Chapter 6: Viscous flows in Ducts

: lam : Tur

- There are three types of flows: (Depending on Re number)
  - 1- laminar flow : At low Re :- The flow is smooth the viscosity is high

    $Re \leq 1000$
  - 2- $1000 \leq Re \leq 10000$ transition to turbulence
  - 3- Turbulent flow : At high Re low. viscosity

    $Re \geq 10000$

Reynolds number:

$$Re = \frac{\rho V d}{\mu}$$

or

$$Re = \frac{V d}{\nu}$$

V: Average velocity

ν: Kinematic viscosity

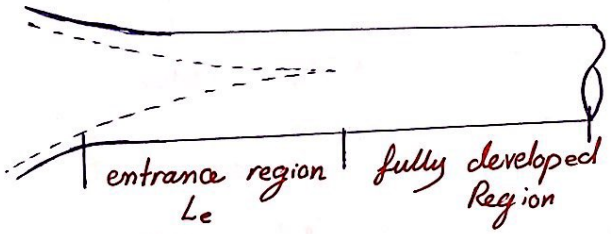
$Re_{critical} = 2300$

- Note: we only study internal flows (Bounded)

Difference between internal and external flow

- Internal: يكون محدود بسطح صلب والمحدود واضحة
- external: يكون غير محدود زي الـ flow حول الطائرة

- In Internal flows :-



- For laminar flow: $\frac{L_e}{d} \approx 0.06\, Re$ (Max $l_e = 138d$ at critical Re)
- For Turbulent flow: $\frac{L_e}{d} \simeq 1.6\, Re^{1/4}$ for $Re \leq 10^7$

Note: Turbulent pressure drops with diameter more than laminar so to decrease pumping pressure use lerger pipes

Head loss The friction factor

$$\left(\frac{P}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_1 = \left(\frac{P}{\rho g} + \frac{\alpha V^2}{2g} + z\right)_2 + h_f \Longrightarrow h_f = \Delta z + \frac{\Delta P}{\rho g}$$

if V is constant

دلي آموجود
إذا كانت Bernoli

$$h_f = f \frac{L}{d} \frac{V^2}{2g}$$

and we can relate head loss to wall shear stress

$$h_f = \frac{4 \tau_w L}{\rho g d}$$

- **For laminar flow**:- $h_f = \frac{32 \mu L V}{\rho g d^2} = \frac{128 \mu L Q}{\pi \rho g d^4}$

$$f = \frac{64}{Re}$$

- **For turbulent flow**: $\frac{1}{\sqrt{f}} = -1.8 \log\left(\left(\frac{\epsilon/d}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right)$

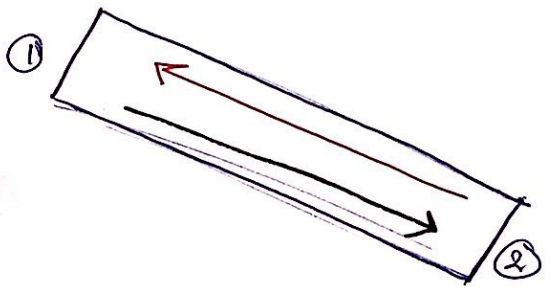
or you can use moody chart

- **Remember**:

$h_f = \Delta HGL$

where $HGL = z + \frac{P}{\rho g}$ if V is constant

flow Goes from higher HGL to low HGL



so if $HGL_① < HGL_②$

Then flow is 2→1

if $HGL_① > HGL_②$

Then flow is 1→2

• four Types of Pipe flow problems

Type 1: Given: d, L, V or Q, ρ, μ and g

Find : h_f

solution: find Re, $\frac{\epsilon}{d}$ and find f then compute h_f

Type 2: Given: d, L, h_f, ρ, μ and g

Find: V or Q

solution: Direct:- calculate ξ, Then use relation

$$Re\sqrt{f} = (-8\xi)^{\frac{1}{2}} \log\left(\frac{\epsilon/d}{3.7} + \frac{1.775}{\sqrt{\xi}}\right)$$

