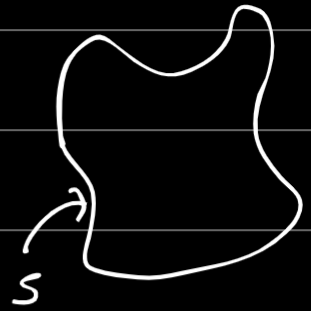


Gauss's Law:

Integral form: $\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$

Differential Form

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$



⌘ Spherical Symmetry

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \int_V \rho(\vec{r}) d\tau$$

$$\rho(\vec{r}) = \rho(r) \quad \leftarrow \text{spherical symmetry}$$

$$\rightarrow \vec{E}(\vec{r}) \rightarrow \vec{E}(r) \rightarrow \vec{E} = E(r) \hat{r}$$

Note: $\rho(\vec{r}) = \begin{cases} \vec{a} \cdot \vec{r} & r < d \\ 0 & r \geq d \end{cases} \neq \rho(r)$

$$\vec{a} = 2\hat{x} + 3\hat{y} + 4\hat{z}$$

$$\vec{r}_1 = 2\hat{x}, \quad \vec{r}_2 = -2\hat{x}$$

$$\rho(\vec{r}) = \frac{\sin(x) \cos(y)}{z}$$

Now,

Inside:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_0^{2\pi} \int_0^\pi E \hat{r} \cdot (r^2 \sin\theta d\theta d\phi \hat{r})$$

$$= \frac{1}{\epsilon_0} \int_0^r \int_0^{2\pi} \int_0^\pi \rho(r) r^2 \sin\theta d\theta d\phi$$

$$4\pi r^2 E = \frac{1}{\epsilon_0} 4\pi \int_0^r \rho(r) r^2 dr$$

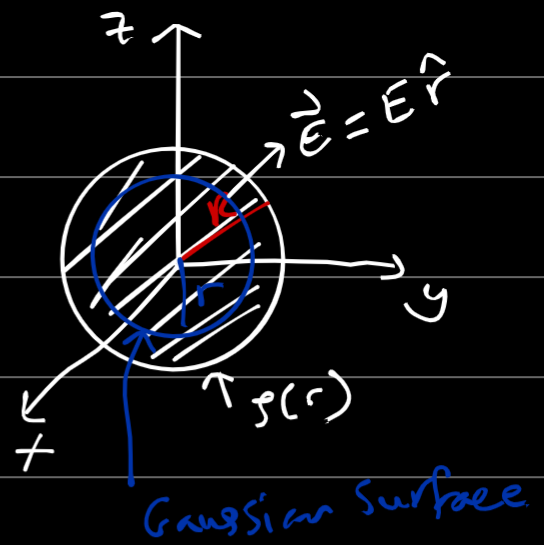
$$E = \frac{1}{\epsilon_0 r^2} \int_0^r r^2 \rho(r) dr \quad r < R$$

outside

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow 4\pi r^2 E = 4\pi \frac{1}{\epsilon_0} \int_0^R r^2 \rho(r) dr$$

$$\Rightarrow E = \frac{1}{\epsilon_0 r^2} \int_0^R r^2 \rho(r) dr$$

$$\vec{E}(\vec{r}) = \begin{cases} \frac{\hat{r}}{\epsilon_0 r^2} \int_0^r r^2 \rho(r) dr & r \leq R \\ \frac{\hat{r}}{\epsilon_0 r^2} \int_0^R r^2 \rho(r) dr & r > R \end{cases}$$



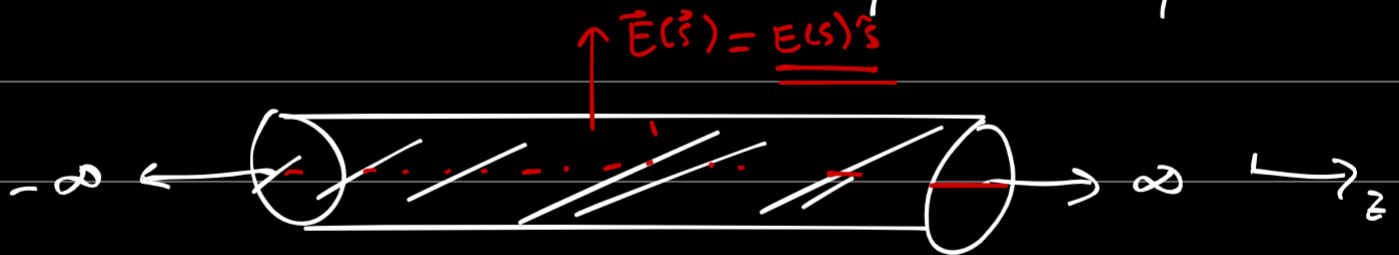
Example: $\rho(r) = \begin{cases} \frac{r}{R} \rho_0 & r \leq R \\ 0 & r > R \end{cases}$

