Fundamentals Physics

Tenth Edition

Halliday

Chapter 9_2

Center of Mass and Linear Momentum

9-4 Collision and Impulse (9 of 9)

Checkpoint 5

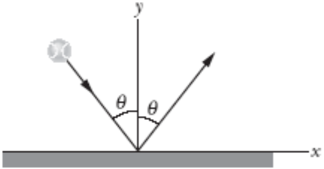
The figure shows an overhead view of a ball bouncing from a vertical wall without any change in its speed. Consider the change $\Delta\vec{p}$ in the ball's linear momentum. (a) Is Δp_x positive, negative, or zero? (b) Is Δp_y positive, negative, or zero? (c) What is the direction of $\Delta\vec{p}$?

Answer:

(a) Zero (the force of impact on the ball is in the y direction, p_x is conserved)

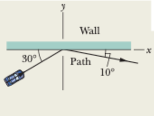
(b) Positive ($P_{yf} - (-P_{yi})$)

(c) along the positive *y*-axis (normal force)



Sample Problem 9.04 Two-dimensional impulse, race car–wall collision

v_i = 70 m/s along a straight line at 30˚, after the collision, he is traveling at speed v_f = 50 m/s at 10˚ from the wall, *m* is 80 kg, (a) What is the impulse $\vec{J}$ on the driver due to the collision?

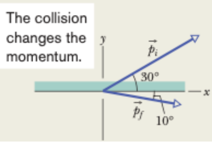


(a)

$$\vec{J} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$$

x component: Along the *x* axis we have:

$$J_x = m(v_{fx} - v_{ix})$$
$$= (80\text{ kg})[(50\text{ m/s})\cos(-10°) - (70\text{ m/s})\cos 30°]$$
$$= -910\text{ kg}\cdot\text{m/s.}$$

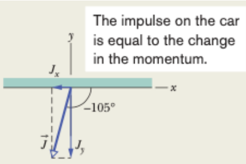


Y component: Along the *y* axis:

$$J_y = m(v_{fy} - v_{iy})$$
$$= (80\text{ kg})[(50\text{ m/s})\sin(-10°) - (70\text{ m/s})\sin 30°]$$
$$= -3495\text{ kg}\cdot\text{m/s} \approx -3500\text{ kg}\cdot\text{m/s.}$$

Impulse: The impulse is then

$$\vec{J} = (-910\hat{i} - 3500\hat{j})\text{ kg}\cdot\text{m/s,} \qquad \text{(Answer)}$$ 4th quadrant



(c)

which means the impulse magnitude is

$$J = \sqrt{J_x^2 + J_y^2} = 3616\text{ kg}\cdot\text{m/s} \approx 3600\text{ kg}\cdot\text{m/s.}$$

The angle of J is given by $\theta = \tan^{-1} J_y/J_x$ = 75.4˚, it is actually 75.4˚+180˚= 255.4˚, which we can also write as θ = –105˚ (Answer)

Sample Problem 9.04 Two-dimensional impulse, race car–wall collision

(b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?

Eq. 9-35 ($J = F_{avg}\,\Delta t$)

$$F_{avg} = \frac{J}{\Delta t} = \frac{3616\ \text{kg}\cdot\text{m/s}}{0.014\ \text{s}}$$
$$= 2.583 \times 10^5\ \text{N} \approx 2.6 \times 10^5\ \text{N}.$$

Note that $F = ma$ with m = 80 kg, a = 3.22 x 10^3 m/s^2 = 329g, which is fatal.

9-5 Conservation of Linear Momentum (2 of 5)

- For an impulse of zero we find:

$$\vec{P} = \text{constant} \quad (\text{closed, isolated system}).$$ (no change in P) **Equation (9-42)**

- Which says that:

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

9-5 Conservation of Linear Momentum (3 of 5)

- This is called the **law of conservation of linear momentum**
- Check the components of the net external force to know if you should apply this

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

9-5 Conservation of Linear Momentum (4 of 5)

- Internal forces can change momenta of parts of the system, but cannot change the linear momentum of the entire system
- Do not confuse momentum and energy

9-5 Conservation of Linear Momentum (5 of 5)

Checkpoint 6

An initially stationary device lying on a frictionless floor explodes into two pieces, which then slide across the floor, one of them in the positive *x* direction. (a) What is the sum of the momenta of the two pieces after the explosion? (b) Can the second piece move at an angle to the *x* axis? (c) What is the direction of the momentum of the second piece?

