

wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

$$\psi = \sin(kx \pm \omega t)$$

$k = \frac{2\pi}{\lambda}$	$\omega = \frac{2\pi}{T}$	$\frac{\omega}{k} = v$
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Phasors and addition of waves

$y_1 = A_1 e^{i\phi_1}$	$y_2 = A_2 e^{i\phi_2}$	$\delta = \phi_2 - \phi_1$
$y_r = y_1 + y_2$	$y_r = y_r e^{i\phi}$	

$ y_r = A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta$	$\tan \phi = \frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}$
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Spherical waves	Cylindrical waves
$\vec{E}(x, y, z, t) = \frac{\vec{E}_0}{r} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$	$\vec{E}(x, y, z, t) = \frac{\vec{E}_0}{\sqrt{r}} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

Electromagnetic theory

$\vec{\nabla} \cdot \vec{D} = \rho$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{\nabla} \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0(1 + \chi) \vec{E} = \epsilon \vec{E} = \epsilon_0 n^2 \vec{E}$	$\vec{B} = \mu_0 \vec{H} + \vec{M}$
$n = \sqrt{\frac{\epsilon}{\epsilon_0}}$	$\vec{j} = \sigma \vec{E}$

Electromagnetic wave equation

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{c}{n}$	$\vec{k} \cdot \vec{E} = 0$	$\vec{k} \times \vec{E} = \omega \vec{B}$	$ \vec{B} = \frac{n}{c} \vec{E} $	$\frac{ \vec{E} }{ \vec{H} } = \sqrt{\frac{\mu}{\epsilon}} = Z$
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Energy and momentum

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{pointing vector}$$

$$I = |\langle \vec{S} \rangle| = \frac{1}{2} E_0 H_0 = \frac{1}{2} c \epsilon_0 E_0^2 = c \langle u \rangle \quad I: \text{irradiance}, \langle u \rangle: \text{average energy density}$$

$\langle \vec{F} \rangle = \langle \frac{d\vec{P}}{dt} \rangle = \hat{k} q \langle v_x B \rangle$	$\langle \frac{dU}{dt} \rangle = c q \langle v_x B \rangle$	$\langle \frac{d\vec{P}}{dt} \rangle = \frac{1}{c} \langle \frac{dU}{dt} \rangle$
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Damped oscillator model

$$m(\ddot{x} + \gamma \dot{x} + \omega_0^2 x) = q E_0 e^{i\omega t}$$

$$x_0 = \frac{q E_0}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$p = qx = \frac{N q^2 E_0 e^{i\omega t}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

dipole moment for N oscillator per unit volume (macroscopic polarization)

$$n = 1 + \frac{N q^2}{2m\epsilon_0(\omega_0^2 - \omega^2 + i\gamma\omega)} = n' + i n''$$

$n' = 1 + \frac{N q^2}{2m\epsilon_0} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$	$n'' = \frac{N q^2}{2m\epsilon_0} \frac{\gamma \omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$
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For $\omega_0 \ll \omega$:

$n' = 1 + \frac{N q^2}{2m\epsilon_0 \omega_0} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + (\frac{\gamma}{2})^2}$	$n'' = \frac{N q^2}{8m\epsilon_0 \omega_0} \frac{\gamma}{(\omega_0 - \omega)^2 + (\frac{\gamma}{2})^2}$
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Refraction

$r_{\perp} = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$	$t_{\perp} = \frac{E_{0t}}{E_{0i}} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t}$
$r_{\parallel} = \frac{E_{0r}}{E_{0i}} = \frac{n_i \cos \theta_t - n_t \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$	$t_{\parallel} = \frac{E_{0t}}{E_{0i}} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i}$

$\theta_{Broster} = \tan^{-1} \frac{n_t}{n_i}$	$\theta_{Critical} = \sin^{-1} \frac{n_t}{n_i}$
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Reflected power	Transmittance power	
$R = \frac{I_r A_r}{I_i A_i} = r^2$	$T = \frac{I_t A_t}{I_i A_i} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t^2$	$I = \frac{n \epsilon_0 c_0}{2} E_0 ^2$

