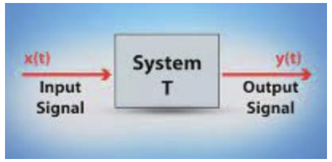
# Systems

## *System and Systems Model*

Definition 1(system): a system is an aggregation of simple physical elements according to certain topologies that achieves a defined task by transforming the input physical excitation signal to the output physical response signal.

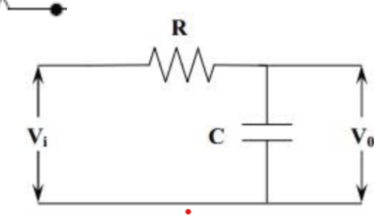
Definition 2 (system model): a system model is a mathematical function that describes the system behavior, and transformation between the input excitation signals models and the output response signals models, *under well-defined operational conditions.*



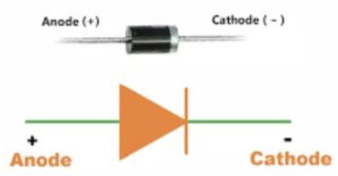

Example of system models:

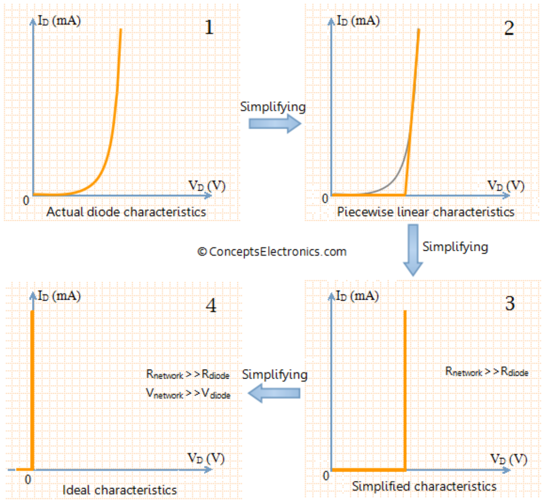
An Ohmic resistor: $v(t) = Ri(t)$


RC circuit: $\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_i(t)$



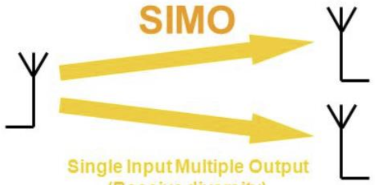

A Diode:

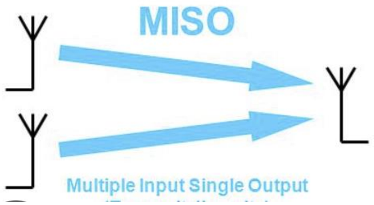


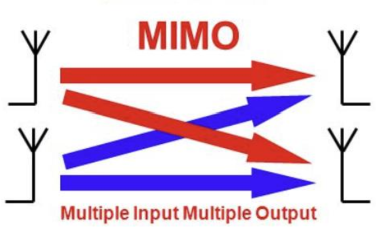



## System and System Models Classification

- <u>Input-output signals:</u>

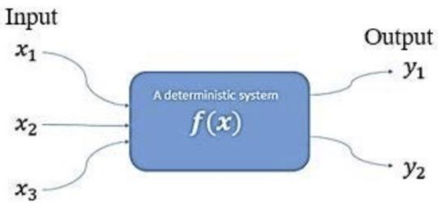




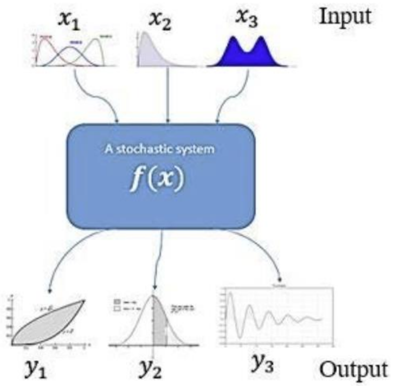



- <u>Variable-Based:</u>

Deterministic




Statistical




- <u>Time-Based:</u>

Analog: $t \in R \rightarrow \mathrm{y(t)} \in R$

Staircase: $t \in R \rightarrow \mathrm{y(t)} \in N$

Discrete-time $t \in N \rightarrow \mathrm{y(t)} \in R$

Digital $t \in N \rightarrow \mathrm{y(t)} \in N$ $(with\ binary\ codes)$

- <u>Model-Based:</u>

Linear/Nonlinear Static/Dynamic Time-Invariant/ Time-Variant Causal/Noncausal

Linear/Nonlinear System:

A system is said to be linear↔ it satisfies the superposition principle that is,

$\forall x_1(t), x_2(t)$ *inputs*, $\alpha_1, \alpha_2$ *parameters*, *and* $\forall$t: *if* $x_1(t) \xrightarrow{T} y_1(t), x_2(t) \xrightarrow{T} y_2(t)$ then for

$x(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t) \xrightarrow{T} y(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$

Linearity Test:

