Exp Find Asy. of $f(x) = \frac{\sin x}{x}$ and sketch

H. Asy $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\sin x}{x} = 0 \Rightarrow y = 0$ is H. Asy.

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sin x}{x} = 0$

$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

$0 \leq \lim_{x \to -\infty} \frac{\sin x}{x} \leq 0$

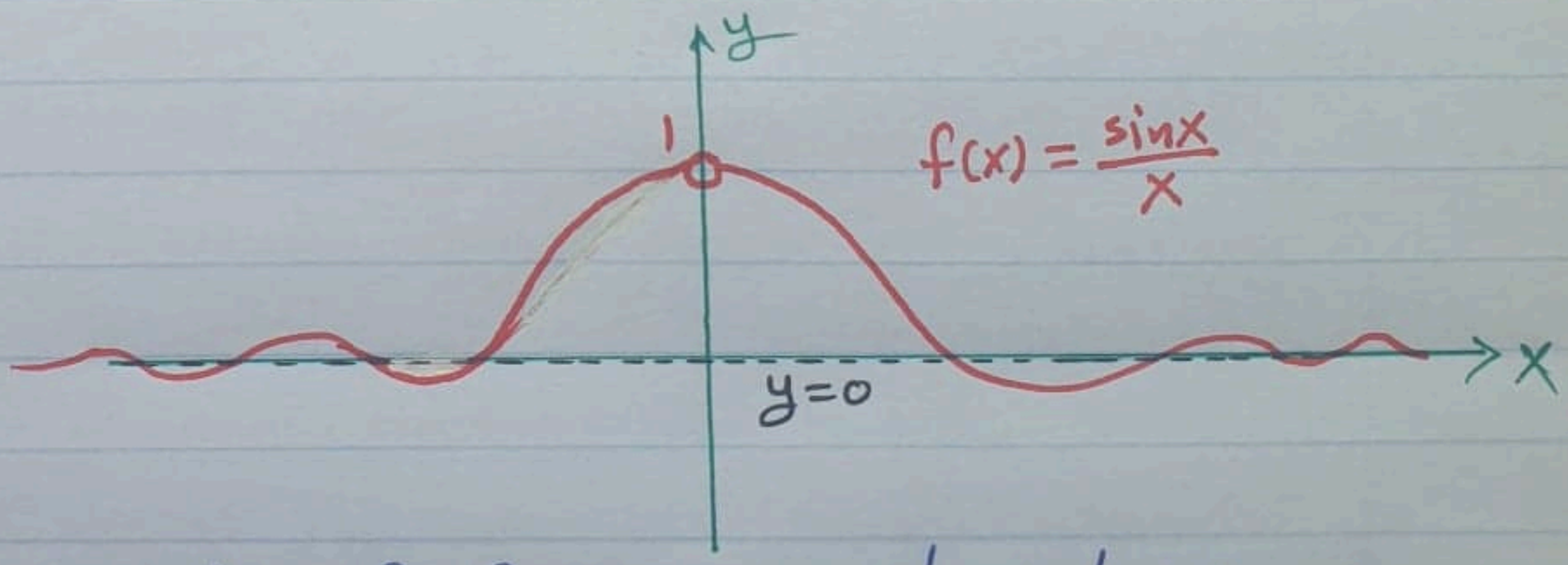
V. Asy check zeros of denominater

check $x = 0$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin x}{x} = 1 \Rightarrow x = 0$ is not V. Asy.

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin x}{x} = 1 \Rightarrow x = 0$ is not V. Asy.

O. Asy: $f(x)$ has no O. Asy since it has H. Asy.



Remark: The graph of $f(x)$ may intersect the Asy.

Exp Let $f(x) = \frac{x^3+1}{x^2-1}$ Find Asy. and sketch

- f has O. Asy since the degree of numerator is one more than the degree of denominator
- Hence, f has no H. Asy.

- To find O. Asy $\Rightarrow$ we use Long Division

$$x^2-1 \;\overline{)\; x^3+1}$$

$$x$$

$$-x^3 + x$$

$$x+1$$

$$f(x) = \frac{x^3+1}{x^2-1} = x + \frac{x+1}{x^2-1}$$

$y = x$ is the O. Asy.

