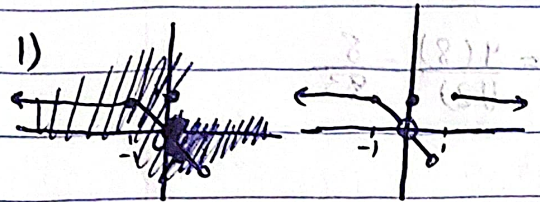
chapter 2 practise exercises

1)



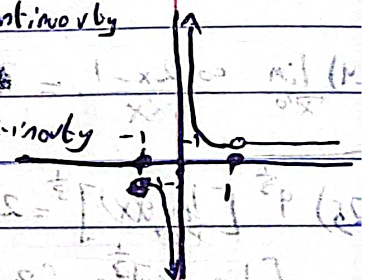
at $x=-1 \Rightarrow \lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x) = \lim_{x\to -1} f(x) = 1$, continuity

at $x=0 \Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 0$, $\lim_{x\to 0} f(x) = 0$, not cont. removable discontinuity

at $x=1 \Rightarrow \lim_{x\to 1^+} f(x) = 1$, $\lim_{x\to 1^-} f(x) = -1$, $\lim_{x\to 1} f(x)$ DNE, not cont. unremovable discontinuity

2) at $x=-1 \Rightarrow \lim_{x\to -1^-} f(x) = 0$, $\lim_{x\to -1^+} f(x) = -1$, $\lim_{x\to -1} f(x)$ DNE, unremovable discontinuity

at $x=0 \Rightarrow \lim_{x\to 0^-} f(x) = -\infty$, $\lim_{x\to 0^+} f(x) = \infty$, $\lim_{x\to 0} f(x)$ DNE, unremovable Discontinuity



at $x=1 \Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1} f(x) = 1$, removable Discontinuity

7) a. $f(x) = x^{\frac{1}{3}}$, $[-\infty, \infty]$ b. $g(x) = x^{\frac{3}{4}}$, $[0, \infty]$ c. $h(x) = x^{-\frac{2}{3}} \Rightarrow [-\infty, \infty] - \{0\}$ D. $h(x) = x^{-\frac{1}{6}} \Rightarrow (0, \infty$

8) a. $f(x) = \tan x$; $\mathbb{R} - \{\frac{\pi}{2} \pm \pi n\}$ $n \in \mathbb{Z}$ b. $g(x) = \csc x$; $\mathbb{R} - \{\pi \pm n\pi\}$, $n \in \mathbb{Z}$

c. $h(x) = \frac{\cos x}{x - \pi}$, $\mathbb{R} - \{\pi\}$ d. $h(x) = \frac{\sin x}{x}$, $\mathbb{R} - \{0\}$

9) a $\lim_{x\to 0} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x}$ = DNE b. $\lim_{x\to 2} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} = \lim_{x\to 2} \frac{(x-2)^2}{x(x+7)\cancel{(x-2)}} = \frac{0}{2\cdot 9} = 0$

10) $\lim_{x\to 0} \frac{x^2 + x}{x^5 + 2x^4 + x} = \lim_{x\to 0} \frac{\cancel{x}(x+1)}{\cancel{x}(x^4 + 2x^3 + 1)} = \frac{1}{1} = 1$ b. $\lim_{x\to -1} \frac{x^2 + x}{x^5 + 2x^4 + x} = \lim_{x\to -1} \frac{x\cancel{(x+1)}}{(x^3 + x^2 - x + 1)\cancel{(x+1)}} = \frac{-1}{2}$

$1 + x \,\overline{)\, x^4 + 2x^3}$
$-(x^4 + x^3)$
$x^3 + 1$

practise (2.6) exercises

11) $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim_{x\to 1} \frac{\cancel{1-\sqrt{x}}}{(\cancel{1-\sqrt{x}})(1+\sqrt{x})} = \frac{1}{2}$

12) $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4} = \lim_{x\to a} \frac{\cancel{x^2-a^2}}{(\cancel{x^2-a^2})(x^2+a^2)} = \frac{1}{2a^2}$

13) $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2}+2hx+h^2-\cancel{x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$

14) $\lim_{x\to 0} \frac{(x+h)^2-x^2}{h} = \frac{h^2-0}{h} = h, h\neq 0$

15) $\lim_{x\to 0} \frac{\frac{1}{2+x}-\frac{1}{2}}{x} = \lim_{x\to 0} \frac{\cancel{2}-x-\cancel{2}}{x(2x+4)} = \frac{-1}{2x+4} = \frac{-1}{4}$

16) $\lim_{x\to 0} \frac{(2+x)^3-8}{x} = \lim_{x\to 0} \frac{(\cancel{2+x-2})((2+x)^2+2(x+2)+4)}{\cancel{x}} = (2)^2+2.2+4 = 12$

