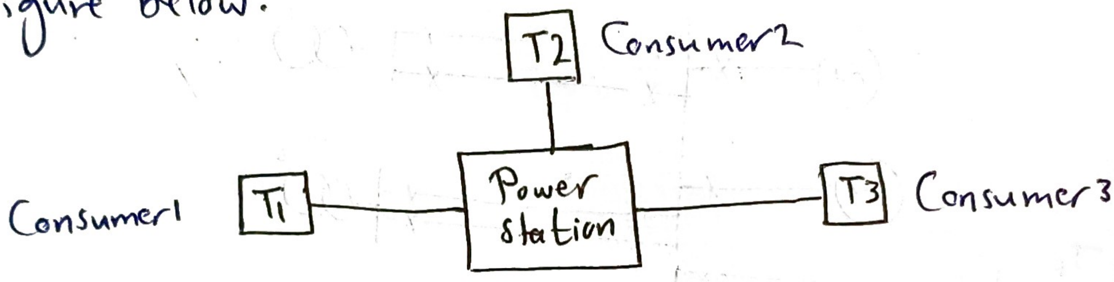
Power Flow Analysis

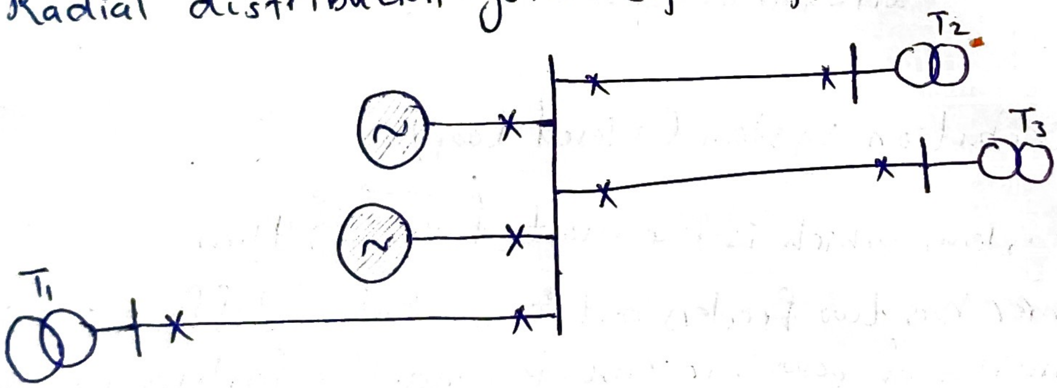
>> The development of simple distribution system [open-loop / closed-loop] Network arrangements

When a consumer requests electrical power from a supply authority, ideally all that is required is a cable and a transformer, shown physically as in Figure below.



A simple distribution system

① Radial distribution system (open loop)



Advantages

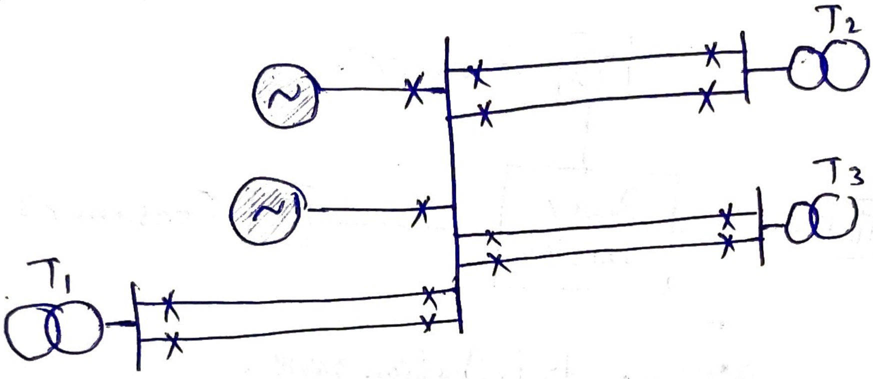
If a fault occurs at T2 then only the protection on one leg connecting T2 is called into operation to isolate this leg. The other consumers are not affected.

Disadvantages

If the conductor to T2 fails, then supply to this particular consumer is lost completely and cannot be restored until the conductor is replaced / repaired.

② Radial distribution system with parallel feeders (open Loop)

This disadvantage (radial) can be overcome by introducing add[illegible] (parallel) feeders (as shown below) connecting each of the consumers radially. However, this requires more cabling and is not always economical.



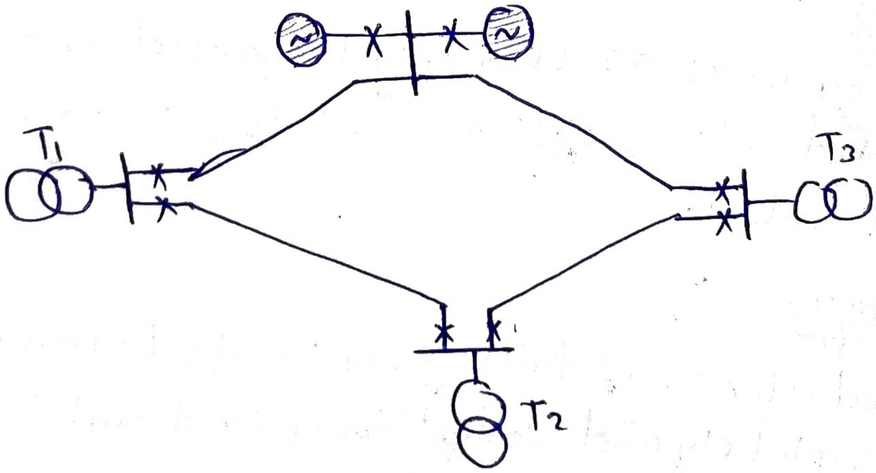
Radial distribution system with parallel feeders

Loop

③ Ring main distribution system (closed Loop)

The Ring main system, which is the most مفضل favored.

Here each consumer has two feeders but connected in different paths to ensure continuity of power, in case of conductor failure in any section.



<u>Advantages</u>:

- Essentially, meets the requirements of two alternative feeds to give 100% continuity of supply, بينما whilst saving in cabling compared to parallel feeds.

