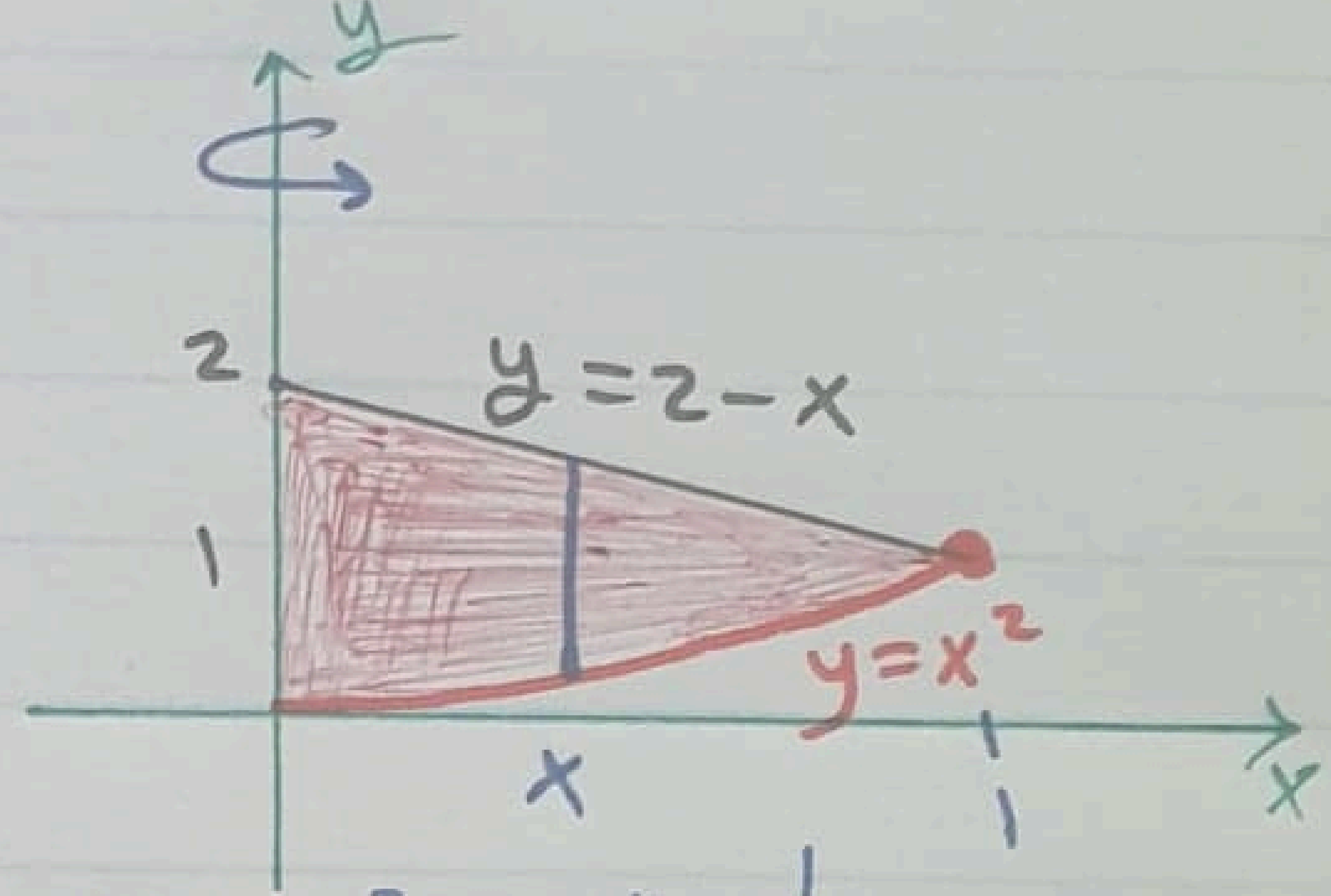
Discussion 6.2

[9] Use shell Method to find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the lines $y = 2 - x$, $x = 0$, $x \geq 0$ about y-axis



$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell} \\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell} \\ \text{height}\end{pmatrix} dx$$

$$= \int_0^1 2\pi \, (x)(2 - x - x^2) \, dx$$

$$= 2\pi \int_0^1 (2x - x^2 - x^3) \, dx = 2\pi \left(x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \frac{5\pi}{6}$$

