

Ch.8 Techniques of Integration:-

Some integral Formulas:-

$$\textcircled{1} \int a \, dx = ax + c \quad \textcircled{2} \int a x^n \, dx = \frac{a x^{n+1}}{n+1} + c, n \neq -1$$

$$\textcircled{3} \int \frac{a}{x} \, dx = a \ln|x| + c \quad \textcircled{4} \int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + c$$

$$\textcircled{5} \int e^{ax} \, dx = \frac{e^{ax}}{a} + c \quad \textcircled{6} \int a^x \, dx = \frac{a^x}{\ln a} + c$$

$$\textcircled{7} \int \sin(ax) \, dx = -\frac{1}{a} \cos(ax) + c \quad \textcircled{8} \int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + c$$

$$\textcircled{9} \int \tan x \, dx = \ln|\sec x| + c \quad \textcircled{10} \int \cot x \, dx = \ln|\sin x| + c$$

$$\textcircled{11} \int \sec x \, dx = \ln|\sec x + \tan x| + c$$

$$\textcircled{12} \int \csc x \, dx = -\ln|\csc x + \cot x| + c = \ln|\csc x - \cot x| + c$$

$$\textcircled{13} \int \sec^2 x \, dx = \tan x + c$$

$$\textcircled{14} \int \csc^2 x \, dx = -\cot x + c$$

$$\textcircled{15} \int \sec x \tan x \, dx = \sec x + c$$

$$\boxed{16} \int \csc x \cot x \, dx = -\csc x + C$$

$$\boxed{17} \int \cosh(ax) \, dx = \frac{\sinh(ax)}{a} + C$$

$$\boxed{18} \int \sinh(ax) \, dx = \frac{\cosh(ax)}{a} + C$$

$$\boxed{19} \int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\boxed{20} \int \operatorname{csch}^2 x \, dx = -\coth x + C$$

$$\boxed{21} \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

$$\boxed{22} \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

$$\boxed{23} \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\boxed{24} \int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\boxed{25} \int \frac{1}{x \sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + C$$