- additivity: *if* $x_1(t) \xrightarrow{T} y_1(t), x_2(t) \xrightarrow{T} y_2(t)$

  $x(t) = x_1(t) + x_2(t) \xrightarrow{T} y(t) = y_1(t) + y_2(t)$

- Proportionality: *if* $\bar{x}(t) \xrightarrow{T} \bar{y}(t)$ then for

  $x(t) = \alpha\bar{x}(t) \xrightarrow{T} y(t) = \alpha\bar{y}(t)$

Example: $v(t) = Ri(t)$

$v_1(t) = Ri_1(t),\ v_2(t) = Ri_2(t)$

Additivity: $i(t) = i_1(t) + i_2(t)$ $v(t) = R(i_1(t) + i_2(t)) = Ri_1(t) + Ri_2(t)) = v_1(t) + v_2(t)$

*Proportionality:* $\bar{v}(t) = R\bar{i}(t) \xrightarrow{T} v(t) = \alpha i(t) = \alpha R\bar{i}(t) = \alpha\,\bar{v}(t)$

Example2: $v(t) = Ri(t) + i_0$

$v_1(t) = Ri_1(t) + i_0 \qquad v_2(t) = Ri_2(t) + i_0$

Additivity: $i(t) = i_1(t) + i_2(t) \qquad v(t) = R(i_1(t) + i_2(t)) + i_0 \neq v_1(t) + v_2(t)$

$= R(i_1(t) + i_2(t)) + 2i_0$ → not satisfied → Nonlinear

*Exercise: determine if the system model* $y(t) = e^{x(t)}$ *is linear, show your proof.*

Time-invariant/ Time-variant system:

Definition: A system is said to be time-invariant if it is invariant with respect to reference shift-operation. That is, if $\forall\ excitation\ x(t), reponse\ y(t)\ and\ time\ shift\ \tau,$ the system response to the shifted $excitation\ \ x(t-\tau)$ is a shifted response form $y(t-\tau)$.

Remark: a system model with constant parameters is a time-invariant model.

For example: the system $y(t) = 10x(t)$ is linear time-invariant and $y(t) = \text{t}\sin(t)\, x(t)$ linear time variant

Time invariance Test:

- Compute the system response $y(t)$ to the excitation $x(t)$ and shift the response by $\tau$,$y_{sh}(t)$.
- Consider the response $\bar{y}(t)$ to the new excitation $\bar{x}(t) = x(t-\tau)$
- Check if $\bar{y}(t) = y_{sh}(t)$, if yes then the system is time-invariant.

Example1:

Determine if the system $y(t) = x(2t)$ is time invariant.

*Test:*

- The shifted form of $y(t)$ is $y_{sh}(t) = \text{x}(2\text{t} - \tau)$
- Consider the new excitation $\bar{x}(t) = x(t-\tau)$, its response is $\bar{y}(t) = x(2(t-\tau)) = x(2t-2\tau)$
- $\bar{y}(t) \neq y_{sh}(t)$, therefore the system is time-variant

Example2:

Determine if the system $y(t) = \sqrt{x(t)})$ is time invariant.

*Test:*

- The shifted form of $y(t)$ is $y_{sh}(t) = \sqrt{x(t-\tau)}$
- Consider the new excitation $\bar{x}(t) = x(t-\tau)$, its response is $\bar{y}(t) = \sqrt{x(t-\tau)}$
- $\bar{y}(t) = y_{sh}(t)$, therefore the system is time-invariant

Static and Dynamic Systems:

A system with transfer relation T is static if its response $y(t)$ occurs at the same time of its excitation $x(t)$. That is $y(t) = T[x(t)].(instatnteneous)$

- A static system is represented by an algebraic equation.
- A static time-invariant system is defined by a proportional relation of the type $y(t) = \alpha x(t)$

A dynamic system has a response that evolves in time based on known history or future information.

- A dynamic system is defined by a differential or integrodifferential equation.
- A dynamic time-invariant system is represented by a linear differential equation with constant coefficients.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 5y(t) = x(t) \qquad \frac{dy}{dt} + 5y(t) = x(t)$$

- If the differential equation is nonlinear then the system is a dynamic nonlinear system.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 5y(t) + 6 = x(t) \qquad \frac{dy^2}{dt} + 5y(t) = x(t)$$

- If the differential equation has variable coefficients then the system is a dynamic time variant system.

$$\frac{dy^2}{dt} + \sin(t)y(t) = x(t) \qquad \frac{dy^2}{dt} + 5ty(t) = x(t)$$

Causal/Noncausal Systems:

Definition: A system is said to be causal if it satisfies the cause-effect principle which asserts that the response can not precede the application of the excitation.

