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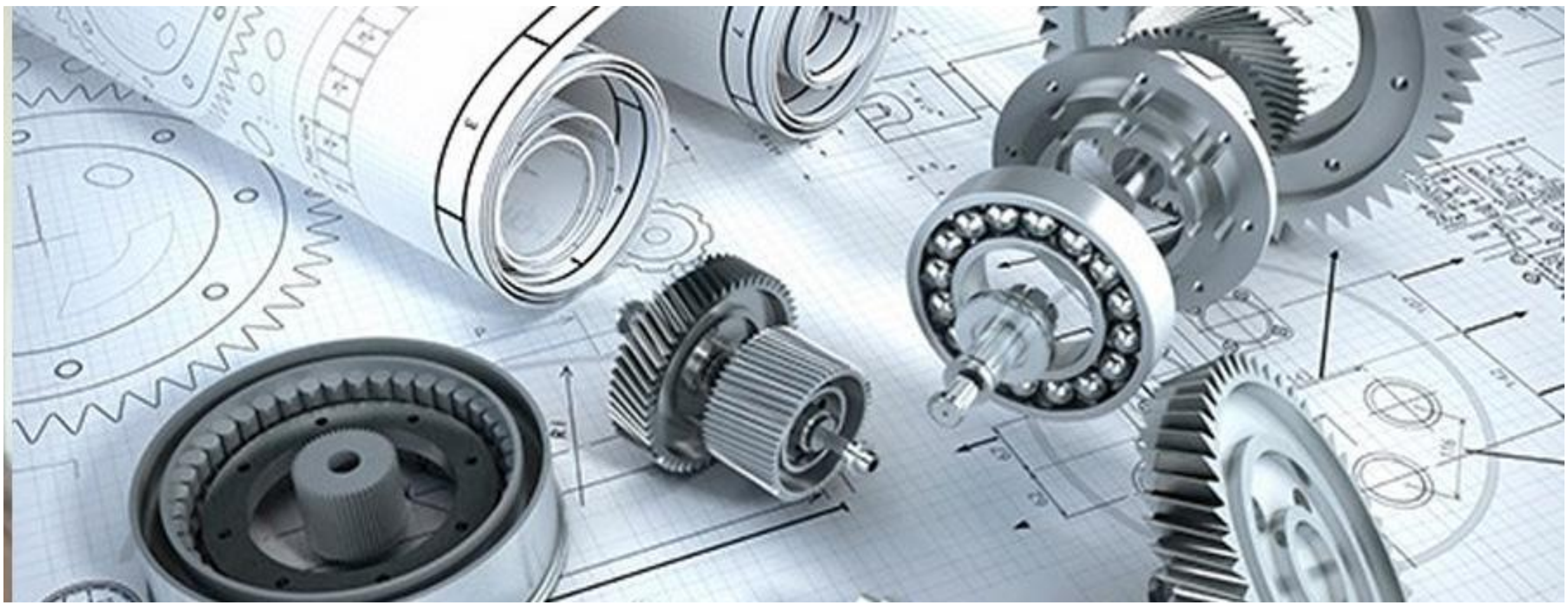
MACHINE DESIGN 1
ENMC 4421