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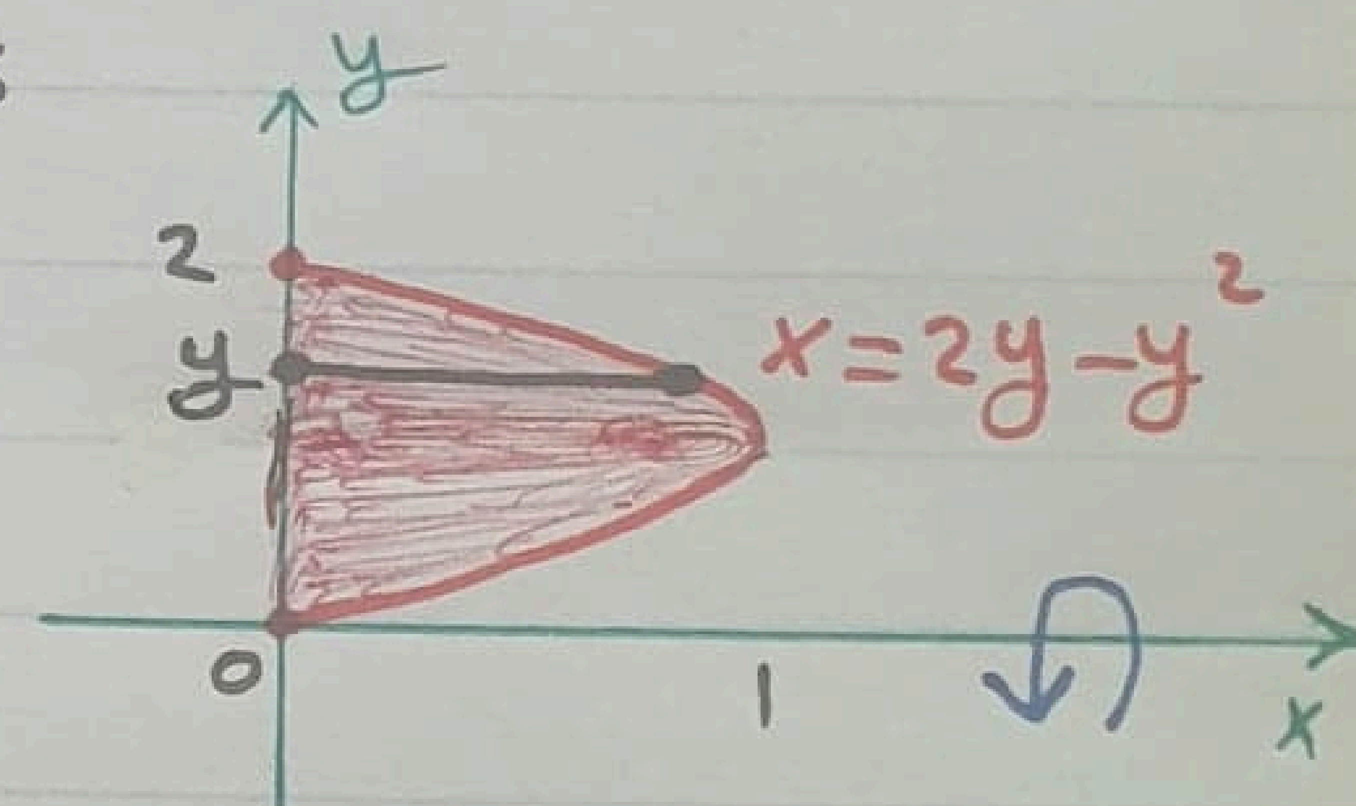
[17] Use shell Method to find the volume of the solid generated by revolving the region bounded by the curv $x = 2y - y^2$ and y-axis about x-axis



