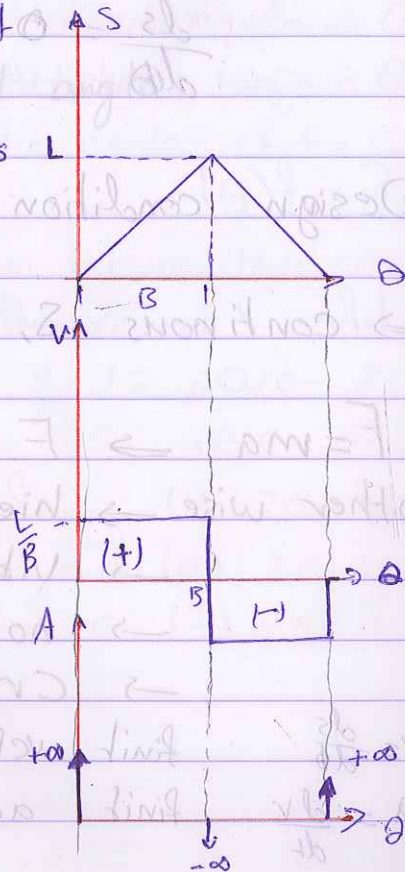
1] Straight line diagram (constant velocity)
gives infinite acceleration at inflection point

$A \rightarrow \infty \Rightarrow$ high stresses and Inertia Force

$$S = V\theta$$

$$\frac{ds}{d\theta} = V$$

$$\therefore V = \frac{L}{B}$$



2] Jerk :- derivative of acceleration
gives indication of impact loading

Jerk $\rightarrow \infty$: high impact loading and high stress and vibration

2] constant acceleration.

$$S = K\theta^2$$

$$\dot{S} = 2K\theta$$

$$\ddot{S} = 2K$$

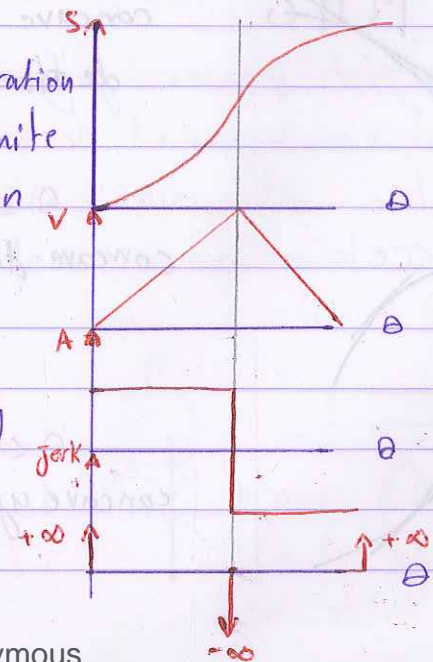
$$S = \frac{1}{2} A \theta^2$$

$$\dot{S} = A\theta$$

$$\ddot{S} = A$$

constant acceleration curve give infinite Jerk at inflection point.

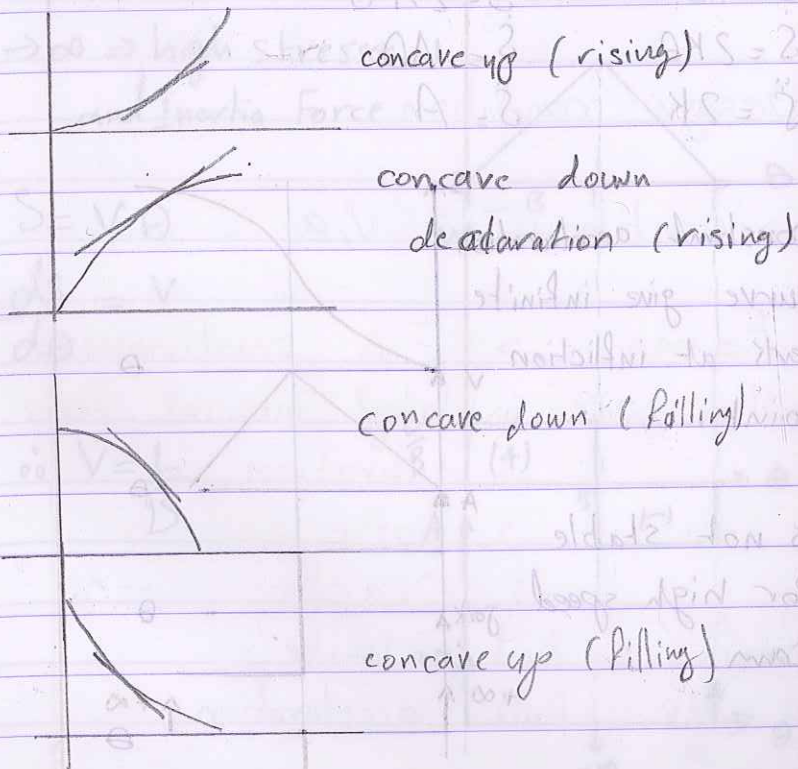
is not stable for high speed Cam



3] Curves that give continuous acceleration and velocity

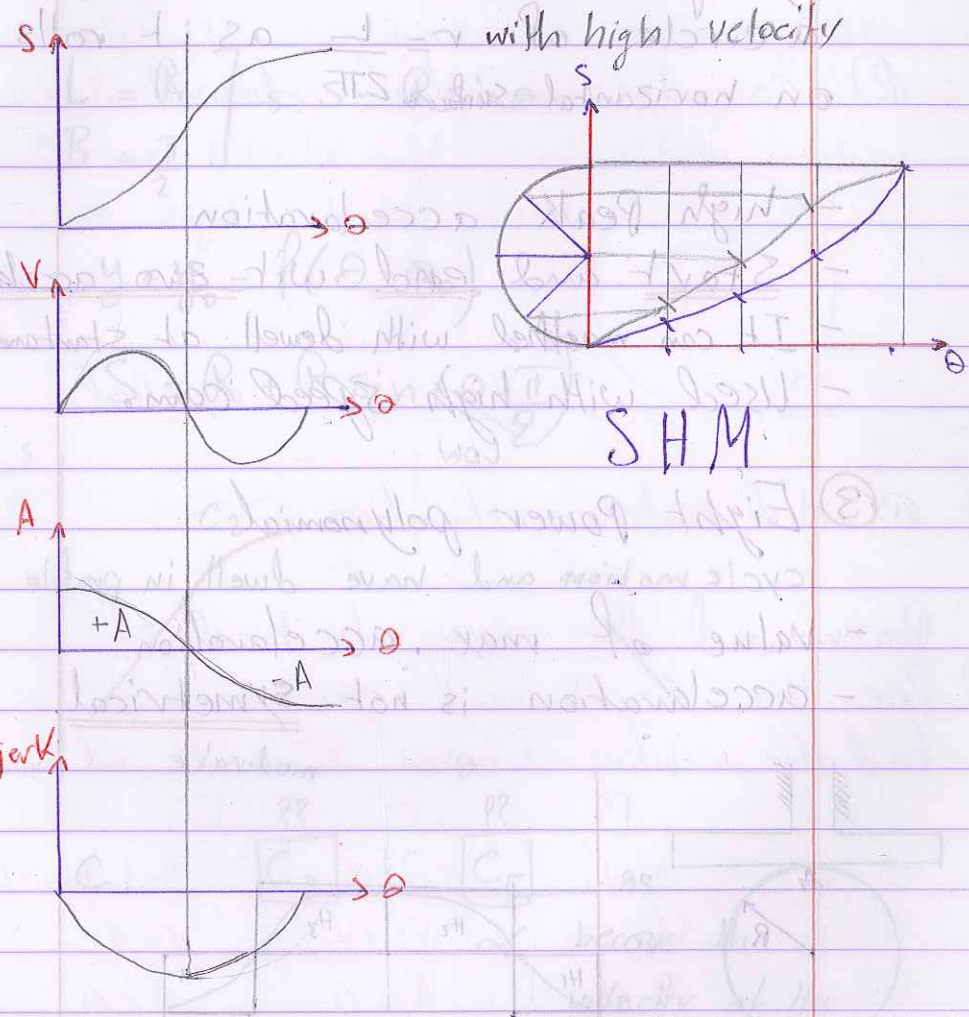
Standard Displacement Diagram

- 1) simple harmonic motion
- 2) cycloidal
- 3) Eight power polynomial



1] Simple Harmonic motion (SHM)

- give lowest peak acceleration
- Used to match acceleration at start and end



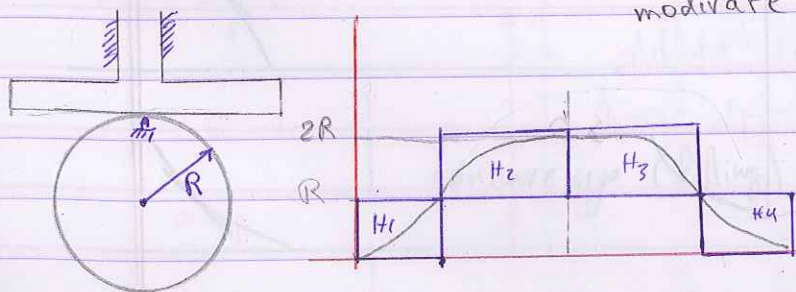
② Cycloidal

The horizontal distance transversed by a point on circumfrances of a circle of $r = \frac{L}{2\pi}$ as it rolls on horizontal surface.

- high Peak acceleration
- Start and end wit zero accel
- It can ~~match~~ with dwell at start and end
- Used with high speed cam.

③ Eight Power polynomial

- cycle motion and have dwell in profile
- value of max acceleration
- acceleration is not symetrical



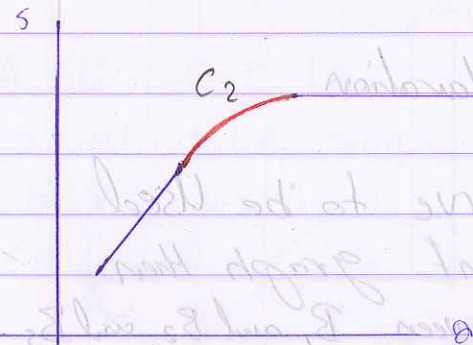
$$h = R - R \cos \theta$$

$$H_2 \Rightarrow S = L \sin \frac{\pi \theta}{2B}$$

$$\left. \begin{array}{l} L = R \\ B = \frac{\pi}{2} \end{array} \right\} S = R \sin \theta$$

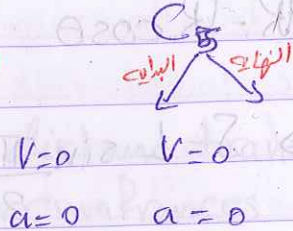
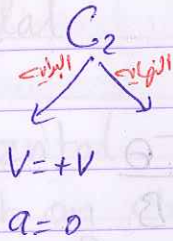
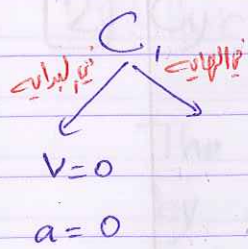
$$y - y_0 = f(\theta - \theta_0)$$

$$S = R + R \sin \left(\theta - \frac{\pi}{2} \right)$$

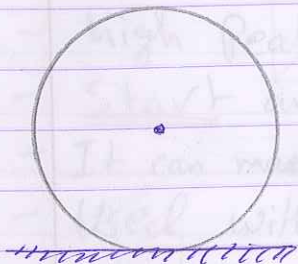


$$C_1 \quad C_2 \quad C_5$$

because the velocity at the begin = 0



② Can be matched do well



$y(t) = \text{cycloid}$

① Modarate acceleration

Recommend the curve to be used for the displacement graph then find the relative between B₁ and B₂ and B₃

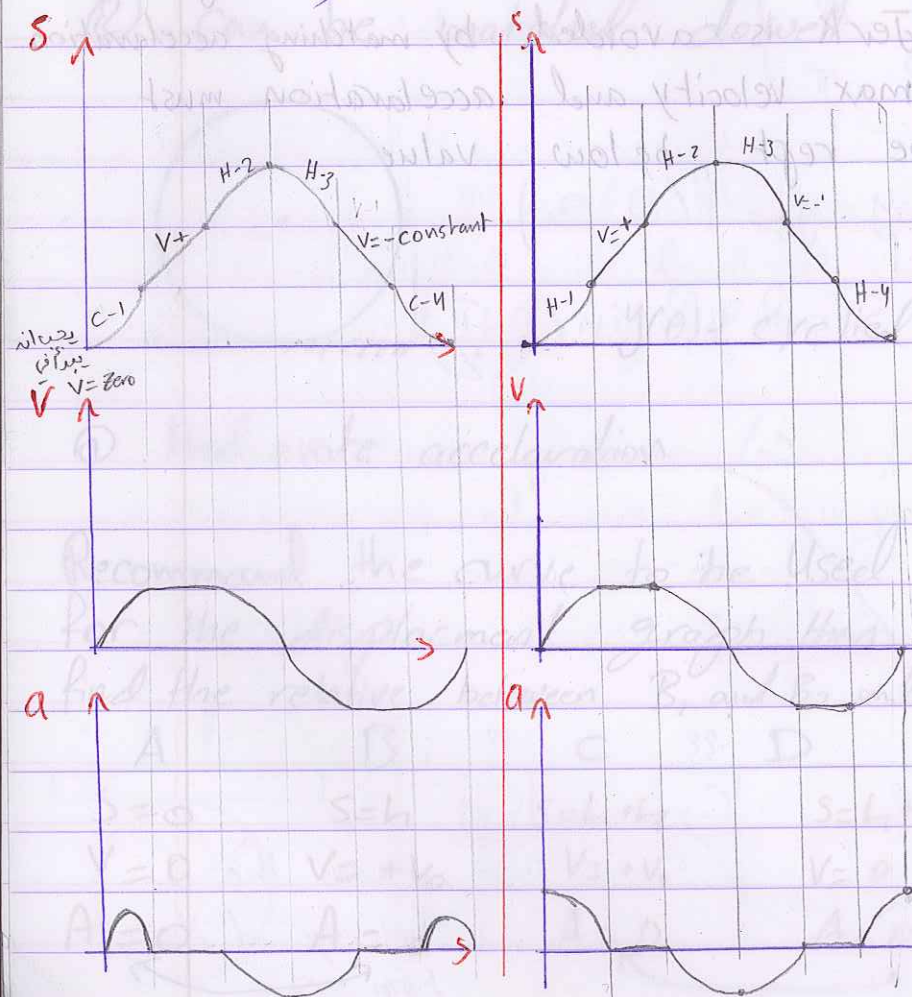
A	B	C	D
$S=0$	$S=L$	$S=L+t_2$	$S=L+t_2+t_3$
$V=0$	$V=+V_0$	$V=+V_0$	$V=0$
$A=0$	$A=0$	$A=0$	$A=0$

Diagram showing connections between points: C₁ connects A and B; C₂ connects B and C; C₃ connects C and D.

✗ Matching displacement Curve:-

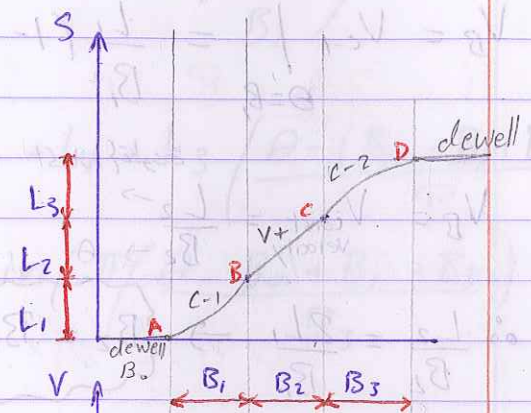
- 1) motion requirement must be met
- 2) displacement, velocity, acceleration must be continuous at control point
→ Jerk is avoided by matching acceleration
- 3) max velocity and acceleration must be kept below value

Ex a follower is to have a period of constant velocity during out ward جانب خارجي and returns travel. Select curve to be matched with these constant velocity curve for infinit jerk.