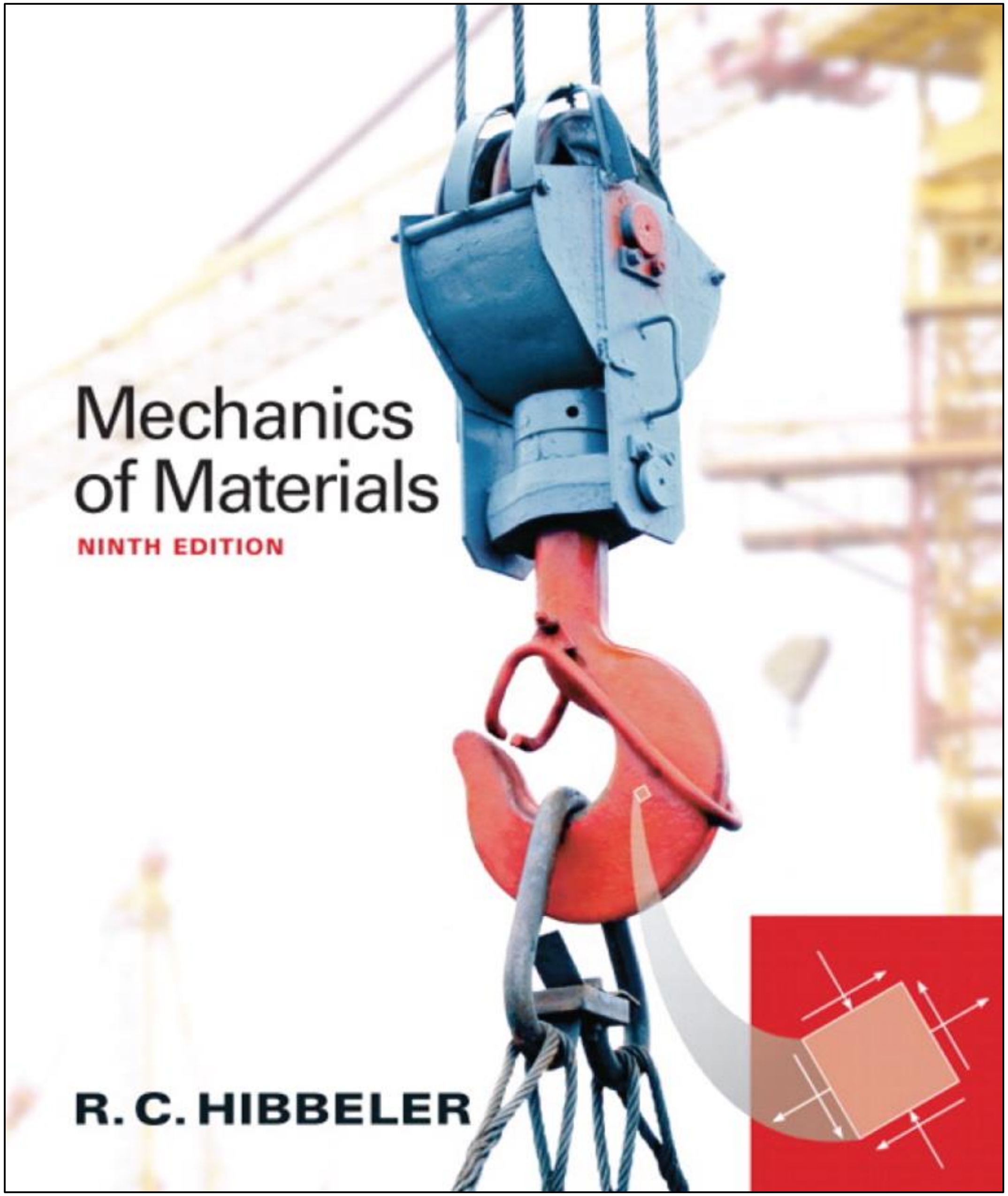
Department of Mechanical and Mechatronics Engineering

Dr. Rashad Mustafa



Design of Machine Elements
ENMC 4421

Part 1: Mechanics of Material



1.1 Introduction

Physics

Mechanics | **Thermodynamics** | **Electricity**

- Statics
- Dynamics
- Mechanics of deformable bodies (Strength/Mechanics of Materials)

- **Stress:** is associated with the strength of materials from which the body is made
- **Strain:** is a measure of the deformation of the body

Mechanics of Materials: Studies the internal effects of Stress and Strain in a solid body that is subjected to an external loading

1.2 Equilibrium of a Deformable Body

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External Loads:

- **Surface Load:** It caused by the direct contact of one body with the surface of another

- **Body Forces:** It is developed when one body exerts a force on another body without direct physical contact between bodies

