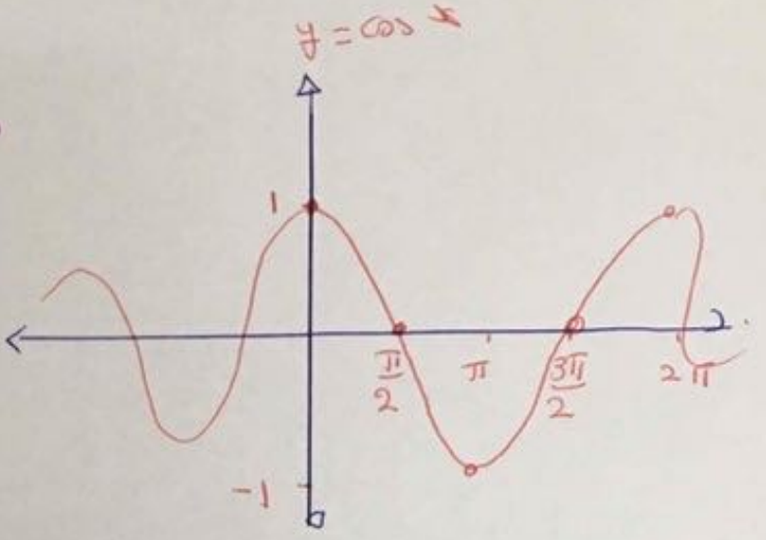
Chapter 1 : Functions : الاقترانات

Reviews:

① $y = \cos x$ "u-tip"



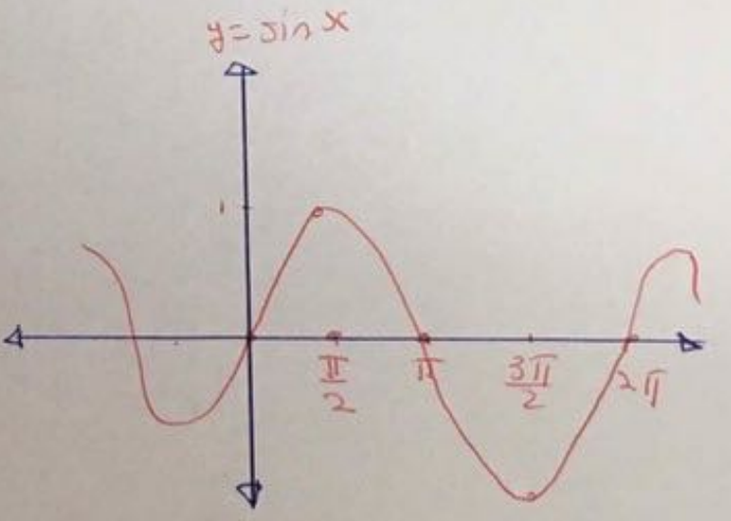
Domain = $\mathbb{R}$

Range = $[-1,1]$

Period = 2π

$\Rightarrow \cos(x+2\pi) = \cos x$

② $y = \sin x$ "u-b"



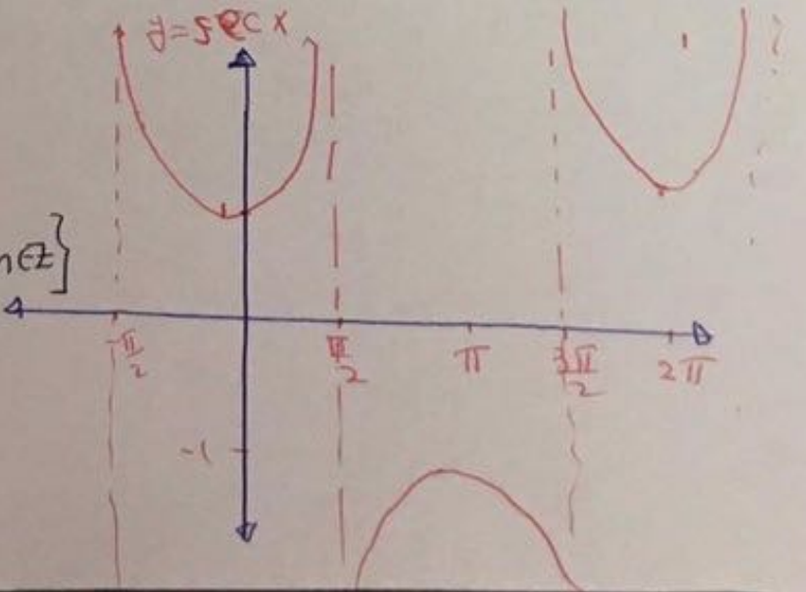
Domain = $\mathbb{R}$

Range = $[-1,1]$

Period = 2π

$\sin(x+2\pi) = \sin x$

③ $y = \sec x$



Domain = $\mathbb{R} - \left\{(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\right\}$