Answer:

(a) Zero ($P_i = P_f$)

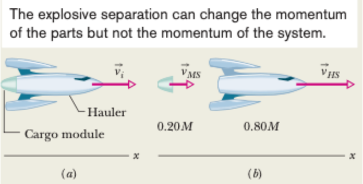
(b) no

(c) the negative *x* direction

Sample Problem 9.05 One-dimensional explosion, relative velocity, space hauler

Figure *a* space hauler and cargo, mass *M*, $\vec{v}_i$ of magnitude 2100 km/h. With a small explosion, the hauler ejects the cargo module, of mass 0.20*M* (Fig. *b*), The hauler then travels 500 km/h faster than the module along the *x* axis, the relative speed *v*rel between the hauler and the module is 500 km/h.

What then is the velocity $\vec{v}_{HS}$ of the hauler relative to the Sun?



$\vec{P}_i = \vec{P}_f$ (note that the momenta of the hauler and module certainly change)
$P_i = Mv_i$
Let v_{MS} be the velocity of the ejected module relative to the Sun

$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}, \qquad (9\text{-}46)$$

$$\begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to Sun}\end{pmatrix} = \begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to module}\end{pmatrix} + \begin{pmatrix}\text{velocity of}\\ \text{module relative}\\ \text{to Sun}\end{pmatrix}$$

Then: $v_{MS} = v_{HS} - v_{rel}$

Substitute this in 9-46, then 9-46 and $P_i = Mv_i$ into $\vec{P}_i = \vec{P}_f$:

$$Mv_i = 0.20M(v_{HS} - v_{rel}) + 0.80Mv_{HS},$$

Which gives:

$$v_{HS} = v_i + 0.20v_{rel},$$
$$v_{HS} = 2100 \text{ km/h} + (0.20)(500 \text{ km/h})$$
$$= 2200 \text{ km/h}.$$

9-6 Momentum and Kinetic Energy in Collisions (2 of 7)

- Types of collisions:
- **Elastic collisions**:
  - Total kinetic energy is unchanged (conserved)
  - A useful approximation for common situations
  - In real collisions, some energy is always transferred
- **Inelastic collisions**: some energy is transferred
- **Completely inelastic collisions**:
  - The objects stick together
  - Greatest loss of kinetic energy

9-6 Momentum and Kinetic Energy in Collisions (3 of 7)

- For one dimension:
- Inelastic collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$ **Equation (9-51)**

- Completely inelastic collision, for target at rest:

$$m_1 v_{1i} = (m_1 + m_2)V$$ **Equation (9-52)**

9-6 Momentum and Kinetic Energy in Collisions (4 of 7)

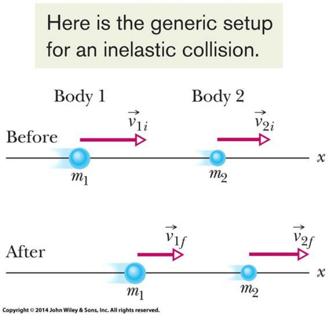


Figure 9-14

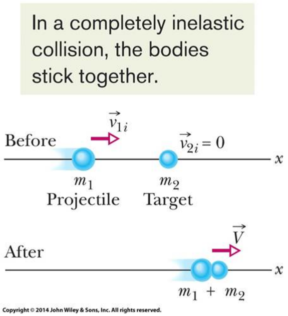


Figure 9-15

9-6 Momentum and Kinetic Energy in Collisions (5 of 7)

- The center of mass velocity remains unchanged:

$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}.$$ **Equation (9-56)**

- Figure 9-16 shows freeze frames of a completely inelastic collision, showing center of mass velocity