Causal system characterization:

A system is causal if $\forall x_1(t), x_2(t)$ and $\forall \tau$: $x_1(t) = x_2(t)\ \forall t\ < \tau\ \rightarrow y_1(t) = y_2(t)\ \forall t\ < \tau$, Or alternatively, if $\forall x(t)$ , $x(t) = 0\ \forall t\ < \tau\ \rightarrow y(t) = 0\ \forall t\ < \tau$

Test:

For causality Check: prove that $t_{excitation} < t_{response}$

For noncausality check: find a case at which $t_{excitation} > t_{response}$

Example1:

Determine if the systems $y(t) = x(t - \tau)$ is causal for:

- $\tau > 0$
- $\tau < 0$
- Solution: for the system to be causal the system model must satisfy t $- \tau \leq t$, that is $-\tau \leq 0 \rightarrow \tau \geq 0$

Example2:

Determine if the system $y(t) = x(\sqrt{t})$is causal.

Solution: $\sqrt{t} \leq t \rightarrow 0 \leq t \leq t^2 \rightarrow 0 < 1 \leq t$, so the system is not causal-->

In fact $y\left(\frac{1}{4}\right) = x(\frac{1}{2})$

## Linear Time Invariant Response (LTI):

Impulse Response:

Definition: The zero-state response $h(t)$ of a linear time-invariant system with transform $\mathrm{y}(t) = T[x(t)]$ for the excitation input $\delta(t)$ is said to be the impulse response of the system.

The impulse response completely characterizes the LTI system. Moreover, the zero state response $y(t)$ of an LTI system, with impulse response $h(t)$ , to any excitation signal $x(t)$ system can be computed using the convolution integral of $x(t), and\ h(t)$( discussed later)

Determination of the impulse response and the solution of a dynamic LTI system to any singularity signal excitation:

Procedure:

- Determine the zero-input response $g(t)$ of the $n^{th}$ order dynamic system for $t \geq 0^+$
- Build the response model using the form: $\mathrm{y}(t) = g(t)u(t) + \sum_{k=0}^{m} \alpha_k u_k(t)$ with $u_m$ the minimum order singularity signal with $n^{th}$ derivative that covers the maximum order excitation singularity signal term.
- Apply the generalized identity of singularity signals to construct the relations that defines $\alpha_k$ in terms of $g(t)$ and its derivatives at $t = 0$.
- Equate the expressions of $g(t)$ and its derivatives to the values obtained from the identity of singularity signals.
- Solve the set of equations to determine the values of the parameters of the solution $g(t)$ and the $\alpha_k$s

Example1:

Determine the impulse response of the system $\frac{dy(t)}{dt}+5y(t)=2x(t)$.

Solution: The impulse response $h(t)$ is obtained to $x(t)=\delta(t)$, thus the equation becomes:\

$\frac{dh(t)}{dt}+5h(t)=2\delta(t)$

The zero input response of the system is the solution of $\frac{dg(t)}{dt}+5g(t)=0$ which is given by:

$g(t)=Ae^{-5t}$

The solution $h(t)$ is constructed as $h(t)=g(t)u(t)$, since its first derivative (system order) has the term $\delta(t)$ that is all the parameters $\alpha_k=0$.

Now compute the derivative of $h(t)$ and apply in the differential equation (**remember that $g(t)\delta(t)=g(0)\delta(t)$**):

$g'(t)u(t)+g(0)\delta(t)+5g(t)u(t)=2\delta(t)\leftrightarrow g(0)=2$.

Applying $g(0^+)=2=Ae^{+0}\rightarrow A=2$.

The solution is: $h(t)=2e^{-5t}u(t)$

**Exercise:** compute the solution of the same system for $x(t)=10\delta(2t-8)$.

Example2: Determine the response of the system $\frac{dy(t)}{dt}+5y(t)=2\dot{x}(t)$ for $x(t)=\delta(t)$.

Solution: The zero input response of the system is the solution of $\frac{dg(t)}{dt}+5g(t)=0$ which is given by: $g(t)=Ae^{-5t}$

The solution $y(t)$ is constructed as $y(t)=g(t)u(t)+B\delta(t)$, since its first derivative (system order) has the term $\dot{\delta}(t)$. In fact with out adding the zero order singularity signal $\delta(t)$ the maximum order derivative (first order in this case) has the maximum singularity order $\delta(t)$ which does not cover the $\dot{\delta}(t)$ term of the right side.

Differentiating $y(t)$ and applying in the differential equation we get

$g'(t)u(t)+g(0)\delta(t)+B\dot{\delta}(t)+5g(t)u(t)+5B\delta(t)=2\dot{\delta}(t)\ \leftrightarrow B=2$ and $g(0)+5B=0\rightarrow g(0)=-10=Ae^{0}=A$.

Thus the solution is given by: $y(t)=-10e^{-5t}u(t)+2\delta(t)$

**Exercise:** Determine the response of the system $\frac{dy(t)}{dt}+5y(t)=2\ddot{x}(t)$ to $x(t)=\delta(t)$.

**Important fact and exercise:** Observe that the zero-state response to the derivative of $\delta(t)$ is the derivative of the response to $\delta(t)$. This result is generalized in the following theorem(show).

