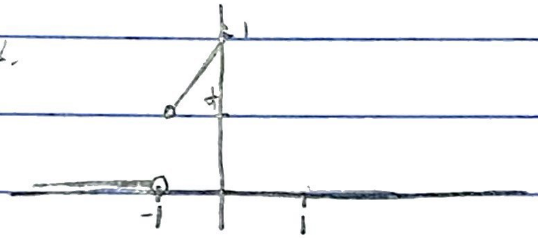
Exercises :

Q1: given the distribution function $F(x) = \begin{cases} 0 & , \quad x < -1 \\ \frac{x+2}{4} & , \quad -1 \leq x < 1 \\ 1 & , \quad 1 \leq x \end{cases}$. Sketch $F(x)$



a. $Pr(-\frac{1}{2} < x \leq \frac{1}{2}) = F(\frac{1}{2}) - F(-\frac{1}{2}) = \frac{\frac{1}{2}+\cancel{2}}{4} - \frac{-\frac{1}{2}+\cancel{2}}{4} = \frac{1}{4}$.

b. $Pr(X=0) = F(0) - F(0^-) = \frac{2}{4} - \frac{2}{4} = 0$

c. $Pr(X=1) = F(1) - F(1^-) = 1 - \frac{3}{4} = \frac{1}{4}$

d. $Pr(2 < X \leq 3) = F(3) - F(2) = 1 - 1 = 0$

discrete

Q13: let $f(x) = \begin{cases} \frac{x}{6} & , x = 1, 2, 3 \\ 0 & , \text{elsewhere} \end{cases}$ be the p.d.f of X.

Find the distribution function and the p.d.f of $Y = X^2$.

c.d.f of X is $F(x) = \sum f(x)$

$$F(x) = \begin{cases} 0 & , x < 1 \\ \frac{1}{6} & , 1 \leq x < 2 \\ \frac{3}{6} & , 2 \leq x < 3 \\ 1 & , 3 \leq x \end{cases}$$

→ we defined $y = x^2$ so y have space $B = \{1, 4, 9\}$ and the c.d.f of Y will be found by

$F(y) = Pr(Y \leq y) = Pr(X^2 \leq y) = Pr(-\sqrt{y} \leq X \leq \sqrt{y})$

but $Pr(-\sqrt{y} \leq X \leq \sqrt{y}) = F_x(\sqrt{y}) - \underline{F_x(-\sqrt{y})} \rightarrow = 0$ elsewhere.

So $F(y) = F(\sqrt{y})$

$$= \left.\begin{cases} 0 & , y < 1 \\ \frac{1}{6} & , 1 \leq y < 4 \\ \frac{3}{6} & , 4 \leq y < 9 \\ 1 & , 9 \leq y \end{cases}\right\} \rightarrow \text{c.d.f of } Y$$

So the p.d.f of y: $f(y) = \begin{cases} \frac{\sqrt{y}}{6} & , x = 1, 4, 9 \\ 0 & , \text{elsewhere} \end{cases}$

continuous

	p.d.f	c.d.f
X	f	F
Y	g	G

Q76: let X have the p.d.f $f(x) = \begin{cases} 4x^3 & , 0 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$

Find the distribution function and p.d.f of $Y = -2\ln X^4 = -8\ln x$.

① $G(y) = \Pr(Y \le y) = \Pr(-8\ln x \le y) = \Pr(\ln x \ge -\frac{y}{8})$

$= \Pr(x \ge e^{-\frac{y}{8}})$.

$= \int_{e^{-\frac{y}{8}}}^{\infty} f(x)\, dx$

so $G(y) = \begin{cases} 0 & , y < 0 \\ \int_{e^{-\frac{y}{8}}}^{1} 4x^3 dx = 1 - e^{-\frac{y}{2}} & , y > 0 \end{cases}$

$\int_{e^{-\frac{y}{8}}}^{1} 4x^3 dx = x^4 \Big|_{e^{-\frac{y}{8}}}^{1} = 1 - (e^{-\frac{y}{8}})^4$
$= 1 - e^{-\frac{y}{2}}$

Now, since it cont. $g(y) = \bar{G}(y)$.

$g(y) = \begin{cases} \frac{1}{2} e^{-\frac{y}{2}} & , y > 0 \\ 0 & , \text{elsewhere}. \end{cases}$

Q79: let $f(x) = \begin{cases} \frac{1}{3} & , -1 < x < 2 \\ 0 & , \text{elsewhere} \end{cases}$ be the p.d.f of X.

Find the distribution function and the p.d.f of $Y = X^2$.

Hint: $Pr(X^2 \leq y)$ for two casses $0 \leq y < 1$ and $1 \leq y < 4$.

① $G(y) = Pr(Y \leq y) = Pr(X^2 \leq y) = Pr(\sqrt{y} \leq X \leq \sqrt{y})$.

$$\Rightarrow \quad G(y) = \begin{cases} \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{3}\,dx = \frac{2}{3}\sqrt{y} & , 0 \leq y < 1 \\ \frac{1}{3} + \int_{1}^{\sqrt{y}} \frac{1}{3}\,dx = \frac{1}{3} + \frac{1}{3}\sqrt{y} & , 1 \leq y < 4 \\ 1 & , 4 \leq y \\ 0 & , y \leq 0 \end{cases}$$

$$\text{So} \quad g(y) = \begin{cases} \frac{1}{3\sqrt{y}} & , 0 \leq y < 1 \\ \frac{1}{6\sqrt{y}} & , 1 < y < 4 \\ 0 & , \text{elsewhere} \end{cases}$$