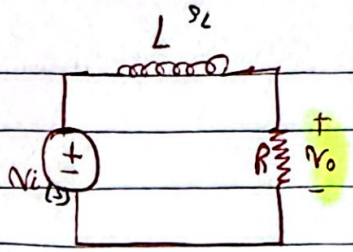
## LPF

### 1 RL




$$H(S) = \frac{R/L}{S + R/L}$$

$S = j\omega$

$\omega_c = \frac{R}{L} = 2\pi f$

$$H(j\omega) = \frac{\omega_c}{j\omega + \omega_c}$$

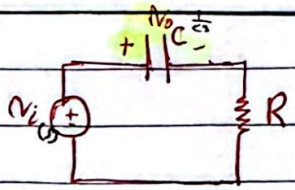
$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$$

$R/L = \omega_c$

$$\theta = -\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

### 2 RC




$$H(s) = \frac{\frac{1}{cs}}{R + \frac{1}{cs}}$$

$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

$S = j\omega$

$\omega_c = \frac{1}{RC} = 2\pi f$

$$H(j\omega) = \frac{\omega_c}{j\omega + \omega_c}$$

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

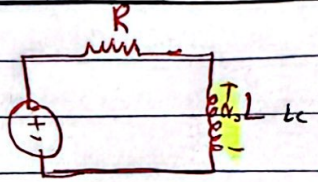
$\frac{1}{RC} = \omega_c$

$$\theta = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

## HPF

### 1 RL




$$H(S) = \frac{S}{S + R/L}$$

$$|H(j\omega)| = \frac{j\omega}{\sqrt{(\omega)^2 + (\omega_c)^2}}$$

$S = j\omega$

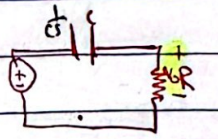
$\omega_c = \frac{R}{L} = 2\pi f$

$$H(S) = \frac{j\omega}{j\omega + \omega_c}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\theta = 90 - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

### 2 RC




$$H(S) = \frac{R}{R + \frac{1}{sC}} = \frac{S}{S + \frac{1}{RC}}$$

$$H(S) = \frac{j\omega}{j\omega + \omega_c}$$

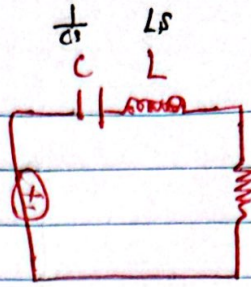
$\omega_c = \frac{1}{RC}$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\theta = 90 - \tan^{-1}\left(\frac{\omega}{\omega_c}\right)$$

# BPF

## (1) series RLC




$$H(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{s\left(\frac{R}{L}\right)}{s^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC}}$$

$|H(j\omega)|_{s=\frac{1}{\sqrt{2}}}$

$s = j\omega$

$\beta = \omega_{c1} - \omega_{c2} = \frac{R}{L}$

$H(s) = \frac{\beta s}{s^2 + \beta s + \frac{1}{LC}}$, $\omega_0^2$

$\omega_{c1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

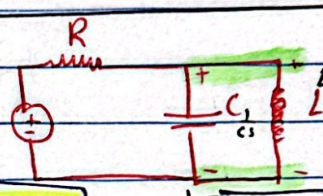
$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \frac{1}{\sqrt{LC}}$

$Q = \frac{\omega_0}{\beta} = \sqrt{\frac{L}{CR^2}}$

Remember: $\frac{R}{L} = \beta$, $\frac{1}{LC} = \omega_0^2$

## (2) Parallel RLC

$Z_{eq} = \frac{1/C}{\frac{1}{sC} + sL}$




$|H(j\omega)|_{s=\frac{1}{\sqrt{2}}}$

$H(s) = \frac{Z_{eq}}{Z_{eq} + R}$

$H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$, $H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}$

$\omega_{c1} = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$

$\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC}$

$\omega_0 = \frac{1}{\sqrt{LC}} \to \sqrt{\omega_{c1}\omega_{c2}}$

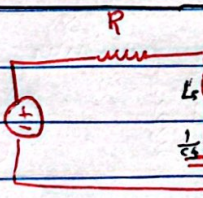
$Q = \frac{\omega_0}{\beta} = \sqrt{\frac{R^2 C}{L}}$