Ex follower motion

O-A :- dwell
A-B :- rise with acc
B-C :- rise with const velocity
C-D :- rise with deceleration
D :- dwell motion

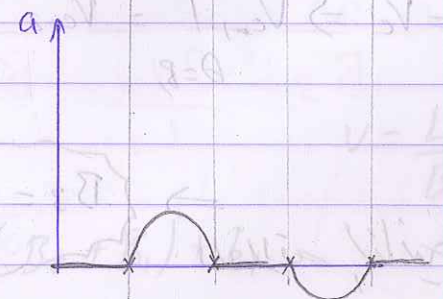


A B C D

$$S=0 \quad S=L_1 \quad S=L_1+L_2 \quad S=L_1+L_2+L_3$$

$$V=0 \quad V=V_1 \quad V=V_1 \quad V=0$$

$$A=0 \quad A=0 \quad A=0 \quad A=0$$



a) find standard curve to be matched (Displacement diagram)??

b) relation between B_1, B_2, B_3 to match velocity at B.C??

$$b) V_B = V_{C-1} \Big|_{\theta=B_1} = \frac{L_1}{B_1} (1 - \cos \pi \frac{\theta}{B_1}) = \frac{2L_1}{B_1}$$

$$V_B = V_{\text{const}} = \frac{L_2}{B_2} \xrightarrow{\text{velocity}} \theta$$

$$\therefore \frac{L_2}{B_2} = \frac{2L_1}{B_1} \Rightarrow B_2 = B_1 \frac{L_2}{2L_1} \checkmark$$

$$V_C = V_{C-2} \Big|_{\theta=0} = \frac{L_3}{B_3} (1 + \cos \pi \frac{\theta}{B_3}) = \frac{2L_3}{B_3}$$

$$V_B = V_C \Rightarrow V_{C-1} \Big|_{\theta=B_1} = V_{C-2} \Big|_{\theta=0}$$

$$\Rightarrow B_3 = \frac{B_1 L_3}{L_1}$$

هذا هو V_d

$$C_1 \quad S_1 = L_1 \left(\frac{\theta - B_0}{B_1} \right) - \frac{1}{\pi} \sin \left(\pi \left(\frac{\theta}{B_1} - B_0 \right) \right)$$

$$V = \frac{L_1}{B_1} \left(1 - \cos \pi \left(\frac{\theta - B_0}{B_1} \right) \right)$$

$$C_2 \Rightarrow S = L_1 + L_2 + L_3 \left(\frac{\theta - (B_0 + B_1 + B_2)}{B_3} \right) + \frac{1}{\pi} \sin \left(\pi \left(\frac{\theta - B_0 + B_1 + B_2}{B_3} \right) \right)$$

$$S = L_1 + L_2$$

$$V = \frac{L_3}{B_3} \left(1 + \cos \left(\pi \left(\frac{\theta - (B_0 + B_1 + B_2)}{B_3} \right) \right) \right)$$

$$B \rightarrow C$$

$$V = \frac{L_2}{B_2}$$

$$S = \frac{L_2}{B_2} (\theta - (B_0 + B_1) + L_1)$$

at B from the right $V_B = \frac{L_2}{B_2}$
 // // left $V_B = \frac{2L_1}{B_1}$

$$\Rightarrow \frac{L_2}{B_2} = \frac{2L_1}{B_1}$$

$$B_1 = \frac{2L_1}{L_2} B_2$$

at A acceleration في اتجاه اليمين
 في البداية، اليمين

at C from the left $V_C = \frac{L_2}{B_2}$

from the right $V_C = \frac{2L_3}{B_3}$

$$B_3 = \frac{2L_3}{L_2} B_2$$

Determine the maximum acceleration

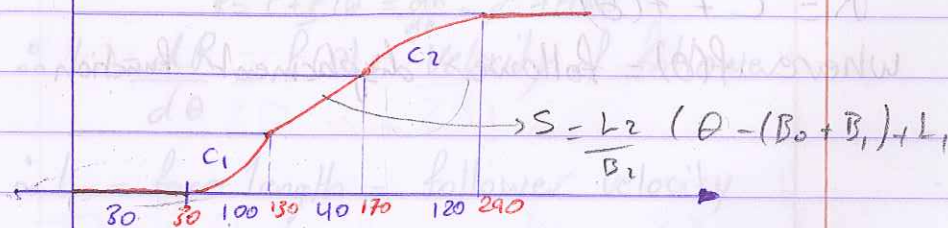
$$a_{\max} \text{ at } \theta = \frac{B_1}{2} = \frac{\pi L_1}{B_1^2} \left(\sin \frac{\pi}{2} \right)$$

$$a_{\min} = -\frac{\pi L_3}{B_3^2}$$

max absolute acceleration

$$|a_{\min}| < |a_{\max}|$$

determine S at $\theta = 120^\circ$



$$\Rightarrow S = L_1 \left(\frac{120 - 30}{100} - \frac{1}{\pi} \sin \left(\frac{\pi (120 - 30)}{100} \right) \right)$$

problem [17, 22, 36, 37, 40, 43, 44]

X Cam profile Design (Analytical method)

II Disk Cam with Radial Flat-faced Follower

we have to determine, [Design Requirement]

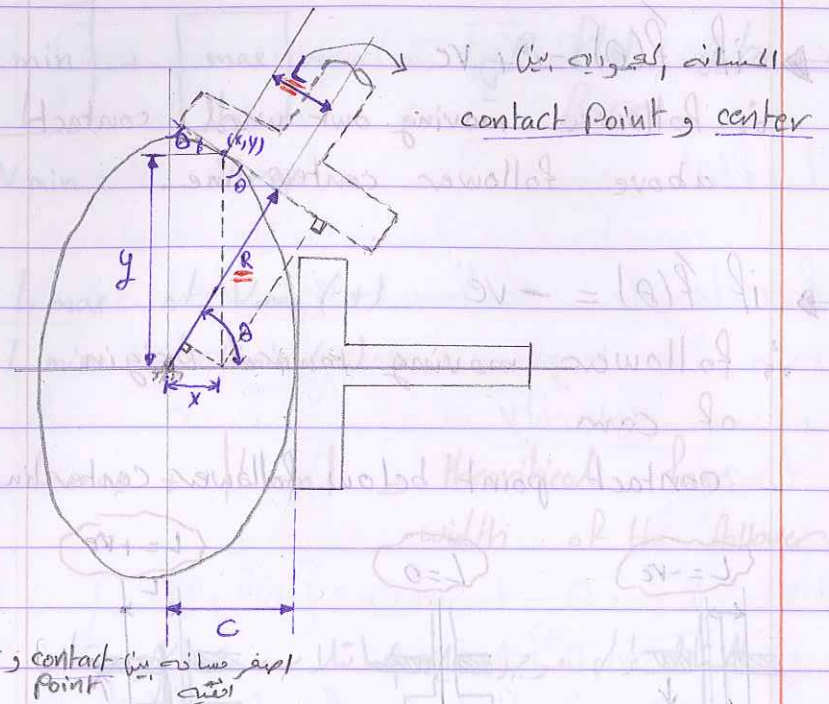
- 1) Profile of the cam [Parametric Eq of cam Profile]
- 2) min Follower Length [contact Point]
- 3) min. cam Radius [base circle]

C : min cam Radius (base circle)

R : Radial displacement at follower

$$R = C + f(\theta)$$

where $f(\theta)$ = follower displacement function



$$R = y \sin \theta + x \cos \theta$$

$$L = y \cos \theta - x \sin \theta$$

$$R = C + f(\theta) = \frac{dR}{d\theta} = \dot{f} + \dot{f}(\theta)$$

$$\therefore L = \frac{dR}{d\theta} = \dot{f}(\theta) = \text{velocity of follower}$$

$$\therefore L = \text{face length} = \text{follower velocity}$$

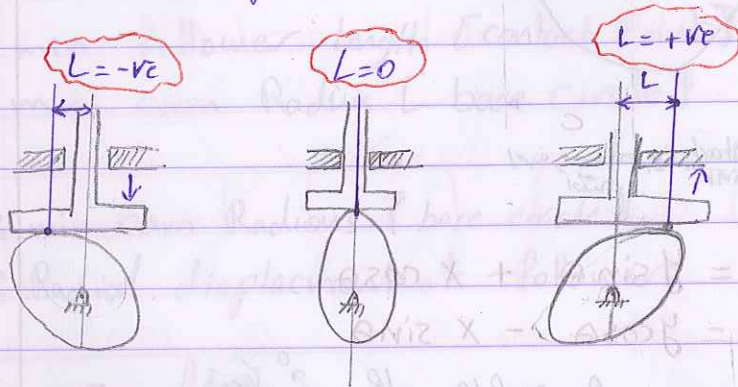
→ if $\dot{f}(\theta) = +ve$

∴ Follower moving out ward contact above follower center line.

→ if $\dot{f}(\theta) = -ve$

∴ Follower moving toward origin of cam

- contact point below follower centerline



$$L = v$$

$$v = +ve \Rightarrow L = +ve$$



the follower is rising

$$\text{dwell} \Rightarrow v = 0, \quad h = 0$$

$$\boxed{\begin{matrix} \min \\ L \end{matrix}} + \boxed{\begin{matrix} \max \\ L \end{matrix}} \quad \begin{matrix} v_{\min} \\ v_{\max} \end{matrix}$$

Flat face force

$$L_{\max} \text{ at } v_{\max} (+)$$

$$L_{\min} \text{ at } v_{\min} (-)$$

$$L_{\max} + |L_{\min}| = \text{theoretical face width of the follower}$$

العرض النظري لوجه المتابع

$$v_{\max} = \frac{5\pi}{2 \left(\frac{120 \times \pi}{180} \right)} = 3.75 = L_{\max}$$

$$v_{\min} = -\frac{\pi L}{2\beta} = -\frac{\pi L}{2\beta} = -5$$

$$\text{Theoretical face width} = 3.75 + |-5| = 8.75 \text{ cm}$$

$C_{min} \Rightarrow [F + \ddot{F}]$ is min in negative

$$L = f(\theta)$$

∴ face length = follower velocity

∴ min face length \rightarrow max velocity
+ min velocity
 $t_{min} \rightarrow$

$$x = R \cos \theta - L \sin \theta$$

$$y = R \sin \theta + L \cos \theta$$

but $R = c + f(\theta)$ and $L = \dot{f}(\theta)$

$$\therefore x = [c + f(\theta)] \cos \theta - \dot{f}(\theta) \sin \theta$$

$$y = [c + f(\theta)] \sin \theta + \dot{f}(\theta) \cos \theta$$

$$R \Leftrightarrow y$$

$$L \Leftrightarrow x$$

$$R = y \sin \theta + x \cos \theta$$

$$L = y \cos \theta - x \sin \theta$$

* Cam Pointing [Cusp]:-

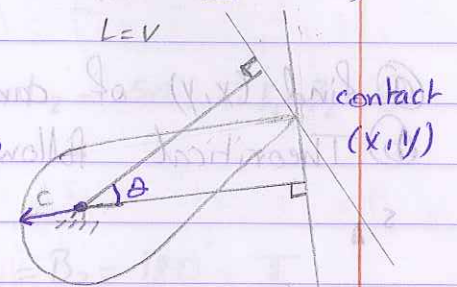
A point on cam surface, for small rotation of cam, the contact point will not change (if the follower not move)

الكام يدير حركته مع ثبات المتابع

$$\frac{dx}{d\theta} = 0, \frac{dy}{d\theta} = 0 \quad (L = \dot{f}(\theta))$$

$$L = V$$

$$\frac{dx}{d\theta} = -[c + f(\theta) + \dot{f}(\theta)] \sin \theta$$



$$\frac{dy}{d\theta} = [c + f(\theta) + \dot{f}(\theta)] \cos \theta$$

$$\frac{dx}{d\theta} = \frac{dy}{d\theta} = 0 \Rightarrow c + f(\theta) + \dot{f}(\theta) = 0$$

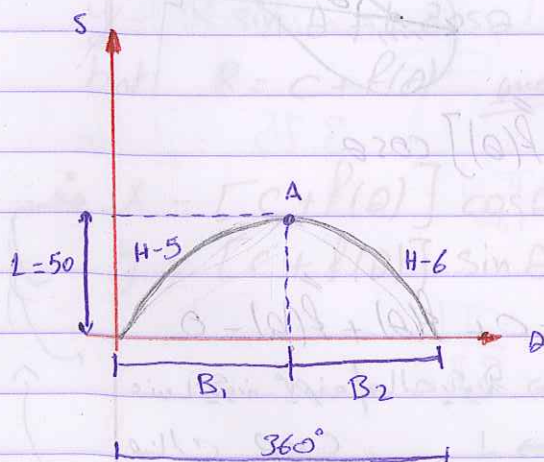
to avoid cusp $\Rightarrow c + f(\theta) + \dot{f}(\theta) > 0$

$\Rightarrow c > -[F + \dot{F}]$ $c = -v_c$ has no significant meaning

- ∴ C largest cam Radius when $[f + \dot{f}]$ is min
- C minimums cam Radius when $[f + \dot{f}]$ is max

Ex: Disk cam with flat-faced follower
 37 Follower moves with (S H M) for rise and return in one revolution of the cam
 $L_{\text{rot}} = 50 \text{ mm}$, $C_{\text{min}} = 25 \text{ mm}$

- Find (x, y) of cam contour
- Theoretical follower face length



we make equality of acceleration

$$\therefore a_{H-5} \Big|_{\theta=0} = a_{H-6} \Big|_{\theta=B_2}$$

constant acceleration

$$\frac{\pi^2 L_1}{2 B_1^2} \left(\cos \frac{\pi(\theta)}{B_1} \right) = \frac{-\pi^2 L_2}{2 B_2^2} \left(\cos \frac{\pi B_2}{B_2} \right)$$

$$\frac{L_1}{B_1^2} = \frac{L_2}{B_2^2} \quad \text{but } L_1 = L_2 = 50 \quad \therefore B_1 = B_2 \quad \text{--- (1)}$$

$$B_1 + B_2 = 360^\circ \quad \therefore B_1 = B_2 = 180^\circ = \pi$$

$$[H-5] \Rightarrow f(\theta) = s = \frac{L}{2} (1 - \cos \frac{\pi \theta}{\pi + B}) = 25(1 - \cos \theta)$$

$$\therefore f(\theta) = 25(1 - \cos \theta)$$

$$f'(\theta) = 25(\sin \theta)$$

$$\Rightarrow x = [c + f(\theta)] \cos \theta - f(\theta) \sin \theta$$

$$= [25 + 25 - 25 \cos \theta] \cos \theta - 25 \sin \theta \sin \theta$$

$$= 50 \cos \theta - 25 \cos^2 \theta - 25 \sin^2 \theta$$

$$= 50 \cos \theta - 25 [\cos^2 \theta + \sin^2 \theta]$$

$$\therefore \cos \theta = \frac{x + 25}{50}$$

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$$y = [c + f(\theta)] \sin \theta + f(\theta) \cos \theta$$

$$= [25 + 25 - 25 \cos \theta] \sin \theta + 25 \sin \theta \cos \theta$$

$$\therefore y = 50 \sin \theta$$

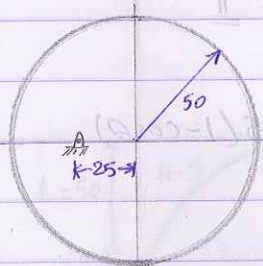
$$\therefore \sin \theta = \frac{y}{50}$$

$$\text{but } \cos^2(\theta) + \sin^2(\theta) = 1$$

$$\left(\frac{y}{50}\right)^2 + \left(\frac{x+25}{50}\right)^2 = 1$$

$$\therefore y^2 + (x+25)^2 = 50^2 \Rightarrow \text{Circle Equation}$$

with radius of 50
and far from center 25



$$b) L_{\min} = f(\theta)_{\max} \rightarrow \text{max velocity}$$

from the Eq of velocity of H-5

$$v = \frac{\pi L_1}{B_1} \left(\sin \frac{\pi \theta}{B} \right) \Rightarrow v_{\max} = \frac{\pi L_1}{2B_1} \text{ when } \theta = \frac{B}{2} = \frac{\pi}{2}$$

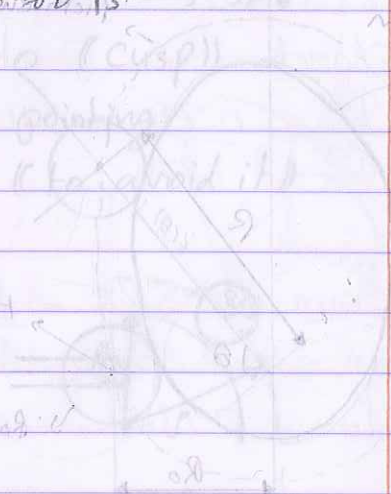
$$\therefore L_{\min} = \frac{\pi L_1}{2B_1 \pi} = 25$$

or from $f(\theta)$

$$\Rightarrow L_{\min} = \max \text{ of } (25 \sin \theta) = 25 \text{ at } \frac{\pi}{2}$$

$$\therefore L_{\text{tot}} = 25 + 25 = 50 \text{ mm}$$

Harmonic موجات دائرية
complete إدريس دائرة
revolution إدريس



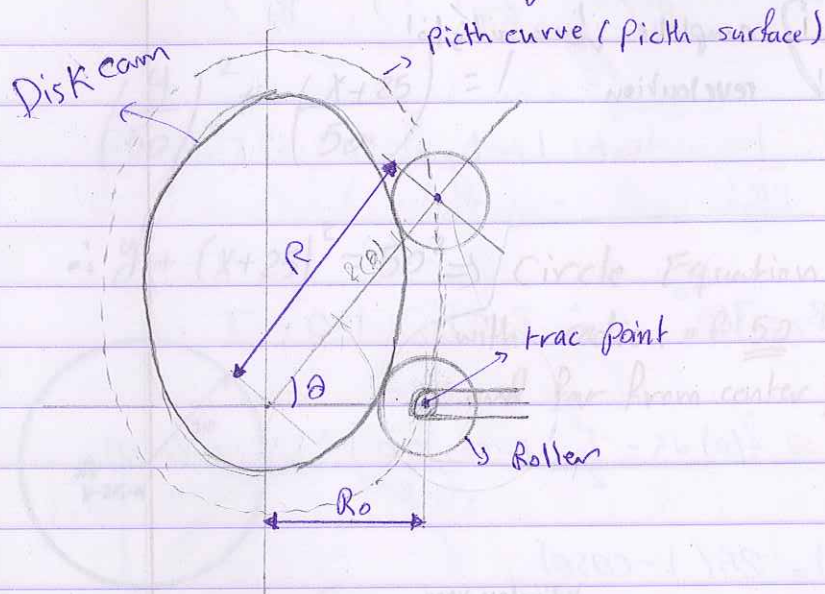
2. Disk cam with Radial Roller follower

R_0 = min pitch radius

R = radial follower displacement

$$R = R_0 + f(\theta)$$

$f(\theta)$ = follower displacement function



⊗ Cam Pointing [Cusp] :-

ρ_c = radius of curvature of cam surface

ρ = radius of curvature of pitch surface

R_r = roller radius

$$\rho = \rho_c + R_r$$

for constant ρ $R_r \uparrow \rightarrow \rho_c \downarrow$

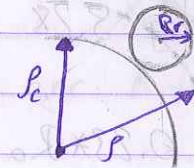
$R_r = \rho \rightarrow \rho_c = 0$ (Cusp)