Disadvantages:-

For faults at T_1 fault current is fed into fault via two parallel paths effectively reducing the impedance from the source to the fault location, and hence the fault current is much higher compared to a radial path. The fault current in particular could vary depending on the exact location of the fault.

Protection must therefore be fast and discriminate correctly. so that other consumers are not interrupted.

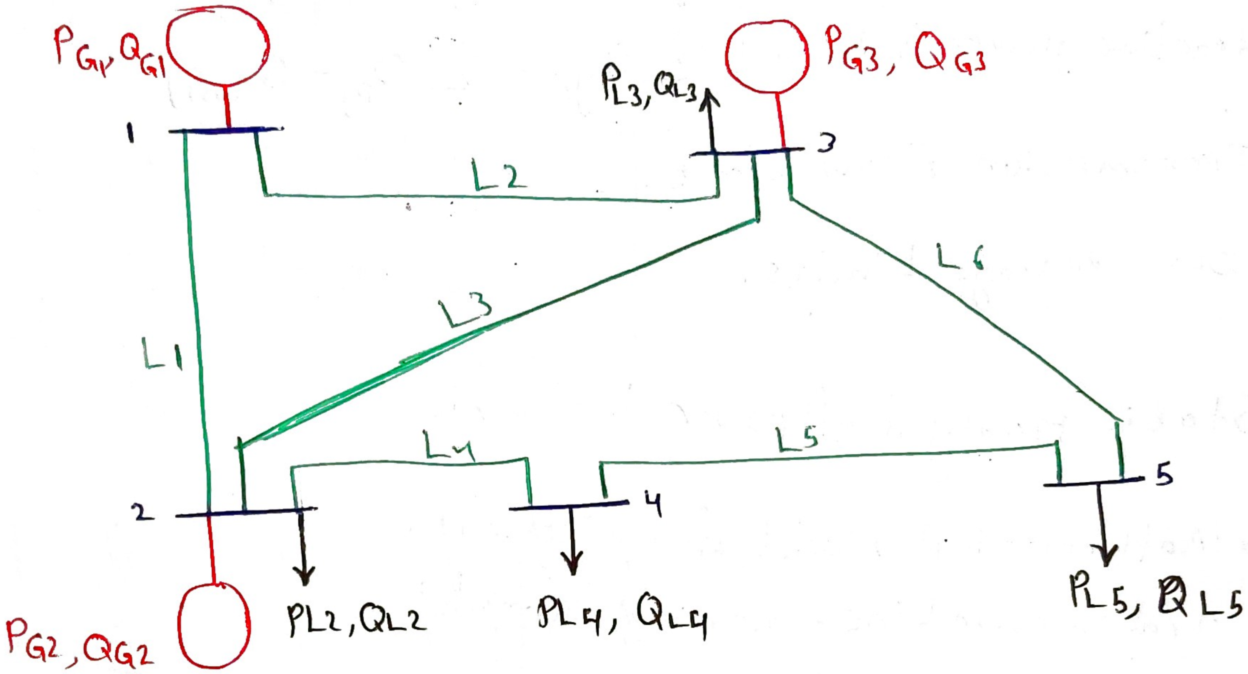
$V = Z I$

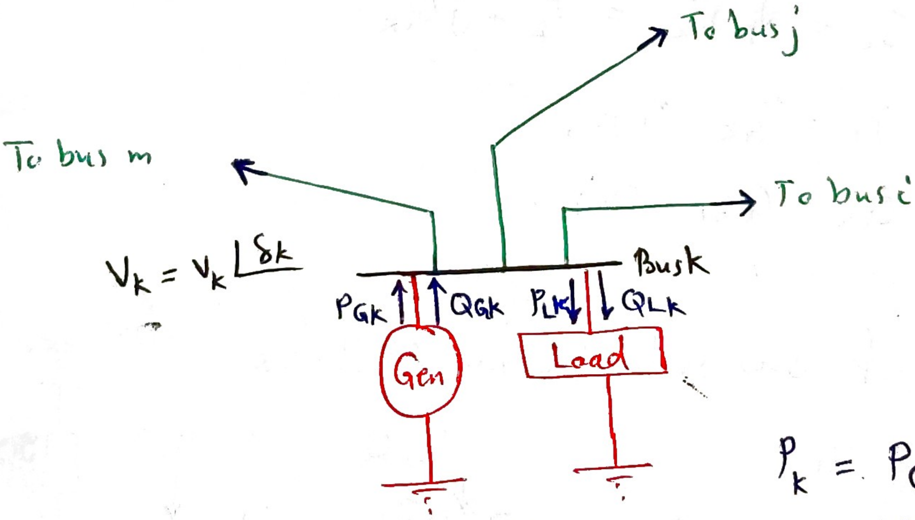
$I = \frac{V}{Z}$

(4) Inter connected, Network system

Power Flow Analysis

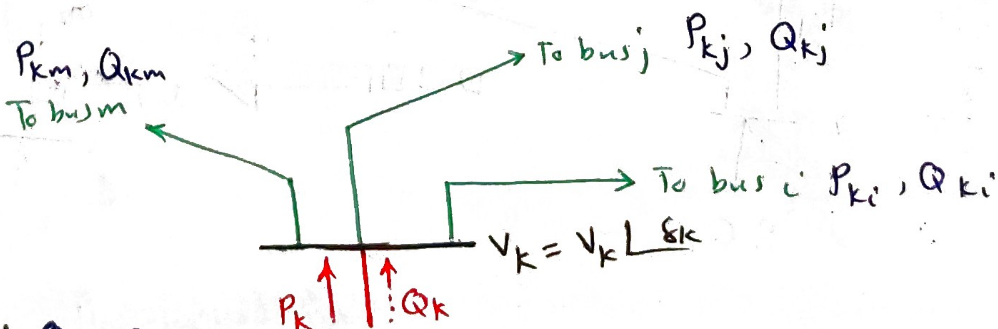
Load Flow Analysis





$$P_k = P_{Gk} - P_{Lk}$$

$$Q_k = Q_{Gk} - Q_{Lk}$$



$P_k = P_{Gk} - P_{Lk}$ | $Q_k = Q_{Gk} - Q_{Lk}$

$P_k = P_{ki} + P_{kj} + P_{km}$ | $Q_k = Q_{ki} + Q_{kj} + Q_{km}$

Power Flow Study:-

- Static Analysis of power Network
- Real power balance $(\sum P_{g_i} - \sum P_{D_j} - P_{loss})$
- Reactive power balance $(\sum Q_{g_i} - \sum Q_{D_j} - Q_{loss})$
- Transmission Flow Limit.
- Bus voltage Limits.