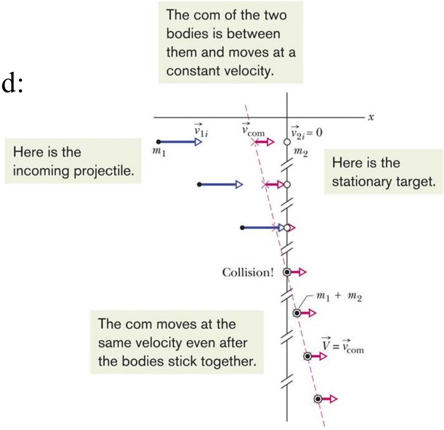


Figure 9-16

9-6 Momentum and Kinetic Energy in Collisions (7 of 7)

Checkpoint 7

Body 1 and body 2 are in a completely inelastic one-dimensional collision. What is their final momentum if their initial momenta are, respectively, (a) $10\ \text{kg}\cdot\text{m/s}$ and 0; (b) $10\ \text{kg}\cdot\text{m/s}$ and $4\ \text{kg}\cdot\text{m/s}$; (c) $10\ \text{kg}\cdot\text{m/s}$ and $-4\ \text{kg}\cdot\text{m/s}$?

Answer:

(a) 10 kg m/s

(b) 14 kg m/s

(c) 6 kg m/s

Sample Problem 9.07 Conservation of momentum, ballistic pendulum

Block of wood of mass M = 5.4 kg, a bullet of mass m = 9.5 g, fired into the block, *block + bullet* then swing upward, their center of mass rising a vertical distance h = 6.3 cm before the pendulum comes momentarily to rest.

What is the speed of the bullet just prior to the collision?

(1) the collision and (2) the rise, during which mechanical energy *is* conserved.

Total momentum before the collision = total momentum after the collision

$$V = \frac{m}{m + M} v.$$

Mechanical energy at bottom = mechanical energy at top

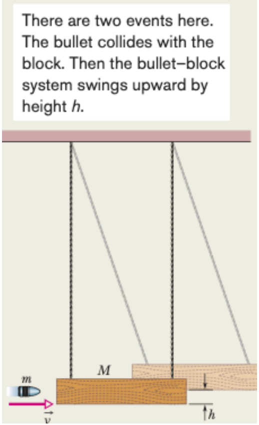
$$\tfrac{1}{2}(m + M)V^2 = (m + M)gh.$$

Combining:

$$v = \frac{m + M}{m}\sqrt{2gh} \qquad (9\text{-}61)$$

$$= \left(\frac{0.0095\ \text{kg} + 5.4\ \text{kg}}{0.0095\ \text{kg}}\right)\sqrt{(2)(9.8\ \text{m/s}^2)(0.063\ \text{m})}$$

$$= 630\ \text{m/s}. \qquad \text{(Answer)}$$



9-7 Elastic Collisions in One Dimension (2 of 8)

- Total kinetic energy is conserved in elastic collisions

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

- For a stationary target, conservation laws give:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{(linear momentum).}$$ **Equation (9-63)**

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{(kinetic energy).}$$ **Equation (9-64)**

9-7 Elastic Collisions in One Dimension (3 of 8)

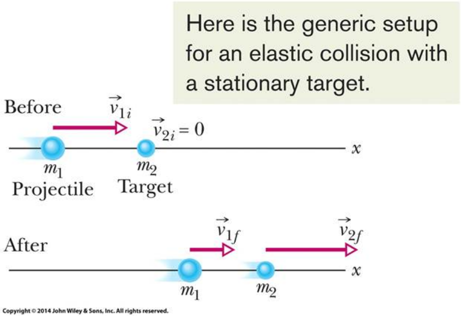


Figure 9-18

9-7 Elastic Collisions in One Dimension (4 of 8)

- With some algebra we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$ **Equation (9-67)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$ **Equation (9-68)**

9-7 Elastic Collisions in One Dimension (5 of 8)

- Results
  - Equal masses: $v_{1f} = 0$, $v_{2f} = v_{1i}$: the first object stops
  - Massive target, $m_2 >> m_1$: the first object just bounces back, speed mostly unchanged
  - Massive projectile: $v_{1f} \approx v_{1i}$, $v_{2f} \approx 2v_{1i}$: the first object keeps going, the target flies forward at about twice its speed