$= \mathbb{R} - \left\{n\pi + \frac{\pi}{2}, n \in \mathbb{Z}\right\}$

Range = $[1,\infty) \cup (-\infty,-1]$

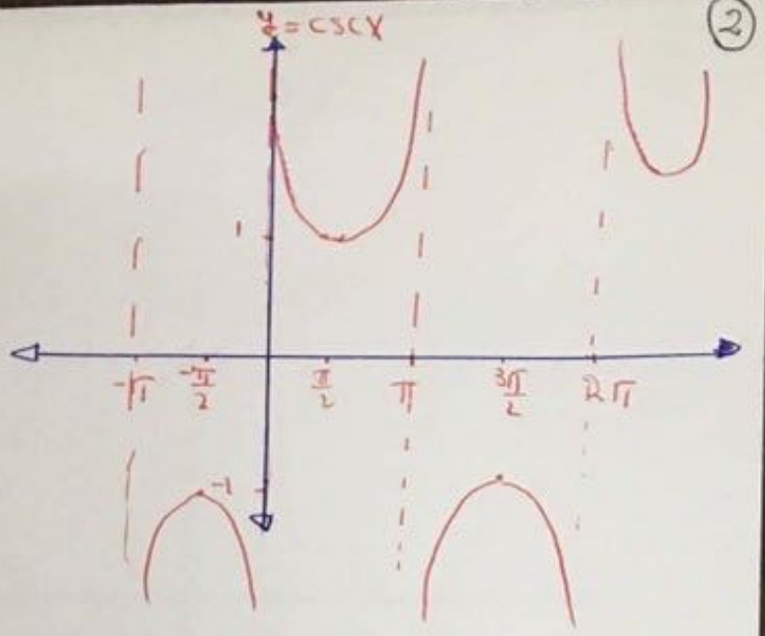
Period = 2π

④ $y = \csc x$

1. Domain $= \mathbb{R} - \{n\pi \; ; n \in \mathbb{Z}\}$

Range $= (-\infty, -1] \cup [1, \infty)$

Period $= 2\pi$



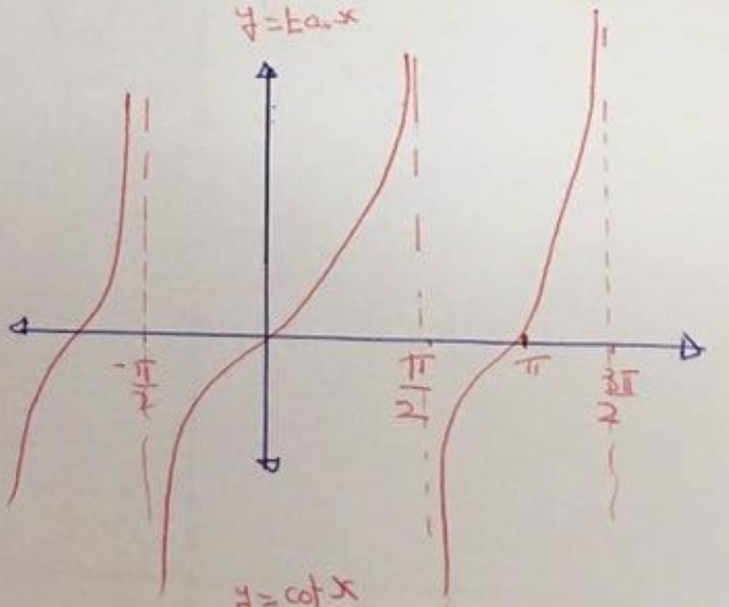
⑤ $y = \tan x$

Domain $= \mathbb{R} - \{(2n+1)\frac{\pi}{2} \; ; n \in \mathbb{Z}\}$

Range $= \mathbb{R}$

Period $= \pi$

$\tan(x + \pi) = \tan x$