» Static Analysis of power Network

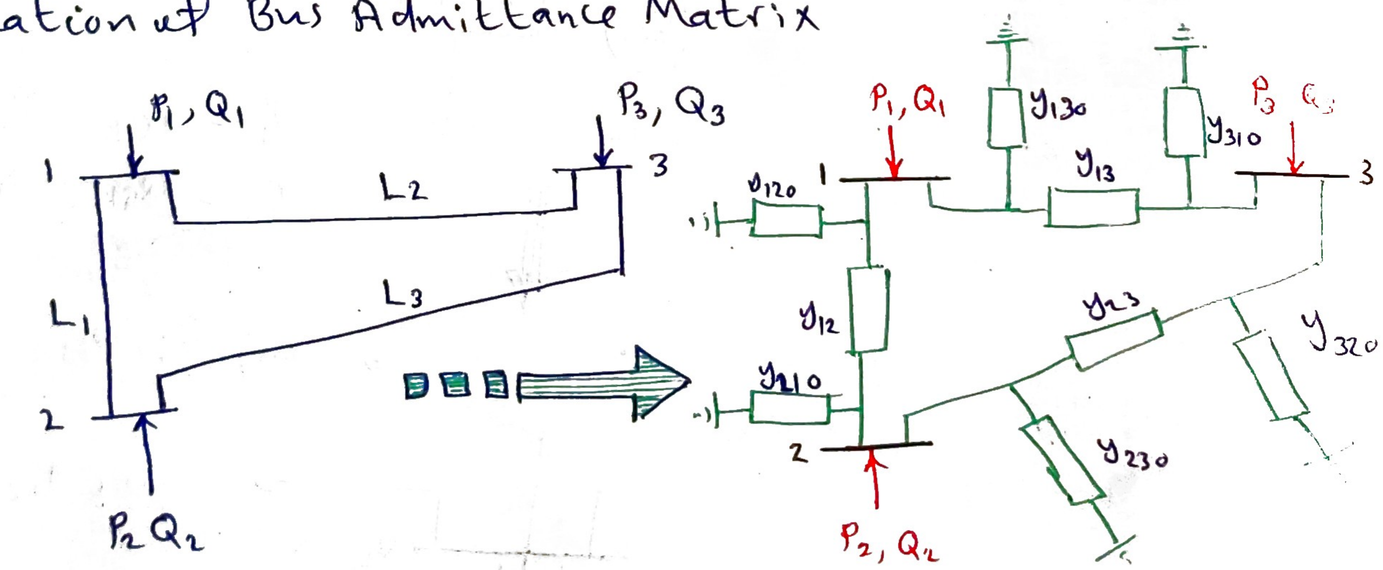
- Mathematical Model of the Network.
- Transmission Line – nominal π model.
- Bus power injections–

$$S_k = V_k I_k^* = P_k + jQ_k$$

$$P_k = P_{Gk} - P_{Lk}.$$

$$Q_k = Q_{Gk} - Q_{Lk}.$$

» Formation of Bus Admittance Matrix



$$I_1 = y_{120}V_1 + y_{12}(V_1 - V_2) + y_{130}V_1 + y_{13}(V_1 - V_3)$$

$$I_2 = y_{210}V_2 + y_{12}(V_2 - V_1) + y_{230}V_2 + y_{23}(V_2 - V_3)$$

$$I_3 = y_{310}V_3 + y_{13}(V_3 - V_1) + y_{320}V_3 + y_{23}(V_3 - V_2)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} (y_{120}+y_{12}+y_{130}+y_{13}) & -y_{12} & -y_{13} \\ -y_{21} & (y_{210}+y_{12}+y_{230}+y_{23}) & -y_{23} \\ -y_{31} & -y_{32} & (y_{310}+y_{13}+y_{320}+y_{23}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$Y_{11} = y_{120} + y_{12} + y_{130} + y_{13}$$

$$Y_{22} = y_{210} + y_{12} + y_{230} + y_{23}$$

$$Y_{33} = y_{310} + y_{13} + y_{320} + y_{23}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

- Y_{ii} is called Self-Admittance (Driving Point Admittance)

  Y_{ij} is called Transfer-Admittance (Mutual Admittance)

$$I_{Bus} = Y_{Bus} V_{Bus} \; ; \; V_{Bus} = Z_{Bus} I_{Bus}$$

Characteristics of Y_{BUS} Matrix :-

- Dimension of Y_{bus} is $(N \times N) \longrightarrow N \equiv$ Number of buses.
- Y_{bus} is symmetric matrix
- Y_{bus} is a sparse (متفرق، متناثر) matrix (up to 90 % to 95 % sparse)
- Diagonal Elements Y_{ii} are obtained as Algebraic sum of all elements Incident to bus 'i'
- Off-diagonal Elements $Y_{ij} = Y_{ji}$ are obtained as negative of admittance Connecting bus i and j

Power Flow Equations :-

$$I_{BUS} = Y_{BUS} V_{BUS}$$

$$I_k = \sum_{n=1}^{N \text{ (\# of buses)}} Y_{kn} V_n \Longrightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$S_k = P_k + jQ_k = V_k I_k^*$$