$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$

Rem. $\frac{1}{RC} = \beta$, $\frac{1}{LC} = \omega_0^2$

# BRF

## (1) series RLC




$H(s) = \frac{s^2 + 1/LC}{s^2 + s\left(\frac{R}{L}\right) + \frac{1}{LC}}$

$H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}$

$\omega_{c1} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

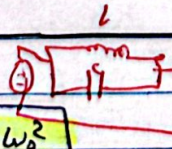
$\beta = \frac{R}{L} = \omega_{c2} - \omega_{c1}$

$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \frac{1}{\sqrt{LC}}$

Rem. $\frac{R}{L} = \beta$, $\frac{1}{LC} = \omega_0^2$

$Q = \frac{\omega_0}{\beta} = \sqrt{\frac{L}{CR^2}}$

## (2) Parallel RLC




$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$

$H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}$

$\omega_{c1} = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$

$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$

$\beta = \omega_{c1} - \omega_{c2} = \frac{1}{RC}$

$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \frac{1}{\sqrt{LC}}$

$Q = \frac{\omega_0}{\beta} = \sqrt{\frac{R^2 C}{L}}$

Rem. $\frac{1}{RC} = \beta$, $\frac{1}{LC} = \omega_0^2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ , $[z]$ is the impedance matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{22} & y_{21} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ , $[Y] = [z]^{-1}$ is the admittance matrix.

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$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$ , $[A]$ is a transmission matrix.

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & -b_{12} \\ b_{21} & -b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$ , $[B] = [A]^{-1}$ is a transmission matrix.

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$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$ , $[H]$ is a hybrid matrix.

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$ , $[G] = [H]^{-1}$ is a hybrid matrix

| | | | |
|---|---|---|---|
| $z_{11} = \left.\frac{V_1}{I_1}\right\|_{I_2=0}$ O.C | $z_{12} = \left.\frac{V_1}{I_2}\right\|_{I_1=0}$ O.C | $y_{11} = \left.\frac{I_1}{V_1}\right\|_{V_2=0}$ | $y_{12} = \left.\frac{I_1}{V_2}\right\|_{V_1=0}$ |
| $z_{21} = \left.\frac{V_2}{I_1}\right\|_{I_2=0}$ O.C | $z_{22} = \left.\frac{V_2}{I_2}\right\|_{I_1=0}$ O.C | $y_{21} = \left.\frac{I_2}{V_1}\right\|_{V_2=0}$ | $y_{22} = \left.\frac{I_2}{V_2}\right\|_{V_1=0}$ |
| $a_{11} = \left.\frac{V_1}{V_2}\right\|_{I_2=0}$ | $a_{12} = -\left.\frac{V_1}{I_2}\right\|_{V_2=0}$ | $b_{11} = \left.\frac{V_2}{V_1}\right\|_{I_1=0}$ | $b_{12} = \left.\frac{-V_2}{I_1}\right\|_{V_1=0}$ |
| $a_{21} = \left.\frac{I_1}{V_2}\right\|_{I_2=0}$ | $a_{22} = -\left.\frac{I_1}{I_2}\right\|_{V_2=0}$ | $b_{21} = \left.\frac{I_2}{V_1}\right\|_{I_1=0}$ | $b_{22} = \left.\frac{-I_2}{I_1}\right\|_{V_1=0}$ |
| $h_{11} = \left.\frac{V_1}{I_1}\right\|_{V_2=0}$ | $h_{12} = \left.\frac{V_1}{V_2}\right\|_{I_1=0}$ | $g_{11} = \left.\frac{I_1}{V_1}\right\|_{I_2=0}$ | $g_{12} = \left.\frac{I_1}{I_2}\right\|_{V_1=0}$ |
| $h_{21} = \left.\frac{I_2}{I_1}\right\|_{V_2=0}$ | $h_{22} = \left.\frac{I_2}{V_2}\right\|_{I_1=0}$ | $g_{21} = \left.\frac{V_2}{V_1}\right\|_{I_2=0}$ | $g_{22} = \left.\frac{V_2}{I_2}\right\|_{V_1=0}$ |