$R_r = \rho \rightarrow$ cam pointing

\Rightarrow for no pointing (to avoid it)

$$\underline{R_r < \rho_{min}}$$

$$\rho = \frac{[R^2 + (\frac{dR}{d\theta})^2]^{3/2}}{R^2 + 2(\frac{dR}{d\theta})^2 - R \frac{d^2R}{d\theta^2}}$$



$\rho_{min} \rightarrow \frac{d\rho}{d\theta} \rightarrow 0$ \Rightarrow لا يوجد قوس في الشكل
لا يوجد قوس في الشكل

Fig 3.27, 28, 29 for SMM, cyc, Power

Fig 3.30

$$\rightarrow \left(\frac{\rho_{min}}{R_0} \text{ vs } B \right) \Rightarrow \frac{L}{R_0}$$

Ex:- Radial roller follower

$$L = 0.75''$$

SHM

$$B = 30^\circ$$

$$R_r = 0.25''$$

$$R_o = 1.875''$$

Check for pointing??

$$\frac{L}{R_o} = \frac{0.75}{1.875} = 0.4$$

$$R_o = 1.875$$

from fig 5.28 $\Rightarrow \frac{f_{min}}{R_o} = 0.23$

$$f_{min} = 0.23 \times R_o = 0.23 \times 1.875 = 0.43$$

$$R_r < f_{min} \therefore \text{No pointing}$$

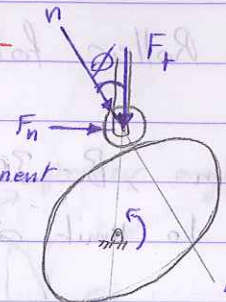
$$x_A = R \cos \theta$$

$$y_A = R \sin \theta$$

$$x_P = x_A - R_r \cos(\alpha + \theta)$$

$$y_P = y_A - R_r \sin(\alpha + \theta)$$

Pressure Angle:-



F_n = undiscarble component

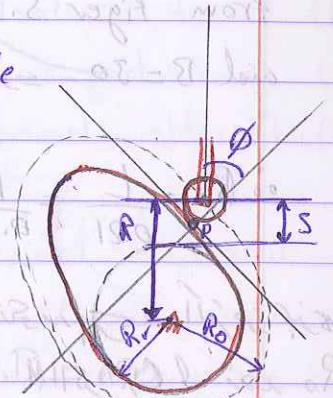
$$\phi \downarrow \rightarrow F_n \downarrow$$

$$F_n \downarrow \rightarrow T \downarrow$$

ϕ must be small as possible

$$R = R_o + f(\theta)$$

$$\frac{dR}{d\theta} = f'(\theta) \Rightarrow \tan \phi = \frac{f'(\theta)}{R_o + R}$$



for small $d\theta \rightarrow$ the arc $R d\theta \approx$ through line

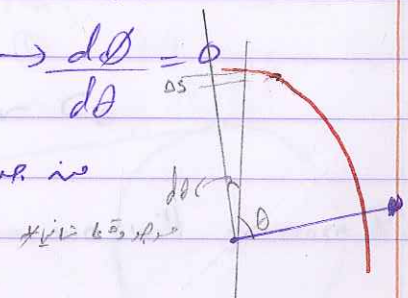
$$\tan \phi = \frac{\Delta s}{R d\theta}$$

$$\Delta s = dR \rightarrow \tan \phi = \frac{dR}{d\theta} \frac{1}{R}$$

$$\rightarrow \text{to find } \phi_{max} \rightarrow \frac{d\phi}{d\theta} = 0$$

5.31

derivative



EX: Radial Roller follower

$$L = 19 \text{ mm}, B = 30, \text{ SHM}$$

Find R_o to limit $\alpha_{\max} = 30^\circ$??
max Presser Angle

From figer 3.31 at $\alpha_{\max} = 30$
and $B = 30 \Rightarrow \frac{L}{R_o} = 0.21$ for SHM

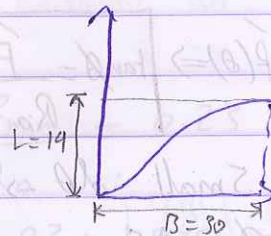
$$\therefore R_o = \frac{L}{0.21} = \frac{19}{0.21} = 90.5 \text{ mm}$$

نکته: در این مسئله، اگرچه از SHM استفاده شده است، اما چون R_o را می‌خواهیم، باید از رابطه $\frac{L}{R_o} = 0.21$ استفاده کنیم.

R_o (Cycloidal - Poly)

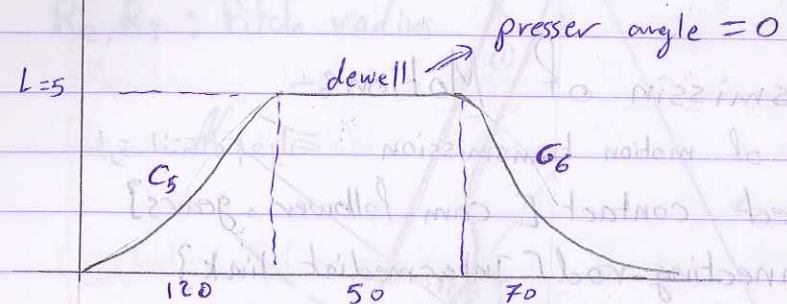
در جزئیات، باید R_o را بزرگتر از R_o انتخاب کنیم.

تکانه نمی‌باشد، زیرا α_{\max} را محدود کرده است.



Presser angle in dwell = zero

Example



$$R_o = 8, R_r = 2$$

for C_5

$$\frac{L}{R_o} = \frac{5}{8} = 0.625, B_1 = 120$$

$$\alpha_{\max} = 26^\circ \text{ for } C_5$$

$$\frac{L}{R_o} = \frac{5}{8} = 0.625, B_2 = 70$$

$$\alpha_{\max} = 38^\circ \text{ for } C_6$$

$\alpha > 30$ so we will increase R_o

GEAR

chapter 7

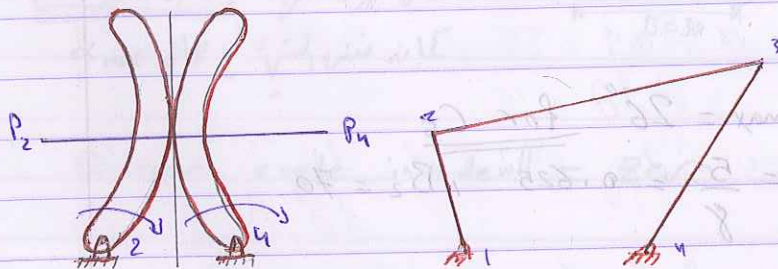
Transmission of Motion:-

ways of motion transmission طرق انتقال الحركة

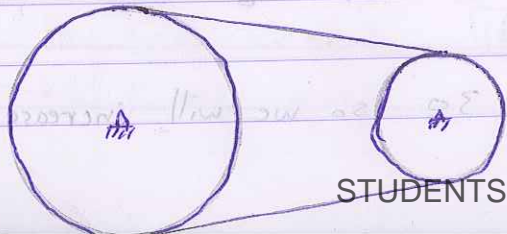
- 1) Direct contact [cam-follower, gears]
- 2) connecting rod [Intermediate link]
- 3) Flexible element [belt, chain]

line of transmission:-

motion is transmitted from driver to driven machine along line of transmission

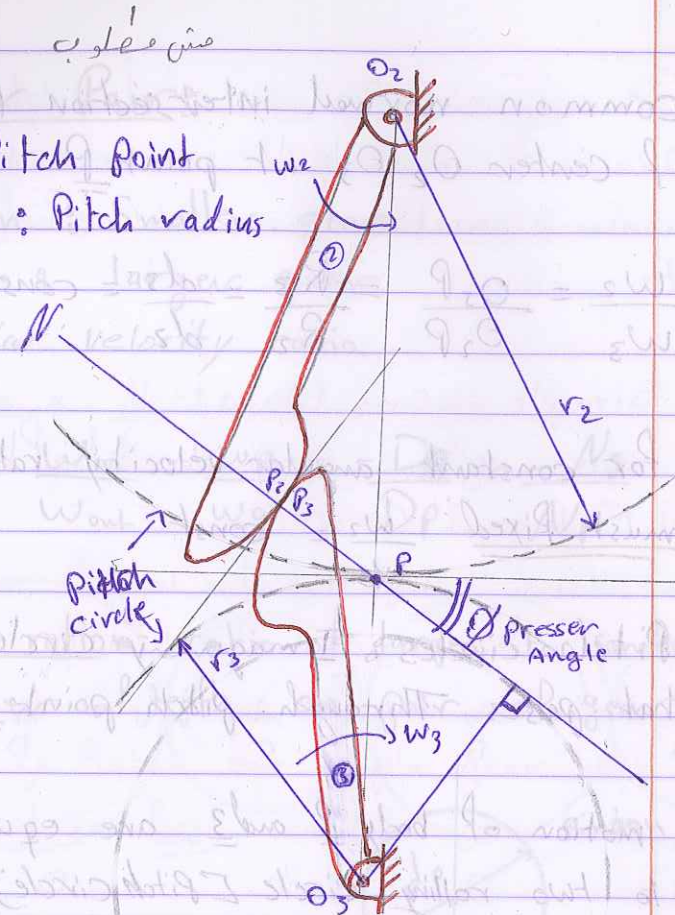


① common normal P_2P_4 ② along connecting rod



③ flexible element

P : Pitch point
 R_2, R_3 : Pitch radius



Gears are used to transmit motion or power between two shaft (rotating shaft)

Common normal intersection line of center O_2 O_3 at point P

$$\frac{\omega_2}{\omega_3} = \frac{O_3 P}{O_2 P} = \frac{R_3}{R_2} = \frac{d_3}{d_2} = \text{constant}$$

For constant angular velocity ratio, P must be fixed $\frac{\omega_2}{\omega_3} = \text{const}$

Pitch circles: Imaginary circles that pass through pitch point P

⇒ motion of body 2 and 3 are equivalent to two rolling circles [Pitch circle]

Gear Pair

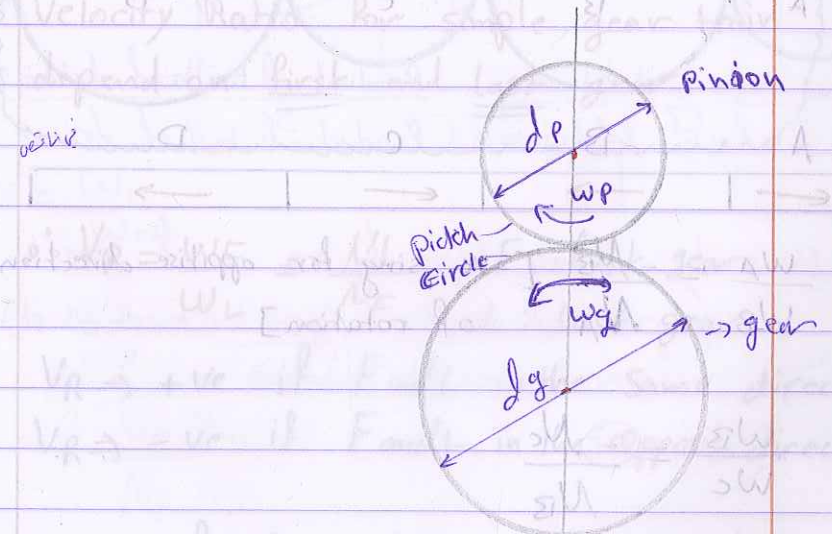
Pinion: Smaller gear

Gear: larger gear

Angular velocity ratio

$$(VR) = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_p}{\omega_g} = -\frac{D_g}{D_p} = -\frac{N_g}{N_p}$$

gear ratio: ratio of larger to smaller number of teeth on pair of gear

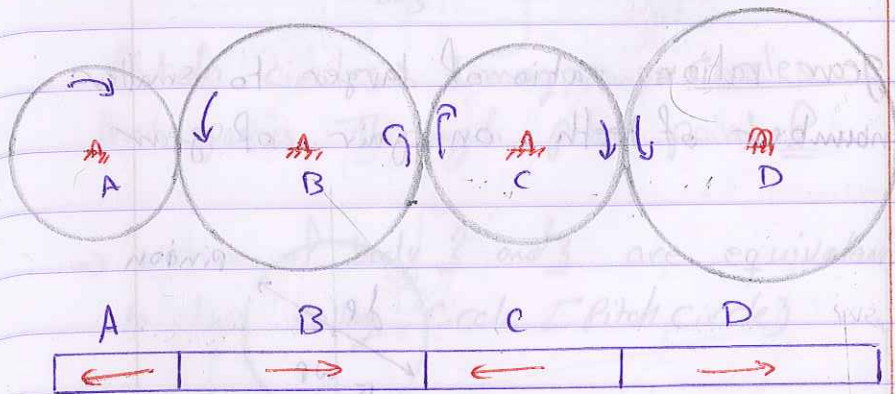


Gear Trains :-

Two or more gear in mesh to transmit motion from one shaft to another

[I] Simple gear train :-
One gear in each shaft

[a] External Gears



$$\frac{\omega_A}{\omega_B} = -\frac{N_B}{N_A} \quad [-ve \text{ sign for opposite direction of rotation}]$$

$$\frac{\omega_B}{\omega_C} = -\frac{N_C}{N_B}$$

$$\frac{\omega_C}{\omega_D} = -\frac{N_D}{N_C}$$

$$\text{Velocity ratio} = \frac{\omega_{in}}{\omega_{out}} = \frac{\omega_A}{\omega_D} = \frac{\omega_A}{\omega_D}$$

$$= \frac{\omega_A}{\omega_B} \times \frac{\omega_B}{\omega_C} \times \frac{\omega_C}{\omega_D} = \frac{\omega_A}{\omega_D}$$

$$= -\frac{N_B}{N_A} \times -\frac{N_C}{N_B} \times -\frac{N_D}{N_C} = -\frac{N_D}{N_A}$$

$V_R = +ve$ if input and output gear rotate in the same direction
 $V_R = -ve$ if input and output gear rotate in the opposite direction

Velocity Ratio for simple gear train depend on first and last gear

$$\therefore V_R = \frac{\omega_F}{\omega_L} = \frac{N_L}{N_F} \quad F: \text{first gear} \quad L: \text{last gear}$$

$V_R \Rightarrow +ve$ if F and L in the same direction
 $V_R \Rightarrow -ve$ if F and L in the opposite direction

tooth no of intermediate gear [Idle gear] are cancel

Idler gear فاصلہ

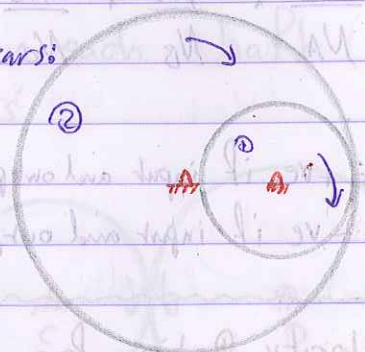
⇒ Purpose of Idler gear:-

- 1) To increase the center distance
- 2) To change direction of rotation

(b) Internal gears:

Pinion with internal gears:

$$V_R = \frac{W_{in}}{W_{out}} = \frac{W_1}{W_2} = + \frac{N_2}{N_1} \quad (2)$$

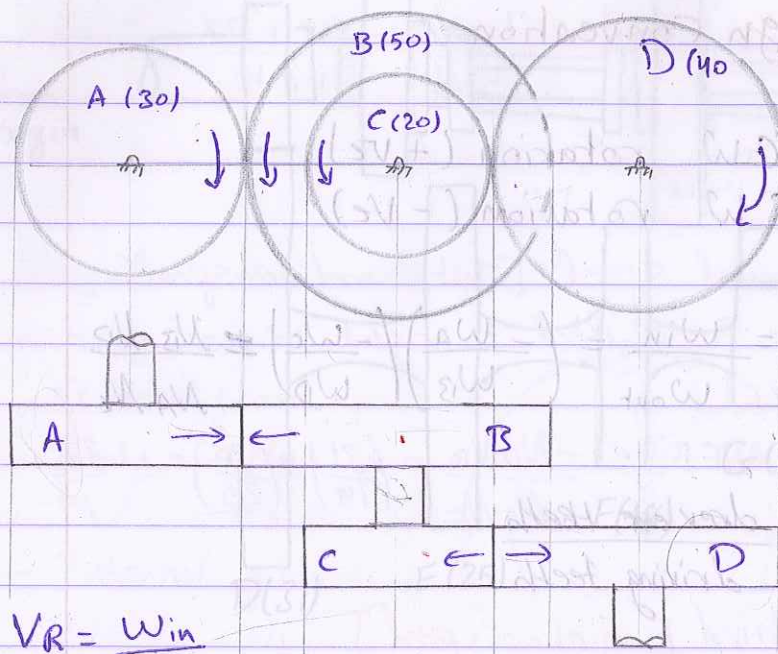


$$V_R = \frac{W_{in}}{W_{out}} = \frac{W_{driver}}{W_{driven}} = \frac{\text{Product of teeth of driven gear}}{\text{Product of teeth of driver gear}}$$

