

37) $\sum_{n=1}^{\infty} (\ln \sqrt{n+1} - \ln \sqrt{n})$

10.2

1

$$S_n = \ln \sqrt{2} + \ln \sqrt{3} - \ln \sqrt{2} + \ln 2 - \ln \sqrt{3}$$

$$S_n = \ln \sqrt{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = \infty$$

Hence, $\sum_{n=1}^{\infty} (\ln \sqrt{n+1} - \ln \sqrt{n})$ div by n^{th} Partial Sum test

51) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n} = \frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \frac{3}{16}$

$$r = -\frac{2}{4} \cdot \frac{2}{3} \Rightarrow r = -\frac{1}{2} \in (-1, 1) \text{ conv}$$

Handwritten: *Handwritten sketch*

$$\text{Sum} = \frac{a}{1-r} = \frac{\frac{3}{2} \cdot \frac{2}{3}}{1 - (-\frac{1}{2})} = 1$$

Hence, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n}$ conv by Geometric Series

64) $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$

$$\lim_{n \rightarrow \infty} \frac{2^n + 4^n}{3^n + 4^n} = \lim_{n \rightarrow \infty} \frac{(\frac{1}{2})^n + 1}{(\frac{3}{4})^n + 1} = \frac{0+1}{0+1} = 1 \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

75) $\sum_{n=0}^{\infty} (-1)^n (x+1)^n$

$$S_n = 1 - (x+1) + (x+1)^2 - (x+1)^3 + \dots$$

$$r = -x-1 \in (-1, 1) \Rightarrow \text{conv}$$

$$-1 < -x-1 < 1$$

$$0 < -x < 2$$

$$0 > x > -2$$

$$\text{Sum} = \frac{a}{1-r} = \frac{1}{1+x+1} = \frac{1}{2+x}$$

$$6] \frac{5}{1 \cdot 2} + \frac{5}{2 \cdot 3} + \frac{5}{3 \cdot 4} + \dots + \frac{5}{n(n+1)} + \dots$$

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$$5 \left\{ \frac{51}{n(n+1)} \right\}$$

$$\frac{51}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$5 \left\{ \left(\frac{1}{n} - \frac{1}{n+1} \right) \right\}$$

$$= \frac{1}{n} - \frac{1}{n+1}$$

$$S_n = 5 \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} \right)$$

$$S_n = 5 \left(1 - \frac{1}{n+1} \right)$$

Hence, $\sum \frac{5}{n(n+1)}$ conv by n^{th} Partial Sum Test

$$\lim_{n \rightarrow \infty} S_n = 1.5 = 5$$

$$14] \sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n} \right) = \sum_{n=0}^{\infty} 2 \cdot \left(\frac{2}{5} \right)^n = 2 + \frac{4}{5} + \frac{8}{25}$$

$$r = \frac{2}{5} \in (-1, 1) \text{ conv}$$

$$\text{Sum} = \frac{a}{1-r} = \frac{2}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \frac{10}{3}$$

Hence, $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n} \right)$ conv by Geometric Series

$$18] \left(\frac{-2}{3} \right)^2 + \left(\frac{-2}{3} \right)^3 + \left(\frac{-2}{3} \right)^4 + \left(\frac{-2}{3} \right)^5 + \left(\frac{-2}{3} \right)^6 + \dots$$

$$\sum_{n=1}^{\infty} \left(\frac{-2}{3} \right)^{n+1}$$

$$r = \frac{-2}{3} \in (-1, 1) \text{ conv}$$

$$\text{Sum} = \left(\frac{-2}{3} \right)^2 \cdot \frac{3}{5} = \frac{4}{9} \cdot \frac{3}{5} = \frac{4}{15}$$

Hence, $\sum_{n=1}^{\infty} \left(\frac{-2}{3} \right)^{n+1}$ conv by Geometric Series

$$24) 1.4\overline{14} = 1.414141414\dots$$

$$= 1 + \left(\frac{414}{1000} + \frac{414}{1000000} + \dots \right)$$

$$r = \frac{1}{1000} \in (-1, 1) \text{ conv}$$

$$\text{Sum} = 1 + \left(\frac{414}{1000} \cdot \frac{1000}{999} \right) = \frac{414}{999} + 1 = \frac{1413}{999}$$

$$32) \sum_{n=0}^{\infty} \frac{e^n}{e^n + n}$$

$$\lim_{n \rightarrow \infty} \frac{e^n}{e^n + n} = \lim_{n \rightarrow \infty} \frac{e^n}{e^n + 1} = \lim_{n \rightarrow \infty} \frac{e^n}{e^n} = 1 \neq 0 \text{ div by } n^{\text{th}} \text{ term test}$$

$$38) \sum_{n=1}^{\infty} (\tan(n) - \tan(n-1))$$

$$S_n = \cancel{\tan 1} - \tan 0 + \cancel{\tan 2} - \cancel{\tan 1} + \tan 3 - \cancel{\tan 2} + \dots$$

$$S_n = \tan(n)$$

$$\lim_{n \rightarrow \infty} S_n = \infty$$

$$\text{Hence, } \sum_{n=1}^{\infty} (\tan(n) - \tan(n-1)) \text{ div by Partial Sum test}$$

$$44) \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$S_n = 1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{9}} + \cancel{\frac{1}{9}} - \frac{1}{16} + \dots$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

$$\lim_{n \rightarrow \infty} S_n = 1$$

$$\text{Hence, } \sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} \text{ conv by } n^{\text{th}} \text{ Partial Sum test}$$

54) $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

$\cos n\pi \Rightarrow (-1)^n$

$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n} = 1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{100} + \dots$

$r = -\frac{1}{5} \in (-1, 1)$ conv

Sum = $1 \cdot \frac{5}{6} = \frac{5}{6}$

Hence, $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$ conv by Geometric Series

62) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$n < \frac{n^n}{n!}$

$\sum_{n=1}^{\infty} n$

$\lim_{n \rightarrow \infty} n = \infty$

Hence. div

63) $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

$r = \frac{1}{2} \in (-1, 1)$

conv

Sum = $\frac{1}{2} \cdot \frac{2}{1} = 1$

$r = \frac{3}{4} \in (-1, 1)$

conv

Sum = $\frac{3}{4} \cdot \frac{4}{1} = 3$

$\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n} = 1 + 3 = 4$

Hence. conv by Geometric Series

$$78) \sum_{n=0}^{\infty} (\ln x)^n = 1 + \ln x + (\ln x)^2 + (\ln x)^3 + \dots$$

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$$r = \ln x \in (-1, 1) \rightarrow \text{conv}$$

$$-1 < \ln x < 1$$

$$\text{Sum} = \frac{a}{1-r} = \frac{1}{1-\ln x}$$

$$e^{-1} < e^{\ln x} < e^1$$

$$\boxed{\frac{1}{e} < x < e}$$

$$90) 1 + e^b + e^{2b} + e^{3b} + \dots = 9$$

$$\sum_{n=0}^{\infty} e^{nb} = 9$$

$$\text{Sum} = \frac{1}{1-e^b} = 9$$

$$\frac{1}{9} = 1 - e^b$$

$$e^b = \frac{8}{9}$$

$$b = \ln\left(\frac{8}{9}\right)$$