17) $\lim_{x\to 1} \frac{x^{\frac{1}{3}}-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(\cancel{x^{\frac{1}{6}}-1})(x^{\frac{1}{6}}+1)}{(\cancel{x^{\frac{1}{6}}-1})(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1)} = \frac{2}{3}$

18) $\lim_{x\to 64} \frac{x^{\frac{2}{3}}-16}{\sqrt{x}-8} = \lim_{x\to 64} \frac{(x^{\frac{1}{3}}-4)(x^{\frac{1}{3}}+4)}{x^{\frac{1}{2}}-8} = \lim_{x\to 64} \frac{(\cancel{x^{\frac{1}{6}}-2})(x^{\frac{1}{6}}+2)(x^{\frac{1}{3}}+4)}{(\cancel{x^{\frac{1}{6}}-2})(x^{\frac{1}{3}}+2x^{\frac{1}{6}}+4)} = \frac{4(8)}{(12)} = \frac{8}{3}$

19) $\lim_{x\to 0} \frac{\tan(2x)}{\tan(\pi x)} = \lim_{x\to 0} \frac{\sin 2x \cos\pi x}{\sin\pi x \cos 2x} = \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}\frac{\cos\pi x}{\pi x}}{\frac{\sin\pi x}{\pi x}\frac{\cos 2x}{2x}} = \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin\pi x}{\pi x}} \cdot \lim_{x\to 0} \frac{\frac{\cos\pi x}{\pi x}}{\frac{\cos 2x}{2x}}$

but $\cos\frac{\pi x}{\pi x} = \frac{\cos 2x}{2x}$ as $x\to 0$

$\Rightarrow 1.1 = 1$

20) $\lim_{x\to\pi^-} \csc x = \infty$

21) $\lim_{x\to\pi} \sin\left(\frac{x}{2}+\sin x\right) = \sin\left(\frac{\pi}{2}+0\right) = 1$

22) $\lim_{x\to\pi} \cos^2(x-\tan x) = \cos^2(\pi-0) = (1)^2 = 1$

23) $\lim_{x\to 0} \frac{8x}{3\sin x - x} \xrightarrow[\text{by } x]{\text{Divide}} \lim_{x\to 0} \frac{8}{\frac{3\sin x}{x}-1} = \frac{8}{3-1} = \frac{8}{2} = 4$

24) $\lim_{x\to 0} \frac{\cos 2x-1}{\sin x} = \frac{1-2\sin^2 x-1}{\sin x} = \lim_{x\to 0} -2\sin x = 0$

25) $4^{\frac{1}{3}}\left[\lim_{x\to 0^+} g(x)\right]^{\frac{1}{3}} = 2$

$\left[\lim_{x\to 0^+} g(x)\right]^{\frac{1}{3}} = \frac{2}{2^{\frac{2}{3}}} = 2^{\frac{1}{3}}$

$\boxed{\lim_{x\to 0^+} g(x) = 2}$

26) $\lim_{x\to\sqrt{5}} \frac{1}{x+g(x)} = 2$

$\frac{1}{\sqrt{5}+\lim_{x\to\sqrt{5}} g(x)} = 2$

$\sqrt{5}+\lim_{x\to\sqrt{5}} g(x) = \frac{1}{2}$

$\boxed{\lim_{x\to\sqrt{5}} g(x) = \frac{1}{2}-\sqrt{5}}$

27) $\lim_{x\to 1} \frac{3x^2+1}{g(x)} = \infty$

$\frac{4}{\lim_{x\to 1} g(x)} = \infty$

$\lim_{x\to 1} g(x) = 0^+$

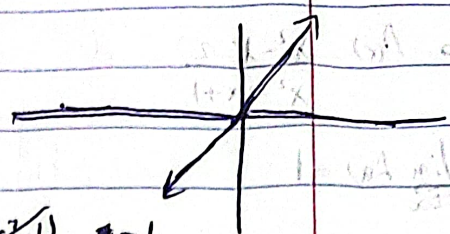
28) $\lim_{x\to -2} \frac{5-x^2}{g(x)} = 0$

$\frac{1}{\lim_{x\to -2} g(x)} = 0$

$\lim_{x\to -2} g(x) = \pm\infty$

Chapter 2 : practise exercises

29) $f(x) = \frac{x(x^2-1)}{|x^2-1|}$



$\lim_{x\to 1} f(x) = \frac{x(x^2-1)}{(x^2-1)} = \lim \frac{x\cancel{(x-1)}\cancel{(x+1)}}{\cancel{(x-1)}\cancel{(x+1)}} = 1$, $\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{x\cancel{(x^2-1)}}{\cancel{(x^2-1)}} = -1$

