Chapter 7.1, Problem 42E

Problem

Let *X* and *Y* be sets, let *A* and *B* be any subsets of *X*, and let *C* and *D* be any subsets of *Y*. Determine which of the properties are true for all functions *F* from *X* to *Y* and which are false for at least one function *F* from *X* to *Y*. Justify your answers.

Exercise

$F(A \cap B) \subseteq F(A) \cap F(B)$

Step-by-step solution

Step 1 of 2

$F(A \cap B) \subseteq F(A) \cap F(B)$

Now we need to show that the statement is true

Let *F* be a function from X to Y

Take $y \in F(A \cap B)$

So we must show that y is in $y \in F(A) \cap F(B)$

Step 2 of 2

Since $y \in F(A \cap B)$

So by the definition we have $y = f(x)$ for some $x \in A \cap B$

This implies that $y = f(x)$ for some $x \in A$ and $x \in B$

So we have $y = f(x)$ for some $x \in A$ and $y = f(x)$ for some $x \in B$

So $y \in f(A)$ and $y \in f(B)$

Thus $y \in f(A) \cap f(B)$

Therefore

$f(A \cap B) \subset f(A) \cap f(B)$