Remark of exp3 :

$$p(E_1) = \frac{\binom{13}{1}\binom{39}{0}}{\binom{52}{1}} = \frac{13}{52} = \frac{1}{4}$$

$$p(E_2) = \frac{\binom{4}{1}\binom{48}{0}}{\binom{52}{1}} = \frac{4}{52} = \frac{1}{13}$$

Exercises :

Q19: $\mathcal{C} =$ H, TH, TTH, TTTH, ...

probability : $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... $\rightarrow \sum_{1}^{\infty} (\frac{1}{2})^x$, $x = 1, 2, 3, \ldots$

1. show $p(\mathcal{C}) = 1$

$$p(\mathcal{C}) = \sum_{1}^{\infty} (\tfrac{1}{2})^x = \frac{a}{1-r} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

2. let $C_1 = \{c : c$ is H, TH, TTH, TTTH, or TTTTH $\}$

$C_2 = \{c : c$ is TTTTH or TTTTTH $\}$, compute

برهنة - $p(C_1) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}$

- $p(C_2) = \frac{2\times 1}{2\times 32} + \frac{1}{64} = \frac{3}{64}$

- $p(C_1 \cap C_2) =$ $C_1 \cap C_2 = \{TTTTH\}$

$p(C_1 \cap C_2) = \frac{1}{32}$

- $p(C_1 \cup C_2) = p(C_1) + p(C_2) - p(C_1 \cup C_2)$

$= \left(\frac{31}{32} + \frac{3}{64}\right) - \frac{1}{32}$

$= \frac{65}{64} - \frac{2}{64} = \frac{63}{64}$

Q21: $\mathcal{C} = \{c : 0 < c < \infty\}$, let $C \subset \mathcal{C}$ be defined by $C = \{c : 4 < c < \infty\}$ and take $P(C) = \int_C e^{-x}\,dx$, evaluate $P(C), P(C^*), P(C \cup C^*)$?

$$\rightarrow P(C) = \int_4^{\infty} e^{-x}\,dx = -e^{-x}\Big|_4^{\infty} = e^{-x}\Big|_{\infty}^{4} = e^{-4} = \frac{1}{e^4}.$$

$\rightarrow P(C^*) \Rightarrow \quad P(C) = 1 - P(C^*)$

$\rightarrow P(C^*) = 1 - P(C)$

$= 1 - e^{-4}.$

$\rightarrow P(C \cup C^*) = P(\mathcal{C}) = 1.$

Q26: A bowl contains 16 chips, of which 6 are Red, 7 are White and 3 Blue. If 4 chips are taken at Random and without replacment, Find the probability:

a. each of the 4 chips is red ? E_1

$$P(E_1) = \frac{\binom{6}{4}\binom{10}{0}}{\binom{16}{4}} = 8.24 \times 10^{-3}$$

15

b. None of the 4 chips is Red ?

$$P(E_2) = \frac{\binom{10}{4}\binom{6}{0}}{\binom{16}{4}} = 0.115$$

c. there is at least 1 chip of each color ?

$$P(E_3) = P(1R, 1W, 2B) + P(1R, 2W, 1B) + P(2R + 1W + 1B)$$

$$= \frac{\binom{6}{1}\binom{7}{1}\binom{3}{2}}{\binom{16}{4}} + \frac{\binom{6}{1}\binom{7}{2}\binom{3}{1}}{\binom{16}{4}} + \frac{\binom{6}{2}\binom{7}{1}\binom{3}{1}}{\binom{16}{4}}$$

$= 0.45$

Q29: Three distinct integer are chosen at Random From the First 20 positive integers, compute the probability:

a. their sum is even?

$$Pr(a+b+c \text{ even}) = Pr(\text{all 3 even}) + Pr(\text{1 even 2 odd})$$

$$= \frac{\binom{10}{3}\binom{10}{0}}{\binom{20}{3}} + \frac{\binom{10}{1}\binom{10}{2}}{\binom{20}{3}}$$

$$= \frac{2}{19} + \frac{15}{38}$$

$$= \frac{1}{2}$$

b. there product is even?

$$Pr(a.b.c \text{ even}) = 1 - Pr(a.b.c \text{ odd})$$

$$= 1 - Pr(\text{all 3 odd})$$

$$= 1 - \frac{\binom{10}{0}\binom{10}{3}}{\binom{20}{3}}$$

$$= 1 - \frac{2}{19}$$

$$= \frac{17}{19} = 0.8947.$$

Q31: in a lot of 50 light bulbs, there are 2 bad bulbs. an inspector examines 5 bulbs, which are selected at Random and without replacement.

a. Find the probability of at least 1 defective bulb among the 5.

Pr(at least one defective bulb) = 1 − Pr(No defective bulbs)

$$= 1 - \frac{\binom{48}{5}\underbrace{\binom{2}{0}}_{\to 1}}{\binom{50}{5}}$$

$$= 1 - \frac{1712304}{2118760}$$

$$= 1 - \frac{198}{245}$$

$$= \frac{47}{245}$$

b. How many bulbs should he examine so that the probability of finding at least 1 bad bulb exeeds $\frac{1}{2}$.

$$\rightarrow \Pr(\text{No defective bulbs}) = \frac{\binom{48}{n}\binom{2}{0}}{\binom{50}{n}}$$

$$= \frac{\frac{48!}{n!(48-n)!}}{\frac{50!}{n!(50-n)!}}$$

$$= \frac{48!}{\cancel{n!}(48-n)!} \cdot \frac{\cancel{n!}(50-n)!}{50!}$$

$$= \frac{48!}{50!} \cdot \frac{(50-n)!}{(48-n)!}$$

$$Pr(\text{No defective} --) = \frac{48!}{50 \cdot 49 \cdot 48!} \cdot \frac{(50-n)(49-n)\cancel{(48-n)!}}{\cancel{(48-n)!}}$$

$$= \frac{(50-n)(49-n)}{(50)(49)}$$

$\rightarrow Pr(\text{at least one defective}) = 1 - Pr(\text{No defective} --)$

$$= 1 - \frac{(50-n)(49-n)}{(50)(49)}$$

$\rightarrow$ We want $Pr(\text{at least one defective}) > \frac{1}{2}$.

$$\Rightarrow \quad \cancel{1} - \frac{(50-n)(49-n)}{(50)(49)} > \frac{1}{2}$$

$$-1 \qquad\qquad -1$$

$$\cancel{-}\left(\frac{(50)(49) - 50n - 49n + n^2}{(50)(49)}\right) > \cancel{-}\frac{1}{2}$$

$$\frac{n^2 - 99n + 2450}{2450} < \frac{1}{2}$$

$$n^2 - 99n + 2450 < 1225$$