***Theorem (zero state response):***

Given an LTI system with zero state response $y(t)$ to the input excitation $x(t)$, then the zero-state response to:

- $\frac{dx(t)}{dt}$ is $\frac{dy(t)}{dt}$
- $\int_0^t x(\sigma)d\sigma$ is $\int_0^t y(\sigma)d\sigma$

Example3:

Determine the response of the system $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = \ddot{x}(t)$, for $x(t) = \delta(t)$.

- The system has characteristic roots of the characteristic algebraic equation: $\sigma_1 = -1\ and \sigma_2 = -2$, therefore the zero input response $g(t) = Ae^{-t} + Be^{-2t}$
- The zero-state solution is $y(t) = g(t)u(t) + C\ \delta(t)$
- $2 \times y(t) = 2 \times (g(t)u(t) + C\ \delta(t))$
- $3 \times \frac{dy(t)}{dt} = 3 \times (g'(t)u(t) + g(0)\delta(t) + C\delta'(t))$
- $\frac{d^2y}{dt^2} = g''(t)u(t) + g'(0)\delta(t) + g(0)\delta'(t) + C\delta''(t)$
- Identity of singularity signals:

  Balance of $\delta''$: $C$=1

  Balance of $\delta'$: $g(0) + 3C = 0 \leftrightarrow g(0) = -3 \times 1 = -3$

  Balance of $\delta$: $g'(0) + 3g(0) + 2C = 0 \leftrightarrow g'(0) = -3 \times -3 - 2 \times 1 = 7$

Computing A and B using $g(0)$ and $g'(0)$:

$$g(0^+) = -3 = A + B$$
$$g'(0) = 7 = -A - 2B$$

Solving the system we obtain $B = -4$ and $A = -3 + 4 = 1$. Thus the solution is:

$y(t) = (e^{-t} - 4e^{-2t})u(t) + \delta(t)$

<u>Exercise:</u>

Compute the response to $x(t) = \dot{\delta}(t)$ and then compute it from the solution of Example 3 using the zero-state response theorem, compare.

Convolution Integral:

Theorem: Impulse and step response of an LTI system:

Given an LTI system with impulse response $h(t)$ and/or a step response $a(t)$, the zero state response of the system to any input $x(t)$ can be determined by the convolution integrals:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\sigma)h(t-\sigma)d\sigma$$

$$y(t) = x'(t) * a(t) = \int_{-\infty}^{\infty} x'(\sigma)a(t-\sigma)d\sigma \quad \text{(Duhamel's Integral)}$$

Proof (convolution with $h(t)$): from the convolution property of $\delta(t)$ we can write:

$x(t) = \int_{-\infty}^{\infty} x(\sigma)\delta(t-\sigma)d\sigma$, applying the linear time invariant transform $T$ to this excitation input we obtain

$$y(t) = T[x(t)] = T[\int_{-\infty}^{\infty} x(\sigma)\,\delta(t-\sigma)d\sigma] = \int_{-\infty}^{\infty} x(\sigma)T[\delta(t-\sigma)]d\sigma$$

$$= \int_{-\infty}^{\infty} x(\sigma)\,h(t-\sigma)d\sigma$$

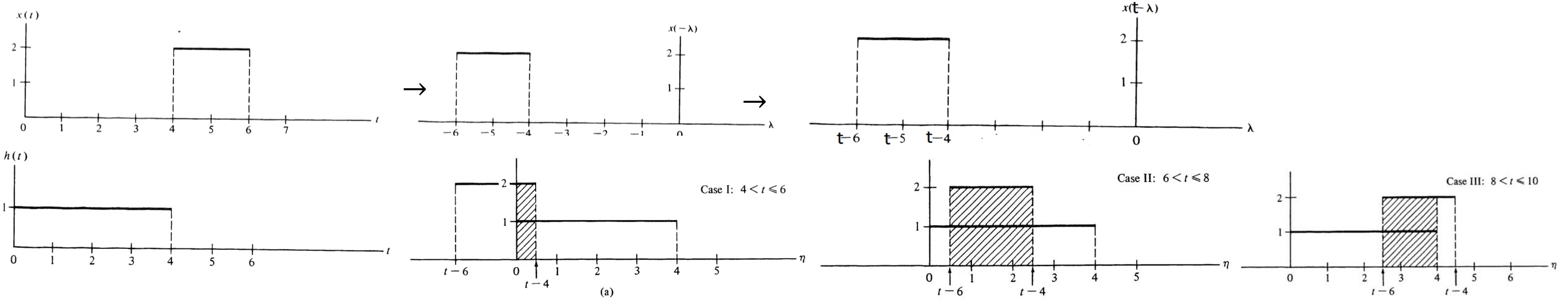
Computation of convolution Integral $x_1(t) * x_2(t)$:

- Transfer of the independent variable from the $t$ space to the $\sigma$ space $x_1(t) \to x_1(\sigma), x_2(t) \to x_2(\sigma)$.
- folding of one of the two inputs of the convolution operator $x_2(\sigma) \to x_2(-\sigma)$.
- Shift of the folded variable by $t$ : $x_2(-\sigma) \to x_2(t-\sigma)$
- Integration of the multiplication using the form $\int_{-\infty}^{\infty} x_1(\sigma)x_2(t-\sigma)d\sigma$ over the integrand definition ranges