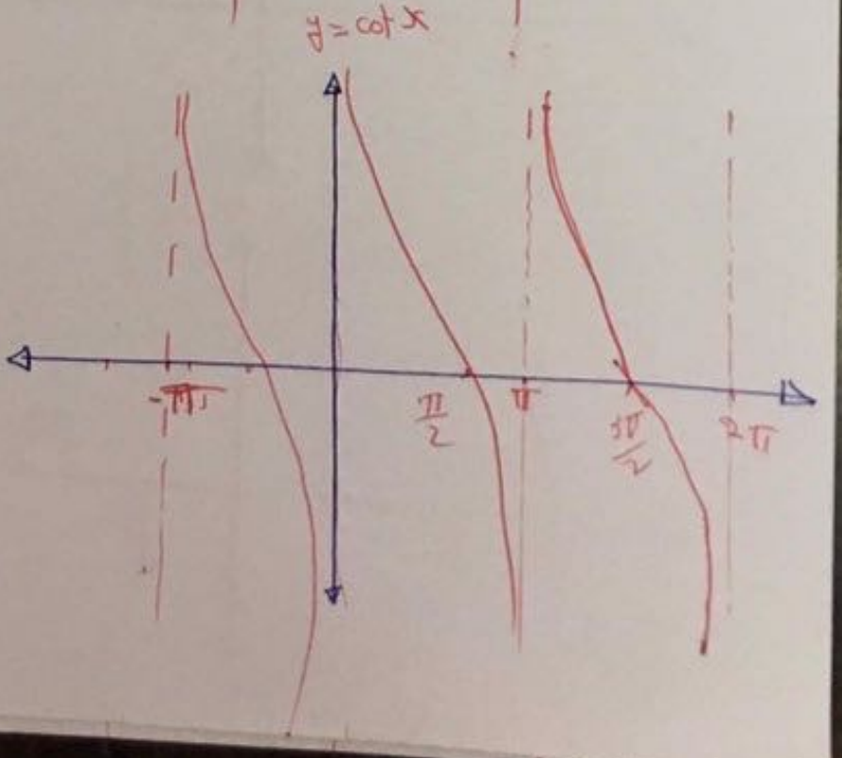


⑥ $y = \cot x$

Domain $= \mathbb{R} - \{n\pi \; ; n \in \mathbb{Z}\}$

Range $= \mathbb{R}$

Period $= \pi$

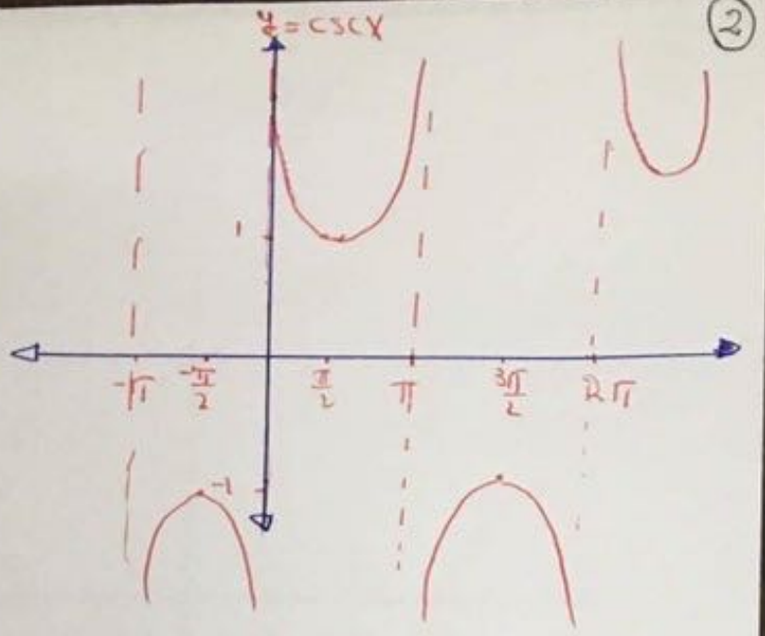


④ $y = \csc x$

1. Domain $= \mathbb{R} - \{n\pi \; ; n \in \mathbb{Z}\}$

Range $= (-\infty, -1] \cup [1, \infty)$

Period $= 2\pi$



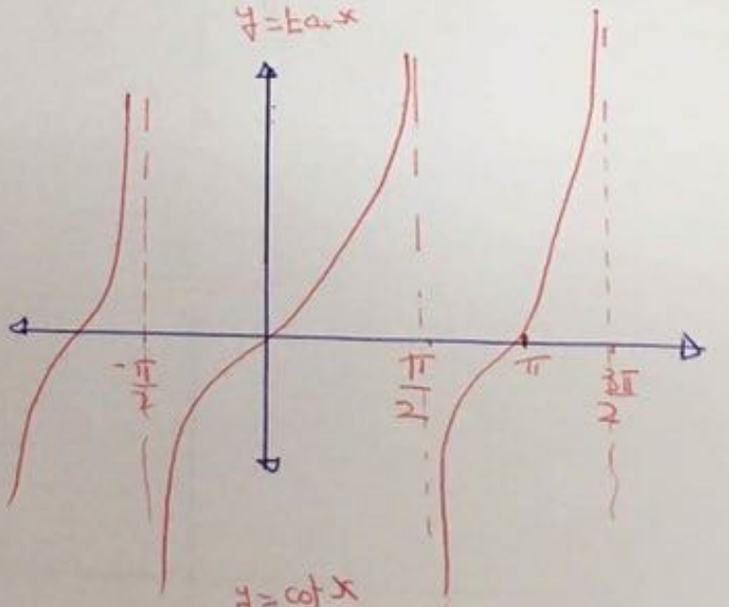
⑤ $y = \tan x$

Domain $= \mathbb{R} - \{(2n+1)\frac{\pi}{2} \; ; n \in \mathbb{Z}\}$

Range $= \mathbb{R}$