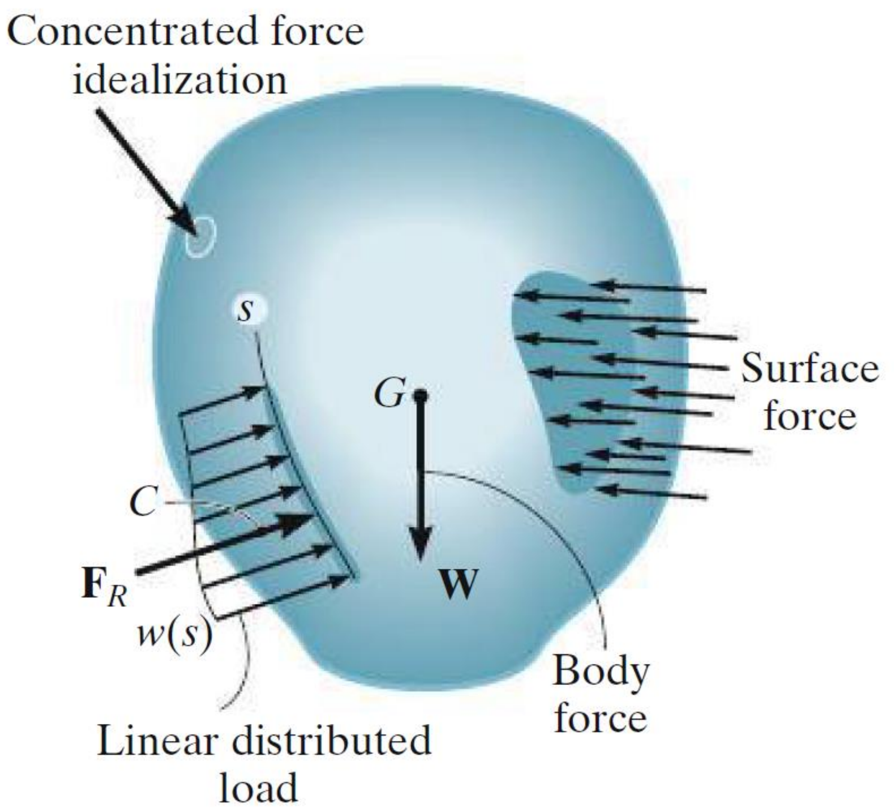


Fig. 1–1

1.2 Equilibrium of a Deformable Body

Surface Force

Distributed area **Line** **concentrated**

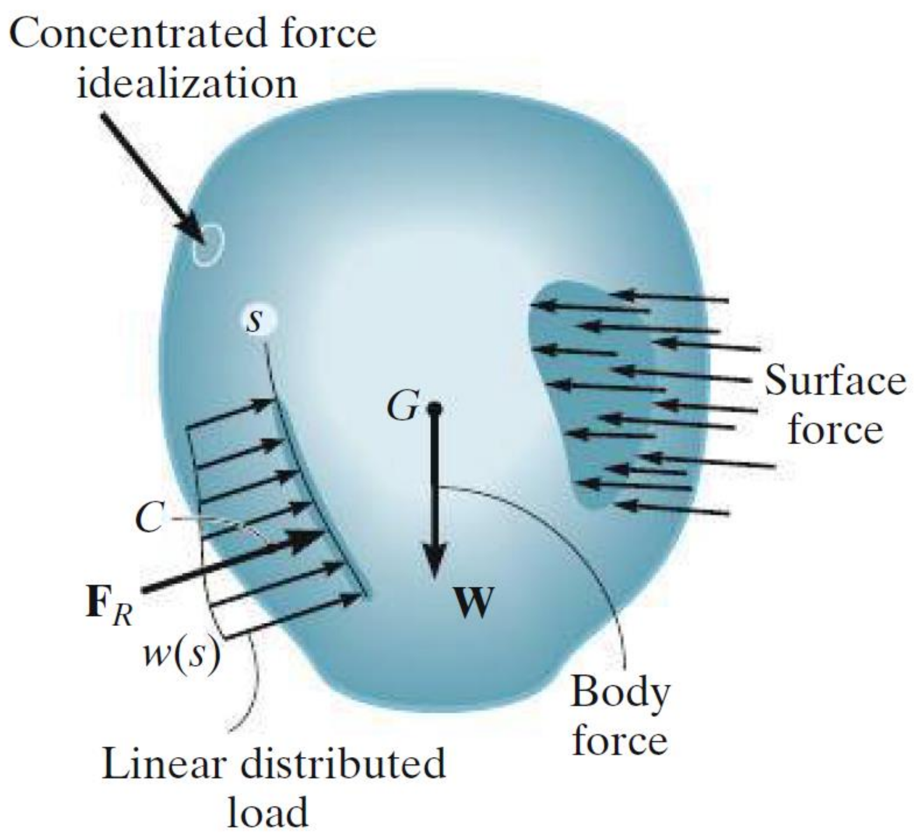
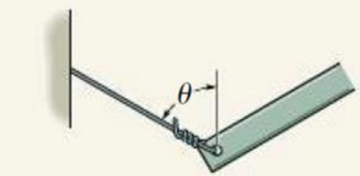
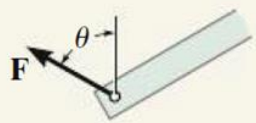
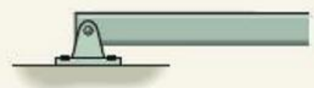
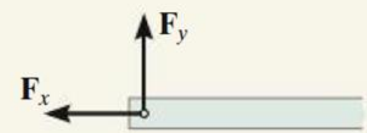
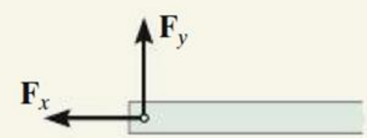
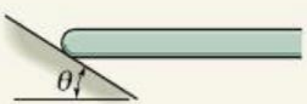
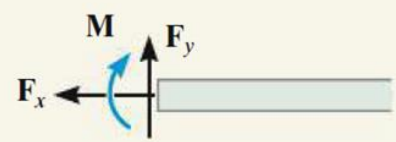