$S_k \to$ bus k

$$P_k + jQ_k = V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^* \qquad k = 1, 2, 3, \ldots, N$$

$$V_n = V_n e^{j\delta_n}$$

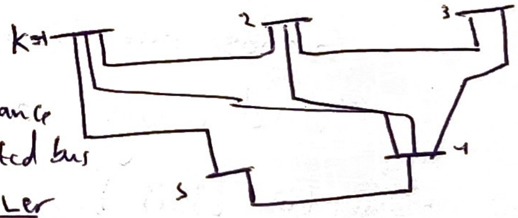
$$Y_{kn} = Y_{kn} e^{j\theta_{kn}} \qquad k, n = 1, 2, 3, \ldots, N$$

θ_{kn} ← angle of the admittance.

$$P_k + jQ_k = V_k \sum_{n=1}^{N} Y_{kn} V_n e^{j(\delta_k - \delta_n - \theta_{kn})}$$

$$\boxed{\begin{aligned} P_k &= V_k \sum_{n=1}^{N} Y_{kn} V_n \cos(\delta_k - \delta_n - \theta_{kn}) \\ Q_k &= V_k \sum_{n=1}^{N} Y_{kn} V_n \sin(\delta_k - \delta_n - \theta_{kn}) \end{aligned}}$$

Y_{kn} → The admittance that connected bus k to all other buses



Characteristics of Power Flow Equations :-

* Power Flow Equations are Algebraic ((There is no driffative or driffrentiance))
  - Static System. ← because we use

* Power Flow Equations are Non-linear (sin, cos) (and multiplication)
  - Iterative Solution

* Relate P, Q in terms of V, δ and Y_{BUS} Elements
  - P, Q → f (V, δ)

Consumers Controlled by (شغب) (إضراب) — and the Power System has no control on them

Characterization of Variables :-

* Load $(P_L, Q_L) \Rightarrow$ Uncontrolled (Disturbance) Variable.
* Generation $(P_G, Q_G) \Rightarrow$ Control Variable. ((depends on the Load))
* Voltage $(V, \delta) \Rightarrow$ State Variable.

Economic dispatch problem

For a Given Operating Condition $\longrightarrow$ Loads and Generations at all buses are known (Specified)

$\Rightarrow$ Find the Voltage Magnitude and Angle (V, δ) at each bus.

Problem in Power Flow $\longrightarrow$

All generation Variables (P_G, Q_G) can not be specified as (بما) Losses are not known a priori (قبلي).

why $\longrightarrow$

Solution $\longrightarrow$

Choose one bus as reference where Voltage Magnitude and angle are specified. The losses are assigned to this bus. This bus is called "Slack Bus".

Classification of Busbars :-

مراجعة مهمة

Each bus k is classified into one of the following three bus types :-

[1] Swing Bus - There is only one swing bus, which for convenience (وسيلة للراحة) is numbered bus 1.

The swing bus is a reference bus for which $V_1 \angle \delta_1$, typically $1.0 \angle 0^\circ$ per unit, is specified (input data).

The power-flow program computes ~~...~~ P_1 and Q_1.

[Load bus - P_k and Q_k are specified (input data).
The power flow program computes V_k and δ_k.

*
[3] Voltage Controlled bus - P_k and V_k are input data.
The power flow program computes Q_k and δ_k.
Examples are buses which generators, switched shunt capacitors, or static var system are connected.
Maximum and minimum var Limits $Q_{Gk,max}$, $Q_{Gk,min}$ that this equipment can supply are also input data. Another examples is a bus to which a tap changing transformer is connected;

6.4 POWER FLOW SOLUTION

Power flow studies, commonly known as *load flow*, form an important part of power system analysis. They are necessary for planning, economic scheduling, and control of an existing system as well as planning its future expansion. The problem consists of determining the magnitudes and phase angle of voltages at each bus and active and reactive power flow in each line.

In solving a power flow problem, the system is assumed to be operating under balanced conditions and a single-phase model is used. Four quantities are associated with each bus. These are voltage magnitude $|V|$, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types.

Slack bus One bus, known as *slack* or *swing bus*, is taken as reference where the magnitude and phase angle of the voltage are specified. This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the network.

Load buses At these buses the active and reactive powers are specified. The magnitude and the phase angle of the bus voltages are unknown. These buses are called P-Q buses.

Regulated buses These buses are the *generator buses*. They are also known as *voltage-controlled buses*. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are to be determined. The limits on the value of the reactive power are also specified. These buses are called P-V buses.

6.4.1 POWER FLOW EQUATION

Consider a typical bus of a power system network as shown in Figure 6.7. Transmission lines are represented by their equivalent π models where impedances have been converted to per unit admittances on a common MVA base.

Application of KCL to this bus results in

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \end{aligned} \quad (6.23)$$