Period $= \pi$

$\tan(x + \pi) = \tan x$

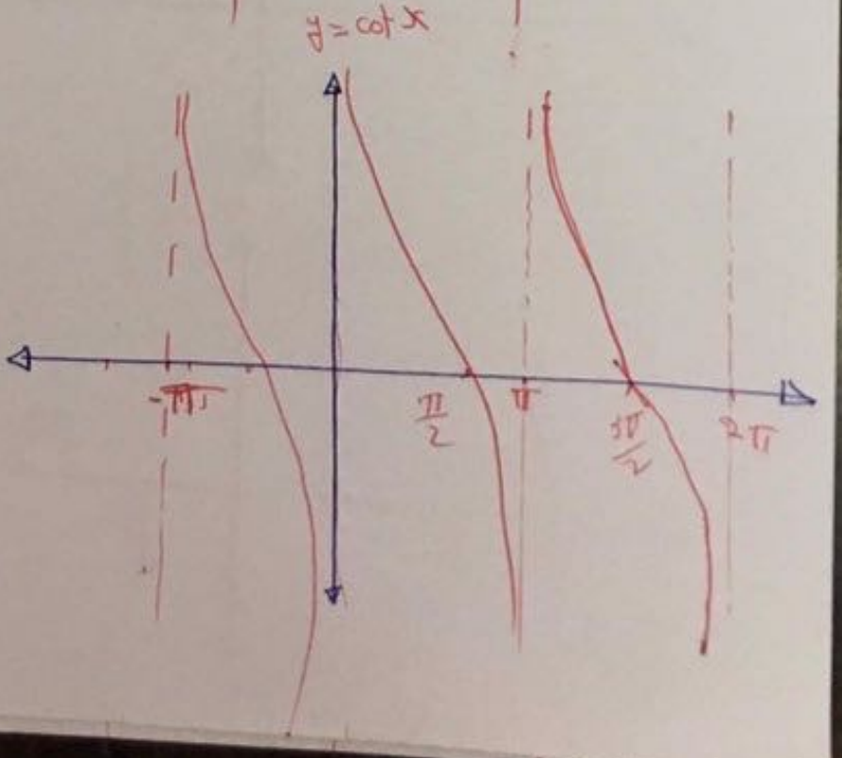


⑥ $y = \cot x$

Domain $= \mathbb{R} - \{n\pi \; ; n \in \mathbb{Z}\}$

Range $= \mathbb{R}$

Period $= \pi$



How to find the period of trigonometric functions

Group 1: $\cos(ax)$, $\sin ax$, $\csc ax$, $\sec ax$

$$\text{Period} = \frac{2\pi}{\text{coefficient of } x} = \frac{2\pi}{a}$$

Group 2: $\tan(bx)$, $\cot(bx)$

$$\text{Period} = \frac{\pi}{\text{coefficient of } x} = \frac{\pi}{b}$$

Also we have.

