

$$1) \sum_{n=1}^{\infty} n \tan^{-1}\left(\frac{2}{n}\right)$$

$$n \tan^{-1}\left(\frac{2}{n}\right) \leq n \frac{\pi}{2}$$

first 1:

$$\lim_{n \rightarrow \infty} n \tan^{-1}\left(\frac{2}{n}\right) = \infty \cdot \frac{\pi}{2} = \infty \neq 0$$

div n^{th} term test

$$2) \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)}{(2n-1)!}$$

$$\lim_{n \rightarrow \infty} \frac{1}{(2n)!} = 0$$

$$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

$$\lim_{n \rightarrow \infty} \frac{1}{(2n+2)!} \cdot (2n)! = \lim_{n \rightarrow \infty} \frac{(2n)!}{(2n+1)(2n)!} = 0 < 1$$

conv

$\therefore$ conv Abs

$$3) \sum \frac{(-1)^n \pi^{2n+1}}{(2n+1)!}$$

$$\sin(\pi) = 0$$

Haytham
Shahadeh

$$4) \sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{(2x+3)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(2x+3)^n} = |2x+3| \sqrt{\lim_{n \rightarrow \infty} \frac{n}{n+1}} = |2x+3| < 1$$

$$\frac{-1}{-3} < 2x+3 < \frac{1}{-3}$$

$$x = -2 \rightarrow \sum (-1)^n \frac{1}{\sqrt{n}} \rightarrow \text{conv}$$

$$x = -1 \rightarrow \sum \frac{1}{\sqrt{n}} \rightarrow \text{div}$$

$$-4 < 2x < -2$$

$$-2 \leq x < -1$$

conv Abs

$$\text{IC conv} = -2 \leq x < -1$$

$$\sum_{n=1}^{\infty} \left(\frac{\sqrt{x-1} - 1}{2} \right)^n$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{x-1} - 1}{2} = \frac{\sqrt{x-1} - 1}{2}$$

$$x=2 \rightarrow \{ 0 = 0 \}$$

$$x=10 \rightarrow \{ 1 = 1 \}$$

$$-1 < \frac{\sqrt{x-1} - 1}{2} < 1$$

$$-2 < \sqrt{x-1} - 1 < 2$$

$$-1 < \sqrt{x-1} < 3$$

$$0 < x-1 < 9$$

$$2 < x < 10$$

conv Abs

$$6) \lim_{n \rightarrow \infty} \frac{\ln(n^2)}{n}$$

$$e^{\lim_{n \rightarrow \infty} \ln n \ln n^2} = e^{\infty} = \infty$$

$$7) a_n = (-1)^n \cos(n\pi)$$

$$\lim_{n \rightarrow \infty} (-1)^n (-1)^n = \lim_{n \rightarrow \infty} (-1)^{2n} = 1 \quad \text{conv}$$

$$8) \sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n 2}{5^n}$$

$$2 \sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{5} \right)^n$$

$$\text{Sum} = 2 \left[-\frac{2}{5} \cdot \frac{5}{7} \right] = -\frac{4}{7}$$

$$9) a_n = e^{(1+\frac{2}{n})^n}$$

$$e^{\lim_{n \rightarrow \infty} (1+\frac{2}{n})^n} = e^{e^2} \text{ conv}$$

$$3) 10) \sum_{n=0}^{\infty} r^{-n} = \frac{5}{4} \quad r?$$

$$\text{Sum} = \frac{1}{1-\frac{1}{r}} = \frac{5}{4}$$

$$? \sum_{n=0}^{\infty} \left(\frac{1}{r}\right)^n$$

$$1 - \frac{1}{r} = \frac{4}{5}$$

$$\frac{1}{r} = \frac{1}{5}$$

$$\boxed{r=5}$$

$$7) 11) e^{-x} = 1 - x + \frac{x^2}{2} \quad 0 < x < 0.4$$

$$e^{-x} = \sum (-1)^n \frac{x^n}{n!}$$

$$e^{-x} \leq 1 - x + \frac{x^2}{2}$$

$$L - P \leq |a_{n+1}|$$

$$12) f(x) = e^x \quad a=9$$

$$e^x = \sum_{n=0}^{\infty} \frac{e^9 (x-9)^n}{n!}$$

$$13) a_n > 0 \quad b_n > 0 \quad n > N$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$$