$$\vec{E}(\vec{r}) = \begin{cases} \frac{\hat{r}}{\epsilon_0 r^2} \int_0^r \frac{r}{R} \rho_0 r'^2 dr' & r \leq R \\ \frac{\hat{r}}{\epsilon_0 r^2} \int_0^R \frac{r}{R} \rho_0 r'^2 dr' & r > R \end{cases}$$

$$\vec{E}(\vec{r}) = \begin{cases} \frac{r^2 \rho_0 \hat{r}}{4 \epsilon_0 R} & r \leq R \\ \frac{R^3 \rho_0}{\epsilon_0 r^2} \hat{r} & r > R \end{cases}$$

③ Cylindrical Symmetry

$\rho(\vec{s}) = \rho(s) \leftarrow$ It does not depend on ϕ and z



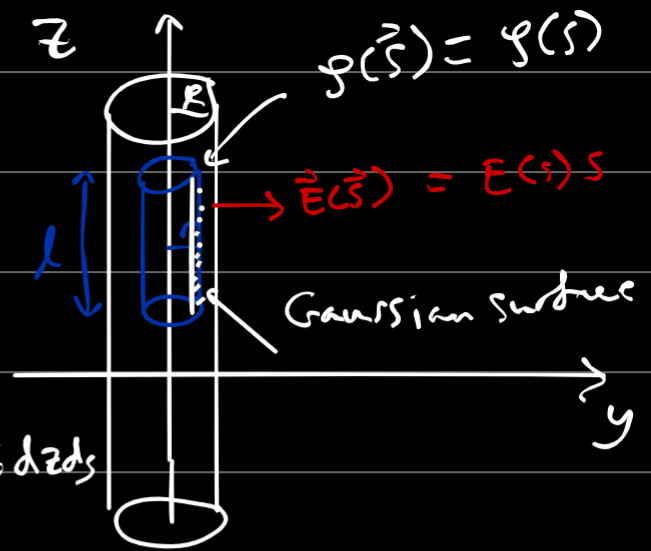
e.g. $\rho(\vec{s}) = \begin{cases} -k s^2 & s \leq R \\ 0 & \text{otherwise} \end{cases} = \rho(s) \quad \checkmark$

$\rho(\vec{s}) = \begin{cases} \vec{a} \cdot \vec{s} & s \leq R \\ 0 & s > R \end{cases} \neq \rho(s) \quad \times$

inside (i.e. $r \leq R$)

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\int_0^{2\pi} E \hat{s} \cdot (l s d\phi \hat{s}) = \frac{1}{\epsilon_0} \int_0^s \int_0^{2\pi} \int_0^l \rho(s) s d\phi dz ds$$



$$2\pi l s E = \frac{2\pi l}{\epsilon_0} \int_0^s s \rho(s) ds$$

$$\Rightarrow \vec{E} = \frac{1}{s \epsilon_0} \int_0^s s \rho(s) ds \hat{s}$$

outside: $r \geq R$

$$\vec{E} = \frac{1}{s \epsilon_0} \int_0^R s \rho(s) ds \hat{s}$$

$$\vec{E} = E(s) \hat{s} = \begin{cases} \frac{\hat{s}}{s \epsilon_0} \int_0^s s \rho(s) ds & s \leq R \\ \frac{\hat{s}}{s \epsilon_0} \int_0^R s \rho(s) ds & s \geq R \end{cases}$$

Example: Consider a cylinder of radius R that have a volume charge density given by

$$\rho(\vec{s}) = -k s^2$$

$$\vec{E}(\vec{r}) = E(s)\hat{s} = \begin{cases} \frac{\hat{s}}{s\epsilon_0} \int_0^s s(-ks^2) ds & s \leq R \\ \frac{\hat{s}}{s\epsilon_0} \int_0^R s(-ks^2) ds & s \geq R \end{cases}$$

$$= \begin{cases} -\frac{k s^3}{4\epsilon_0} \hat{s} & s \leq R \\ -\frac{k R^4}{4\epsilon_0 s} \hat{s} & s \geq R \end{cases}$$