7.7 Hyperbolic Functions

Def: The hyperbolic functions are defined as :-

$$[1] \sinh x = \frac{e^x - e^{-x}}{2}$$

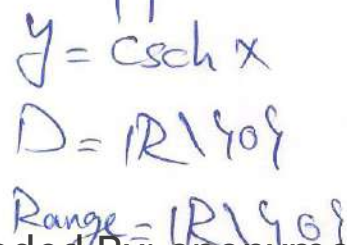
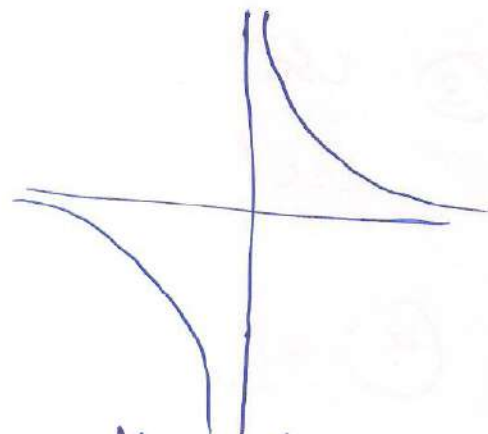
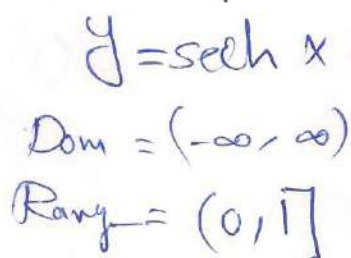
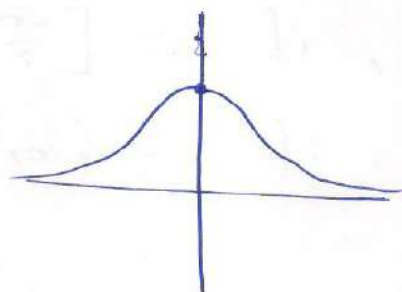
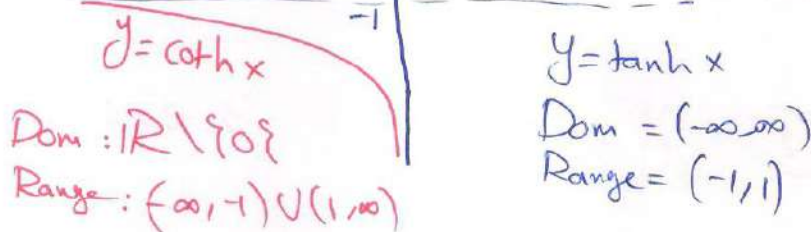
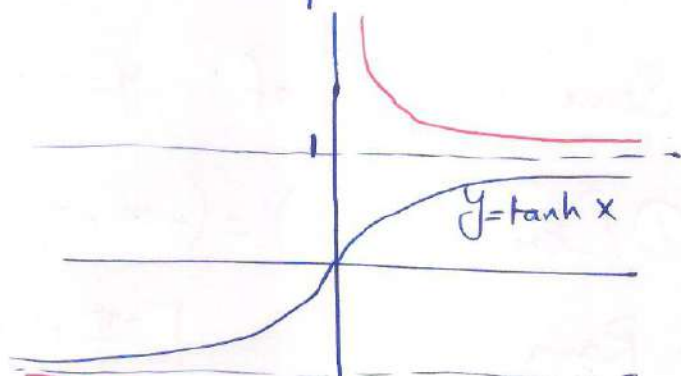
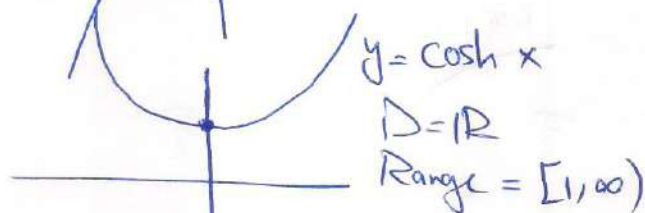
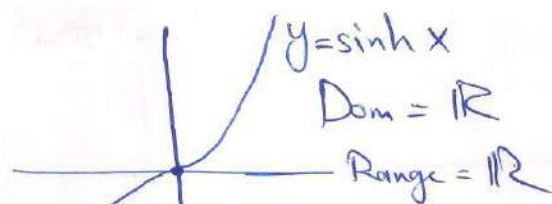
$$[2] \cosh x = \frac{e^x + e^{-x}}{2}$$

$$[3] \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$[4] \coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

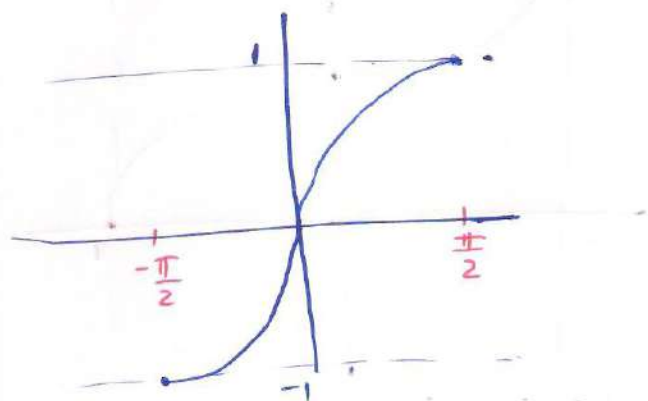
$$[5] \operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$[6] \operatorname{csch} x = \frac{2}{e^x - e^{-x}}$$



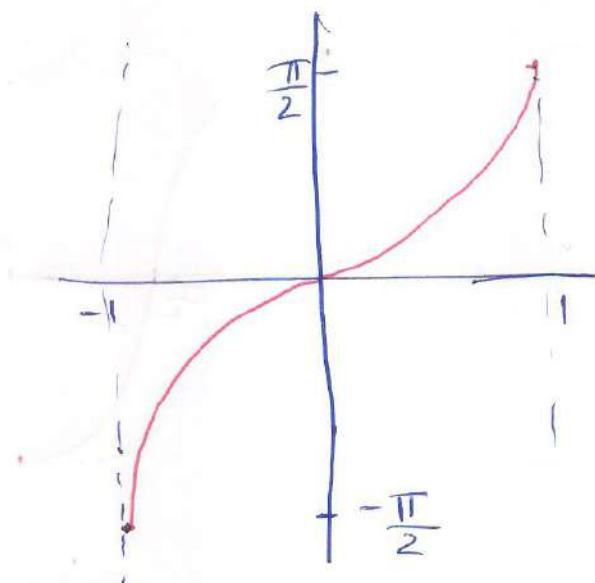
7.6 Inverse Trigonometric function

$$\square y = \sin x$$



$$y = \sin x, \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y = \sin^{-1}(x) = \arcsin(x)$$