- V. Asy : check zeros of $x^2-1=0$

$$(x-1)(x+1)=0$$

$\boxed{x=1}$ or $\boxed{x=-1}$

check $\boxed{x=1}$

$$\lim_{x\to 1^+} \frac{x^3+1}{x^2-1} = \frac{2}{\text{small}+} = \infty \Rightarrow x=1 \text{ is V. Asy}$$

$$\lim_{x\to 1^-} \frac{x^3+1}{x^2-1} = \frac{2}{\text{small}-} = -\infty$$

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check $\boxed{x=-1}$

$$\lim_{x\to -1^+} \frac{x^3+1}{x^2-1} = \lim_{x\to -1^+} \frac{3x^2}{2x} = -\frac{3}{2}$$

$$\lim_{x\to -1^-} \frac{x^3+1}{x^2-1} = -\frac{3}{2}$$

$\Rightarrow$ $x=-1$ is not V. Asy but $x=-1$ is removable discontinuity $(\frac{0}{0})$

Holes (0, -1)

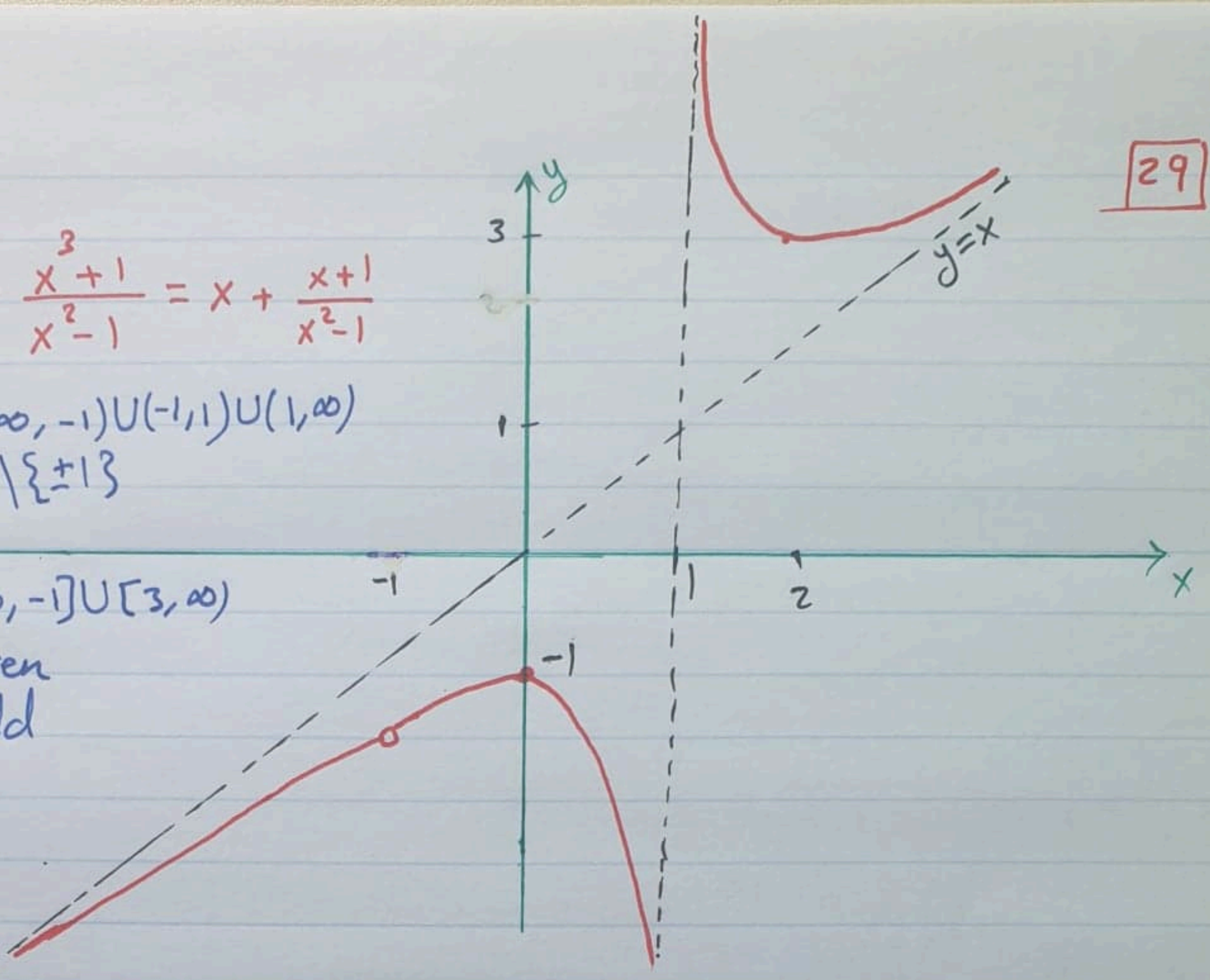
$$f(x) = \frac{x^3+1}{x^2-1} = x + \frac{x+1}{x^2-1}$$

$D = (-\infty,-1) \cup (-1,1) \cup (1,\infty)$
$= \mathbb{R} \setminus \{\pm 1\}$