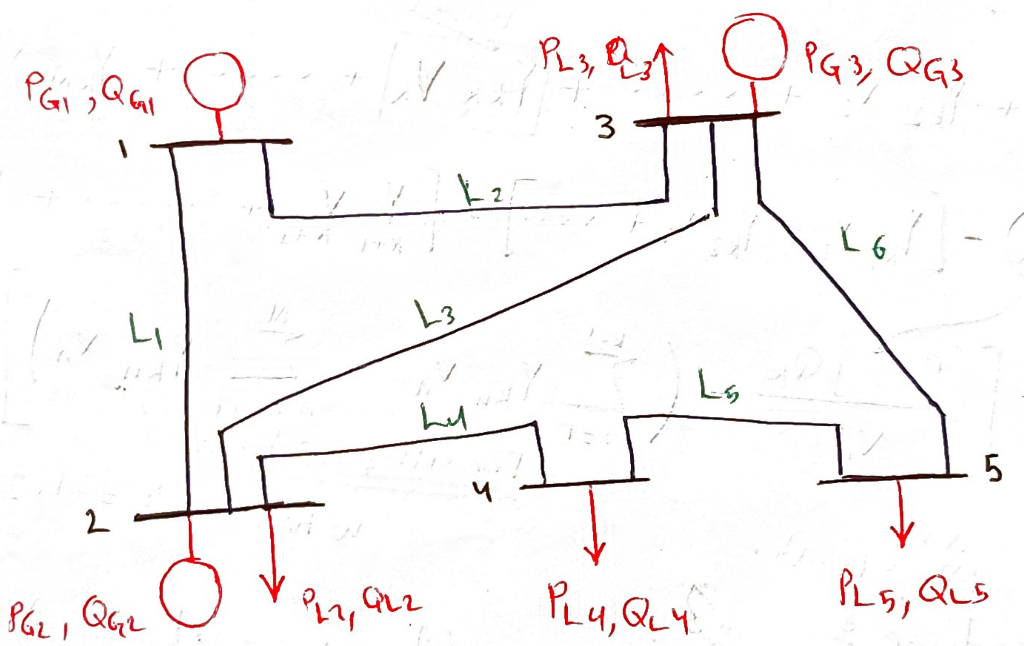
or

Classification of Busbars :-

1. Swing Bus (V_i, δ_i)
2. PV Bus (Voltage Control Bus)
3. PQ Bus (Load Bus)

With each bus i, 4 variables (P_i, Q_i, V_i, and δ_i) are associated. Depending on the type of bus two variables are specified (known) and two unknown variables are obtained from power flow solution.

5 - Bus Power System



Bus Data

Bus	Type	V per unit	δ deg	P_G per unit	Q_G per unit	P_L per unit	Q_L per unit	Q_{Gmax} per unit	Q_{Gmin} per unit
1	Swing	1.03	0	–	–				
2									
3									
4									
5									

Power Flow Solution by Gauss-Seidel Method

Non-linear, Algebraic

Iterative Solution

$$I_{BUS} = Y_{BUS} V_{BUS}$$

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n$$

$$S_k = P_k + jQ_k = V_k I_k^*$$

$$P_k + jQ_k = V_k \left[\sum_{n=1}^{N} Y_{kn} V_n \right]^* \quad , k = 1, 2, \ldots, N$$

$$(P_k + jQ_k)^* = V_k^* I_k$$

$$P_k - jQ_k = V_k^* I_k$$

$$I_k = \frac{P_k - jQ_k}{V_k^*}$$

$$I_k = \frac{P_k - jQ_k}{V_k^*} \quad , \text{ Also}$$

$$I_k = \sum_{n=1}^{N} Y_{kn} V_n \quad , \text{ or}$$

leave this

$$I_k = Y_{k1} V_1 + Y_{k2} V_2 + \cdots + \boxed{Y_{kk} V_k} + \cdots + Y_{kN} V_N$$

$$Y_{kk} V_k = I_k - \left[Y_{k1} V_1 + Y_{k2} V_2 + \cdots \right] - \left[Y_{k+1} V_{k+1} + \cdots + Y_{kN} V_N \right]$$

Find ??!

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n + \sum_{n=k+1}^{N} Y_{kn} V_n \right) \right]$$

For all P-Q buses

where $k = 1, 2, 3, \ldots N$

PQ, Slack bus (V, δ), ~~PV~~ (X no) → later

Iterative Procedure :

1. Make an initial guess (تخمين) $|V_i|^{(0)}$ and $\delta_i^{(0)}$ (10-30)
   - Flat Start $|V_i|^{(0)} = 1.0$ and $\delta_i^{(0)} = 0.0$

2. Use this solution in PFE (Power Flow Eq) to obtain a better first solution.

3. First Solution is used to obtain a better second solution and so on.

$$V_k^{i+1} = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i} \right) \right]$$

updating the buses values

$k = 1, 2, \ldots, N$, i is iteration count

Continue iteration till $|V_k^{i+1} - V_k^i| \leq \varepsilon$

Algorithm Steps :-

1. With P_{gi}, Q_{gi}, P_{di}, and Q_{di} known Calculate bus injections P_i, Q_i

delivered

2. Form Y_{BUS} Matrix

3. Set initial voltage $V_i^{(0)}$, $\delta_i^{(0)}$

4. Iteratively solve equation

updating value

the value which is available

$$V_k^{i+1} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i}\right)\right]$$

to obtain new values of bus voltages.

Algorithm Modification when PV Buses are also Present

$P_k + jQ_k = V_k I^*$

$P_k - jQ_k = V_k^* I_k$

$Q_k = -\text{Im}[V_k^* I_k]$

$$Q_i = -\text{Im}\left[V_i^* \underbrace{\sum_{k=1}^{n} Y_{ik} V_k}_{I}\right]$$

$$Q_i^{(r+1)} = -\text{Im}\left[(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)}\right]$$

The revised value of δ_i is obtained from immediately following step 1. Thus

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

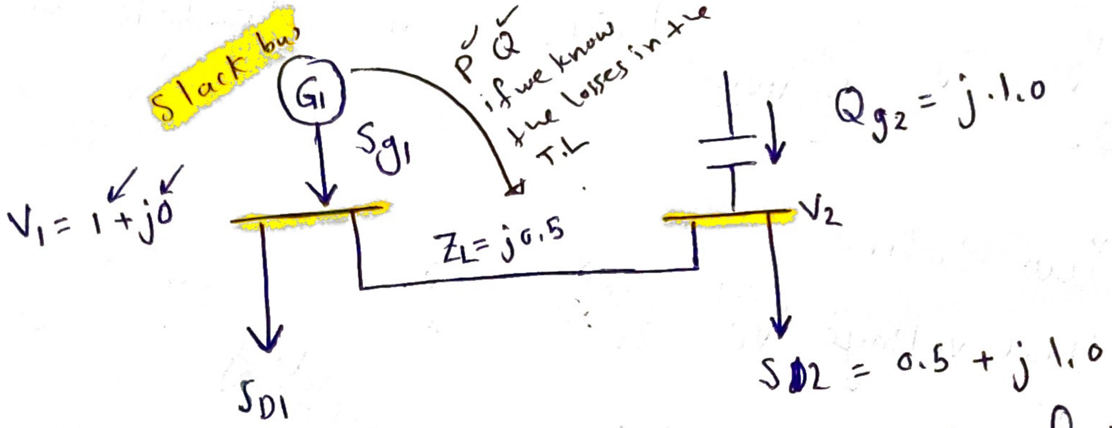
$$= \text{Angle of}\left[\frac{A_i^{(r+1)}}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)}\right]$$

Where

$$A_i^{(r+1)} = \frac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$$

The algorithm for PQ buses remains unchanged

Example:- For the system shown, $Z_L = j\,0.5$, $V_1 = 1\angle 0^\circ$, $S_{G2} = j1.0$ and $S_{D2} = 0.5 + j1.0$. Find V_2 using Gauss-Seidel iteration technique.



