(2.2) exercises

1) a. $\lim_{x\to 1} g(x) \Rightarrow \lim_{x\to 1^+} g(x) = 0$, $\lim_{x\to 1^-} g(x) = 1 \Rightarrow \lim_{x\to 1^+} g(x) \neq \lim_{x\to 1^-} g(x)$ so limit DNE.

b. $\lim_{x\to 2} g(x) = 1$ } c. $\lim_{x\to 3} g(x) = 0$ { d. $\lim_{x\to 2.5} g(x) = 2.5$ because $g(x) = -x+3 \ \forall x \in [2,3]$

2) a. $\lim_{t\to 2} f(t) = 0$ { b. $\lim_{t\to -1} f(t) = -1$ } c. $\lim_{t\to 0} f(t) \Rightarrow \left(\lim_{t\to 0^+} f(t) = 1\right) \neq \left(\lim_{t\to 0^-} f(t) = -1\right)$ limit DNE.

d. $\lim_{t\to 0.5} f(t) = 1$, because $f(t) = 1 \ \forall t \in [0,\infty]$

3) a. true , b. true , c. false , d. False , e. false , f. True , g. True

4) a. False, b. False , c. True , D. True , E. True

5) a. $\lim_{x\to 0} \frac{x}{|x|} \Rightarrow \lim_{x\to 0^+} \frac{x}{x} = 1$, $\lim_{x\to 0^-} \frac{x}{-x} = -1 \Rightarrow \lim_{x\to 0^+} \frac{x}{x} \neq \lim_{x\to 0^-} \frac{x}{x} \Rightarrow$ limit DNE.

6) $\lim_{x\to 1} \frac{1}{x-1} \Rightarrow \lim_{x\to 1^+} \frac{1}{x-1} = \infty$, $\lim_{x\to 1^-} \frac{1}{x-1} = -\infty \Rightarrow \lim_{x\to 1^+} \frac{1}{x-1} \neq \lim_{x\to 1^-} \frac{1}{x-1} \Rightarrow$ lim DN

7) it may exist or may not

8) it May exist or may not

9) it may be 5 or not

10) no

Find limits

11) $\lim_{x\to -7} (2x+5) = -14+5 = -11$ } 12) $\lim_{x\to 2} (-x^2+5x-2) = -4+10-2 = 4$

13) $\lim_{t\to 6} 8(t-5)(t-7) = 8(1)(-1) = -8$ } 14) $\lim_{x\to -2} (x^3-2x^2+4x+8) = -8-2\times 4 + -8+8 = -$

15) $\lim_{x\to 2} \frac{x+3}{x+6} = \frac{5}{8}$ } 16) $\lim_{s\to \frac{2}{3}} (3s)(2s-1) = 2\left(\frac{1}{3}\right) = \frac{2}{3}$ } 17) $\lim_{x\to -1} 3(2x-1)^2 = 3(-3)^2 = 2$

18) $\lim_{y\to 2} \frac{y+2}{y^2+5y+6} = \frac{4}{4+10+6} = \frac{1}{5}$ | 19) $\lim_{y\to -3} (5-y)^{\frac{4}{3}} = \sqrt[3]{(8)^4} = 2^4 = 16$

20) $\lim_{z\to 0} (z-8)^{\frac{1}{3}} = (-8)^{\frac{1}{3}} = -2$ | 21) $\lim_{h\to 0} \frac{3}{\sqrt{3h+1}+1} = \frac{3}{2}$

22) $\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{(\sqrt{5h+4}+2)}{(\sqrt{5h+4}+2)} = \lim_{h\to 0} \frac{5h+\cancel{4}-\cancel{4}}{h(\sqrt{5h+4}+2)} = \lim_{h\to 0} \frac{5}{\sqrt{5h+4}+2} = \frac{5}{4}$

2.2 exeercsices

23) $\lim\limits_{x\to 5} \frac{x-5}{x^2-25} = \lim\limits_{x\to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(x+5)} = \frac{1}{5+5} = \frac{1}{10}$