Some properties of $y = \sin^{-1}(x)$

$$\textcircled{1} \text{Dom}(\sin^{-1}(x)) = [-1, 1] \quad , \quad \text{Range}(\sin^{-1}(x)) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

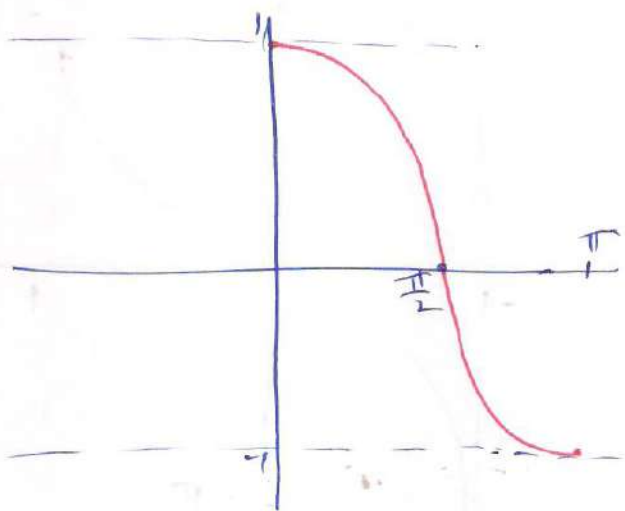
$$\textcircled{2} y = \sin^{-1}(x) \text{ is equivalent to } \sin y = x, \text{ if } x \in \text{Dom}(\sin^{-1}(x))$$

$$\textcircled{3} \sin^{-1}(\sin x) = x \quad \text{if } x \in \text{Dom}(\sin x) \\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\textcircled{4} \lim_{x \rightarrow 1^-} \sin^{-1}(x) = \frac{\pi}{2}$$

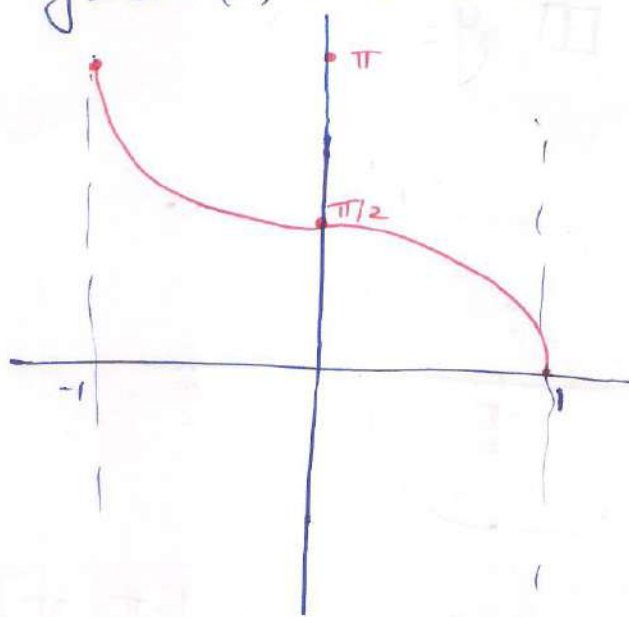
$$\lim_{x \rightarrow -1^+} \sin^{-1}(x) = -\frac{\pi}{2}$$

② $y = \cos x$



$y = \cos x, x \in [0, \pi]$

$y = \cos^{-1}(x) = \arccos x$



$y = \cos^{-1}(x), x \in [-1, 1]$

Some properties $y = \cos^{-1}(x)$

① $\text{Dom}(\cos^{-1}x) = [-1, 1]$ / $\text{Range}(\cos^{-1}(x)) = [0, \pi]$

② $y = \cos^{-1}x$ is equivalent to $\cos y = x$, if $x \in \text{Dom}(\cos^{-1}x)$