Fig. 1–1

1.2 Equilibrium of a Deformable Body

Support Reactions: A force that develop at the supports or points at contact between bodies are called reaction

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

1.2 Equilibrium of a Deformable Body

Support Reactions: Example 1

1.2 Equilibrium of a Deformable Body

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Support Reactions: Example 2

1.2 Equilibrium of a Deformable Body

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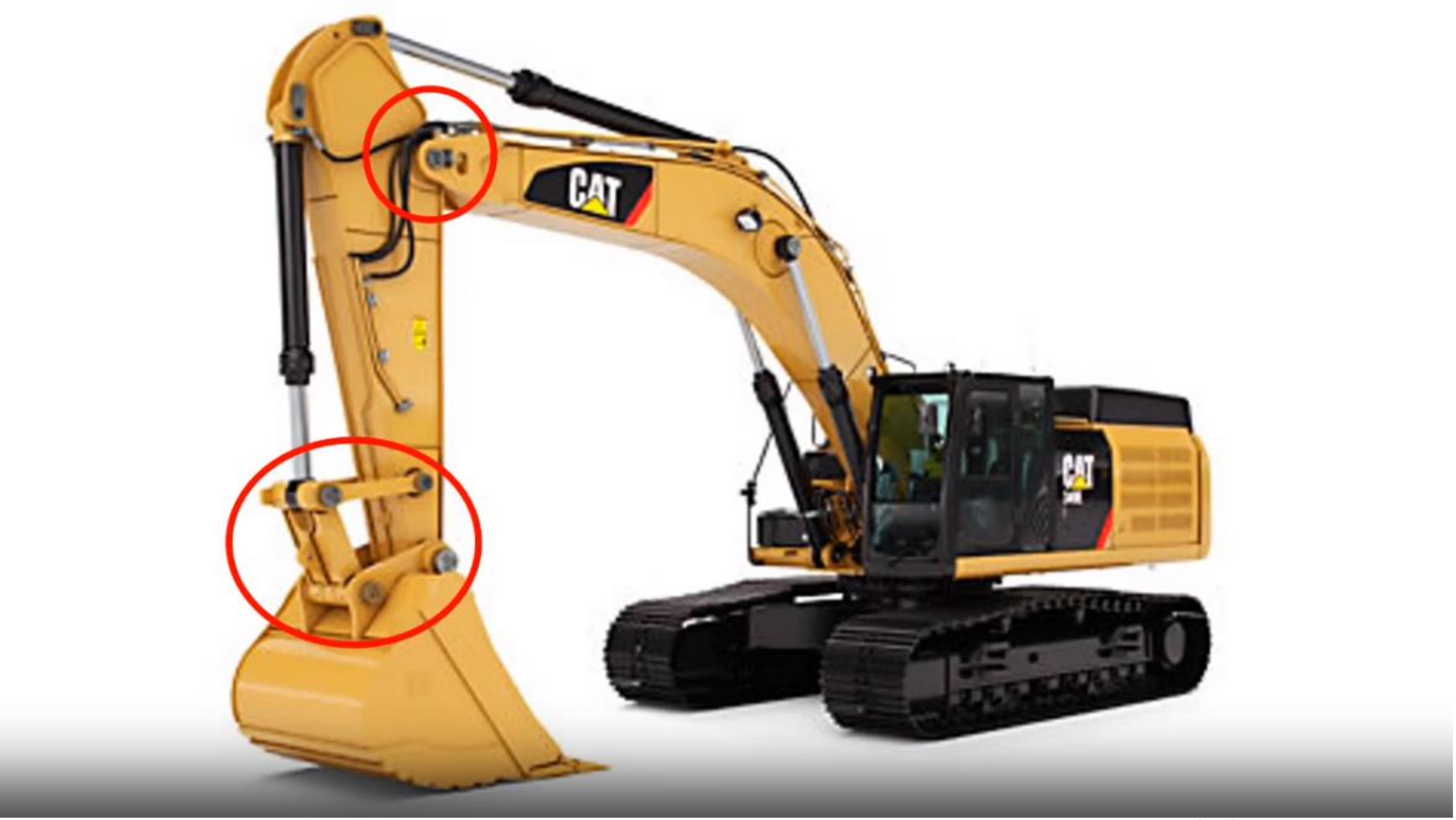
Support Reactions: Example 3