$$x = -\left[y^2 - 2y\right] = -\left[(y-1)^2 - 1\right] = 1 - (y-1)^2$$

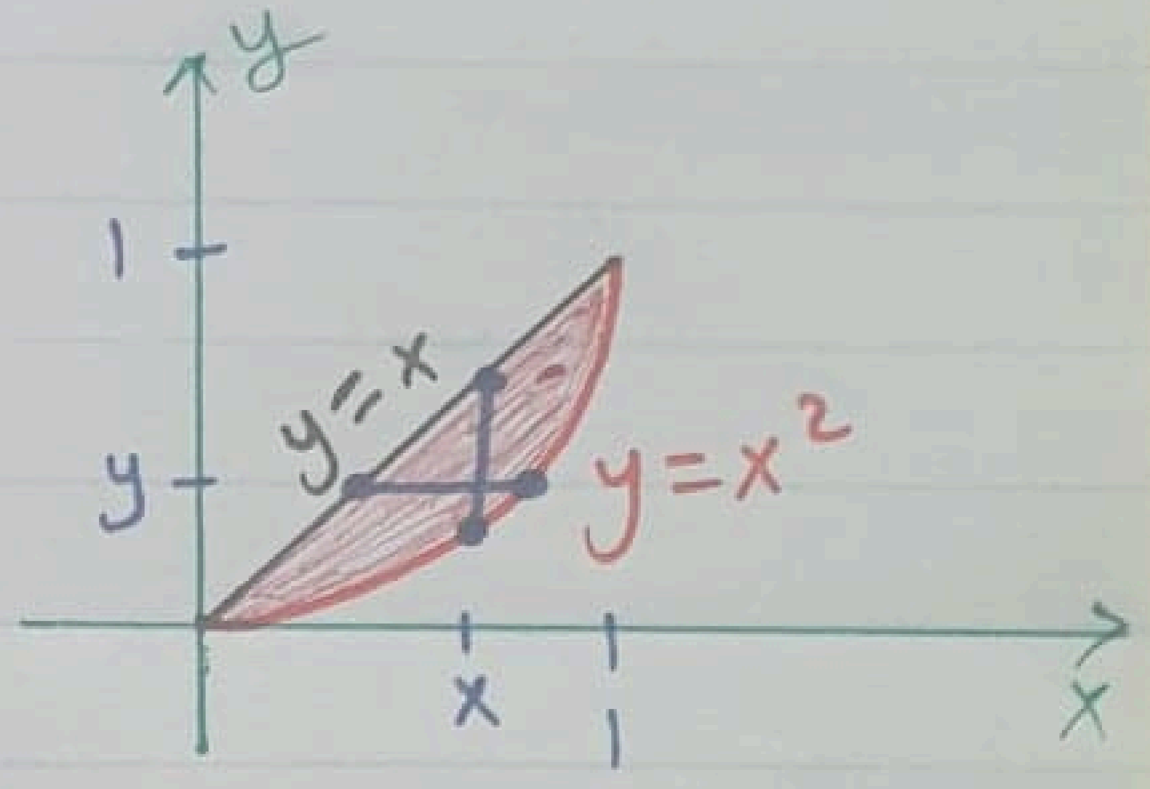
$$V = \int_c^d 2\pi \begin{pmatrix}\text{shell} \\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell} \\ \text{length}\end{pmatrix} dy$$

$$= \int_0^2 2\pi \, (y)(2y - y^2) \, dy = 2\pi \int_0^2 (2y^2 - y^3) \, dy$$

$$= 2\pi \left(\frac{2y^3}{3} - \frac{y^4}{4}\right)\Big|_0^2 = 2\pi\left[\left(\frac{16}{3} - \frac{16}{4}\right) - (0 - 0)\right] = \frac{8\pi}{3}$$

$\boxed{29}$ Compute the volume of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about x-axis and y-axis using

[1] Shell Method [2] Washer Method



$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, \ x = 1$

$y = 0, \ y = 1$

[1] Shell Method

→ about x-axis

$$V = \int_c^d 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{length}\end{pmatrix} dy$$

$$= \int_0^1 2\pi\,(y)(\sqrt{y} - y)\,dy = \frac{2\pi}{15}$$

→ about y-axis

$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix} dx$$

$$= \int_0^1 2\pi(x)(x - x^2)\,dx = \frac{\pi}{6}$$

[2] Washer Method

→ about x-axis $\Rightarrow \boxed{R(x) = x \quad \text{and} \quad r(x) = x^2}$

$$V = \int_a^b \pi\,[R^2(x) - r^2(x)]\,dx = \int_0^1 \pi(x^2 - x^4)\,dx = \frac{2\pi}{15}$$

→ about y-axis $\Rightarrow \boxed{R(y) = \sqrt{y} \quad \text{and} \quad r(y) = y}$

$$V = \int_c^d \pi\,[R^2(y) - r^2(y)]\,dy = \int_0^1 \pi\,(y - y^2)\,dy = \frac{\pi}{6}$$