Velocity ratio depend on no. of teeth of indep end out put gear (First, last)

2) Compound Gear train:-

Two or more gear have the same axis (on same shaft)



$$V_R = \frac{W_{in}}{W_{out}}$$

$$= \frac{W_A}{W_D} = \frac{\text{Product of no. driven}}{\text{Product of no. driver}}$$

$$= + \frac{N_B N_D}{N_A N_C}$$

$$= \frac{(50)(40)}{(30)(20)} = + \frac{20}{6}$$

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if $\omega_A = 1600 \text{ rpm (ccw)}$

$$\therefore \omega_B = \frac{+6}{20} (1600) = +480 \text{ rpm C.C.W}$$



Sign Convention

CCW rotation (+ve)

CW rotation (-ve)

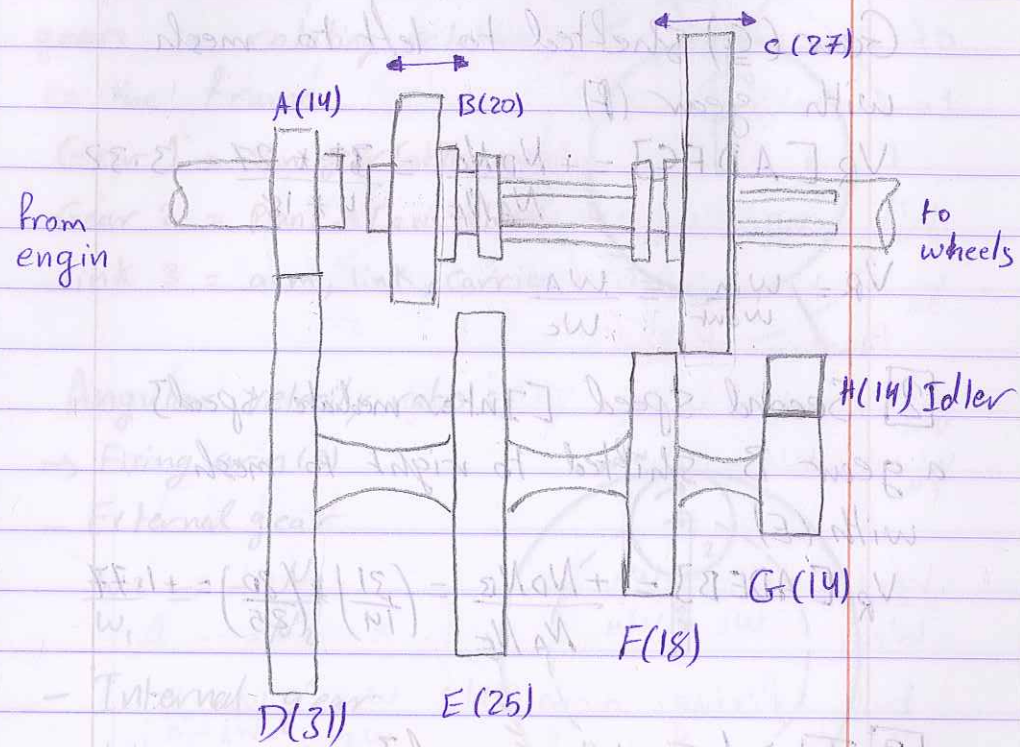
$$V_R = \frac{\omega_{in}}{\omega_{out}} = \left(\frac{-\omega_A}{\omega_B} \right) \left(\frac{-\omega_C}{\omega_D} \right) \pm \frac{N_B N_D}{N_A N_C}$$

$$= \frac{\text{driven teeth}}{\text{driving teeth}}$$

$$\text{Train Value (e)} = \frac{\omega_{out}}{\omega_{in}} = \frac{\text{Product of teeth driving}}{\text{Product of teeth driven}}$$

$$e = \frac{1}{V_R}$$

Automotive Transmission :-



Gear A : driven by engine

Gear B, E, F, G : rotat as one Unit

Gear H : Idler that mesh with G

Gears A, D, E, F, G, H are in motion if gear A in motion

[1] First speed [low speed]

Gear (C) shifted to left to mesh with gear (F)

$$V_R [ADFC] = + \frac{N_D N_C}{N_A N_F} = \frac{31 \times 27}{14 \times 18} = 3.32$$

$$V_R = \frac{w_{in}}{w_{out}} = \frac{w_A}{w_c}$$

[2] Second speed [Intermediate speed]

a gear B shifted to right to mesh with (E)

$$V_R [ADEB] = + \frac{N_D N_B}{N_A N_E} = \left(\frac{31}{14} \right) \times \left(\frac{20}{25} \right) = +1.77$$

[3] Third [higher speed]

Gear (B) shifted to left to engage (contact) with gear A by mean of clutch teeth $V_R = 1$

[4] Reverse :-

gear (c) shifted to right to mesh with Idler (H)

$$V_R [ADGHC] = - \left(\frac{31}{14} \right) \left(\frac{14}{14} \right) \left(\frac{27}{14} \right) = -4.27$$

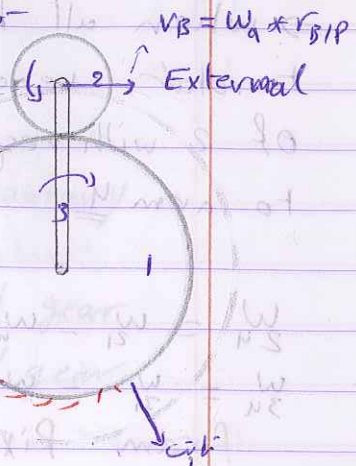
[3] Planetary Gear trains :-

The axis of one or more gears is rotating relative to the frame

Gear 1 = sun gear [at the center]

Gear 2 = Planet → [axis rotates]

link 3 = arm, link, carrier



Angular velocity ratio :-

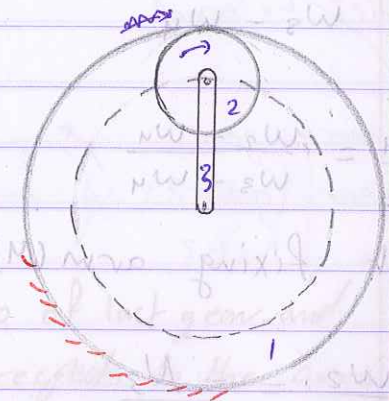
⇒ Fixing arm (3)

- External gear

$$\frac{w_2}{w_1} = - \frac{N_1}{N_2}$$

- Internal gear

$$\frac{w_2}{w_1} = + \frac{N_1}{N_2}$$



⇒ when all member are free to rotate velocity of 2 with respect to arm 4

$$\frac{W_2 - W_4}{W_3 - W_4} = \frac{N_1}{N_2}$$

Frame Fixed

$$\frac{W_2}{34} = W_3 - W_4$$

but fixing arm (4): $\frac{W_{24}}{W_{34}} = \frac{W_2 - 0}{W_3 - 0}$

Frain value = (e) = $-\frac{N_3}{N_2}$

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⇒ applied only for planetary

→ for any planetary gears:-

$L = \text{last}$

$\frac{W_{LA}}{W_{FA}}$ = velocity ratio of last gear and first with respect to the arm

W_F = velocity of the first gear with respect fixed link

$W_L = 1 \quad 1 \quad 1 \quad \text{last } 5 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$

$W_A = \omega_1$ Arm with respect to fixed link.

$$\frac{W_L}{W_F} = \frac{W_L - W_A}{W_F - W_A} = e$$

sig CCW (+)
CW (-)

$$e = \frac{\text{Product of driver gear}}{\text{Product of driven gear}}$$

$e = +ve$ if F and L rotate in the same direction

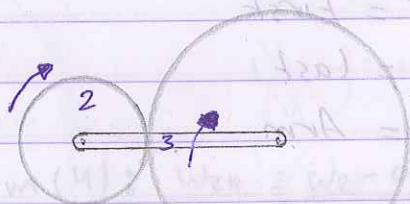
$e = -ve$ if F and L " " " opposite " "

Example: External

First gear = gear 1

last gear = gear 2

Arm = link 3



Fixed

$$\therefore \frac{W_L}{W_F} = \frac{W_L - W_A}{W_F - W_A} = -\frac{N_1}{N_2}$$

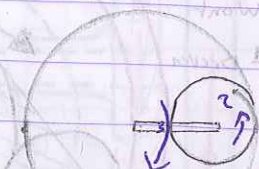
$$\left. \begin{array}{l} W_L = W_2 = W_2 \\ W_F = W_1 = 0 \\ W_A = W_3 = W_3 \end{array} \right\} \Rightarrow -\frac{N_1}{N_2} = \frac{W_2 - W_3}{0 - W_3} \Rightarrow W_2 = W_3 \left(1 + \frac{N_1}{N_2} \right)$$

Internal

First = 1

last = 2

Arm = 3

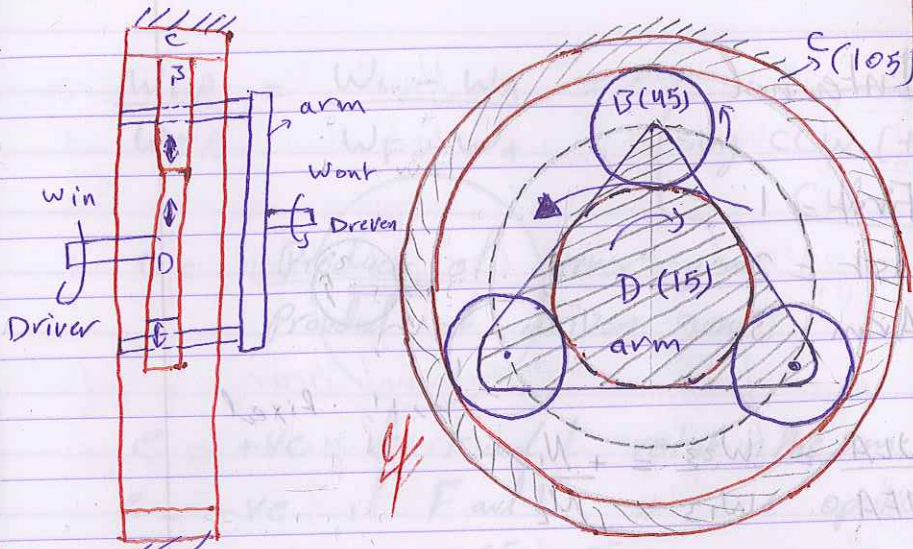


Fixed

$$\frac{W_L}{W_F} = \frac{W_2}{W_1} = +\frac{N_1}{N_2}$$

$$\text{but } \frac{W_2}{W_1} = \frac{W_2 - W_3}{W_1 - W_3}$$

$$\frac{N_1}{N_2} = \frac{W_2 - W_3}{0 - W_3} \Rightarrow W_2 = W_3 \left(1 - \frac{N_1}{N_2} \right)$$



Find W_{out} if $W_{in} = 1000 \text{ rpm}$

$\omega_{in} = 1000$

$$F = D, L = C$$

$$\frac{W_L A}{W_F A} = \frac{W_C - W_A}{W_D - W_A} = \frac{-N_D}{N_C}$$

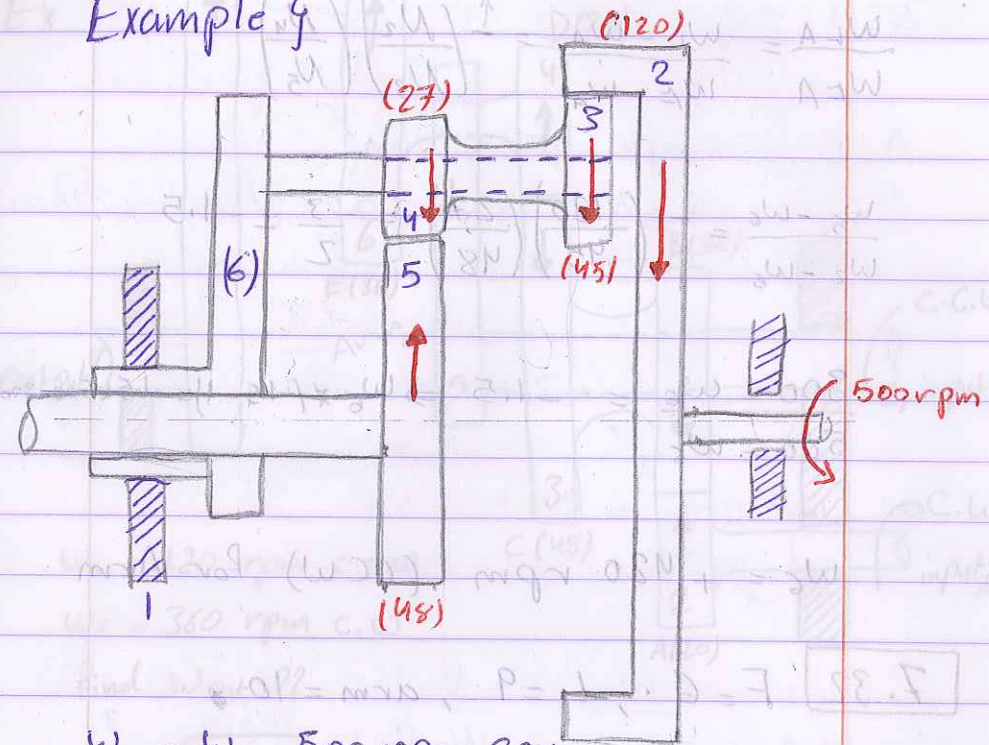
$$\frac{0 - W_A}{W_D - W_A} = \frac{-15}{105} \Rightarrow W_A \left(1 + \frac{15}{105}\right) = W_D \left(\frac{15}{105}\right)$$

$$W_A = W_D \frac{\frac{N_D}{N_C}}{\left(1 + \frac{N_D}{N_C}\right)} = 0.125 W_D = +125 \text{ rpm}$$

in the same direction W_D

$\therefore W_{in}$ and W_{out} in the same direction

Example 2



$$W_{21} = W_2 = 500 \text{ rpm CCW}$$

$$W_{51} = W_5 = 300 \text{ rpm CW}$$

Find $W_6 = ??$

$$F = 2$$

$$L = 5$$

$$\text{Arm} = 6$$

$$\frac{W_L - A}{W_F - A} = \frac{W_L - W_A}{W_F - W_A} = \frac{W_F - W_L}{W_F - W_A} = \frac{N_2}{N_3} \left(\frac{N_4}{N_5} \right)$$

$$\frac{W_5 - W_6}{W_2 - W_6} = - \left(\frac{120}{45} \right) \left(\frac{27}{48} \right) = -\frac{3}{2} = -1.5$$

$$\frac{300 - W_6}{500 - W_6} = -1.5 \Rightarrow W_6 \times (-1.5 - 1) = -1.5 \times 500 - 300$$

$$W_6 = +420 \text{ rpm (CCW) For Arm}$$

$$\boxed{7.32} \quad F=6, L=9, \text{ arm}=10$$

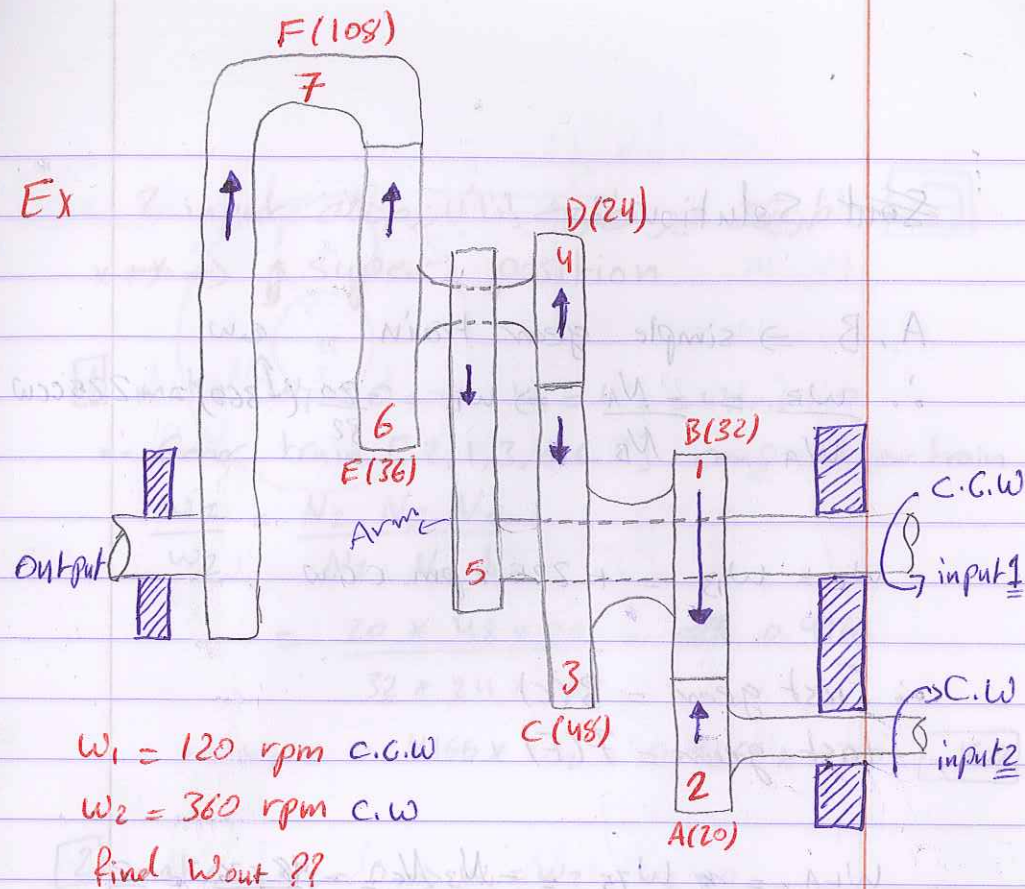
$$\frac{W_9 - W_{10}}{W_6 - W_{10}} = + \frac{N_6 N_8}{N_7 N_9} \Rightarrow W_9 = W_{10}$$

but W_6 and W_{10} is unknown

2, 3, 4, 5 compound gear

relative to arm $\therefore \frac{W_5 - W_6}{W_2 - W_6} = + \frac{N_2 N_4}{N_3 N_5} \Rightarrow W_5 = W_{10} \checkmark$

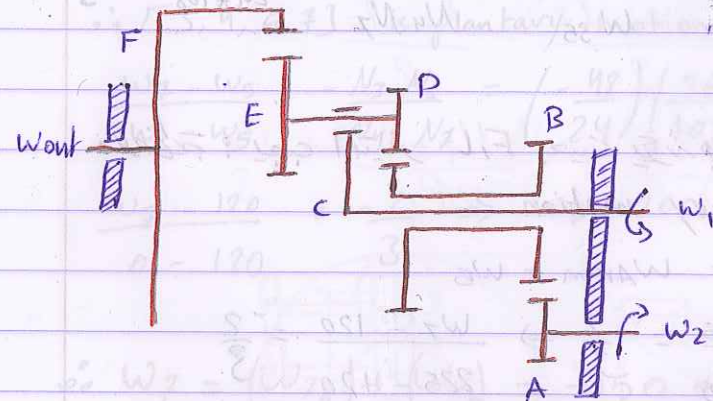
$$\therefore W_6 = v \checkmark$$



$$W_1 = 120 \text{ rpm C.C.W}$$

$$W_2 = 360 \text{ rpm C.W}$$

find W_{out} ??



