

Exp

$$f(x) = 3x - 2$$

Find

$$\frac{d}{dx} 3x - 2$$

$$m = 3$$

$$f'(x) = 3$$

$$① f(0) = 3(0) - 2 = 0 - 2 = -2$$

$$② f(1) = 3(1) - 2 = 3 - 2 = 1$$

$$③ f\left(-\frac{5}{3}\right) = 3\left(-\frac{5}{3}\right) - 2 = -5 - 2 = -7$$

$$④ f(1+h) = 3(1+h) - 2 = 3 + 3h - 2 = 1 + 3h$$

$$⑤ \frac{f(1+h) - f(1)}{h} = \frac{1 + 3h - 1}{h} = \frac{3h}{h} = 3, \quad h \neq 0$$

Operations with Function

Given f and g be any functions of x

$$① (f + g)(x) = f(x) + g(x)$$

$$② (f - g)(x) = f(x) - g(x)$$

$$③ (f \cdot g)(x) = f(x) \cdot g(x)$$

$$④ \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

$$\underline{\text{Exp}} \quad f(x) = \sqrt{x}, \quad g(x) = x^2 - 1 \quad \text{Find}$$

$$\begin{aligned}
 \textcircled{1} \quad (f + g)(4) &= f(4) + g(4) \\
 &= \sqrt{4} + (4^2 - 1) \\
 &= 2 + (16 - 1) \\
 &= 17
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad (f \cdot g)(1) &= f(1) \cdot g(1) \\
 &= \sqrt{1} \cdot (1^2 - 1) \\
 &= 1 \cdot 0 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \sqrt{x} \\
 g(x) &= x^2 - 1
 \end{aligned}$$

$$\textcircled{3} \quad \left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{\sqrt{4}}{4^2 - 1} = \frac{2}{16 - 1} = \frac{2}{15}$$

$$\textcircled{4} \quad (f \cdot f)(4) = f(4) \cdot f(4) = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4$$

Composition

f and g any functions

The composition function g of f is

$$(g \circ f)(x) = g(f(x))$$

(Note: In the original image, there are arrows indicating the flow of composition: an orange arrow from x to f , a pink arrow from f to g , and a pink arrow from f to $f(x)$. A small orange arrow also points to the $\circ$ symbol, which is labeled 'circle'.)

The composition function f of g

$$(f \circ g)(x) = f(g(x))$$

Ex 1 $f(x) = 1 - 2x$, $g(x) = 3x^2$ Find

$$\begin{aligned} \textcircled{1} (g - f)(x) &= g(x) - f(x) \\ &= 3x^2 - (1 - 2x) \\ &= 3x^2 - 1 + 2x \\ &= 3x^2 + 2x - 1 \end{aligned} \quad \left| \quad \begin{aligned} (g - f)(1) &= 3(1)^2 + 2(1) - 1 \\ &= 3 + 2 - 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \textcircled{2} (f \cdot g)(1) &= f(1) \cdot g(1) \\ &= (1 - 2(1)) \cdot 3(1)^2 \\ &= (1 - 2) \cdot 3 \\ &= -1 \cdot 3 \\ &= -3 \end{aligned}$$

$$f(x) = 1 - 2x$$

$$g(x) = 3x^2$$

$$\begin{aligned} \textcircled{3} (f \circ g)(x) &= f(g(x)) = f(3x^2) \\ &= 1 - 2(3x^2) \\ &= 1 - 6x^2 \end{aligned}$$

$$\textcircled{4} (f \circ g)(0) = \underline{1 - 6 \cdot (0)^2} = 1 - 0 = 1$$

$$f(x) = 1 - 2x$$

$$g(x) = 3x^2$$

$$\textcircled{5} (g \circ f)(x) = g(\underline{f(x)}) = g(\underline{1 - 2x})$$

$$= 3(1 - 2x)^2$$

$$\textcircled{6} (g \circ f)(0) = 3(1 - 2(0))^2 = 3(1 - 0)^2 = 3(1)^2 = 3$$

ليس دائماً

$$(g \circ f)(x) \neq (f \circ g)(x)$$

$$\left| \begin{array}{l} f(x) = 1 - 2x \\ g(x) = 3x^2 \end{array} \right.$$

$$\textcircled{7} (f \circ f)(x) = f(\underline{f(x)}) = f(\underline{1 - 2x})$$

$$= 1 - 2(1 - 2x)$$

$$= 1 - 2 + 4x$$

$$= -1 + 4x$$

$$f(x) = 3x^2 - 5x + 1$$

$$f(1) = 3(1)^2 - 5(1) + 1 = 3 - 5 + 1 = -1$$

$$f(2) = 3(2)^2 - 5(2) + 1 = 3(4) - 10 + 1 = 3$$

$$f(x) = 3(x)^2 - 5(x) + 1 = \dots$$