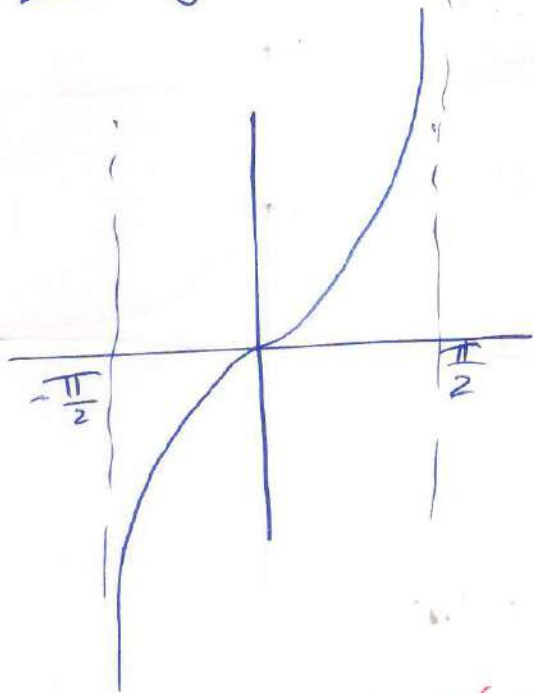
③ $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$

$\cos(\cos^{-1}x) = x$, if $x \in [-1, 1]$

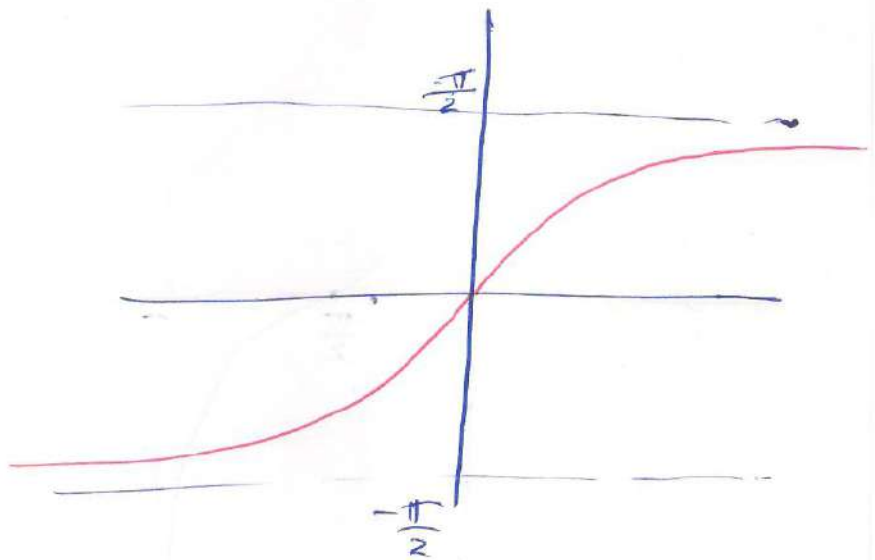
④ $\lim_{x \rightarrow -1^+} \cos^{-1}(x) = \pi$

$\lim_{x \rightarrow 1^-} \cos^{-1}(x) = 0$

$$\boxed{3} \quad y = \tan x$$



$$y = \tan^{-1} x = \arctan x$$



$$y = \tan x, \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Some properties of $y = \tan^{-1}(x)$

$$\textcircled{1} \quad \text{Dom}(\tan^{-1} x) = (-\infty, \infty) \quad \text{Range}(\tan^{-1} x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

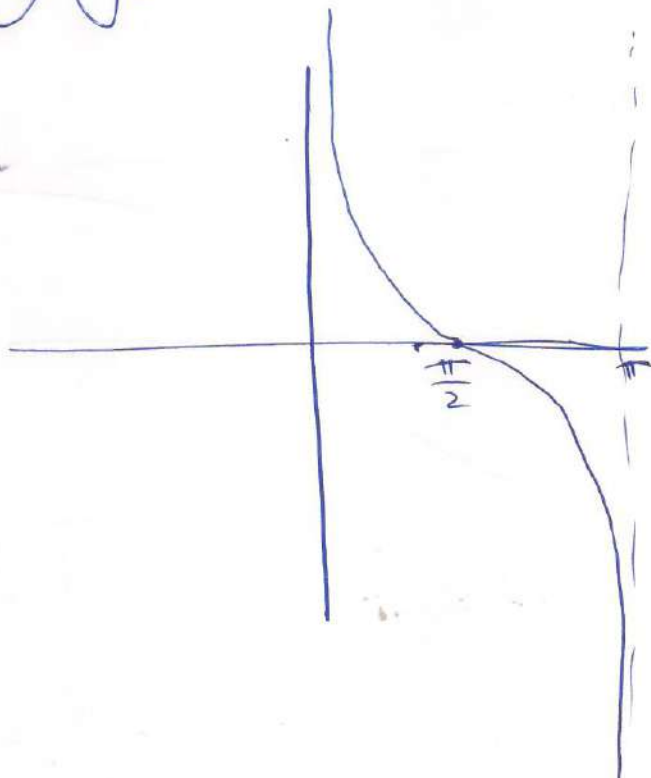
$$\textcircled{2} \quad y = \tan^{-1} x \text{ is equivalent to } \tan y = x, \text{ if } x \in (-\infty, \infty)$$