Solution: Firstly, we calculate the elements of the Y_{BUS}

For $Z_L = j0.5$, we have

$Y_{11} = -j2$

$Y_{12} = j2 = -y_{12}$

$Y_{21} = j2 = -y_{21}$

$Y_{22} = -j2$

We iterate on V_2 using the equation given

$$V_2^{n+1} = \frac{1}{Y_{22}}\left[S_2^* / (V_2^n)^* - Y_{21} * V_1 \right] \quad \text{---- (1)}$$

Given $V_1 = 1\angle 0^\circ$

$S_2 = S_{G2} - S_{D2} = -0.5$

Putting the values of V_1, S_2, Y_{22} and Y_{21} in equation (1), we get

$$V_2^{n+1} = -j\left[0.25/(V_2^n)^*\right] + 1.0 \quad \text{--- (2)}$$

Start with a guess, taking $V_2^0 = 1\angle 0^\circ$ and iterate using equation (2).

We have, $V_2^0 = 1 + j0$

Putting in equation (2), and iterating for V_2, we get

$$V_2^1 = -j[0.25/(1+j0)^*] + 1.0$$

$$= 1.0 - j0.25$$

$$V_2^1 = 1.030776\angle -14.036243^\circ$$

$$V_2^2 = -j[0.25/(1.0-j0.25)^*] + 1.0$$

$$= 1.0 - j0.25/(1.0+j0.25)^*]$$

$$= 1.0/(1.0+j0.25)$$

$$= 0.970143\angle -14.036249^\circ$$

Similarly, we can iterate it further. The results of the iteration, are tabulated below

	Iteration #	V_2
	0	$1\angle 0^\circ$
0.030776	1	$1.030776\angle -14.036243^\circ$
0.060633	2	$0.970143\angle -14.036249^\circ$
0.000118	3	$0.970261\angle -14.931409^\circ$
0.004026	4	$0.966235\angle -14.931416^\circ$
0.000001	5	$0.966236\angle -14.995078^\circ$
0.000756	6	$0.965948\angle -14.995072^\circ$

① V_1 ✓ V_2 ✓
Y ✓
② I ✓
③ loss ✓
④ P, Q ✓

Since, the difference in the values for the voltage doesn't change much between the 5th and 6th iteration, we can stop after the 6th.

Hence, we can see that starting with the value $V_2 = 1\angle 0^\circ$, convergence is reached in six steps.

6

unit. The scheduled loads at buses 2 and 3 are as marked on the diagram. Line impedances are marked in per unit on a 100-MVA base and the line charging susceptances are neglected.

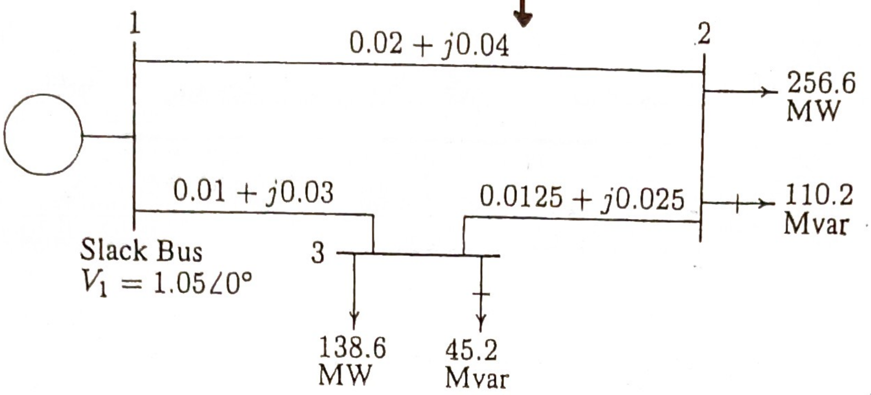


FIGURE 6.9
One-line diagram of Example 6.7 (impedances in pu on 100-MVA base).

(a) Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places.
(b) Find the slack bus real and reactive power.
(c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The admittances are marked on the network shown in Figure 6.10.
At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Since the actual admittances are readily available in Figure 6.10, for hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, V_2 and V_3 are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

$$\boxed{V_k^{i+1} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^{*(i)}} - \left(\sum_{n=1}^{k-1} Y_{kn} V_n^{i+1} + \sum_{n=k+1}^{N} Y_{kn} V_n^{i}\right)\right]}$$

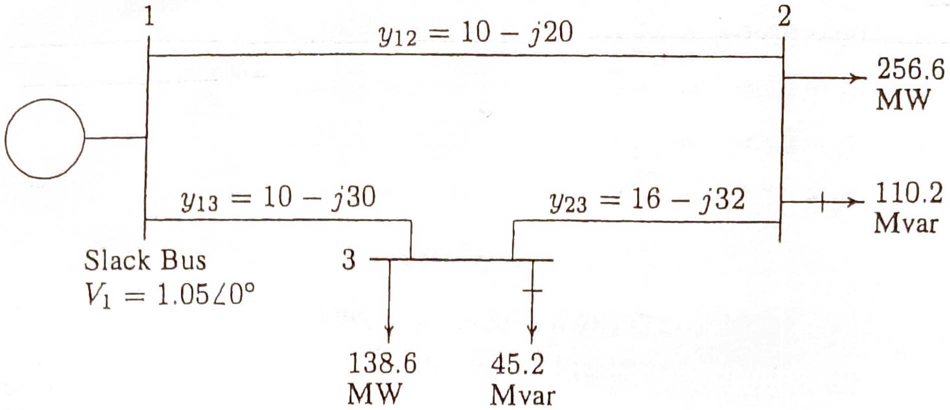


FIGURE 6.10
One-line diagram of Example 6.7 (admittances in pu on 100-MVA base).