Properties of the convolution operator:

- $x_1(t) * x_2(t) = x_2(t) * x_1(t)$
- $x_1(t) * [\alpha x_2(t)] = \alpha[x_1(t) * x_2(t)]$
- $x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$
- $x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$
- If $x_1(t)$ is time limited to $[a\ b]$, $x_2(t)$ is time limited to $[c\ d]$, then $x_1(t) * x_2(t)$ is time limited to $[a+c\ \ b+d]$
- If the area under $x_1(t)$ is $A_1$ and the area under $x_2(t)$ is $A_2$, then the area under $x_1(t) * x_2(t)$ is $A_1 \times A_2$

Example: Compute the response of the LTI system with impulse response $h(t) = \pi(\frac{t-2}{4})$ to the input $x(t) = 2\pi(\frac{t-5}{2})$

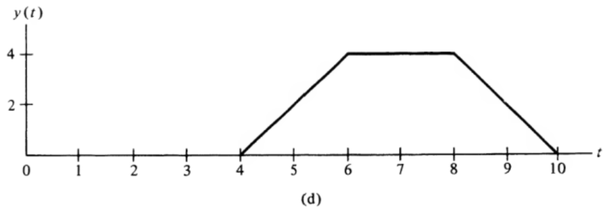



$$y(t) = 0 \ for \ t \leq 4 \ and \ t \geq 10$$

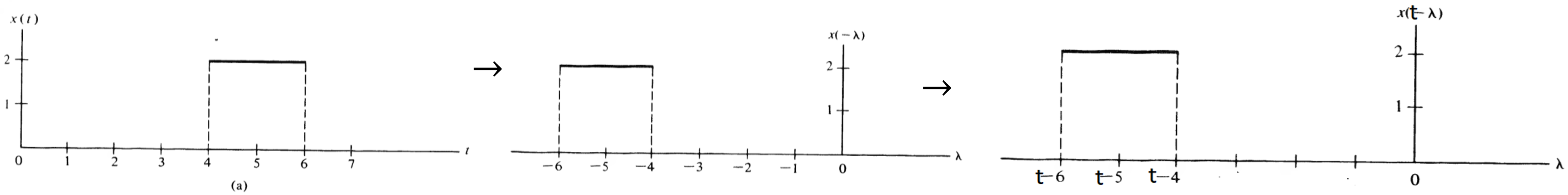
$$y(t) = \int_0^{t-4} 2 \cdot 1 \, d\sigma = 2(t-4) \ \ for \ t-4 \leq 4 \ and \ t-4 \geq 0 \ and \ t-6 \leq 0 \rightarrow t \in [4,6]$$

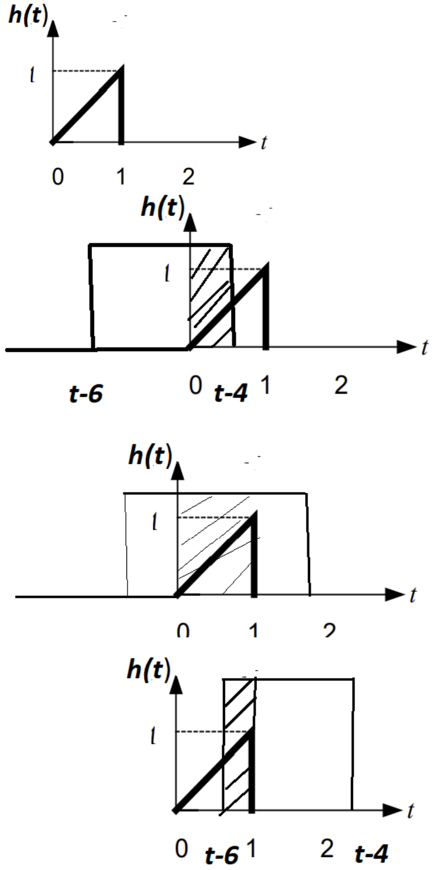
$$y(t) = \int_{t-6}^{t-4} 2 \cdot 1 \, d\sigma = 4 \ \ for \ t-4 \leq 4 \ and \ t-4 \geq 0 \ and \ t-6 \geq 0 \rightarrow t \in [6, 8]$$

$$y(t) = \int_{t-6}^{4} 2 \cdot 1 \, d\sigma = 2(10-t) \ \ for \ t-4 \geq 4 \ and \ t-6 \geq 0 \ and \ t-6 \leq 4 \rightarrow t \in [8, 10]$$