1.2 Equilibrium of a Deformable Body

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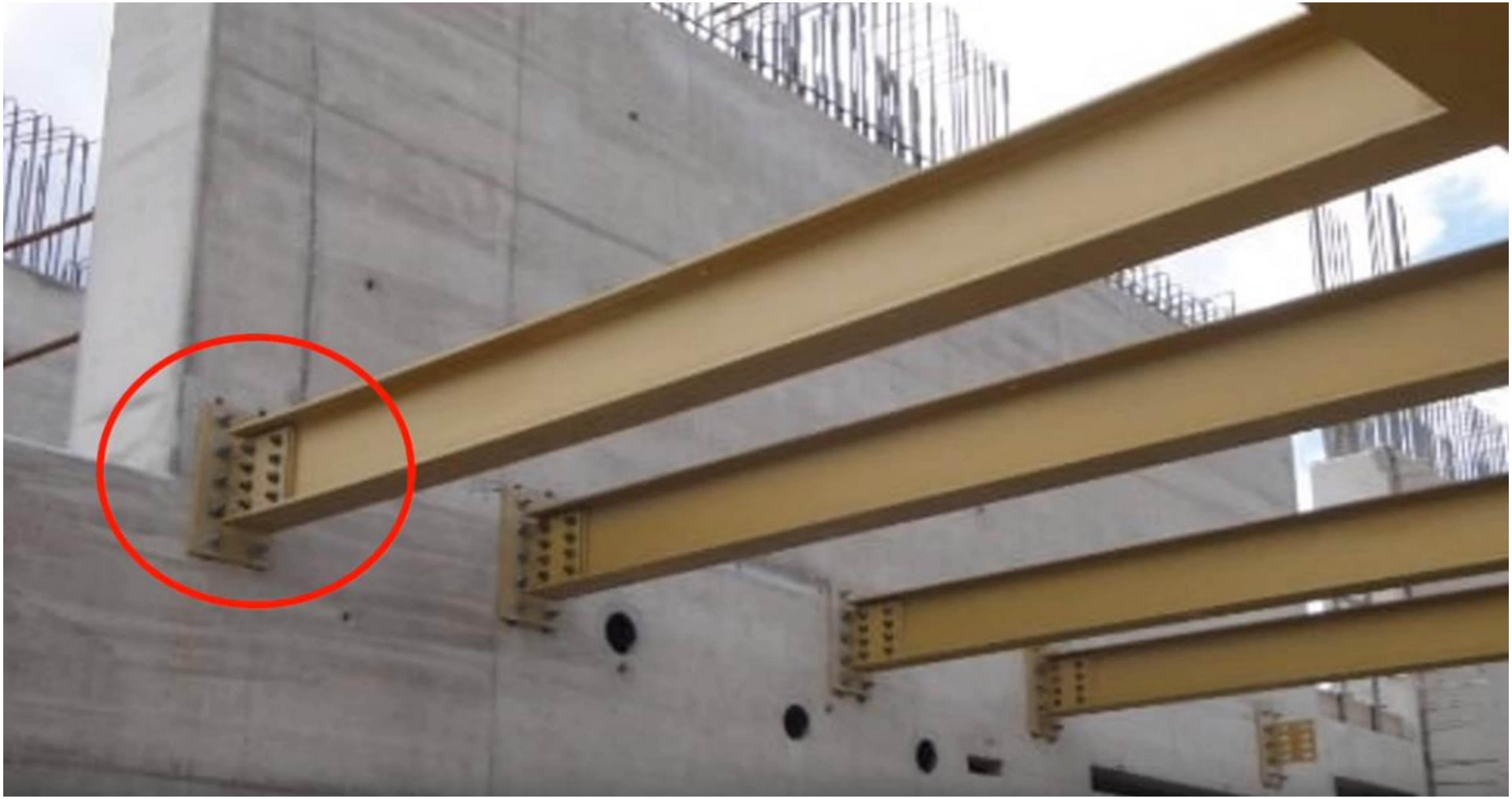
Support Reactions: Example 4