$$V_2^{(1)} = \frac{\frac{-2.566+j1.102}{1.0-j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0 + j0)}{(26 - j52)}$$

$$= \underline{0.9825} - j\underline{0.0310}$$

and $\longrightarrow$ To four decimal places.

$$V_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-1.386+j0.452}{1-j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9825 - j0.0310)}{(26 - j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0011 - j0.0353)}{(26 - j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011+j0.0353} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9816 - j0.052)}{(26 - j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035°} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624°} \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$\begin{aligned} I_{12} &= y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8 \\ I_{21} &= -I_{12} = -1.9 + j0.8 \\ I_{13} &= y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0 \\ I_{31} &= -I_{13} = -2.0 + j1.0 \\ I_{23} &= y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48 \\ I_{32} &= -I_{23} = 0.64 - j0.48 \end{aligned}$$

The line flows are

$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \;\; \text{pu} \\ &= 199.5 \;\; \text{MW} + j84.0 \;\; \text{Mvar} \\ S_{21} = V_2 I_{21}^* &= (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \;\; \text{pu} \\ &= -191.0 \;\; \text{MW} - j67.0 \;\; \text{Mvar} \\ S_{13} = V_1 I_{13}^* &= (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \;\; \text{pu} \\ &= 210.0 \;\; \text{MW} + j105.0 \;\; \text{Mvar} \end{aligned}$$

$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu}$$
$$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu}$$
$$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu}$$
$$= 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

The power flow diagram is shown in Figure 6.11, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

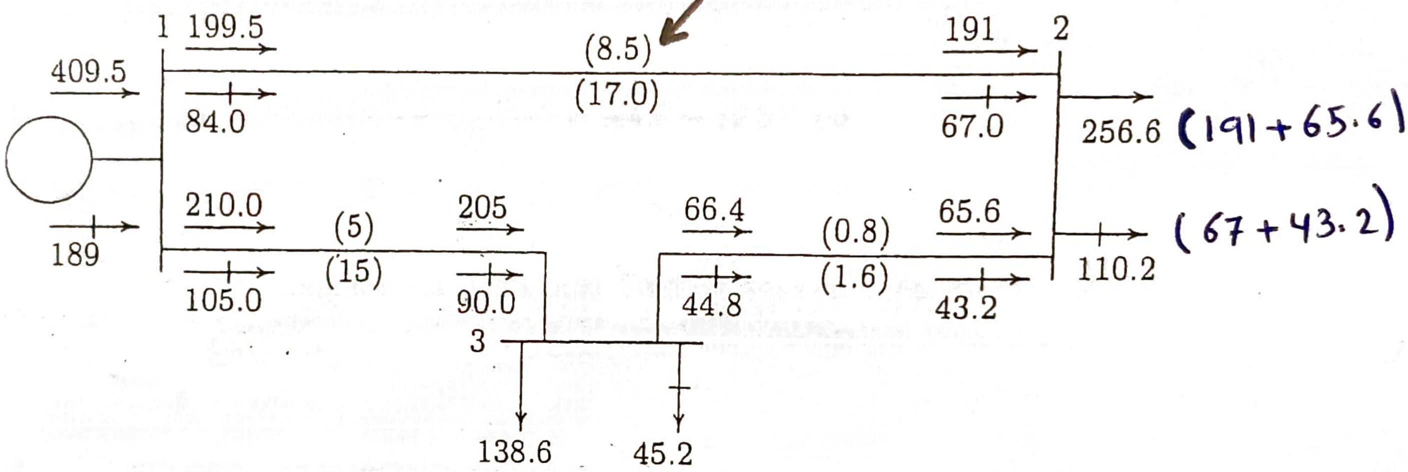


FIGURE 6.11
Power flow diagram of Example 6.7 (powers in MW and Mvar).

Example 6.8 (chp6ex8)

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

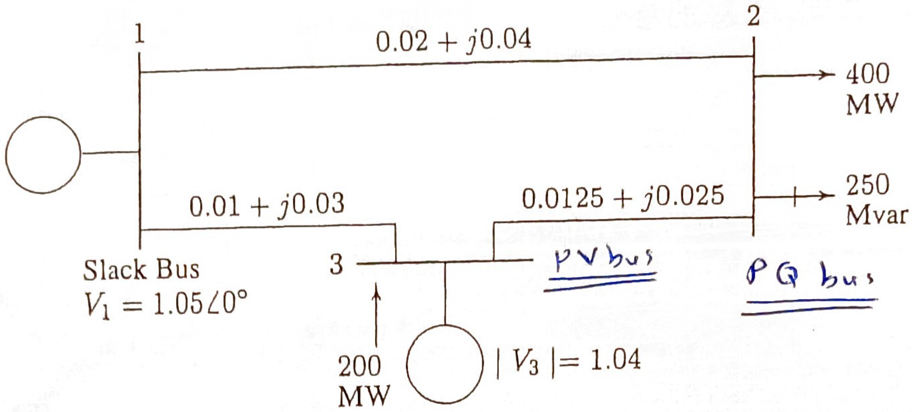


FIGURE 6.12
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The load and generation expressed in per units are

$$\text{(Load)}\quad S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$\text{(gen.)}\quad P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$, V_2 and V_3 are computed from (6.28).

$$\begin{aligned}
V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\
&= \frac{\frac{-4.0 + j2.5}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\
&= 0.97462 - j0.042307
\end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned}
Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\
&= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\
&\quad (16 - j32)(0.97462 - j0.042307)]\} \\
&= 1.16
\end{aligned}$$

$$Q_i^{(r+1)} = -\mathrm{Im}\left[(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)} \right]$$

$$Q_i = -\mathrm{Im}\left[V_i^* \underbrace{\sum_{k=1}^{n} Y_{ik} V_k}_{I} \right]$$

The value of $Q_3^{(1)}$ is used as Q_3^{sch} for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)}$$

$$= 1.03783 - j0.005170$$ ثبت

غير

يحتفظ به

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$|V_3| = 1.04$

$|\ |^2 = (\text{real})^2 + (\text{Img})^2$

$\text{real} = \sqrt{|\ |^2 - (\text{Img})^2}$

real part $= e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}}$$

$$= \frac{\frac{-4.0 + j2.5}{.97462 + j.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 + j0.005170)}{(26 - j52)}$$

$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$

$$= -\Im\{(1.039987 + j0.005170)[(1.039987 - j0.005170)(26 - j62) -$$

$$(10 - j30)(1.05 + j0) - (16 - j32)(0.971057 - j0.043432)]\}$$

$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{2.0 - j1.38796}{1.039987 + j0.00517} + (10 - j30)(1.05) + (16 - j32)(.971057 - j.043432)}{(26 - j62)}$$

$$= 1.03908 - j0.00730$$

غير ثبت

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of 5×10^{-5} pu in seven iterations as given below.