$R = (-\infty,-1] \cup [3,\infty)$

Not Even

Not Odd



Exp sketch $f(x) = \frac{x+1}{x-1}$

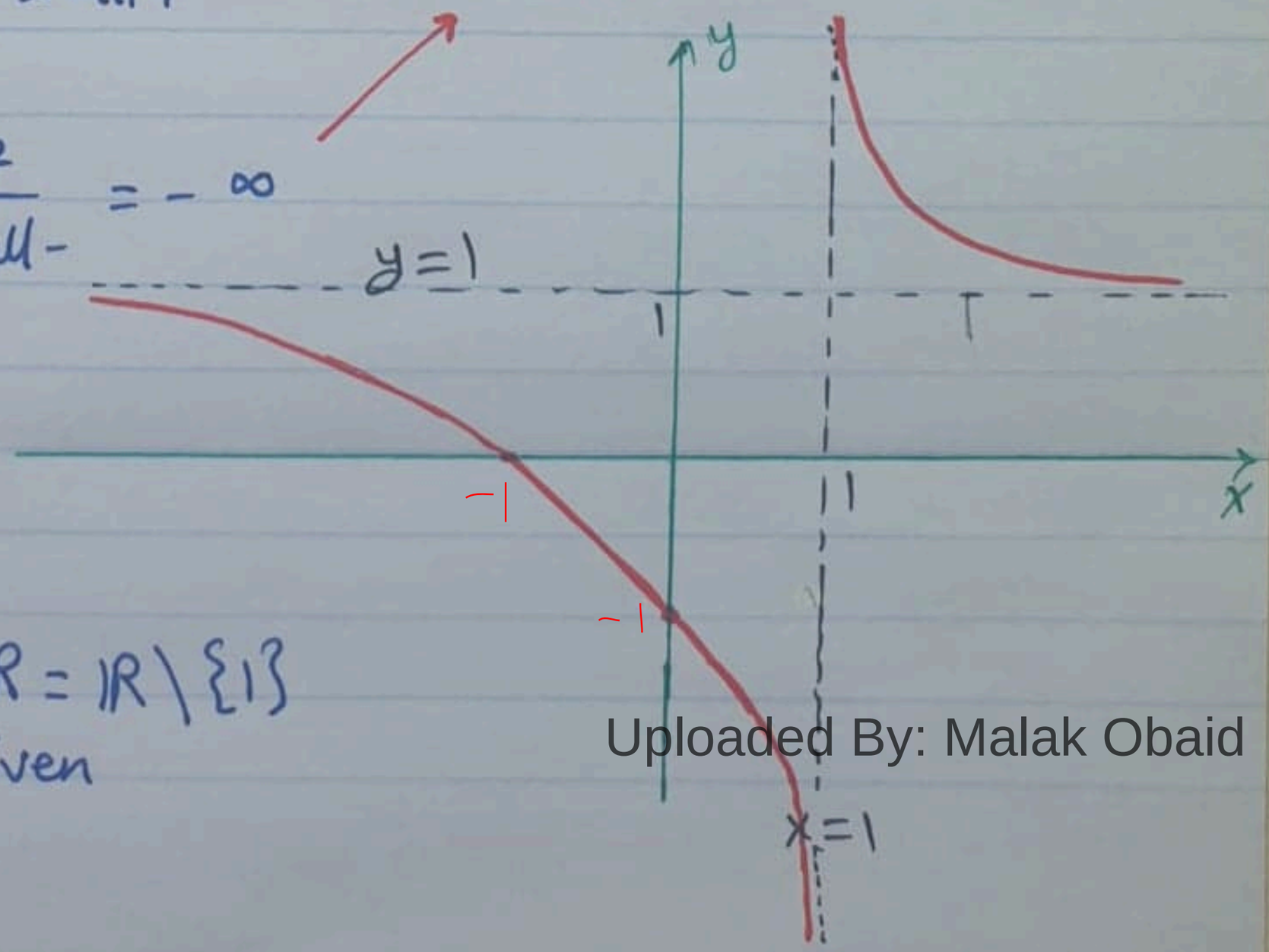
- f has no O.Asy $\Rightarrow$ f has H. Asy $y=1$ since

$$\lim_{x\to\infty} \frac{x+1}{x-1} = 1 = \lim_{x\to-\infty} \frac{x+1}{x-1}$$

- V. Asy : check $x=1$

$$\lim_{x\to 1^+} \frac{x+1}{x-1} = \frac{2}{\text{small}+} = \infty \Rightarrow x=1 \text{ is V. Asy.}$$

$$\lim_{x\to 1^-} \frac{x+1}{x-1} = \frac{2}{\text{small}-} = -\infty$$



- Keypoints $(0,-1)$
$(-1,0)$

- $D = \mathbb{R} \setminus \{1\}$ and $R = \mathbb{R} \setminus \{1\}$
- Not Odd and Not Even