Soln Solution :-

A, B \Rightarrow simple gear train

$$\therefore \frac{W_B}{W_A} = \frac{-N_A}{N_B} \Rightarrow W_B = \frac{-20}{32} (-360) = +225 \text{ ccw}$$

$$W_C = W_B = +225 \text{ rpm C.C.W}$$

First gear = 3 (C)

last gear = 7 (F)

$$\frac{W_{FA}}{W_{FA}} = \frac{W_{FS}}{W_{FS}} = \frac{-N_3 N_6}{N_4 N_7} = \frac{-48 \times 36}{24 \times 108} = \frac{-2}{3}$$

Planetary motion

$$W_1 = W_{arm} = W_5$$

$$\frac{W_7 - W_5}{W_3 - W_5} = \frac{-2}{3} \Rightarrow \frac{W_7 - 120}{225 - 120} = \frac{-2}{3}$$

$$W_7 = 50 \text{ rpm C.C.W}$$

2 input \Rightarrow super position

[1] make $W_1 = 0 \Rightarrow W_1 = W_5 = W_{arm} = 0$

\therefore Gear train [2, 1, 3, 4, 6, 7] compound gear train

$$\frac{W_7}{W_2} = \frac{N_2 N_3 N_6}{N_7 N_4 N_1}$$

$$= \frac{20 \times 48 \times 36}{32 \times 24 \times 108} = 0.4166$$

$$\therefore W_7 = 0.4166 \times W_2 = 0.4166 \times -360 = -150$$

[2] make $W_2 = 0 \Rightarrow W_1 = W_2 = W_3 = 0$

\therefore [3, 4, 6, 7] is planetary motion

$$\frac{W_7 - W_5}{W_2 - W_5} = \frac{-N_3 N_6}{N_4 N_7} = \left(\frac{-48}{24} \right) \left(\frac{36}{108} \right) = \frac{-2}{3}$$

$$\frac{W_7 - 120}{0 - 120} = \frac{-2}{3} \Rightarrow W_7 = 200 \text{ rpm CCW}$$

$$\therefore W_7 = (W_{7a}) + (W_{7b}) = -150 + 200 = +50 \text{ rpm C.C.W}$$

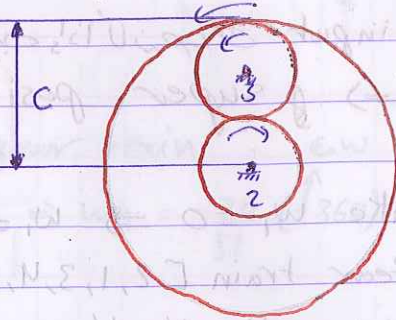
Ex $N_2 = 20$

$N_3 = 15$

$w_2 = 100 \text{ C.W}$

Find w_4 ??

Compound gear



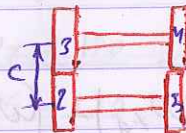
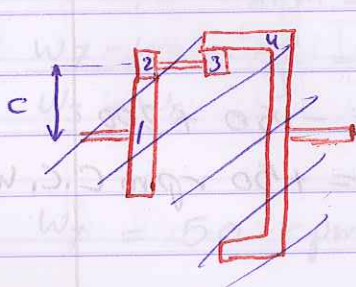
$C = \frac{d_2}{2} + \frac{d_3}{2} = \frac{d_4}{2}$ but same diameter pitch

$\therefore \frac{N_2}{2P} + \frac{N_3}{P} = \frac{N_4}{2P}$

$w_4 = -w_2 \left(\frac{N_2}{N_4} \right)$

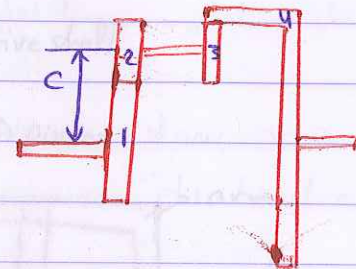
Ex $C = \frac{d_2 + d_3}{2} = \frac{d_4 + d_5}{2}$

$\frac{N_2 + N_3}{2P_1} = \frac{N_4 + N_5}{2P_2}$



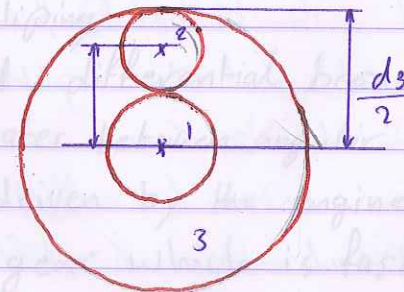
Ex $\frac{d_1 + d_2}{2} = \frac{d_4}{2} - \frac{d_5}{2}$

$\frac{N_1 + N_2}{2P_1} = \frac{N_4 - N_5}{2P_2}$



Ex $C = \frac{d_1 + d_3}{2} = \frac{N_1 + N_2}{2P}$

$\frac{d_3}{2} = \frac{d_1}{2} + d_2 \Rightarrow \frac{N_3}{2P} = \frac{N_1}{2P} + \frac{N_2}{P}$



Ex) $\frac{2b}{5} - \frac{ab}{5} = \frac{ab+ab}{5} \times 3$

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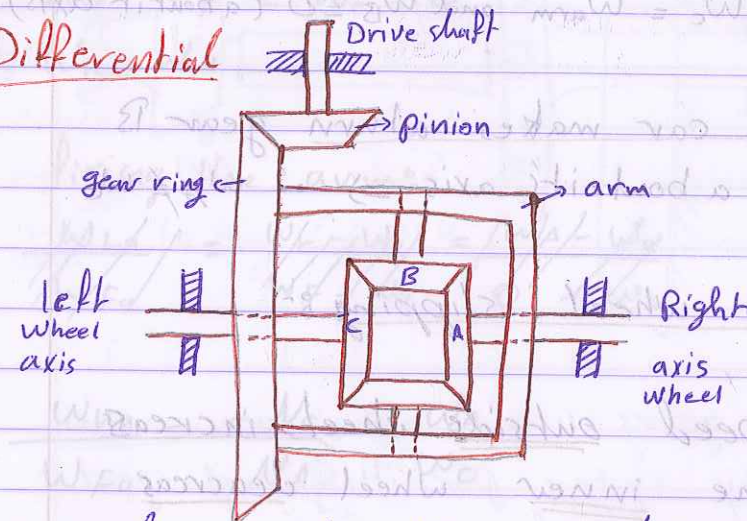
$\frac{2b}{5} - \frac{ab}{5} = \frac{ab+ab}{5} \times 3$

$\frac{2b}{5} - \frac{ab}{5} = \frac{ab+ab}{5} \times 3$

* Application of planetary Motion

- 1- air craft Propeller (speed reducer)
- 2- automobile differential
- 3- automotive gear transmission

* Differential



- * used in automobile rear drive
- * permit the car to turn corner without wheel slipping
- * It is called differential because it differentiates between angular velocity of gear ^{right left} A, C
- * pinion driven by the engine. Pinion drives the ring gear which is fastened to the arm

Differential Function :-

- In straight line motion of car A, C, B, ring rotat as a unit with the arm. (no relative motion between A, C.)

$\therefore W_A = W_C = W_{arm}$ and $W_B = 0$ (about its axis)

- when car makes a turn gear B rotat about its axis $\Rightarrow \therefore W_A \neq W_C$

- for no wheel slipping :-

The speed outside wheel increas and the inner wheel deacreas by the same amount

\Rightarrow If left wheel held fixed (dry pavement) and the right wheel free to rotat (on mud or snow)

$\therefore W_C = 0 \Rightarrow W_A = \left(1 + \frac{N_C}{N_A}\right) W_{arm}$

\Rightarrow it is a disadvantage of differential

\Rightarrow what if fixed?

$$\frac{W_A}{W_F} = \frac{W_C - W_a}{W_F - W_a} = \frac{W_A - W_a}{0 - W_a}$$

but $W_F = W_C = 0$

$W_A = W_a$

fixing the arm :-

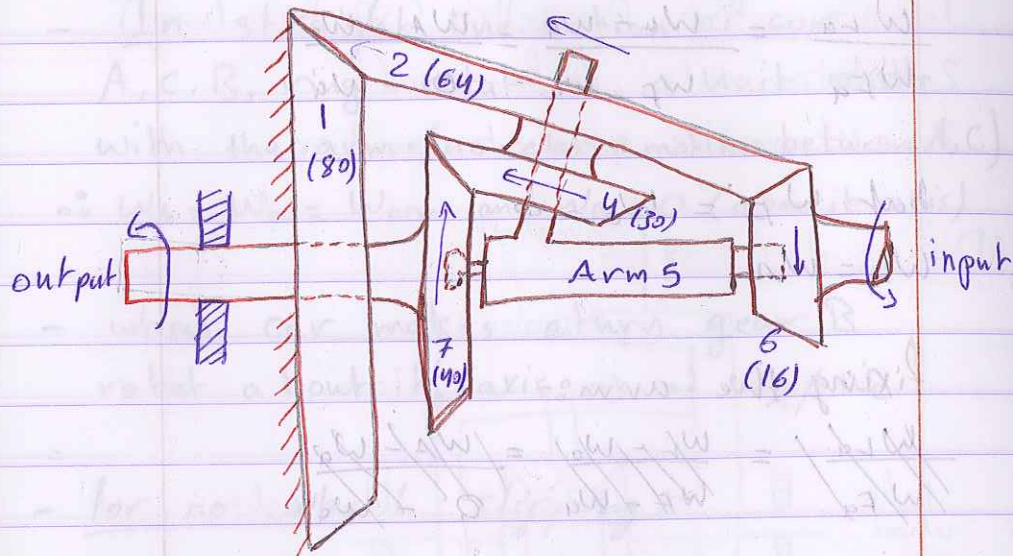
$$\frac{W_A}{W_F} = \frac{W_C - W_a}{W_F - W_a} = \frac{W_A - W_a}{0 - W_a}$$

$$\frac{W_A}{W_F} = \frac{-N_C}{N_A} = \frac{W_A}{W_C}$$

$\therefore \frac{-N_C}{N_A} = \frac{W_A - W_a}{0 - W_a} \Rightarrow W_A = W_{arm} \left(1 + \frac{N_C}{N_A}\right)$

for $N_A = N_C \Rightarrow W_A = 2 W_{arm}$

[Ex] Epicyclic Bevel gear train



Bevel gears used to make more compact planetary system permitting high speed reduction with few gear

gear 1 = fixed

gear 2, 4 is turns freely on arm 5

find $\frac{w_{out}}{w_{in}}$??

$$\frac{w_{out}}{w_{in}} = \frac{w_7}{w_6}$$

$$F = 6, L = 7, \text{Arm} = 5$$

$$\therefore \frac{w_7 - w_5}{w_6 - w_5} = - \frac{N_6 N_4}{N_2 N_7} = \left(- \frac{16}{64} \right) \times \left(\frac{30}{40} \right) = - \frac{3}{16}$$

$$\Rightarrow \frac{w_7 - w_5}{w_6 - w_5} = - \frac{3}{16}$$

since w_5 is unknown \Rightarrow we can't solve this eq or we can find w_5 with respect to w_6 by taking $\{6, 2, 4, 1\}$ and arm planetary motion

$$\therefore F = 6, L = 1, \text{Arm} = 5$$

$$\frac{w_1 - w_5}{w_6 - w_5} = - \frac{1}{5} \Rightarrow - \frac{1}{5} w_6 + \frac{1}{5} w_5 = - w_5$$

$$\therefore \frac{0 - w_5}{w_6 - w_5} = - \frac{1}{5} \Rightarrow - \frac{1}{5} w_6 + \frac{1}{5} w_5 = - w_5$$

$$\therefore \frac{1}{5} w_6 = \frac{6}{5} w_5 \Rightarrow \boxed{w_6 = 6 w_5}$$

$$\therefore \boxed{w_5 = \frac{w_6}{6}} \text{ substituting in first Eq}$$

$$\frac{\omega_7 - \frac{\omega_6}{6}}{\omega_6 - \frac{\omega_6}{6}} = -\frac{3}{16} \Rightarrow \frac{\omega_7 - \frac{\omega_6}{6}}{\omega_6 - \frac{\omega_6}{6}} = -\frac{5}{32} \omega_6$$

$$\frac{\omega_7}{\omega_6} = \frac{1}{96}$$

* Power Screw : [Translation Screw]

* Lead :- The distance the screw advance axially by one true rotation (complete rotation)

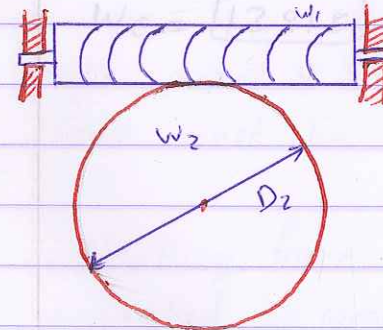
* for single thread : $L = P$

for Double thread : $L = 2P$

for Triple thread : $L = 3P$

* RH : CW

* LH : CCW



$$\frac{\omega_2}{\omega_1} = \frac{L}{\pi D_2}$$

7.25

7.14

7.34

$$W_A = 300 \text{ rpm} = W_7$$

$$W_B = 600 \text{ rpm} = W_2$$

$$W_A = C.W$$

$$W_B = C.W$$

$$W_C = ??, W_5 = ??$$

$$\frac{W_9}{W_7} = \frac{N_7}{N_9}$$

$$W_9 = \frac{30}{24} (300)$$

$$= 375 \text{ C.W}$$

$$W_9 = W_6 = 375 \text{ C.W}$$

$$L = B \quad (2)$$

$$F = C \quad (5)$$

$$\frac{W_{L \text{ Arm}}}{W_{F \text{ Arm}}} = \frac{\pi N_{\text{Driver}}}{\pi N_{\text{Driven}}}$$

$$\frac{W_L - W_{\text{Arm}}}{W_F - W_{\text{Arm}}} = \frac{N_5 \times N_3}{N_4 \times N_2}$$

$$\frac{600 - 375}{W_C - 375} = \frac{18 \times 22}{42 \times 38} = 0.248$$

$$W_C = 375$$

$$W_C = 375$$

$$W_C = 375$$

$$W_C = \boxed{1281.5} \quad \boxed{\text{C.W}} \leftarrow \text{direction}$$

Force must be balance

shaking force can be

eliminated by producing

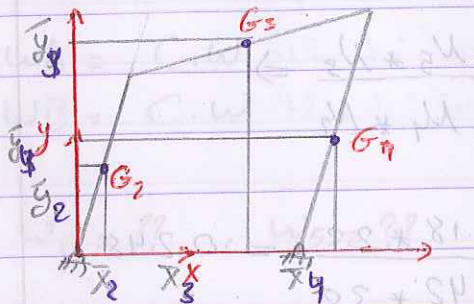
force in opposite

direction that can be

it effect

"Balancing of Mechanism"

- ① Balancing of rotating masses (rotating shaft)
- ② Balancing of mechanism

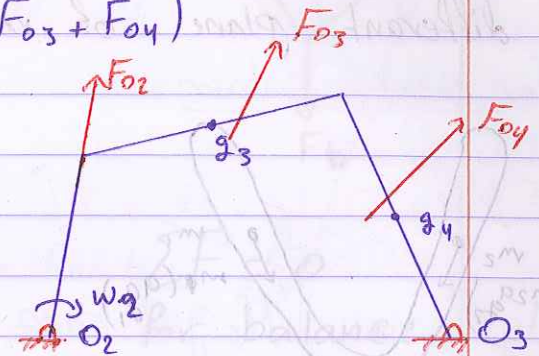


$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i}$$

Shaking Force: Σ of Inertia Force

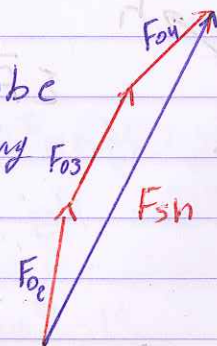
$$F_{sh} = \Sigma (F_{O2} + F_{O3} + F_{O4})$$



→ Shaking force causes vibration if transfer to frame.