$g(x)$ (continous extension) $= \begin{cases} f(x), & x \neq \{-1, 1\} \ (f_x = x) \\ 1, & x = 1 \\ -1, & x = -1 \end{cases}$

30) $f(x) = \sin\left(\frac{1}{x}\right)$ $\quad -1 \leq \sin\frac{1}{x} \leq 1$

$\lim_{x\to 0} -1 = -1$, $\lim_{x\to 0} 1 = 1$

$-1 \leq \lim_{x\to 0} \sin\frac{1}{x} \leq 1$

no specific L so there cant be a continous extension

37) $\lim_{x\to\infty} \frac{2x+3}{5x+7} = \frac{2}{5}$ 38) $\lim_{x\to-\infty} \frac{2x^2+3}{5x^2+7} = \frac{2}{5}$ 39) $\lim_{x\to-\infty} \frac{x^2-4x+8}{3x^3} = 0$ 40) $\lim_{x\to\infty} \frac{1}{x^2-7x+1} = 0$

41) $\lim_{x\to\infty} \frac{x^2-7x}{x+1} = \lim_{x\to\infty} \frac{x-7}{1+\frac{1}{x}} = \infty$ 42) $\lim_{x\to\infty} \frac{x^4+x^3}{12x^3+128} = \infty$ 43) $\lim_{x\to\infty} \frac{\sin x}{\lfloor x \rfloor} \Rightarrow -1 \leq \sin x \leq 1$

$\frac{-1}{\lfloor x \rfloor} \leq \frac{\sin x}{\lfloor x \rfloor} \leq \frac{1}{\lfloor x \rfloor}$

44) $\lim_{\theta\to\infty} \frac{\cos\theta - 1}{\theta} \Rightarrow = \lim_{\theta\to 0} \frac{\cancel{1} - \sin^2\frac{\theta}{2} - \cancel{1}}{\theta}$

$\lim_{x\to\infty} \frac{-1}{\lfloor x \rfloor} = \lim_{x\to\infty} \frac{1}{\lfloor x \rfloor} = 0$

by sandwich theorem

$\lim_{\theta\to\infty} \frac{-\sin\frac{\theta}{2}}{\theta} \cdot \sin\frac{\theta}{2} = 0 \cdot \lim_{\theta\to\infty} \sin\frac{\theta}{2} = 0$

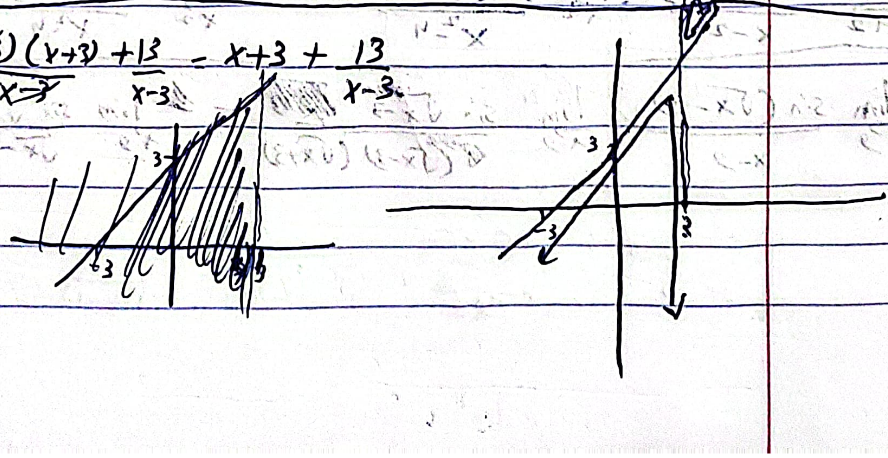
$\lim_{x\to\infty} \frac{\sin x}{\lfloor x \rfloor} = 0$

45) $\lim_{x\to\infty} \frac{x+\sin x+2\sqrt{x}}{x+\sin x} = \lim_{x\to\infty} 1 + \frac{2\sqrt{x}}{x+\sin x} = \lim_{x\to\infty} \frac{1 + 2x^{-\frac{1}{2}}}{1+\frac{\sin x}{x}} = \frac{1+0}{1+0} = 1$

46) $\lim_{x\to\infty} \frac{x^{\frac{2}{3}} + x^{-1}}{x^{\frac{2}{3}} + \cos^2 x} = \lim_{x\to\infty} \frac{1 + x^{-\frac{5}{3}}}{1 + \frac{\cos^2 x}{x^{\frac{2}{3}}}} = \frac{1+0}{1+0} = 1$

47) a. $y = \frac{x^2+4}{x-3} = \frac{x^2-9+13}{x-3} = \frac{\cancel{(x-3)}(x+3)+13}{\cancel{x-3}} = x+3+\frac{13}{x-3}$