$$\{b_n \text{ div}\} \rightarrow \{a_n \text{ div}\}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} = -\frac{1}{\ln 2} + \frac{1}{\ln 3} - \frac{1}{\ln 4} + \dots$$

$$\lim_{n \rightarrow \infty} U_n = \lim_{n \rightarrow \infty} \frac{1}{\ln(n+1)} = 0 \quad \text{conv cond}$$

$$\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$$

$$\frac{1}{\ln(n+1)} < \frac{1}{\ln n}$$

$$\frac{1}{n} < \frac{1}{\ln n}$$

$\downarrow$ div $\downarrow$ div

$$\lim_{n \rightarrow \infty} \frac{1}{\ln(n+1)} \cdot \ln n = 1$$

$$15) \sum_{n=1}^{\infty} \frac{n}{(\ln n + 10)^n}$$

$$\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{n}}}{(\ln n + 10)} = \frac{1}{\infty} = 0 < 1 \quad \text{conv Root test}$$

$$16) a_n = \ln \left(1 + \frac{1}{n}\right)^n$$

$$\ln \left(\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \right) = \ln e = 1 \quad \text{conv}$$

$$17) f(x) = x^3 e^x$$

$$= x^3 \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \right)$$

$$= x^3 + x^4 + \frac{x^5}{2} + \frac{x^6}{3!} + \frac{x^7}{4!} + \dots$$

$$\sum_{n=0}^{\infty} \frac{x^{n+3}}{n!}$$

$$18) \lim_{n \rightarrow \infty} \frac{8 + (-1)^n}{n}$$

$$\frac{7}{n} < \frac{8 + (-1)^n}{n} < \frac{9}{n}$$

$\downarrow$ $\lim_{n \rightarrow \infty} \frac{7}{n} = 0$

$$\frac{9}{n}$$

$\downarrow$ $\lim_{n \rightarrow \infty} \frac{9}{n} = 0$

$$\therefore \lim_{n \rightarrow \infty} \frac{8 + (-1)^n}{n} = 0 \quad \text{by Sandwich Th}$$

$$19) \sum_{n=1}^{\infty} \frac{5}{n(n+1)}$$

$$f(x) = \frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$\sum_{n=1}^{\infty} \left(\frac{5}{n} - \frac{5}{n+1} \right) = \left(5 - \frac{5}{2} \right) + \left(\frac{5}{2} - \frac{5}{3} \right) + \left(\frac{5}{3} - \frac{5}{4} \right) + \dots = \frac{5}{n} - \frac{5}{n+1}$$

$$S_n = 5 - \frac{5}{n+1} = \frac{5n+5-5}{n+1} = \frac{5n}{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = 5 - 0 = 5 \quad \text{conv}$$

20) If $\{a_n\}$ and $\{b_n\}$ conv $\Rightarrow \{a_n b_n\}$ conv false?

$$21) a_n = \frac{2n}{n+1}$$

$$\lim_{n \rightarrow \infty} \frac{2n}{n+1} = \frac{2}{1} = 2 \quad \text{conv}$$

$$a_1 = \frac{2}{2} = 1, \quad a_2 = \frac{4}{3}, \quad a_3 = \frac{6}{4}$$

non decreasing
increasing

$$22) \frac{1}{2}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{20}$$

$$\frac{(-1)^{n+1}}{n^2+n}$$

$$n = 1, 2, 3, 4, \dots, \infty$$

$$23) a_1 = 3, \quad a_{n+1} = \frac{a_n}{3}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{1}{3} = \frac{1}{3} < 1 \quad \text{conv}$$

conv to 0

$$a_2 = 1$$

$$a_3 = \frac{1}{3}$$

$$a_4 = \frac{1}{9}$$

$$\vdots$$

$$0$$

$$24) 5.\bar{4} = 5.44444\dots = 5 + \frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \dots$$

$$5 + \frac{a}{1-r} = 5 + \left[\frac{4}{10} \cdot \frac{10}{9} \right]$$

$$= 5 + \frac{4}{9} = \frac{49}{9}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$$

$$n^2 - n^{\frac{1}{2}} = n^{\frac{3}{2}}$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{n^2+1} \cdot n^{\frac{3}{2}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n+1} n^{\frac{3}{2}}}{n^2+1} = 1$$

$$b_n = \frac{1}{n^{\frac{3}{2}}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}} \text{ conv p-series}$$