1.2 Equilibrium of a Deformable Body

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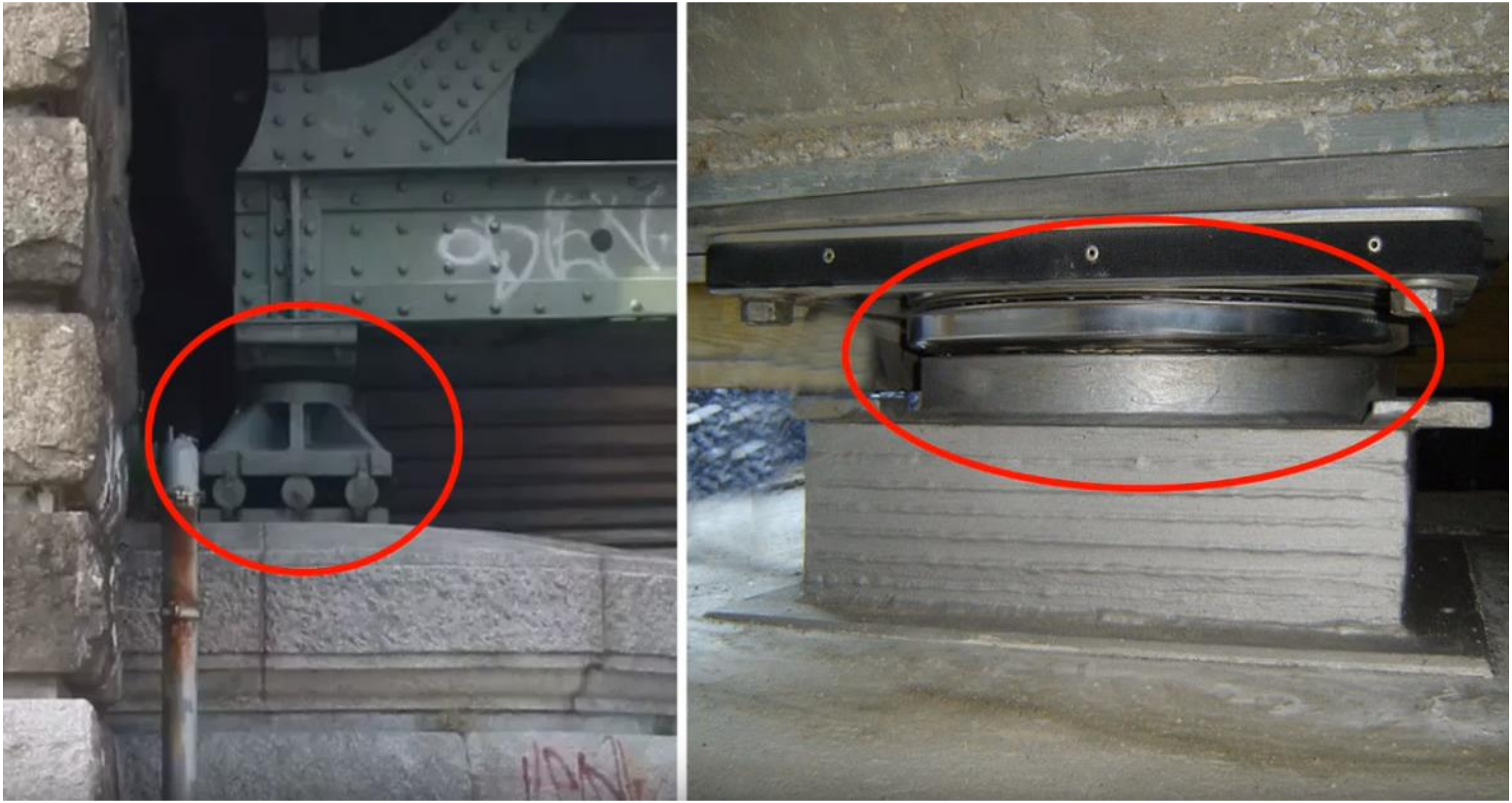
Support Reactions: Example 5