① $f(x)$ is an even function if $f(x)$ is symmetric about the y-axis

$$f(x) = f(-x)$$

② $f(x)$ is an odd function if $f(x)$ is symmetric about the ~~x-axis~~ origin

$$f(-x) = -f(x)$$

1.4 Exercises "page 6"

① Find the domain and the range of the following function

(a) $f(x) = \frac{1}{\sqrt{x}}$

$\frac{1}{\sqrt{x}} \neq 0$

Note: $\frac{a}{b} = 0$ only if $a = 0$

Domain: $x > 0$

Range $= (0, \infty)$

$\frac{1}{\sqrt{0.0001}} = \frac{1}{0.01} = 100$

$\frac{1}{\sqrt{10000}} = \frac{1}{100} = 0.01$

(b) $f(x) = \tan(\pi x)$

$= \frac{\sin \pi x}{\cos \pi x}$

Domain: $\cos \pi x \neq 0$

$\pi x \neq (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$

$x \neq \frac{(2n+1)}{2}$

$x \neq n + \frac{1}{2}$, $n \in \mathbb{Z}$

so Domain $= \mathbb{R} - \{n + \frac{1}{2} : n \in \mathbb{Z}\}$

Range $= (-\infty, \infty)$

(c) $f(x) = 1 + |x|$

$0 \leq |x|$

$1 \leq |x| + 1$

Domain $= \mathbb{R}$

Range $= [1, \infty)$

(d) $f(x) = \sec^2 x$ " $\frac{1}{\cos^2 x}$ "

Domain $= \mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} : n \in \mathbb{Z} \right\}$

$\mathbb{R} - \left\{ n\pi + \frac{\pi}{2} : n \in \mathbb{Z} \right\}$

Range $= [1, \infty)$

(e) $g(x) = \frac{1}{x^2}$

Domain $= \mathbb{R} - \{0\}$

Range $= (0, \infty)$

(f) $h(x) = \frac{1}{\sqrt{1-x^2}}$

Domain: $1 - x^2 > 0$

$1 > x^2$

$|x| < 1$

$-1 < x < 1$

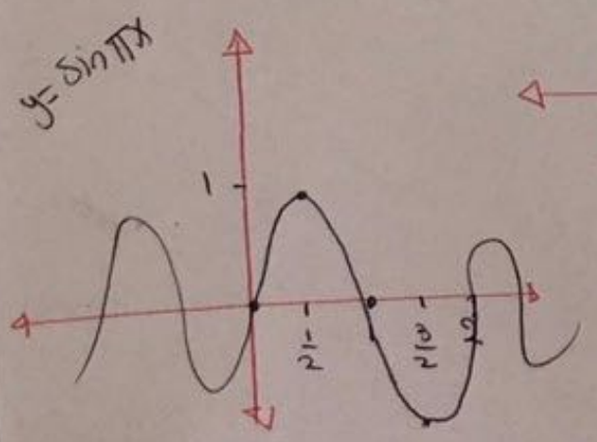
Domain $= (-1, 1)$

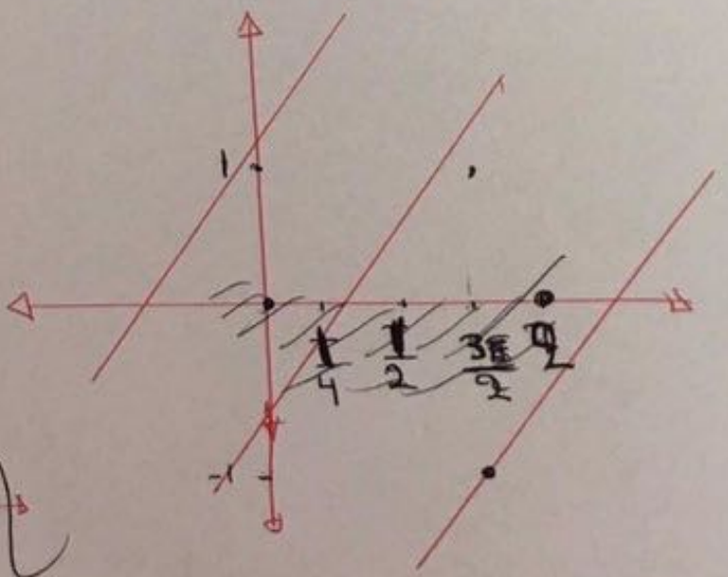
Range $= [1, \infty)$

(2) sketch the following functions:

(a) $y = \sin \pi x$

Period $= \frac{2\pi}{\pi} = 2$





$a > 0$

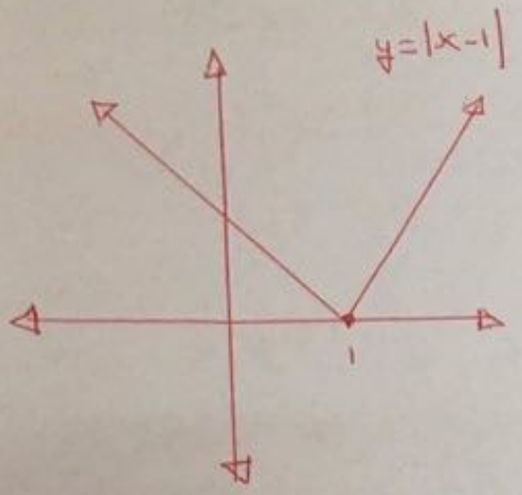
$f(x) + a$ ↑

$f(x) - a$ ↓

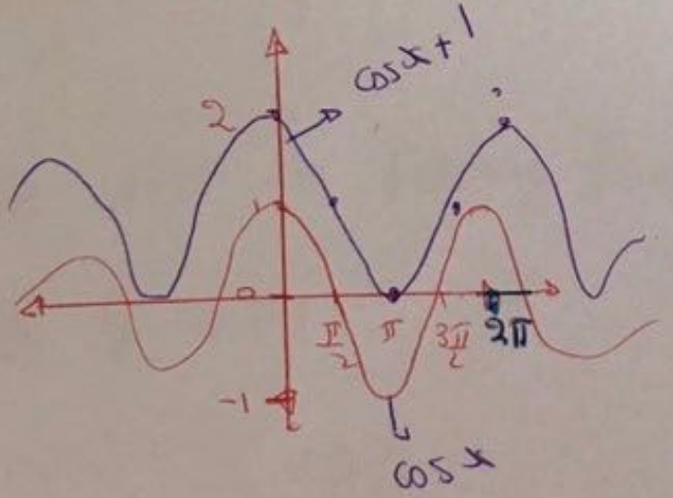
$f(x+a)$ ←

$f(x-a)$ →

(d) $y = |x-1|$



(e) $y = \cos(x) + 1$



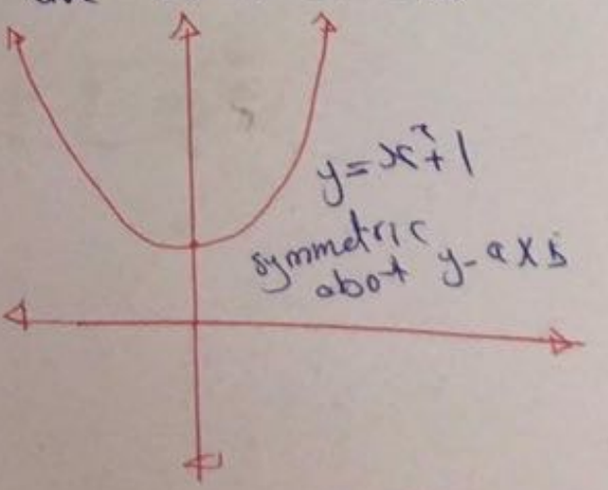
(3) Determine whether the following are even or odd

(a) $f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1 = x^2 + 1$

So $f(x) = f(-x)$

even



(b) $f(x) = x^3 + x$

$f(-x) = (-x)^3 + -x$

$= -x^3 - x$

$= -(x^3 + x)$

$= -f(x)$

ODD

(c) $g(t) = \frac{1}{t-1}$ $\neq$

$g(-t) = \frac{1}{-t-1} = \frac{-1}{t+1}$ $\neq$

$-g(t) = -\left(\frac{1}{t-1}\right) = \frac{-1}{t-1}$

Neither

(d) $h(x) = \frac{x}{x^2-1}$

$h(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -h(x)$

ODD

(b) If $f(x)$ is even and $g(x)$ is odd the $\frac{f(x)}{g(x)}$ is odd

(9)

$f(-x) = f(x)$

$g(-x) = -g(x)$

$\frac{f(x)}{g(x)}$ is odd ?

$$\frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f(x)}{g(x)}\right)$$

so its odd