$$26) \sum_{n=1}^{\infty} \left(\frac{\ln n}{n} \right)^n$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 < 1 \text{ conv by Root test}$$

$$27) \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} = -1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \dots$$

$$E > 0, E < \frac{1}{24}$$

$$28) \sum_{n=0}^{\infty} n! (x-2)^n$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)! (x-2)^{n+1}}{n! (x-2)^n} = |x-2| \lim_{n \rightarrow \infty} (n+1) = \infty > 1$$

radius conv $\rightarrow 0$

$$\begin{array}{c} \leftarrow \text{div} \rightarrow \\ \hline 2 \end{array}$$

$$29) \sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n}$$

$$\lim_{n \rightarrow \infty} \frac{4^{n+1} x^{2n+2}}{n+1} \cdot \frac{n}{4^n x^{2n}} = \lim_{n \rightarrow \infty} \frac{n x^2 4}{n+1} = |4x^2| \lim_{n \rightarrow \infty} \frac{n}{n+1} = |4x^2| < 1$$

$$-1 < 4x^2 < 1$$

$$-\frac{1}{4} < x^2 < \frac{1}{4}$$

$$-\frac{1}{2} \leq x < \frac{1}{2}$$

$$x = -\frac{1}{2}$$

$$x = -\frac{1}{2} \rightarrow \sum_{n=1}^{\infty} \frac{4^n}{n} \left(-\frac{1}{2}\right)^{2n} \rightarrow \sum_{n=1}^{\infty} \frac{4^n}{n} \rightarrow \text{div}$$

$$x = \frac{1}{2} \rightarrow \sum_{n=1}^{\infty} \frac{4^n}{n} \left(\frac{1}{2}\right)^{2n} = \sum_{n=1}^{\infty} \frac{4^n}{n} \rightarrow \text{div}$$

$$\lim_{n \rightarrow \infty} \frac{4^{n+1} \left(\frac{1}{2}\right)^{2n+2}}{n+1} \cdot \frac{n}{4^n \left(\frac{1}{2}\right)^{2n}} = \lim_{n \rightarrow \infty} \frac{4n}{n+1} = 4 > 1$$

$$\text{conv by Ratio Test}$$

$$30) \sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{4^n}$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n (-2)^{-1}}{4^n} = -\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n = -\frac{1}{2} \left[-\frac{1}{2} \cdot \frac{2}{3} \right] = \frac{1}{6}$$

$$31) \sum_{n=1}^{\infty} \frac{n^n}{n!} \quad \lim_{n \rightarrow \infty} \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n} = \lim_{n \rightarrow \infty} \frac{(n+1)^{n+1}}{(n+1)n^n} = \lim_{n \rightarrow \infty} \left(\frac{n+1}{n}\right)^n = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e^1 > 1 \quad \text{div}$$

$$32) \sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n}}$$

$$\sum_{n=0}^{\infty} \left(\frac{98}{9}\right)^n$$

$$\text{Sum} = 1 \cdot \frac{9}{1} = 9$$

$$33) \sum_{n=1}^{\infty} \frac{1}{n^{\ln 2}}$$

div by p-series

$$\ln 2 < 1$$

Question 2:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^{n+1}}{n 4^n}$$

$$\lim_{n \rightarrow \infty} \frac{(x-2)^{n+2}}{(n+1) 4^{n+1}} \cdot \frac{n 4^n}{(x-2)^{n+1}} = \frac{1}{4} |x-2| \lim_{n \rightarrow \infty} \frac{n}{n+1} = \frac{1}{4} |x-2| < 1$$

$$-1 < \frac{1}{4} (x-2) < 1$$

$$-4 < x-2 < 4$$

$$\boxed{-2 < x < 6} \quad \text{conv Abs}$$

$$x = -2 \rightarrow \sum \frac{(-1)^{n+1} (-4)^{n+1}}{n 4^n} \rightarrow \sum \frac{4}{n} \quad \text{div}$$

$$x = 6 \rightarrow \sum \frac{(-1)^{n+1} (4)^{n+1}}{n 4^n} \rightarrow \sum (-1)^{n+1} \frac{4}{n} \quad \text{conv}$$

$$\text{IC conv} \rightarrow -2 < x \leq 6$$

$$\text{conv cond at } x = 6$$

$$\text{Center} = 2$$

$$\text{Radius} = 4$$