$$\textcircled{3} \quad \tan^{-1}(\tan x) = x, \text{ if } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

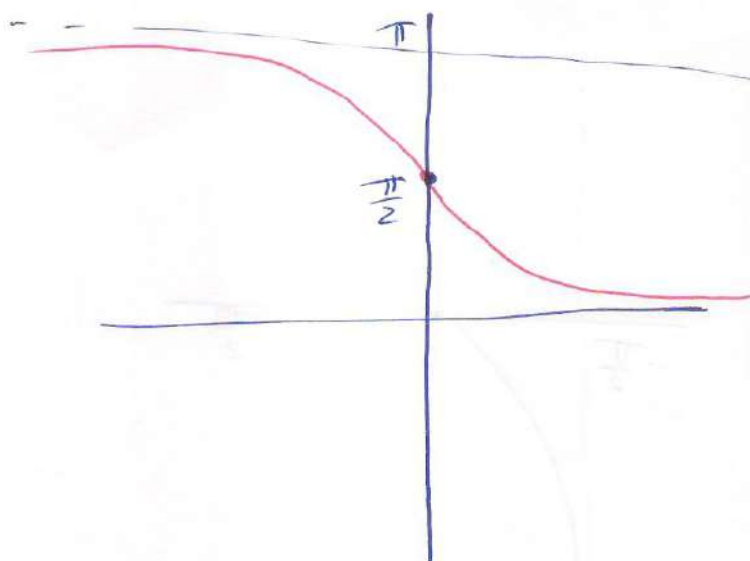
$$\tan(\tan^{-1}(x)) = x, \text{ if } x \in (-\infty, \infty)$$

$$\textcircled{4} \quad \lim_{x \rightarrow \infty} \tan^{-1} x = \frac{\pi}{2} \quad \lim_{x \rightarrow -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

④ $y = \cot x$



$y = \cot^{-1}(x) = \text{arc cot } x$



$y = \cot x, x \in (0, \pi)$

$y = \cot^{-1}(x), x \in (-\infty, \infty)$

Some properties of $y = \cot^{-1}(x)$

① $\text{Dom}(\cot^{-1} x) = (-\infty, \infty)$ $\text{Range}(\cot^{-1} x) = (0, \pi)$