24) $\lim\limits_{x\to -3} \frac{x+3}{x^2+4x+3} = \lim\limits_{x\to -3} \frac{\cancel{x+3}}{\cancel{(x+3)}(x+1)} = \frac{1}{-3+1} = \frac{-1}{2}$

25) $\lim\limits_{x\to -5} \frac{x^2+3x-10}{x+5} = \lim\limits_{x\to -5} \frac{\cancel{(x+5)}(x-2)}{\cancel{(x+5)}} = -5-2 = -7$

26) $\lim\limits_{x\to 2} \frac{x^2-7x+10}{x-2} = \lim\limits_{x\to 2} \frac{(x-5)\cancel{(x-2)}}{\cancel{(x-2)}} = 2-5 = -3$

27) $\lim\limits_{t\to 1} \frac{t^2+t-2}{t^2-1} = \lim\limits_{t\to 1} \frac{(t+2)\cancel{(t-1)}}{\cancel{(t-1)}(t+1)} = \frac{3}{2}$

28) $\lim\limits_{t\to -1} \frac{t^2+3t+2}{t^2-t-2} = \lim\limits_{t\to -1} \frac{(t+2)\cancel{(t+1)}}{(t-2)\cancel{(t+1)}} = \frac{1}{-3}$

29) $\lim\limits_{x\to -2} \frac{-2x-4}{x^3+2x^2} = \lim\limits_{x\to -2} \frac{-2\cancel{(x+2)}}{x^2\cancel{(x+2)}} = \frac{-2}{4} = \frac{-1}{2}$

30) $\lim\limits_{y\to 0} \frac{5y^3+8y^2}{3y^4-16y^2} = \lim\limits_{y\to 0} \frac{\cancel{y^2}(5y+8)}{\cancel{y^2}(3y^2-16)} = \frac{8}{-16} = \frac{-1}{2}$

31) $\lim\limits_{x\to 1} \frac{\frac{1}{x}-1}{x-1} = \lim\limits_{x\to 1} \frac{\cancel{1-x}(-1)}{x\cancel{(x-1)}} = \frac{-1}{1} = -1$

32) $\lim\limits_{x\to 0} \frac{\frac{1}{x-1}+\frac{1}{x+1}}{x} = \lim\limits_{x\to 0} \frac{x+1+x-1}{x(x^2-1)} = \lim\limits_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(x^2-1)} = -2$

33) $\lim\limits_{v\to 1} \frac{v^4-1}{v^3-1} = \lim\limits_{v\to 1} \frac{(v^2+1)\cancel{(v-1)}(v+1)}{\cancel{(v-1)}(v^2+v+1)} = \frac{2\times 2}{3} = \frac{4}{3}$

34) $\lim\limits_{v\to 2} \frac{v^3-8}{v^4-16} = \lim\limits_{v\to 2} \frac{\cancel{(v-2)}(v^2+2v+4)}{(v^2+4)\cancel{(v-2)}(v+2)} = \frac{4+4+4}{(4+4)(4)} = \frac{3}{8}$

35) $\lim\limits_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim\limits_{x\to 9} \frac{\cancel{(\sqrt{x}-3)}}{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

36) $\lim\limits_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}} = \lim\limits_{x\to 4} \frac{x(4-x)}{2-\sqrt{x}} = \lim\limits_{x\to 4} \frac{x\cancel{(2-\sqrt{x})}(2+\sqrt{x})}{\cancel{(2-\sqrt{x})}} = 4(2+\sqrt{4}) = 16$

37) $\lim\limits_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} = \lim\limits_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{(\sqrt{x+3}+2)}{(\sqrt{x+3}+2)} = \lim\limits_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x+3-4)}} = 4$