9-7 Elastic Collisions in One Dimension (6 of 8)

- For a target that is also moving, we get:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$ **Equation (9-75)**

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$ **Equation (9-76)**

9-7 Elastic Collisions in One Dimension (7 of 8)

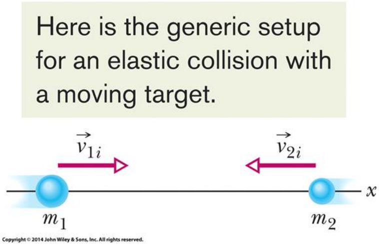
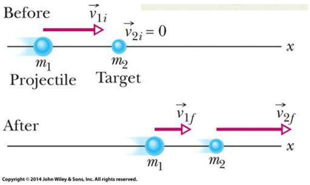


Figure 9-19

9-7 Elastic Collisions in One Dimension (8 of 8)

Checkpoint 8

What is the final linear momentum of the target in the Figure if the initial linear momentum of the projectile is 6 kg·m/s and the final linear momentum of the projectile is (a) $2\ \text{kg}\cdot\text{m/s}$ and (b) $-2\ \text{kg}\cdot\text{m/s}$? (c) What is the final kinetic energy of the target if the initial and final kinetic energies of the projectile are, respectively, 5 J and 2 J?



Answer:

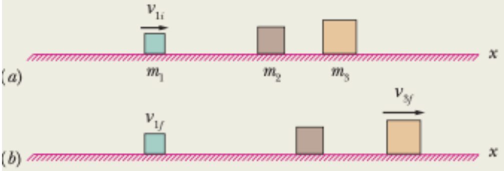
(a) 4 kg m/s ($P_i = P_f$, $P_{f2} = P_i - P_{f1}$)

(b) 8 kg m/s ($P_i = P_f$)

(c) 3 J ($K_{i1} = K_{i1} - K_{f1}$)

Sample Problem 9.08 Chain reaction of elastic collisions

Block 1 approaches a line of two stationary blocks with a velocity of v_{1i} = 10 m/s, collides with 2, which then collides with 3, $m3$ = 6.0 kg. After the second collision, 2 is again stationary and 3 has velocity v_{3f} = 5.0 m/s (Fig. *b*), the collisions are elastic.
What are the masses of 1 and 2?
What is the final velocity v_{1f} of 1?



E_{mec} conserved (energy losses negligible), conserve linear momentum along the *x* axis.
Start with the second collision in which block 2 stops because of its collision with block 3,

$$v_{2f} = \frac{m_2 - m_3}{m_2 + m_3} v_{2i}, \quad (9\text{-}67), \; v_{2f} = 0$$

$m_2 (v_{2i}) - m_3 (v_{2i}) = 0$, $m_2 = m_3 = 6.00$ kg

$$v_{3f} = \frac{2m_2}{m_2 + m_3} v_{2i}, \quad (9\text{-}68) \qquad \Rightarrow v_{2i} = v_{3f} = 5.0 \text{ m/s}$$

The first collision: v_{1i} = 10 m/s, using (9-68):

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i},$$

$$5.0 \text{ m/s} = \frac{2m_1}{m_1 + m_2} (10 \text{ m/s})$$

$$m_1 = \tfrac{1}{3} m_2 = \tfrac{1}{3}(6.0 \text{ kg}) = 2.0 \text{ kg}$$

Using (9-67)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -5.0 \text{ m/s}$$

9-8 Collisions in Two Dimensions (2 of 4)

- Apply the conservation of momentum along each axis
- Apply conservation of energy for elastic collisions

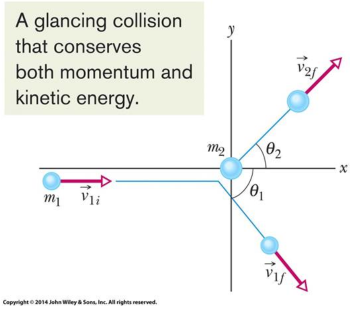


Figure 9-21

9-8 Collisions in Two Dimensions (3 of 4)

Example For Figure 9-21 for a stationary target:

- Along x: $m_1v_{1i} = m_1v_{1f}\cos\theta_1 + m_2v_{2f}\cos\theta_2$, **Equation (9-79)**
- Along y: $0 = -m_1v_{1f}\sin\theta_1 + m_2v_{2f}\sin\theta_2$. **Equation (9-80)**
- Energy: $\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$ **Equation (9-81)**
- These 3 equations for a stationary target have 7 unknowns (since $v_{2i} = 0$) : if we know 4 of them we can solve for the remaining ones.