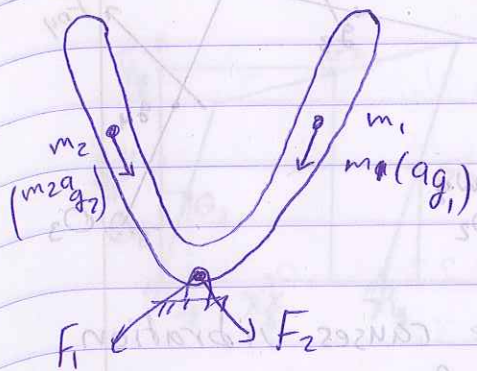
→ to avoid vibration → shaking force must be **balance**

→ shaking force can be eliminated by producing a force in opposite direction (that cancels its effect)

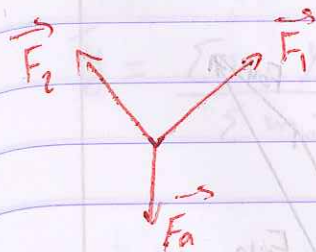


a) all masses are on the same plane of rotation

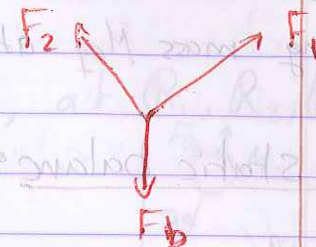
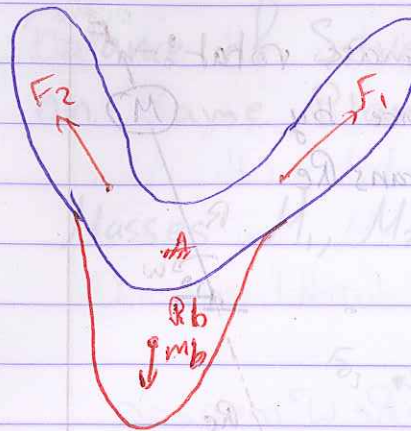
b) different plane of rotation



Different plane of rotation



$$\sum \vec{F} = 0$$



$$\sum F = 0$$

for balance

$$-m_1 R_1 \omega^2 - m_2 R_2 \omega^2 = m_b R_b \omega^2 = 0$$

$$m_b \vec{R}_b = -\sum m_i \vec{R}_i$$

Balance of single mass rotat^o:-

This mass can be balance by adding mass M_{eq} at distans R_e

For static balance:-

$$\Sigma M_o = 0$$

$$Mg R \cos \theta = M_{eq} g R_e \cos \theta$$

$$MR = M_{eq} R_{eq}$$

for dynamic balance:-

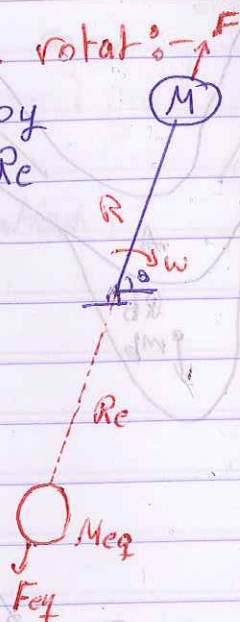
$$\omega = \text{constant}$$

$$F = F_{eq}$$

$$\text{but } F = M \omega^2 R \text{ (Inertia Force)}$$

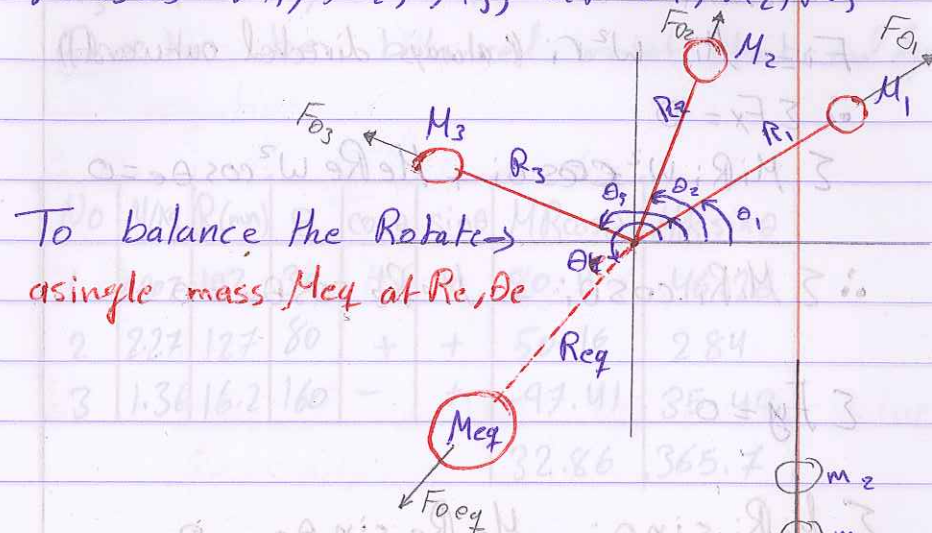
$$\therefore M \omega^2 R = M_{eq} \omega^2 R_{eq}$$

$$\therefore MR = M_{eq} R_{eq}$$



Balance of Several masses laying on Same Plane:-

Masses M_1, M_2, M_3 , at R_1, R_2, R_3



To balance the Rotate^o asingle mass M_{eq} at R_e, θ_e

Static balance:-

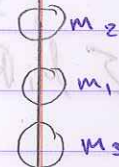
$$\Sigma M_o = 0$$

$$\Sigma$$

$$\Sigma M_i R_i g \cos \theta_i + M_e R_e g \cos \theta_e = 0$$

$$\Sigma M_i R_i \cos \theta_i + M_e R_e \cos \theta_e = 0$$

on the same Plane



Dynamic balance

Inertia force if rotat with constant ω

$$F_i = m_i \omega^2 r_i \text{ (always directed outward)}$$

$$\therefore \sum F_x = 0$$

$$\sum m_i R_i \omega^2 \cos \theta_i + m_e R_e \omega^2 \cos \theta_e = 0$$

$$\therefore \sum m_i R_i \cos \theta_i + m_e R_e \cos \theta_e = 0$$

$$\sum F_y = 0$$

$$\sum m_i R_i \sin \theta_i + m_e R_e \sin \theta_e = 0$$

For static and dynamic balance \approx

$$\sum m_i R_i \cos \theta_i + m_e R_e \cos \theta_e = 0$$

$$\sum m_i R_i \sin \theta_i + m_e R_e \sin \theta_i = 0$$

for several masses on the same plane

Ex] for the rotat with mass m_1, m_2, m_3 at R_1, R_2, R_3 and angular position $\theta_1, \theta_2, \theta_3$. Find m_e at $R_e = 88.9 \text{ mm}$ and θ_e to be balanced rotat static and dynamic

No	M(kg)	R(mm)	θ	$\cos \theta$	$\sin \theta$	$MR \cos \theta$	$MR \sin \theta$
1	0.907	102	30	+	+	80.11	46.26
2	2.27	127	80	+	+	50.16	284
3	1.36	16.2	160	-	+	-97.41	35.49
						32.86	365.7

$$\sum m_i R_i \cos \theta_i + m_e R_e \cos \theta_e = 0$$

$$32.86 + m_e R_e \cos \theta_e = 0$$

$$\textcircled{1} - m_e R_e \cos \theta_e = -32.86 \Rightarrow (\cos \theta_e = -ve)$$

$$\sum m_i R_i \sin \theta_i + m_e R_e \sin \theta_e = 0$$

$$365.7 + m_e R_e \sin \theta_e = 0$$

$$\textcircled{2} - m_e R_e \sin \theta_e = -365.7 \Rightarrow (\sin \theta_e = -ve)$$

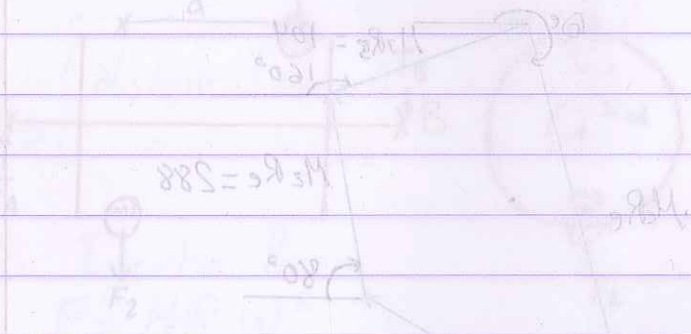
From ① and ② $\tan \theta_c$

$$\tan \theta_c = \frac{-365.7}{-32.86} = 84.86^\circ$$

but because the $\sin \theta_c$ and $\cos \theta_c$ is negative
 $\therefore \theta_c$ is in 3rd quadrant

$$\therefore \theta_c = 84.86 + 180 = 264.86$$

$$M_c = \frac{-365.7}{R \times \sin \theta_c} = 4.13 \text{ Kg (Positive value)}$$



rotat is in 385 N plane
 Inertia Force $F_1 = F_2 = 385 \text{ N}$
 Couple $T = F \cdot a$, will cause reaction A, B

Graphical Solution الحل البياني

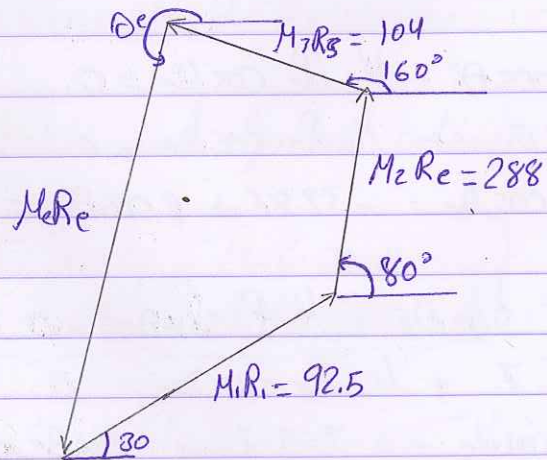
Inertia Force \Rightarrow

$F_i = M_i R_i \omega^2$ but $\omega = \text{constant}$
 F_i is always directed **outward**
 in the direction of θ_i

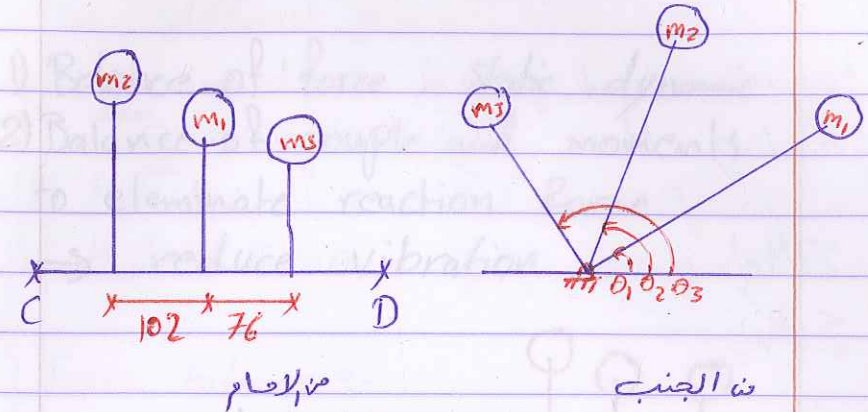
$$\sum F_i + F_e = 0$$

$$\sum M_i R_i \omega^2 + M_e R_e \omega^2 = 0$$

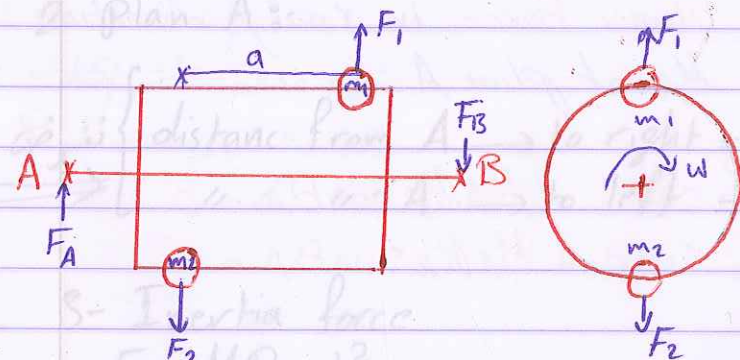
$$\sum M_i R_i + M_e R_e = 0$$



Balance of masses in different planes:-



Rotor composed of two equivalent masses on different planes.

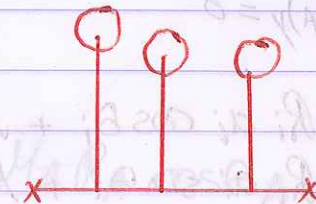


rotor is in static balance
 Inertia Force $F_1 = F_2 = MR \omega^2$

Couple $T = F a$, will cause reaction A, B

Balancing of rotor :-

- 1) Balance of force, static, dynamic
- 2) Balance of couple and moments to eliminate reaction force
 \Rightarrow reduce vibration



كيفية الحل

1. Choose any transverse plan A, B
2. Plan A :-

نفسه $\left\{ \begin{array}{l} \text{distance from A} \rightarrow \text{to right} = +ve \\ \text{" " " A} \rightarrow \text{to left} = -ve \end{array} \right.$

3- Inertia force
 $F = MR\omega^2$

to balance Inertia force

to balance Inertia force :-

Moment about plan A is balance by adding Mass M_B at plane B

$$\Sigma (M_A)_y = 0$$

$$\Sigma (M_A)_x = 0$$

$$\Sigma M_i R_i a_i \cos \theta_i + M_B R_B a_B \cos \theta_B = 0$$

$$\Sigma M_i R_i a_i \sin \theta_i + M_B R_B a_B \sin \theta_B = 0$$

4- To balance Force in x and y direction add M_A at plane A

$$\Sigma M_i R_i \cos \theta_i + M_A R_A \cos \theta_A = 0$$

$$\Sigma M_i R_i \sin \theta_i + M_A R_A \sin \theta_A = 0$$

For balance several masses in different plane

moment balance :-

mass on B

$$\begin{cases} \Sigma M_i R_i a_i \cos \theta_i + M_B R_B a_B \cos \theta_B = 0 \\ \Sigma M_i R_i a_i \sin \theta_i + M_B R_B a_B \sin \theta_B = 0 \end{cases}$$

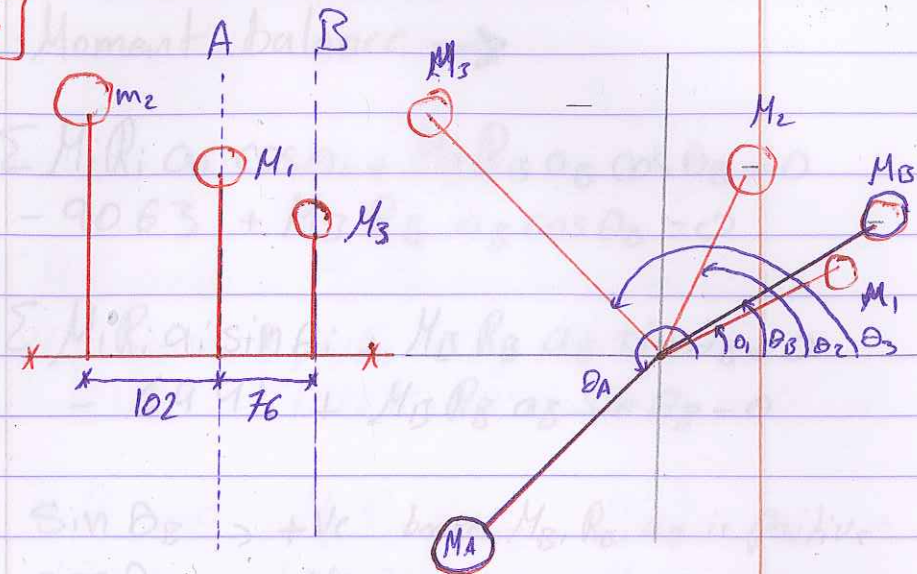
Force balance :-

mass on A

$$\begin{cases} \Sigma M_i R_i \cos \theta_i + M_A R_A \cos \theta_A = 0 \\ \Sigma M_i R_i \sin \theta_i + M_A R_A \sin \theta_A = 0 \end{cases}$$

نضع هذه التأثيرات
الجديدة التوضيحية في B

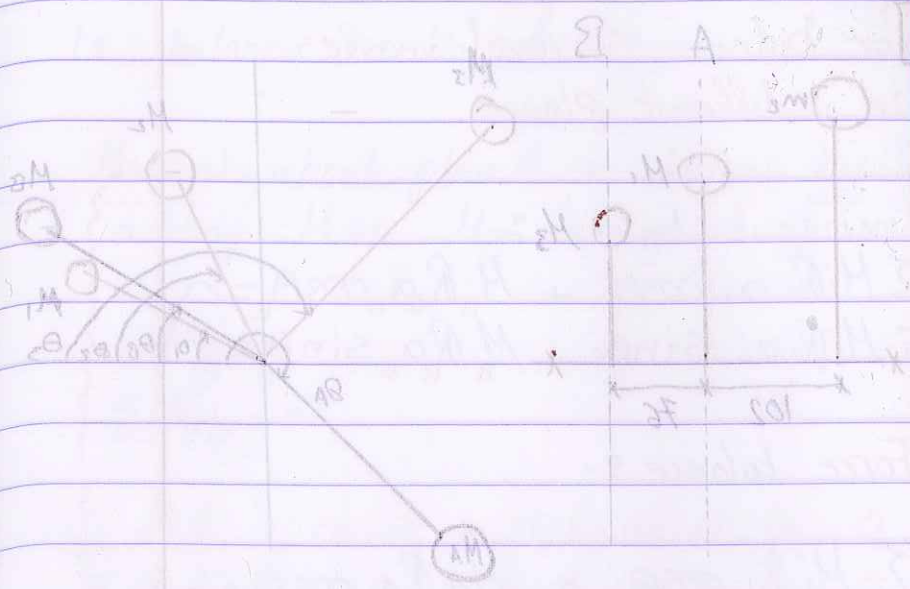
Ex]



Find M_A, M_B at $R_A = R_B = 76 \text{ mm}$ to balance M_1, M_2, M_3 ??