Lenses

Refracting prisms	Spherical	Convex lens
$\frac{n_{prism}}{n_0} = \sin \frac{\frac{1}{2}(A+D)}{\sin \frac{1}{2}A}$	$\frac{n_1}{s_{01}} + \frac{n_2}{s_{i1}} = \frac{n_2 - n_1}{R_1}$	$\frac{1}{s_{01}} + \frac{1}{s_{i1}} = \left(\frac{n_i}{n_m} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$

Mirror

$-\frac{h_0}{h_i} = \frac{d_0}{d_i}$	$\frac{1}{d_0} + \frac{1}{d_i} = \frac{2}{R} = \frac{1}{f}$	$magnification = \frac{h_i}{h_0}$
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Simple telescope

$$M = \frac{\theta'}{\theta} = \frac{f_0}{f_e}$$

Compound lens

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Ray tracing

Translation	Refraction
$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$	$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & \frac{n_2}{n_1} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$

Curved lens

n_1 to n_2	n_2 to n_3
$\begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_1 - n_2} & \frac{0}{n_1} \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$	$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_2 - n_3} & \frac{0}{n_2} \\ \frac{n_2 - n_3}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} x_2 \\ \gamma_2 \end{bmatrix}$

Thin lens

$$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_2 - n_3} & \frac{0}{n_2} \\ \frac{n_2 - n_3}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} + \begin{bmatrix} \frac{1}{n_1 - n_2} & \frac{0}{n_1} \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$$

For $n_1 = n_3 = 1$ and $R_1 = -R_2$:

$$\begin{bmatrix} x_3 \\ \gamma_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \gamma_1 \end{bmatrix}$$

$$f = \frac{R}{2(n-1)}$$

Imaging by thin lens

$$\begin{bmatrix} x_i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} 1 - \frac{s_i}{f} & s_0 + s_i - \frac{s_0 s_i}{f} \\ -\frac{1}{f} & 1 - \frac{s_0}{f} \end{bmatrix} \begin{bmatrix} x_0 \\ \gamma_0 \end{bmatrix}$$

$$s_0 + s_i - \frac{s_0 s_i}{f} = 0 \rightarrow \frac{1}{s_0} + \frac{1}{s_i} = \frac{1}{f}$$

magnification	angular magnification
$\beta_l = 1 - \frac{s_i}{f}$	$\beta_v = 1 - \frac{s_0}{f}$

$\beta_l \beta_v = 1$	$x_i \gamma_i = x_0 \gamma_0$
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Thick lens and lenses system

$$M = \begin{bmatrix} \frac{1}{n_2 - n_3} & \frac{0}{n_2} \\ \frac{n_2 - n_3}{n_3 R_2} & \frac{n_2}{n_3} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{n_1 - n_2} & \frac{0}{n_1} \\ \frac{n_1 - n_2}{n_2 R_1} & \frac{n_1}{n_2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$A = 1 + \frac{n_1 - n_2}{n_2 R_1} d$$

$$B = \frac{n_1}{n_2} d$$

$$C = \frac{1}{n_3} \left(\frac{n_2 - n_3}{R_2} + \frac{n_1 - n_2}{R_1} + \frac{n_1 - n_2}{n_2 R_1} \frac{n_2 - n_3}{R_2} d \right)$$

$$D = \frac{n_1}{n_3} \left(1 + \frac{n_2 - n_3}{n_2 R_2} d \right)$$

Thin lens

Take: $n_1 = n_3$ and $d = 0$

$A = 1$	$B = 0$	$C = \frac{1}{n_1} \left(\frac{n_2 - n_1}{R_2} + \frac{n_1 - n_2}{R_1} \right)$	$D = 1$
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At $-R_1 = R_2 = -R$:

$$C = -\frac{2}{n_1} \left(\frac{n_2 - n_1}{R} \right)$$

$$\text{At } n_1 = 1: C = -\frac{1}{f}$$