Example: Compute the response of the LTI system with impulse response $h(t) = r(t)\,\pi(\frac{t-\frac{1}{2}}{2})$ to the input $x(t) = 2\pi(\frac{t-5}{2})$




$$y(t) = 0 \; for \; t \le 4 \; and \; t \ge 7$$

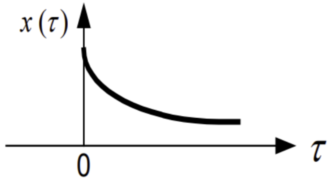
$$y(t) = \int_0^{t-4} 2 \times \sigma d\sigma = (t-4)^2 \quad for \begin{cases} t-4 \ge 0 \\ t-4 \le 1 \\ t-6 \le 0 \end{cases} \rightarrow 4 \le t \le 5$$

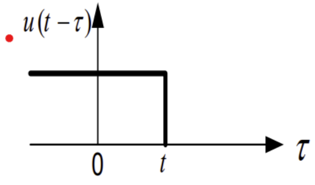
$$y(t) = \int_0^1 2 \times \sigma d\sigma = 1 \quad for \begin{cases} t-4 \ge 0 \\ t-4 \ge 1 \\ t-6 \le 0 \end{cases} \rightarrow 5 \le t \le 6$$

$$y(t) = \int_{t-6}^1 2 \times \sigma d\sigma = 1-(t-6)^2 \quad for \begin{cases} t-4 \ge 1 \\ t-6 \ge 0 \\ t-6 \le 1 \end{cases} \rightarrow 6 \le t \le 7$$

Example: Compute the response of the LTI system with impulse response $h(t) = u(t)$ to the input $x(t) = e^{-2t}u(t)$

$$y(t) = 0 \; for \; t \le 0$$




$$y(t) = \int_0^t e^{-2\tau} d\tau = \frac{e^{-2t}-1}{-2} = \frac{1-e^{-2t}}{2} \; for \; t \ge 0$$

Sinusoidal Steady State Response:

Theorem: Given an LTI system with impulse response h(t), the response of the system to a sinusoidal input $x(t) = Xcos(\omega_0 t + \varphi)$ is sinusoidal with the same input frequency $y(t) = Ycos(\omega_0 t + \theta)$ with:

$Y = X \cdot |H(\omega)|_{\omega=\omega_0}, \quad \theta = \varphi + < H(\omega)|_{\omega=\omega_0}$

Proof:

let $x(t) = Xe^{j(\omega_0 t+\varphi)}$ then $y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot Xe^{j(\omega_0(t-\tau)+\varphi)} d\tau = Xe^{j(\omega_0 t+\varphi)} \int_{-\infty}^{\infty} h(\tau) \cdot e^{-j(\omega_0 \tau)} d\tau = Xe^{j(\omega_0 t+\varphi)} \cdot H(\omega)|_{\omega_0}$

Where $H(\omega) = \int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega\tau} d\tau$ is the frequency response of the system the characterizes the spectral response of the linear time invariant system. $H(\omega)$ is a complex function of the real variable $\omega$ that represents the Fourier transform of the impulse response $h(t)$.

$Re(x(t)) = Xcos(\omega_0 t + \varphi) \rightarrow Re(y(t)) = Ycos(\omega_0 t + \theta)$ *which proves the assertion of the theorem*

Example1: compute the frequency response of the system with impulse response $h(t) = 10e^{-2t}u(t)$

Solution: $H(\omega) = \int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega\tau} d\tau = \int_{0}^{\infty} 10e^{-2\tau} \cdot e^{-j\omega\tau} d\tau = \int_{0}^{\infty} 10e^{-(2+j\omega)\tau} d\tau = 10 \frac{e^{-(2+j\omega)\tau}}{-(2+j\omega)}|_0^{\infty} = 10 \frac{1}{(2+j\omega)}$

$$|H(\omega)| = \frac{10}{\sqrt{4+\omega^2}}, \qquad < H(\omega) = -tan^{-1}(\frac{\omega}{2})$$

Example2(sinusoidal steady-state response): compute the steady-state response of the system in example1 to the input signal $x(t) = 2\cos(4t + \frac{\pi}{3}) + 5\sin(6t + \frac{\pi}{4})$

Solution: the input is composed of two sinusoidal signals so we can apply superposition and compute the sinusoidal steady-state response of each sinusoid using the theorem.

$$y(t) = 2 \cdot \frac{10}{\sqrt{4+4^2}} \cos(4t + \frac{\pi}{3} - tan^{-1}\left(\frac{4}{2}\right)) + 5 \cdot \frac{10}{\sqrt{4+6^2}} \sin(6t + \frac{\pi}{4} - tan^{-1}\left(\frac{6}{2}\right) =$$

$= \frac{20}{\sqrt{20}} \cos(4t + \frac{\pi}{3} - tan^{-1}(2)) + \frac{50}{\sqrt{40}} \sin(6t + \frac{\pi}{4} - tan^{-1}(3))$ (compute the final form, note that the argument of the $tan^{-1}$ is in radiant)

<u>System Stability:</u>

<u>Definition:</u> An LTI system is said to be asymptotically stable if its transient response goes to zero and a steady state response is reached for t goes to infinity.