جدول مقادير

No	M_i	R_i	θ	a	$MR \cos \theta$	$MR \sin \theta$	$MR \cos$	$MR \sin$
1	0.454	50.8	30	0	20	11.5	0	0
2	1.36	76	60	-102	51.7	89.5	-5271	-9130
3	0.907	63.5	150	76	-49.9	28.8	-3492	2189
Σ					21.8	129.8	-9063	-6941



1) Moment balance \rightarrow

$$\sum M_i R_i a_i \cos \theta_i + M_B R_B a_B \cos \theta_B = 0$$

$$-9063 + M_B R_B a_B \cos \theta_B = 0$$

$$\sum M_i R_i a_i \sin \theta_i + M_B R_B a_B \sin \theta_B = 0$$

$$-6941 + M_B R_B a_B \sin \theta_B = 0$$

$\sin \theta_B \rightarrow +ve$ because M_B, R_B, a_B is positive
 $\cos \theta_B \rightarrow +ve$ " " " " " "

θ in first quadrant

$$\tan \theta_B = \frac{6941}{9063} \Rightarrow \theta_B = 37.4^\circ$$

$$\Rightarrow M_B = \frac{6941}{R_B a_B \sin \theta_B} = \frac{6941}{76 \times 76 \times \sin 37.4} = 1.98$$

$$M_B = -1.98 \text{ kg}$$

2) force balance \Rightarrow

M_B = weight of block B, acting vertically downwards

$$\sum M_i R_i \cos \theta_i + M_A R_A \cos \theta_A = 0$$

$$\sum M_i R_i \cos \theta_i + M_B R_B \cos \theta_B + M_A R_A \cos \theta_A = 0$$

$$21.8 + 1.98 \times 76 \times 0.794 + M_A R_A \cos \theta_A = 0$$

$$\sum M_i R_i \sin \theta_i + M_B R_B \sin \theta_B + M_A R_A \sin \theta_A = 0$$

$$129.8 + 1.98 \times 76 \times 0.007 + M_A R_A \sin \theta_A = 0$$

$\cos \theta_A \rightarrow -ve$
 $\sin \theta_A \rightarrow -ve$ } Third quadrant

$$\tan \theta_A = \frac{-221.1}{-141.3} \Rightarrow \theta = 57.4$$

$$\therefore \theta_A = 180 + 57.4 = 237.3$$

$$M_A = \frac{-129.8 - (1.98 \times 76 \times 0.007)}{R_A \sin \theta_A}$$

Graphical solution

Inertia Force :-

$$F_i = M_i \omega^2 R_i$$

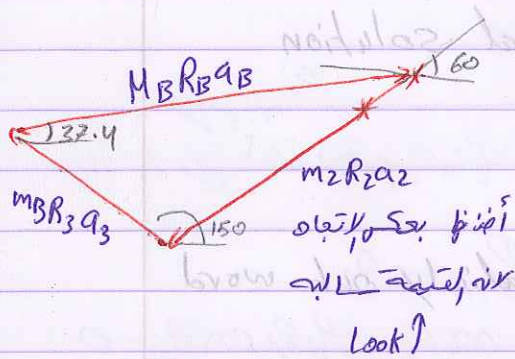
$\Rightarrow F_i \propto M_i R_i$ radially out word

Moment of Inertia force :-

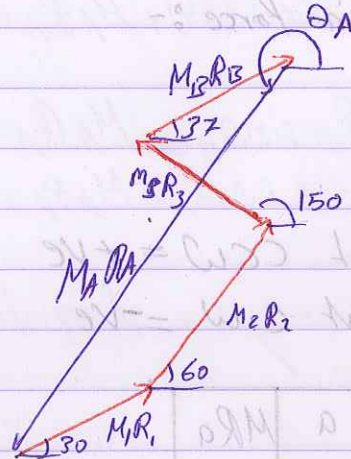
$$m_i = M_i R_i a_i$$

assume $\Rightarrow \begin{cases} \text{moment CCW} = +ve \\ \text{moment CW} = -ve \end{cases}$

No	M	R	MR	a	MRa
1	2.454	50.8	23.1	0	0
2	1.36	76	103.4	-102	-10500
3	0.407	63.5	25.6	57.6	14800



Moment balance



Force balance

Crank shaft balance from book

Fly WHEEL

Fly wheel :- rotating mass used to store energy on flywheel

$$\Rightarrow E = \frac{1}{2} I \omega^2$$

$\omega \uparrow \rightarrow E \uparrow \rightarrow$ Energy stored on flywheel
 $\omega \downarrow \rightarrow E \downarrow \rightarrow$ Energy lost from flywheel

Application of fly wheel :-

- 1- Punch press Machines (see S11)
 if the engine is used without flywheel, a large motor must be used.
 Flywheel is used to store energy after manufacturing process
 rise the velocity from $\omega_{min} \rightarrow \omega_{max}$
 then **Small motor is needed**

$$\omega_1 = \omega_{max}$$

$$\omega_2 = \omega_{min}$$

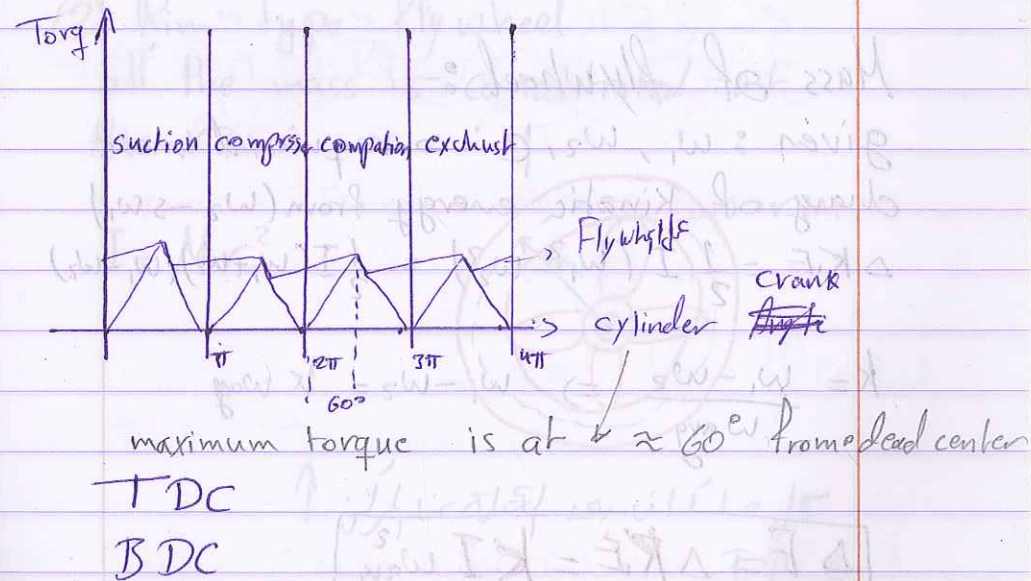
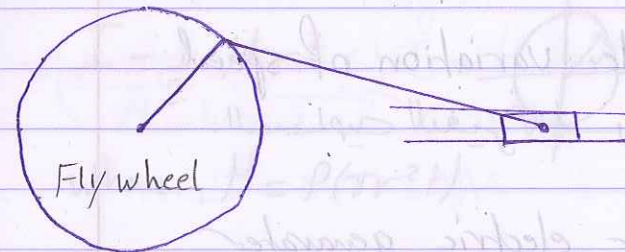
$$\omega_{avg} = \frac{\omega_1 + \omega_2}{2}$$

2- Engines

Stroke engine (Diesel, Petrol)

Steam engine

Single cylinder (Piston)



coefficient of fluctuation:-

$$K = \frac{W_{\max} - W_{\min}}{W_{\text{avg}}}$$

K: Permissible variation of speed
التغير المسموح به في السرعة

$K = 0.001$ for electric generator

Mass of flywheel:-

given: $w_1, w_2, C \rightarrow$ required mass
change of kinetic energy from $(w_2 \rightarrow w_1)$

$$\Delta KE = \frac{1}{2} I (w_1^2 - w_2^2) = \frac{1}{2} I (w_1 + w_2)(w_1 - w_2)$$

$$K = \frac{w_1 - w_2}{W_{\text{avg}}} \Rightarrow w_1 - w_2 = K W_{\text{avg}}$$

$$\Delta E = \Delta KE = K I W_{\text{avg}}^2$$

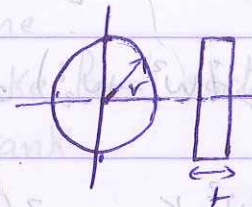
where: ΔE : energy required to rise velocity from w_2 to w_1

Type of follower

① Disk type fly wheel

$$I = \frac{1}{2} M r^2$$

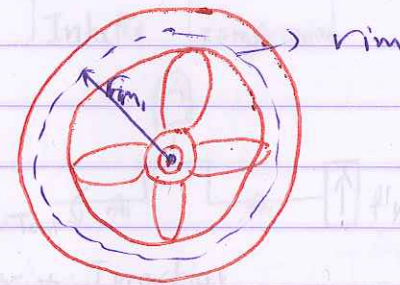
where $M = \rho (\pi r^2 t)$



② Rim type fly wheel

all the mass is concentrated at the rim

$$I = M r_m^2$$



↑
Total Angle
I ↑

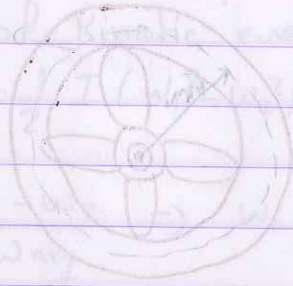
$e \equiv$ maximum fluctuation of energy
(Energy supplied by the flywheel)

Flywheel \Rightarrow $\frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$

$$= \frac{1}{2} I (\omega_1^2 - \omega_2^2) = I \frac{(\omega_1 + \omega_2)}{2} \left(\frac{\omega_1 - \omega_2}{\omega} \right) \omega$$

$$I \omega^2 K = 2 \left(\frac{1}{2} I \omega^2 \right) K = 2 E K$$

$$e = 2 E K \Rightarrow K = \frac{e}{2 E} = \frac{e}{I \omega^2}$$

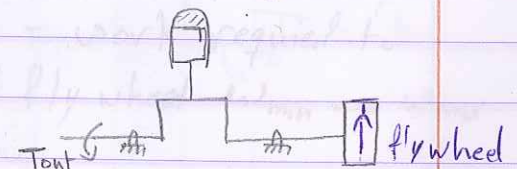
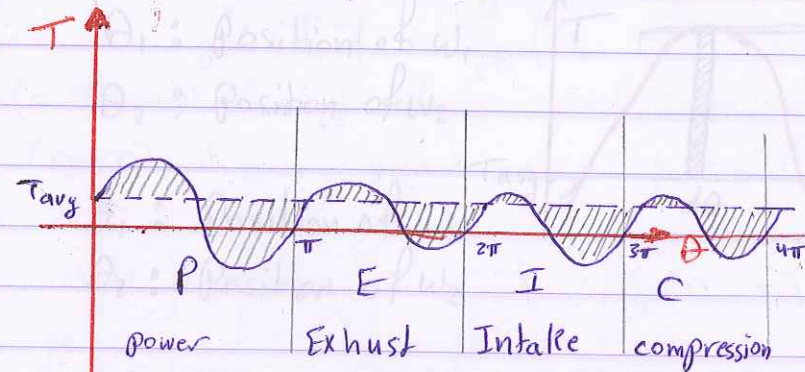


Single cylinder engine:-

Fly wheel of internal combustion engine:-

4 - stroke engine

one power stroke for each two revolution of crank



$$T_{out} = T_{gas\ presser} + Inertia$$

$$T_{avg} = \frac{\sum \text{area under } T-\theta \text{ curve}}{\text{Total Angle}} = \frac{\sum A_i}{4\pi}$$

$$\text{Power} = T \times \omega$$

T_L = Load Torque

T_L = T_{avg}

$T > T_{avg} \Rightarrow$ Energy **stored** on flywheel

$T < T_{avg} \Rightarrow$ Energy **lost** by flywheel

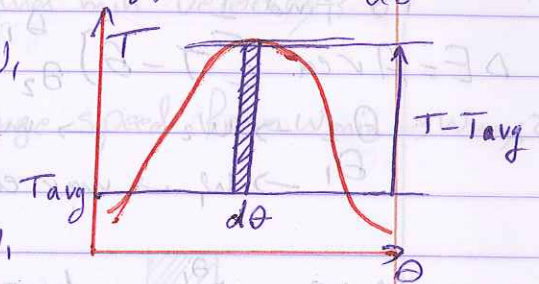
$$T - T_{avg} = I\alpha = I \frac{d\omega}{dt} = I\omega \frac{d\omega}{d\theta}$$

θ_1 : Position of ω_1

θ_2 : Position of ω_2

θ_1 : Position of ω_1

θ_2 : Position of ω_2



$\Delta K.E [\omega_2 - \omega_1] =$ work required to
rise velocity of fly wheel $\omega_{min} \rightarrow \omega_{max}$

$$\Delta K.E [\omega_2 - \omega_1] = \text{Area Under } [T - \theta]_{\theta_1}^{\theta_2}$$

$$\frac{1}{2} I \omega_{avg}^2 = \text{Area } [\theta_2 - \theta_1]$$

To find θ_2 and θ_1 :-

$W_1 = W_{max} \Rightarrow$ Energy max $[E_{max}]$

\Rightarrow Area under $[T-\theta]_{max}$ (A_{max})

* Area represent Energy stored on flywheel

To find I

$$\Delta E = \text{Area } [T-\theta]_{\theta_2}^{\theta_1} = KI W_{avg}^2$$

$\theta_2 \rightarrow W_2 \rightarrow$ min energy \rightarrow min Area $[T-\theta]$

$\theta_1 \rightarrow W_1 \rightarrow$ max energy \rightarrow max Area $[T-\theta]$

$$A [T-\theta]_{\theta_2}^{\theta_1} = KI W_{avg}^2$$

$$P = T_{avg} \cdot W_{avg}$$

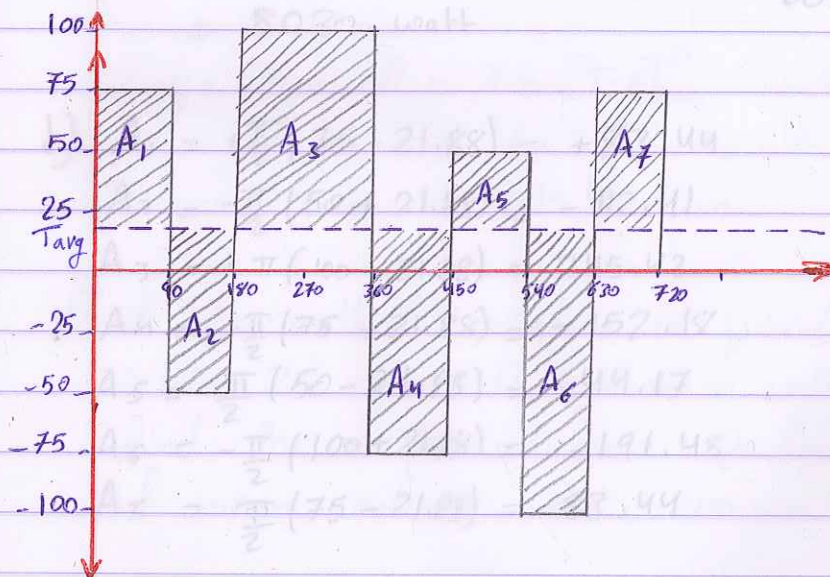
9.11.11

Ex output torque of single cylinder engine. find

a) average output torque and average power if $W_{avg} = 3500 \text{ rpm}??$

b) location of max and min velocity??