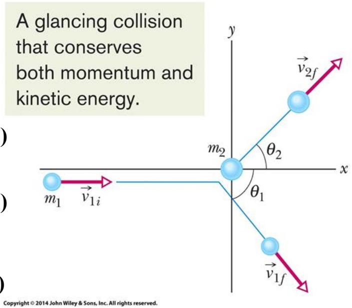


Figure 9-21

9-8 Collisions in Two Dimensions (4 of 4)

Checkpoint 9

In Fig. 9-21, suppose that the projectile has an initial momentum of $6\ \text{kg} \cdot \text{m/s}$, a final x component of momentum of $4\ \text{kg} \cdot \text{m/s}$, and a final y component of momentum of $-3\ \text{kg} \cdot \text{m/s}$. For the target, what then are (a) the final x component of momentum and (b) the final y component of momentum?

Answer:

(a) 2 kg m/s (conserve momentum along x)

(b) 3 kg m/s (conserve momentum along y)

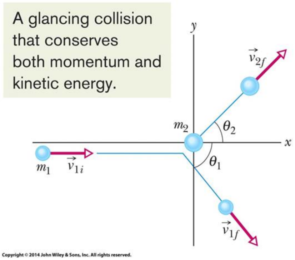


Figure 9-21

9-9 Systems with Varying Mass: A Rocket (2 of 3)

- Rocket and exhaust products form an isolated system
- Conserve momentum $P_i = P_f$
- Rewrite this as:

$$Mv = -dM\,U + (M + dM)(v + dv), \quad \textbf{Equation (9.83)}$$

- We can simplify using relative speed, defined as:
  $(v+dv) = v_{rel} + U$, *then:*

$$U = v + dv - v_{\text{rel}}. \quad \textbf{Equation (9.84)}$$

The ejection of mass from the rocket's rear increases the rocket's speed.

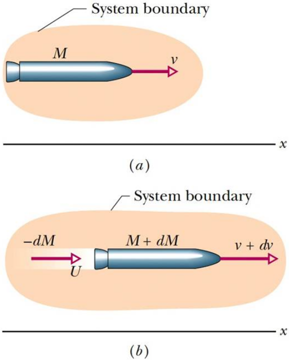


Figure 9-22

9-9 Systems with Varying Mass: A Rocket (3 of 3)

- The first rocket equation:

$$Rv_{\text{rel}} = Ma \qquad \textbf{Equation (9-87)}$$

- R is the mass rate of fuel consumption (dM/dt)
- The left side of the equation is **thrust**, T (=Rv_{rel}, in N)
- Derive the velocity change for a given consumption of fuel as the second rocket equation:

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f} \qquad \textbf{Equation (9-88)}$$

Sample Problem 9.09 Rocket engine, thrust, acceleration

A rocket that is losing mass, M_i is 850 kg consumes fuel at the rate R = 2.3 kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s.

(a) What thrust does the rocket engine provide?

$$T = Rv_{rel} = (2.3 \text{ kg/s})(2800 \text{ m/s}) = 6440 \text{ N} \approx 6400 \text{ N}. \quad \text{(Answer)}$$

(b) What is the initial acceleration of the rocket?

$$a = \frac{T}{M_i} = \frac{6440 \text{ N}}{850 \text{ kg}} = 7.6 \text{ m/s}^2$$

Note that the thrust T of the rocket must exceed the initial F_g on the rocket: (850 kg)(9.8 m/s2) = 8330 N > T
The rocket cannot be launched from Earth's surface!