(Not always right, it can be wrong if there are losses)

Iterative: Assume value of f and use the relation between f and V using $f = h_f \frac{d}{L}\frac{2g}{V^2}$

f is at the fully rough so ϵ/d is calculated then use eq*

fully rough

* $\frac{1}{\sqrt{f}} = -2\log\frac{\epsilon/d}{3.7}$

Type 3: Given: Q, L, h_f, ρ, μ and g

Find : diameter d

solution: • find d in terms of f using $f = \frac{\pi^2}{8}\frac{g h_f d^5}{L Q^2}$

• find Re in terms of d

→ using $Re = \frac{4Q}{\pi d \nu}$

• find Ration ϵ/d

• Guess f then find new Re, new Ratio (use these to find f_{new})

• stop at convergance

Type 4: Given Q, d, h_f, ρ, μ and g

Find: pipe length L

Solution: If minor losses are neglected & pipe is horizontal

$$h_{pump} = \frac{Power}{\rho g Q} = h_f = f\frac{L}{d}\frac{V^2}{2g}$$

Flow in Noncircular ducts

Hydraulic diameter

- $D_H = \frac{4 \times \text{Area}}{\text{wetted perimeter}}$ → بدي

→ If the flow is between Parallel plates

$D_H = 4h$ where $h = \frac{\text{distance apart}}{2}$

→ If the flow is laminar:

$\tau_w = \frac{3\mu V}{h}$

$h_f = \frac{\Delta P}{\rho g} = \frac{3\mu L V}{\rho g h^2}$, $h_f = f \frac{L}{D_h} \frac{V^2}{2g}$

$f_{lam} = \frac{96}{Re_{D_h}}$, $Re = \frac{\rho V D_h}{\mu}$

→ If the flow is turbulent

use D_h as laminar for reasonable accuracy

or use D_{eff} for more accuracy

$D_{eff} = \frac{61}{96} D_h$

Minor or local losses in Pipe systems

$$\Delta h_{tot} = h_f + \Sigma h_m = f\frac{V^2}{2g}\frac{L}{d} + \underbrace{\Sigma K}_{\text{Given or found from curves}}\frac{V^2}{2g}$$

Additional Notes:

In eq: $\left(\frac{P}{\rho g} + \text{---}\right)_1 = \left(\frac{P}{\rho g} + \text{----}\right)_2 + h_f$

- P is either Gauge or absolute for Both sides

6.26 ✓
6.34
6.36
6.109 Pdf

- In pumps

$$P = \frac{\rho g Q h_p}{\eta}$$

- original equation of losses

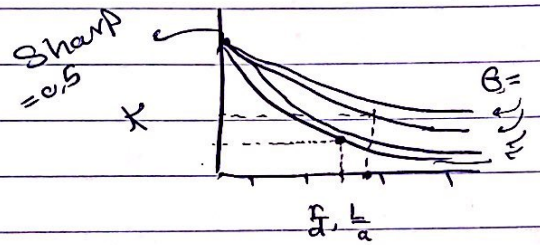
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f + \Sigma h_m - h_p$$

(Σh_m = $\Sigma \frac{K V^2}{2g}$; h_p: Given or found)

How to finel K ?

Sharp exit : $K = 1$

Sharp enbrance : go to Fig 6.21 , $K = 0.5$
If not sharp , cal $\frac{r}{d}$ or $\frac{r}{t}$ and then go vertical to the angle Given

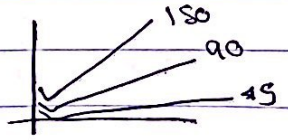


open valve, elbows, Tees → Go to table 6.5 anel
convert d (in m) to inch
choose the right diameter anel read the value of K

1 m ≈ 39.37 inch

Benelings : Fig 6.20 , Calculate $\frac{R}{d}$ ← radius of bend / ↳ diameter

Choose right angle (90, 180, 45) anel obtain K



half open valves , Gate or disk or Gelobe
go to Fig 6.18b
obtain: fractional opening $\frac{h}{D}$ then choose type anel obtain K