$y = x+3$ oblique asymptote



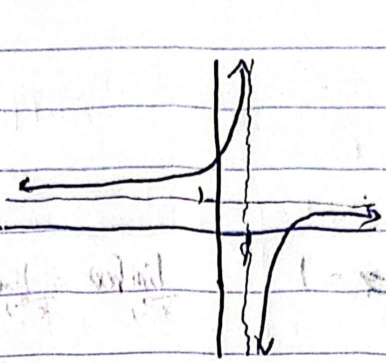
$\lim_{x\to 3^+} \frac{x^2+4}{x-3} = \infty$, $\lim_{x\to 3^-} \frac{x^2+4}{x-3} = -\infty$

the equations are:

- $x = 3$
- $y = x+3$

chapter 2 (Practise exercises)

b. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 2x + 1}$



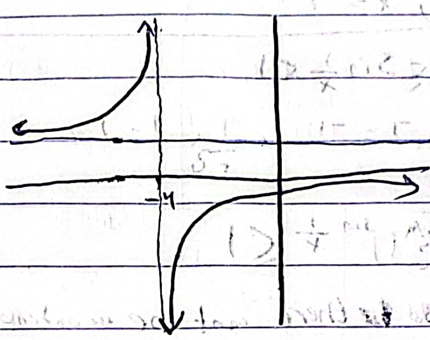
equations: $y = 1$, $x = 1$

$\lim_{x \to \pm\infty} f(x) = 1$

$\lim_{x \to 1^+} f(x) = -\infty$, $\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to -1} f(x) = 0$

c. $y = \dfrac{x^2 + x - 6}{x^2 + 2x - 8}$



$\lim_{x \to \pm\infty} y = 1$

$= \dfrac{(x+3)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$, $x \neq 2$

$\lim_{x \to -4^+} \dfrac{x+3}{x+4} = -\infty$, $\lim_{x \to -4^-} \dfrac{x+3}{x+4} = \infty$

rule to solve these additional exercises: $\lim_{f(x) \to 0} \dfrac{\sin(f(x))}{f(x)} = 1$

25) $\lim_{x \to 0} \dfrac{\sin(1-\cos x)}{x} = \lim_{x \to 0} \dfrac{\sin(1-\cos x)(1-\cos x)}{(1-\cos x)\, x} = \lim_{f(x) \to 0} \dfrac{\sin(f(x))}{f(x)} \cdot \lim_{x \to 0} \dfrac{\cancel{1} - \cancel{1} + \sin^2 \frac{x}{2}}{x}$

$= 1 \cdot \lim_{x \to 0} \dfrac{\sin^2 \frac{x}{2}}{\frac{x}{2}} \cdot \dfrac{1}{2} = \dfrac{1}{2} \cdot \sin \dfrac{0}{2} \cdot \dfrac{1}{2} = 0$

26) $\lim_{x \to 0^+} \dfrac{\sin x}{\sin\sqrt{x}} = \lim_{x \to 0^+} \dfrac{\frac{\sin x}{x}}{\frac{\sin\sqrt{x}}{\sqrt{x}\cdot\sqrt{x}}} = \dfrac{\sqrt{x}\cdot\sqrt{x}}{\sin\sqrt{x}} = 1 \cdot 0 = 0$

27) $\lim_{x \to 0} \dfrac{\sin(\sin x)}{x} = \lim_{x \to 0} \dfrac{\sin(\sin x)\sin x}{x \sin x} = \lim_{x \to 0} \dfrac{\sin x}{x} \cdot \lim_{x \to 0} \dfrac{\sin(\sin x)}{\sin(x)} = 1 \cdot 1 = 1$

28) $\lim_{x \to 0} \dfrac{\sin(x^2 + x)}{x} = \lim_{x \to 0} \dfrac{\sin(x^2+x)(x+1)}{x^2+x} = \lim_{x \to 0} \dfrac{\sin(x^2+x)}{x^2+x} \cdot \lim_{x \to 0} (x+1) = 1 \cdot 1 = 1$

29) $\lim_{x \to 2} \dfrac{\sin(x^2 - 4)}{x - 2} = \lim_{x \to 2} \dfrac{\sin(x^2-4)(x+2)}{x^2-4} = \lim_{x \to 2} \dfrac{\sin(x^2-4)}{x^2-4} \cdot \lim_{x \to 2}(x+2) = 1 \cdot 4 = 4$

30) $\lim_{x \to 9} \dfrac{\sin(\sqrt{x} - 3)}{x - 9} = \lim_{x \to 9} \dfrac{\sin\sqrt{x-3}}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \to 9} \dfrac{\sin\sqrt{x-3}}{\sqrt{x}-3} \cdot \lim_{x \to 9} \dfrac{1}{\sqrt{x}+3} = 1 \cdot \dfrac{1}{6} = \dfrac{1}{6}$