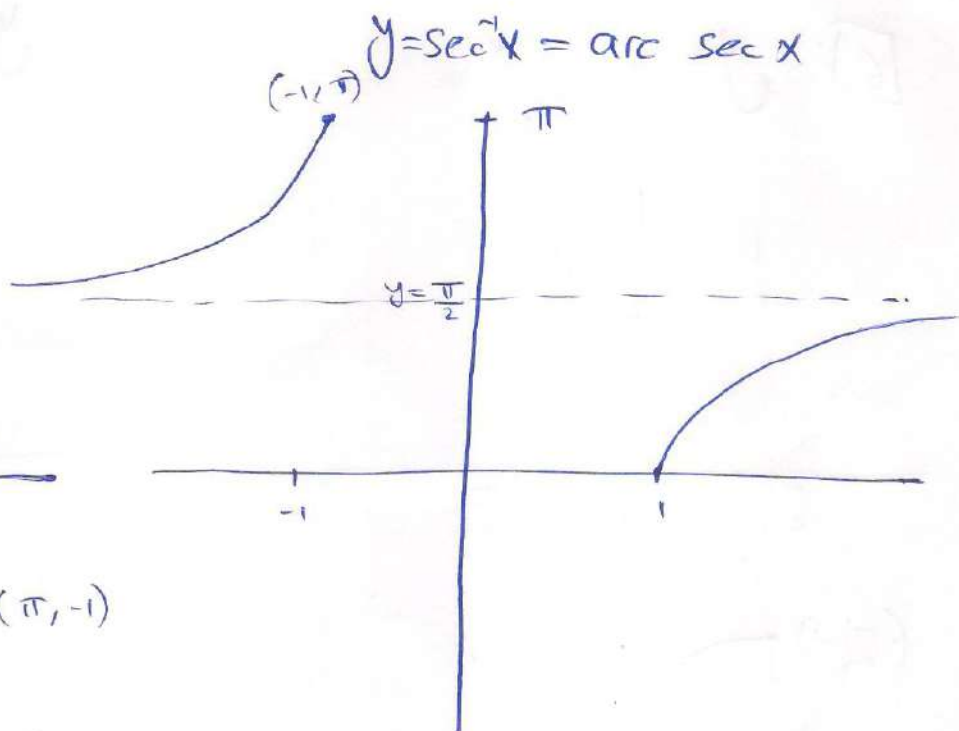
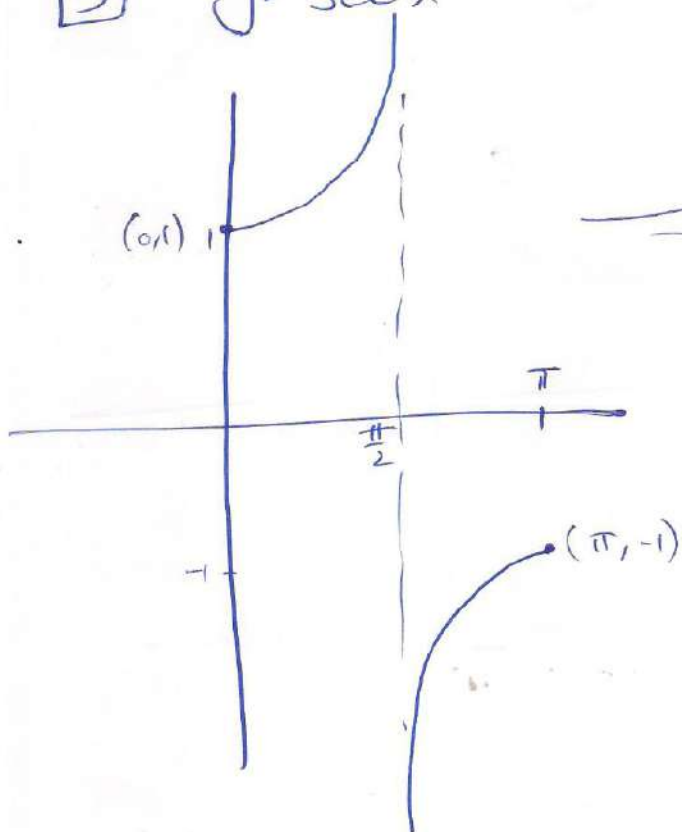
② $y = \cot^{-1} x$ is equivalent to $\cot y = x$, if $x \in (-\infty, \infty)$

③ $\cot(\cot^{-1} x) = x$, if $x \in (-\infty, \infty)$

$\cot^{-1}(\cot x) = x$, if $x \in (0, \pi)$

④ $\lim_{x \rightarrow \infty} \cot^{-1} x = 0$ $\lim_{x \rightarrow -\infty} \cot^{-1} x = \pi$

5 $y = \sec x$



$$y = \sec x, \quad x \in [0, \pi] \setminus \left\{ \frac{\pi}{2} \right\}$$

$$y = \sec^{-1} x, \quad x \in (-\infty, -1] \cup [1, \infty)$$

Some properties of $y = \sec^{-1} x$

① $\text{Dom}(\sec^{-1} x) = (-\infty, -1] \cup [1, \infty)$

$$\text{Range}(\sec^{-1} x) = [0, \pi] \setminus \left\{ \frac{\pi}{2} \right\}$$

② $y = \sec^{-1} x$ is equivalent to $\sec y = x$, if $x \in \text{Dom}(\sec^{-1} x)$

③ $\sec^{-1}(\sec x) = x$, if $x \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