(2.2) exercises

38) $\lim\limits_{x\to -1} \dfrac{\sqrt{x^2+8}-3}{x+1} \cdot \dfrac{(\sqrt{x^2+8}+3)}{(\sqrt{x^2+8}+3)} = \lim\limits_{x\to -1} \dfrac{(x^2+8-9)}{(x+1)(\sqrt{x^2+8}+3)} = \lim\limits_{x\to -1} \dfrac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(\sqrt{x^2+8}+3)} = \dfrac{-2}{6} = \dfrac{-1}{3}$

39) $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2+12}-4}{x-2} \cdot \dfrac{(\sqrt{x^2+12}+4)}{(\sqrt{x^2+12}+4)} = \lim\limits_{x\to 2} \dfrac{(x^2+12-16)}{(x-2)(\sqrt{x^2+12}+4)} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(\sqrt{x^2+12}+4)} = \dfrac{4}{8} = \dfrac{1}{2}$

40) $\lim\limits_{x\to -3} \dfrac{2-\sqrt{x^2-5}}{x+3} \cdot \dfrac{(2+\sqrt{x^2-5})}{(2+\sqrt{x^2-5})} = \lim\limits_{x\to -3} \dfrac{4-(x^2-5)}{(x+3)(2+\sqrt{x^2-5})} = \lim\limits_{x\to -3} \dfrac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}\,2+\sqrt{x^2-5}} = \dfrac{-6}{4} = \dfrac{-3}{2}$

41) $\lim\limits_{x\to -2} \dfrac{x+2}{\sqrt{x^2+5}-3} \cdot \dfrac{(\sqrt{x^2+5}+3)}{(\sqrt{x^2+5}+3)} = \lim\limits_{x\to -2} \dfrac{(x+2)(\sqrt{x^2+5}+3)}{(x^2+5-9)} = \lim\limits_{x\to -2} \dfrac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{(x-2)\cancel{(x+2)}} = \dfrac{6}{-4} = \dfrac{-3}{2}$

42) $\lim\limits_{x\to 4} \dfrac{4-x}{5-\sqrt{x^2+9}} \cdot \dfrac{(5+\sqrt{x^2+9})}{(5+\sqrt{x^2+9})} = \lim\limits_{x\to 4} \dfrac{(4-x)(5+\sqrt{x^2+9})}{25-(x^2+9)} = \lim\limits_{x\to 4} \dfrac{\cancel{(4-x)}(5+\sqrt{x^2+9})}{\cancel{(4-x)}(4+x)} = \dfrac{10}{8} = \dfrac{5}{4}$

43) $\lim\limits_{x\to 0} (2\sin x - 1) = 2\sin 0 - 1 = 0 - 1 = -1$

44) $\lim\limits_{x\to 0} \sin^2 x = (\sin 0)^2 = 0$

45) $\lim\limits_{x\to 0} \sec x = 1$

46) $\lim\limits_{x\to 0} \tan x = 0$

47) $\lim\limits_{x\to 0} \dfrac{1+x+\sin x}{3\cos x} = \dfrac{1+0+0}{3} = \dfrac{1}{3}$

48) $\lim\limits_{x\to 0} (x^2-1)(2-\cos x) = -1 \times (2-1) = -1$

49) $\lim\limits_{x\to \pi} \sqrt{x+4}\,\cos(x+\pi) = \sqrt{4+\pi}\,\cos 2\pi = \sqrt{4+\pi}$

50) $\lim\limits_{x\to 0} \sqrt{7+\sec^2 x} = \sqrt{7+1} = 2\sqrt{2}$

skip 51, 52 ~~too easy~~, 53, 54, 55, 56 too easy

63) $\lim\limits_{x\to 0} \sqrt{5-2x^2} = 5$, $\lim\limits_{x\to 0} \sqrt{5-2x^2} = 5$

by sandwich theorem: $\lim\limits_{x\to 0} f(x) = 5$

64) $\lim\limits_{x\to 0} 2-x^2 = 2$, $\lim\limits_{x\to 0} 2\cos x = 2 \Rightarrow$ by sandwich theorem $\lim\limits_{x\to 0} g(x) = 2$

65) it doesnot have acertain limit DNE