<u>Theorem1:</u> an LTI system with impulse response $h(t)$ is asymptotically stable $\leftrightarrow \lim_{t\to\infty} h(t) = 0$.

<u>Theorem2:</u> adynamic LTI system is asymptotically stable↔ all the roots of its characteristic equation/ the poles of its transfer function have a negative real part (located in the left semi plan of the complex plan)

<u>Theorem3:</u> an LTI system is unstable if it has at least a positive real-part root or a repeated root with zero real part.

<u>Definition:</u>(BIBO stability) an LTI system is said to be BIBO (Bounded Input/Bounded Output)↔ $\forall\, input\ x(t) with\ |x(t)| \leq N,\ \ \exists M < \infty$ so that the respons $|y(t)| \leq M, \forall t$ (weak stability)

<u>Theorom4:</u> a system is BIBO stable ↔ $\int_{-\infty}^{\infty} |h(t)|dt < \infty$ that is if its impulse response is absolutely integrable.

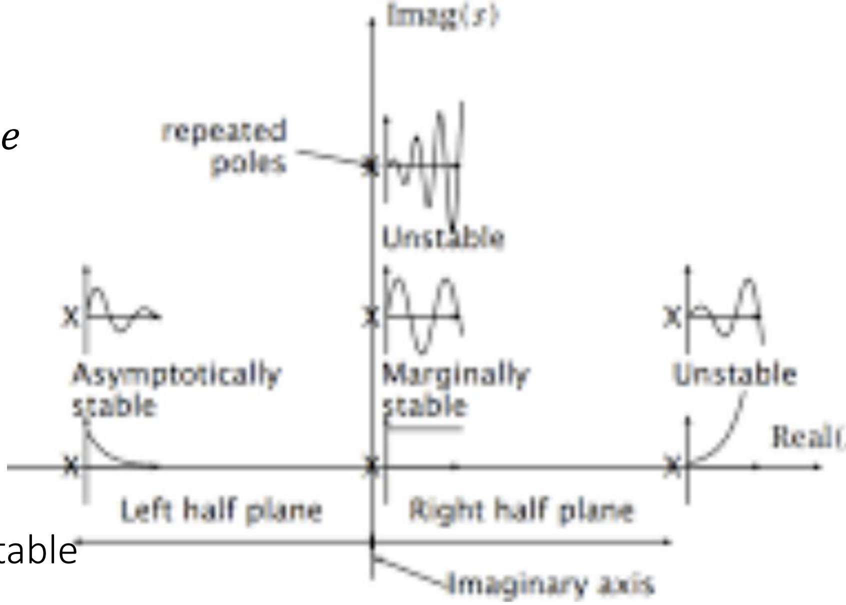
<u>Exercise:</u> prove this theorem.

<u>Theorom5:</u> a system is BIBO stable if it has no roots with positive real parts and all the roots with zero real part are not repeated roots.

<u>Example1</u>: discuss the stability of the following dynamic systems:

- $\frac{d^2y}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = x(t)$, the roots of are $\sigma_1 = -1, \sigma_2 = -2 \rightarrow asymptotically\ stable$
- $H(s) = \frac{(s+4)}{(s^2+4)(s+1)} \rightarrow BIBO\ stable$
- $H(s) = \frac{(s+4)}{(s^2+4)^{\,2}(s+1)} \rightarrow Unstable$
- $H(s) = \frac{(s+4)}{(s+2)(s-1)} \rightarrow Unstable$

Asymptotically stable →BIBO Stable




<u>Example2</u>: Prove that the system with the following $h(t)$ achieves the BIBO stability theorem

$h(t) = 10e^{-3t}u(t) \rightarrow \int_{-\infty}^{\infty} |10e^{-3t}u(t)|dt \rightarrow \int_{0}^{\infty} 10e^{-3t}dt = \frac{10e^{-3t}}{-3}\Big|_0^{\infty} = \frac{10}{3} < \infty$ →BIBO stable

## Modeling and Simulation of an LTI System:

Modeling a system for simulation and prototyping purposes means constructing an internal representation (state space representation) for a given external model representation (differential equation/ Laplace transform). While the external model is unique, the internal model is not. The selected internal topology should serve the simulation or prototyping objectives.

Simulation: Using computer packages (such as Matlab, Mathcad, LabView,…) to analyze system characteristics and its response to various excitation input signals.

Prototyping: Building a system model using hardware components for testing and analysis objectives.

## Observer Representation Model:

As an example of modeling, we consider the observer representation which can be defined by separate and integrate processes.