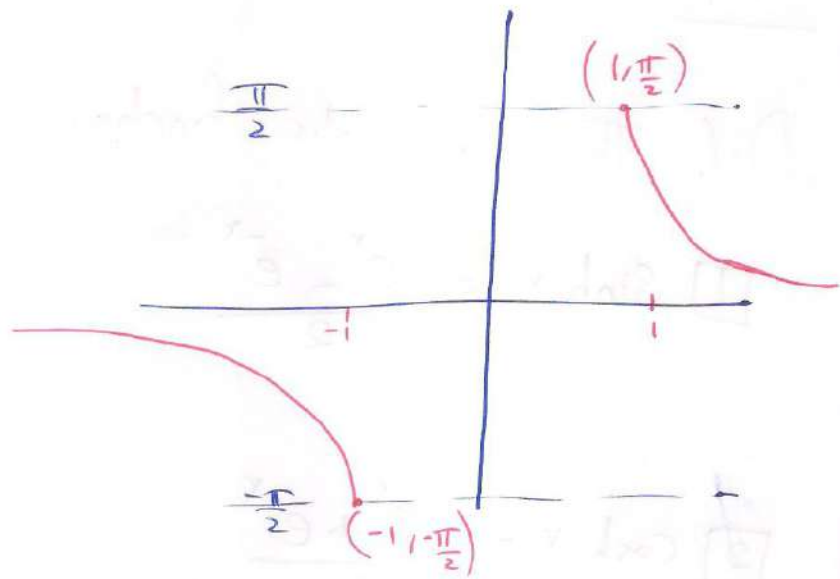
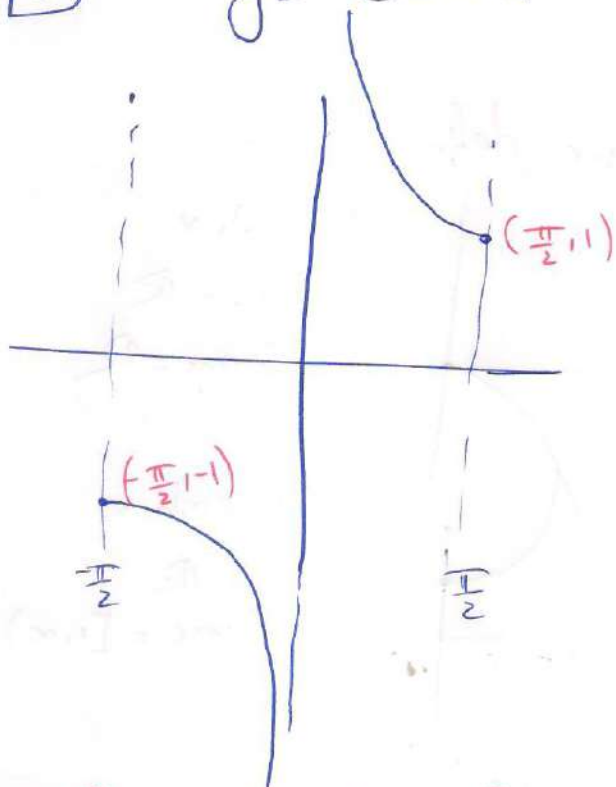
$$\sec(\sec^{-1} x) = x, \quad \text{if } x \in (-\infty, -1] \cup [1, \infty)$$

④ $\lim_{x \rightarrow \infty} \sec^{-1} x = \lim_{x \rightarrow -\infty} \sec^{-1} x = \frac{\pi}{2}$

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$$y = \csc x$$

$$y = \csc^{-1} x = \arccsc(x)$$



Some properties of $y = \csc^{-1} x$

$$\textcircled{1} \text{Dom}(\csc^{-1} x) = (-\infty, -1] \cup [1, \infty)$$

$$\text{Range}(\csc^{-1} x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$$

$$\textcircled{2} y = \csc^{-1} x \text{ is equivalent to } \csc y = x, \text{ if } x \in \text{Dom}(\csc^{-1} x)$$

$$\textcircled{3} \csc^{-1}(\csc x) = x, \text{ if } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$$

$$\csc(\csc^{-1} x) = x, \text{ if } x \in (-\infty, -1] \cup [1, \infty)$$

$$\textcircled{4} \lim_{x \rightarrow \infty} \csc^{-1} x = \lim_{x \rightarrow -\infty} \csc^{-1} x = 0$$