c) work done to change speed from $W_{min} \rightarrow W_{max}??$



Positive area represent work done to increase flywheel speed
negative area represent work done to decrease flywheel speed

$$a) T_{avg} = \frac{A_{net}}{\theta}$$

$$+ve A = \frac{\pi}{2} (75) + \pi (100) + \frac{\pi}{2} (50) + \frac{\pi}{2} (75) = 628.3 \text{ N.m}$$

$$-ve A = \frac{\pi}{2} (50) + \frac{\pi}{2} (75) + \frac{\pi}{2} (100) = 353.4 \text{ N.m}$$

$$A_{net} = 628.3 - 353.4 = 274.9 \text{ N.m}$$

$$T_{avg} = \frac{A_{net}}{4\pi} = \frac{274.9}{4\pi} = \boxed{21.88 \text{ N.m}}$$

$$P_{avg} = T_{avg} \cdot \omega_{avg} = 21.88 \times (3500 \times \frac{2\pi}{60})$$

$$= 8030 \text{ watt}$$

$$b) A_1 = +\frac{\pi}{2} (75 - 21.88) = +83.44$$

$$A_2 = -\frac{\pi}{2} (50 + 21.88) = -112.41$$

$$A_3 = +\pi (100 - 21.88) = 245.42$$

$$A_4 = -\frac{\pi}{2} (75 + 21.88) = -152.18$$

$$A_5 = \frac{\pi}{2} (50 - 21.88) = 44.17$$

$$A_6 = -\frac{\pi}{2} (100 + 21.88) = -191.45$$

$$A_7 = \frac{\pi}{2} (75 - 21.88) = 83.44$$

$$\frac{\tan A}{A} = \frac{p \cdot v \cdot T}{A}$$

$$m \cdot V \cdot 88.15 - (25) \frac{\pi}{5} + (100) \frac{\pi}{5} + (100) \frac{\pi}{5} + (25) \frac{\pi}{5} = A \cdot 91 +$$

$$m \cdot V \cdot 88.15 = (100) \frac{\pi}{5} + (25) \frac{\pi}{5} + (100) \frac{\pi}{5} = A \cdot 91$$

$$m \cdot V \cdot 88.15 - N \cdot 225 - 5.882 = \tan A$$

$$[m \cdot V \cdot 88.15] = \frac{p \cdot v \cdot T}{\pi N} = \frac{\tan A}{\pi N}$$

$$\left(\frac{\pi \times 0.025}{100} \right) \times 88.15 = \frac{p \cdot v \cdot T}{\pi N} = \frac{\tan A}{\pi N}$$

$$80.30 \text{ Watt}$$

$$N \cdot 58 + = (88.15 - 25) \frac{\pi}{5} = A \quad (d)$$

$$N \cdot 511 = (88.15 + 25) \frac{\pi}{5} = A$$

$$5N \cdot 242 = (88.15 - 100) \frac{\pi}{5} = A$$

$$N \cdot 151 = (88.15 + 25) \frac{\pi}{5} = A$$

$$5N \cdot 113 = (88.15 - 25) \frac{\pi}{5} = A$$

$$2N \cdot 191 = (100 + 88.15) \frac{\pi}{5} = A$$

$$N \cdot 88 = (88.15 - 25) \frac{\pi}{5} = A$$

$$A_1 \quad 83.44$$

$$A_1 + A_2 \quad -29.47$$

$$A_1 + A_2 + A_3 \quad W_{\max, \theta=360} \Rightarrow 215.45$$

$$A_1 + A_2 + A_3 + A_4 \quad 63.77$$

$$A_1 + A_2 + A_3 + A_4 + A_5 \quad 107.94$$

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 \quad W_{\min, \theta=630} - 83.51$$

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 \quad 0$$

c) Energy required to rise speed from

$$W_{\min} \rightarrow W_{\max} = (258) \times T \times 1.90$$

$$\text{Energy required} = \text{Area} [T - \theta]_{\theta_2}^{\theta_1} = \text{work} [\theta_2 - \theta_1]$$

$$= A_7 + A_1 - A_2 + A_3$$

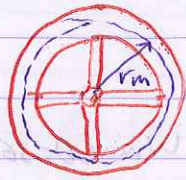
$$= 299.4 \text{ N.m}$$

d) if permitted variation in speed

$$\Delta W = 35 \text{ rpm} \parallel \text{fluctuation in velocity} = 35 \text{ rpm}$$

find the weight of rim type fly wheel
of mean radius $r_m = 200 \text{ mm}$??

$$C = \frac{\max E - \min E}{\text{fluctuation of speed if the speed of engine is } 1500 \text{ rpm}}$$



$$K = \frac{W_1 - W_c}{W_{avg}} = \frac{\Delta W}{W_{avg}} = \frac{35}{3500} = 0.01$$

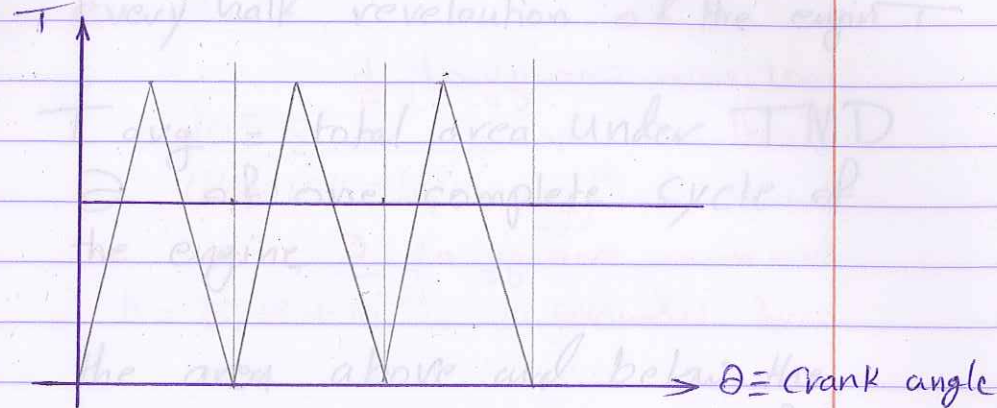
$$\Delta E = K I W_{avg}^2 = \text{Area} [T - \bar{T}]_{\theta_1}^{\theta_2}$$

$$0.01 * I * (367)^2 = 299.4$$

$$I = 0.22 \text{ kg.m}^2$$

$$I = M r_m^2 \Rightarrow M = 5.56 \text{ kg}$$

Problem 13-21



The turning-moment diagram for a petrol engine is drawn to a vertical scale of 1 mm to 6 N.m and a horizontal scale of 1 mm to 1°. The turning moment repeats itself after every half revolution of engine. The Area above and below the mean torque line are: 305, 710, 50, 350, 980 and 275 mm²

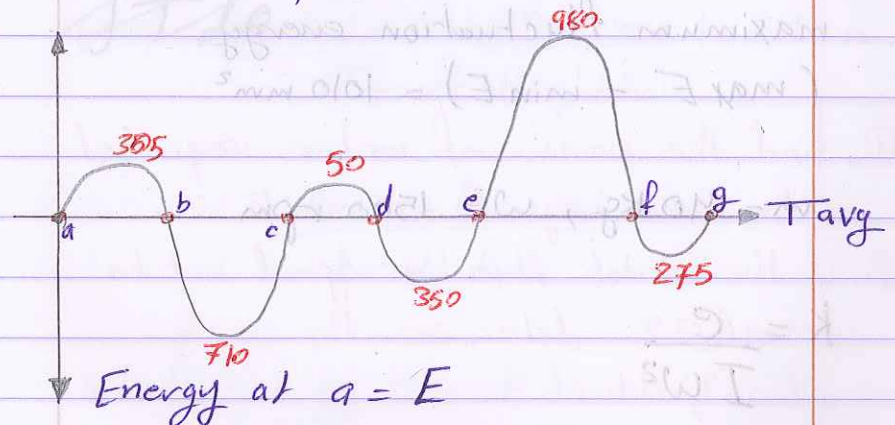
The rotating part amount to a mass of 40 kg at a radius of gyration of 140 mm. Calculate the coefficient of fluctuation of speed if the speed of engine is 1500 rpm

Uploaded By: anonymous

The turning moment repeat it self every half revolution of the engine

T_{avg} = total area under TMD θ of one complete cycle of the engine

the area above and below the mean torque line are as follow



Energy at a = E

" " b = E + 305

" " c = E + 305 - 710 = E - 405

" " d = E - 405 + 50 = E - 355

" " e = E - 355 + 350 = E - 705

" " f = E + 275

$$a = g = E$$

maximum energy at b
and w_{max}

minimum energy at c
and w_{min}

~~maximum absolute energy~~

maximum fluctuation energy
(max E - min E) = 1010 mm²

$$m = 40 \text{ kg}, \omega = 1500 \text{ rpm}$$

$$k = \frac{e}{I\omega^2}$$

$$e = 1010 \text{ mm}^2 \times \left(\frac{6 \text{ N.m}}{1 \text{ mm}} \right)$$

$$\times \left(\frac{1}{1 \text{ mm}} \times \frac{\pi}{180} \right) = 105.8 \text{ N.m}$$

$$\frac{\text{rev}}{\text{min}} \times \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60 \text{ s}}$$

$$k = \frac{e}{I\omega^2}$$

$$= \frac{105.8}{(40) \times (0.140)^2 \times \left(\frac{1500 \times 2\pi}{60} \right)^2}$$

$$k = 5.45 \times 10^{-3} = 0.545 \%$$

$\int T d\theta$



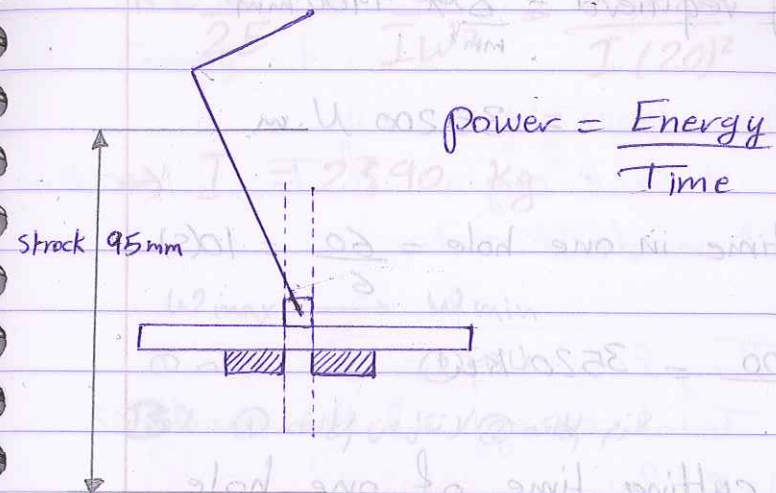
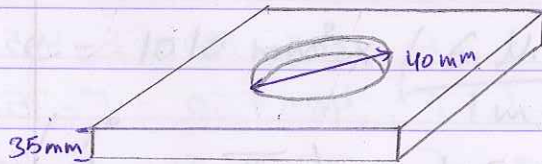
Shear area = $\pi \times \text{radius} \times \text{thickness} = \pi/4 \times d \times t$

coefficient of fluctuation Energy = $\frac{E}{\text{total work in one cycle}}$

Example 13.12??

Punch machine carries out six holes per ~~min~~ minute each hole of 40 mm in 35 mm thick plate that requires 8 N/mm of energy/mm² of sheared area the punch has a stroke of 95 mm

- ① find the power of motor required if the mean speed = 20 m/s
- ② if the total fluctuation speed not to exceed 3% determine the mass of fly wheel



$$\text{power} = \frac{\text{total Energy for 6 hole}}{\text{Time of cutting 6 hole}}$$

$$= \frac{\text{Energy for 1 cut hole}}{\text{total time one hole}}$$

$$\text{total Energy to cut one hole} = \text{area sheared of the hole} \times \frac{\text{Energy required}}{\text{mm}^2}$$

$$\text{sheared area} = \pi d \times \text{thickness} = \pi (40 \text{ mm}) \times 35 = 4400 \text{ mm}^2$$

$$\text{Energy required} = \frac{8.4 \times 10^3 \text{ mm}^2}{\text{mm}^2}$$

$$= 35200 \text{ N.m}$$

$$\text{the time in one hole} = \frac{60}{6} = 10 \text{ (s)}$$

$$\frac{35200}{10} = 3520 \text{ kW}$$

actual cutting time of one hole

$$= \frac{35}{2 \times 95} \times 10 \text{ (s)} = 1.89 \text{ (s)}$$

$$\text{motor gives } 1.89 \times 3520 = 6480 \text{ N.m}$$

hole want 35200 N.m to cut it

$$35200 - 6480 = 28720$$

$$e = 28720 \text{ N.m} = 2 \text{ KE}$$

الطاقة التي يجب ان يحفظها Flywheel

$$K = \frac{e}{2E} = \frac{e}{I\omega^2} = \frac{28720}{I(20)^2}$$

$$\Rightarrow I = 2390 \text{ kg}$$

$$\omega_{\text{max}} \rightarrow \omega_{\text{min}}$$

① قبل

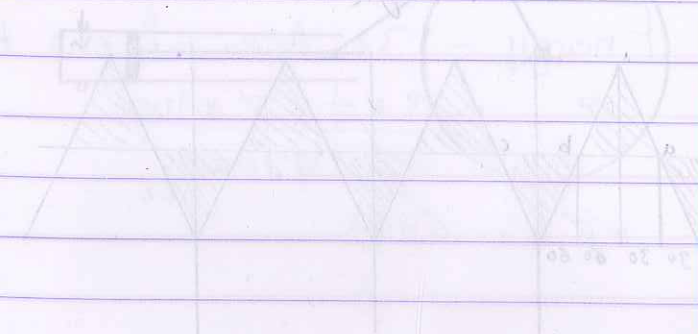
② بعد

13% ① قبل / 15% ② بعد

Steam Engines

① Single overhung crankshaft engine

$$\omega_{\text{max}} = \frac{\pi \times 1000}{\pi \times 1} = \frac{\pi \times 1000}{\pi \times 1} = 1000 \text{ rad/s}$$



Steam engine

Remark

ex for 4 stroke engine if $T_{max} = 1000 \text{ N.m}$
at 60° from dead center

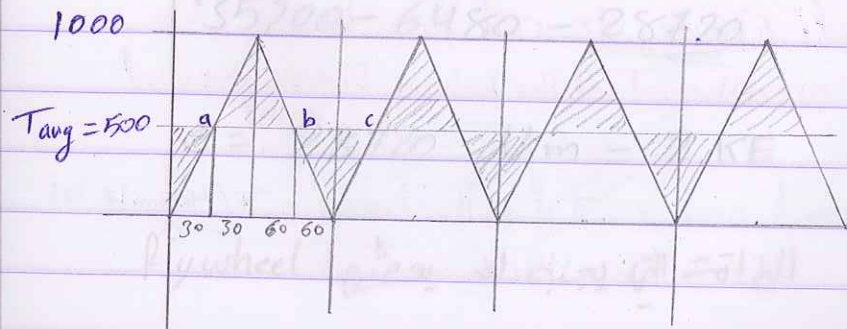
find ① $T_{avg} ??$

② Total energy of the mach??

③ $e ??$

Total Energy = area the curve
 $= 4 \times \frac{1}{2} \times \pi (1000) = 2000 \pi \text{ N.m}$

$$T_{avg} = \frac{\text{total Area}}{\text{total Angle}} = \frac{2000 \pi}{4 \pi} = 500 \text{ N.m}$$



assum

Energy at $a = E$

$$b = E + \frac{1}{2} \times \pi (500) = 125 \pi$$

$$c = E$$

$$d = E + 125 \pi$$

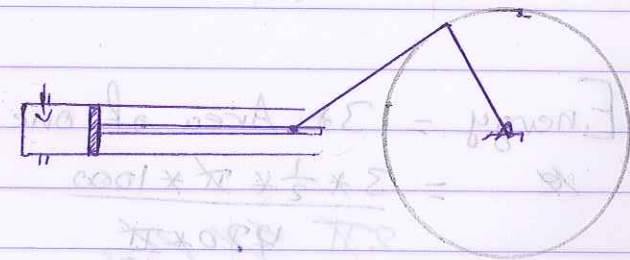
$$e = E$$

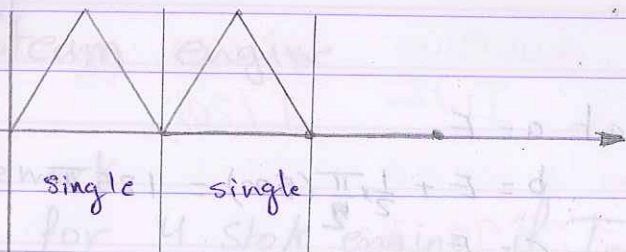
$$e = 125 \pi$$

Steam Engines

- ① Single acting cylinder
 - ② Double acting cylinder
- } cycle 2π

①

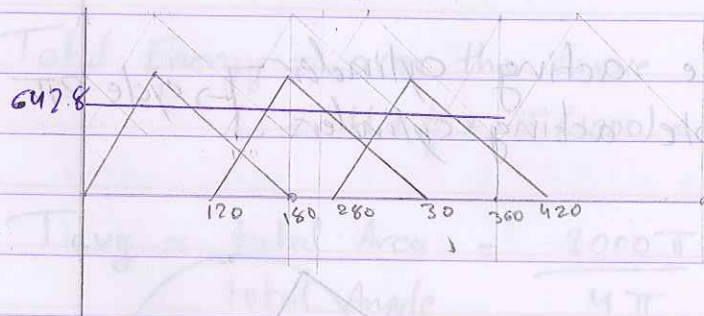




2 piston

$$180 = \frac{360}{2} \Rightarrow 2 \text{ piston } \text{از هر دور یک بار کار می کند}$$

$$120 = \frac{360}{3} \Rightarrow 3 \text{ piston } \text{از هر دور دو بار کار می کند}$$



Energy = 3 * Area of one triangle

$$= \frac{3 * \frac{1}{2} * \pi * 1000}{2\pi}$$

$$= \frac{4800 * \pi}{180}$$

$$= 642.8$$

$$675$$