$V_2^{(3)} = 0.97073 - j0.04479 \quad Q_3^{(3)} = 1.42904 \quad V_3^{(3)} = 1.03996 - j0.00833$

$V_2^{(4)} = 0.97065 - j0.04533 \quad Q_3^{(4)} = 1.44833 \quad V_3^{(4)} = 1.03996 - j0.00873$

$V_2^{(5)} = 0.97062 - j0.04555 \quad Q_3^{(5)} = 1.45621 \quad V_3^{(5)} = 1.03996 - j0.00893$

$V_2^{(6)} = 0.97061 - j0.04565 \quad Q_3^{(6)} = 1.45947 \quad V_3^{(6)} = 1.03996 - j0.00900$

$V_2^{(7)} = 0.97061 - j0.04569 \quad Q_3^{(7)} = 1.46082 \quad V_3^{(7)} = 1.03996 - j0.00903$

The final solution is

$$V_2 = 0.97168\angle -2.6948^\circ \quad \text{pu}$$

$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$
$$V_3 = 1.04\angle -.498^\circ \quad \text{pu}$$
$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$

Line flows and line losses are computed as in Example 6.7, and the results expressed in MW and Mvar are

$S_{12} = 179.36 + j118.734 \quad S_{21} = -170.97 - j101.947 \quad S_{L\,12} = 8.39 + j16.79$

$S_{13} = 39.06 + j22.118 \quad S_{31} = -38.88 - j\,21.569 \quad S_{L\,13} = 0.18 + j0.548$

$S_{23} = -229.03 - j148.05 \quad S_{32} = 238.88 + j167.746 \quad S_{L\,23} = 9.85 + j19.69$

The power flow diagram is shown in Figure 6.13, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

* for slack bus.

$$P_1 - jQ_1 = V_1^* \left[V_1 (y_{12} + y_{13}) - (y_{12} V_2 + y_{13} V_3) \right]$$

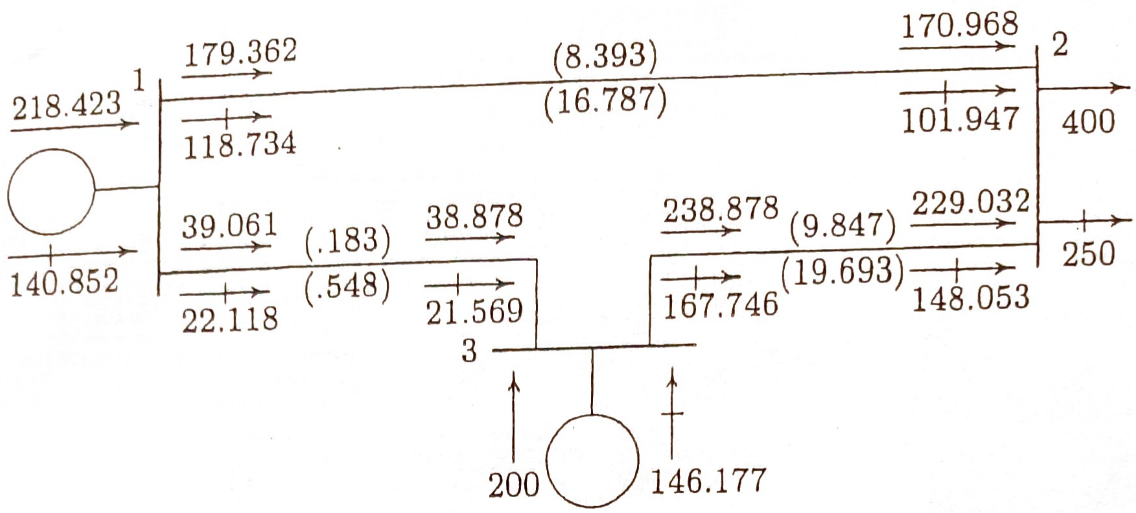


FIGURE 6.13
Power flow diagram of Example 6.8 (powers in MW and Mvar).

6.7 TAP CHANGING TRANSFORMERS

In Section 2.6 it was shown that the flow of real power along a transmission line is determined by the angle difference of the terminal voltages, and the flow of reactive power is determined mainly by the magnitude difference of terminal voltages. Real and reactive powers can be controlled by use of tap changing transformers and regulating transformers.

In a tap changing transformer, when the ratio is at the nominal value, the transformer is represented by a series admittance y_t in per unit. With off-nominal ratio, the per unit admittance is different from both sides of the transformer, and the admittance must be modified to include the effect of the off-nominal ratio. Consider a transformer with admittance y_t in series with an ideal transformer representing the off-nominal tap ratio 1:a as shown in Figure 6.14. y_t is the admittance in per unit based on the nominal turn ratio and a is the per unit off-nominal tap position allowing for small adjustment in voltage of usually ± 10 percent. In the case of phase shifting transformers, a is a complex number. Consider a fictitious bus x between the turn ratio and admittance of the transformer. Since the complex power on either side of the ideal transformer is the same, it follows that if the voltage goes through a positive phase angle shift, the current will go through a negative phase angle shift. Thus, for the assumed direction of currents, we have

$$V_x = \frac{1}{a}V_j \tag{6.43}$$

$$I_i = -a^* I_j \tag{6.44}$$

The current I_i is given by

$$I_i = y_t(V_i - V_x)$$