Example1: Determine the observer model of the system definedby:

$$\frac{d^4y}{dt^4} - 5\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = 6\frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 2x$$

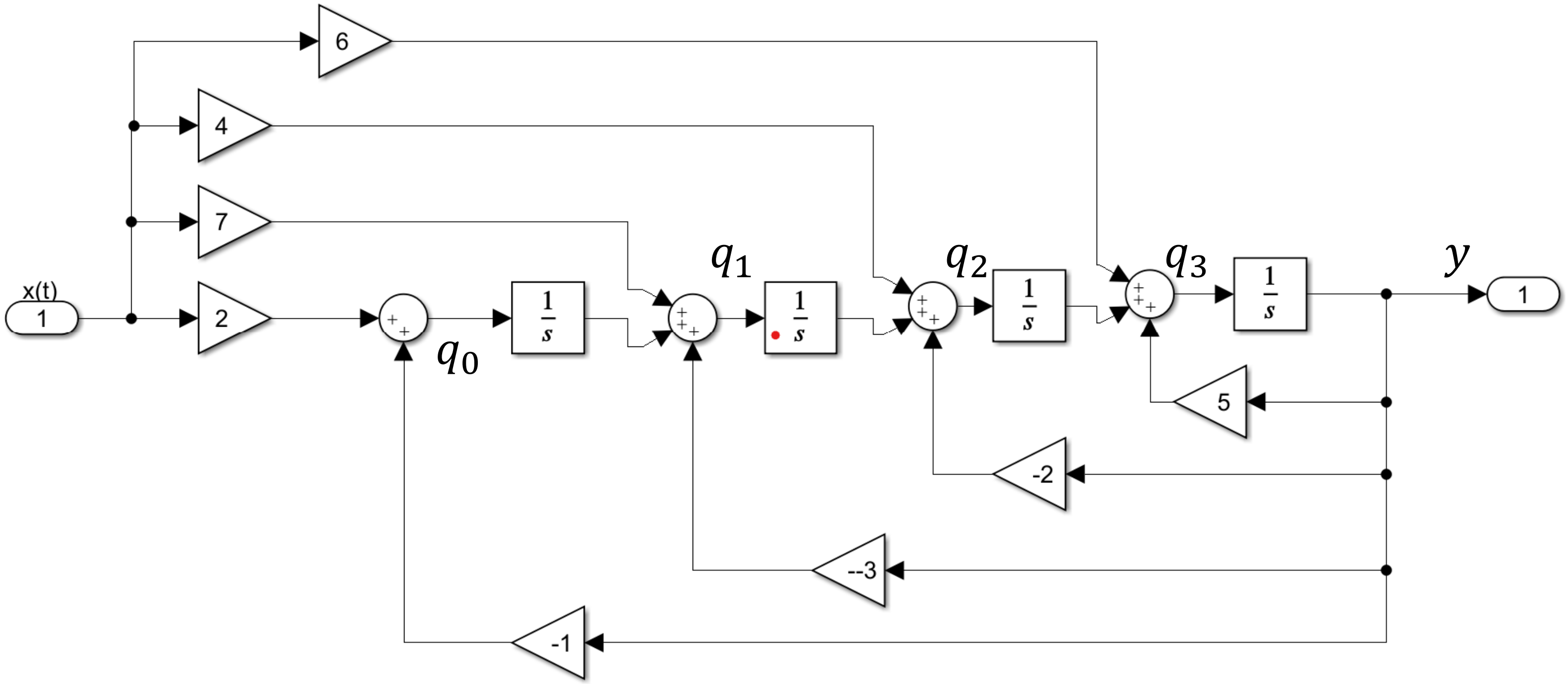
Separate: $$\frac{d^4y}{dt^4} - 5\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 6\frac{d^3x}{dt^3} - 4\frac{d^2x}{dt^2} - 7\frac{dx}{dt} = 2x - y = q_0$$

Integrate + Separate: $$\frac{d^3y}{dt^3} - 5\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 6\frac{d^2x}{dt^2} - 4\frac{dx}{dt} = \int_0^t q_0 d\sigma + 7x - 3y = q_1$$

Integrate + Separate: $$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} - 6\frac{dx}{dt} = \int_0^t q_1 d\sigma + 4x - 2y = q_2$$

Integrate + Separate: $$\frac{dy}{dt} = \int_0^t q_2 d\sigma + 6x + 5y = q_3$$

Integrate: $$y = \int_0^t q_3 d\sigma$$



Example2: Determine the observer model of the system defined by:

$$\frac{d^4y}{dt^4}+2\frac{d^2y}{dt^2}+3\frac{dy}{dt}+5y=3\frac{d^3x}{dt^3}+4\frac{dx}{dt}+2x$$

Separate: $\frac{d^4y}{dt^4}-5\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}+3\frac{dy}{dt}-6\frac{d^3x}{dt^3}-4\frac{d^2x}{dt^2}-7\frac{dx}{dt}=2x-5y=q_0$

Integrate + Separate: $\frac{d^3y}{dt^3}+3\frac{dy}{dt}-6\frac{d^2x}{dt^2}=\int_0^t q_0 d\sigma+4x-3y=q_1$

Integrate + Separate: $\frac{d^2y}{dt^2}-6\frac{dx}{dt}=\int_0^t q_1 d\sigma-2y=q_2$

Integrate + Separate: $\frac{dy}{dt}=\int_0^t q_2 d\sigma+3x=q_3$

Integrate: $y=\int_0^t q_3 d\sigma$