1.2 Equilibrium of a Deformable Body

Support Reactions: Example 6



1.2 Equilibrium of a Deformable Body

Equation of Equilibrium

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M}_O = \mathbf{0}$$

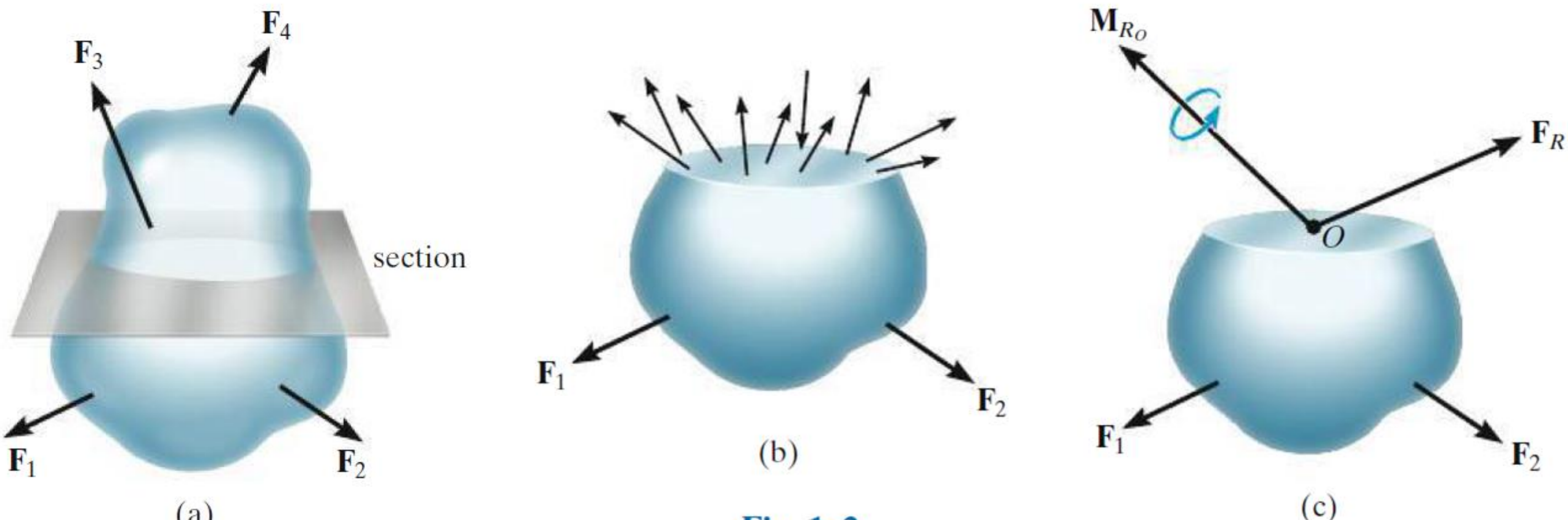


Fig. 1–2

1.2 Equilibrium of a Deformable Body

Equation of Equilibrium

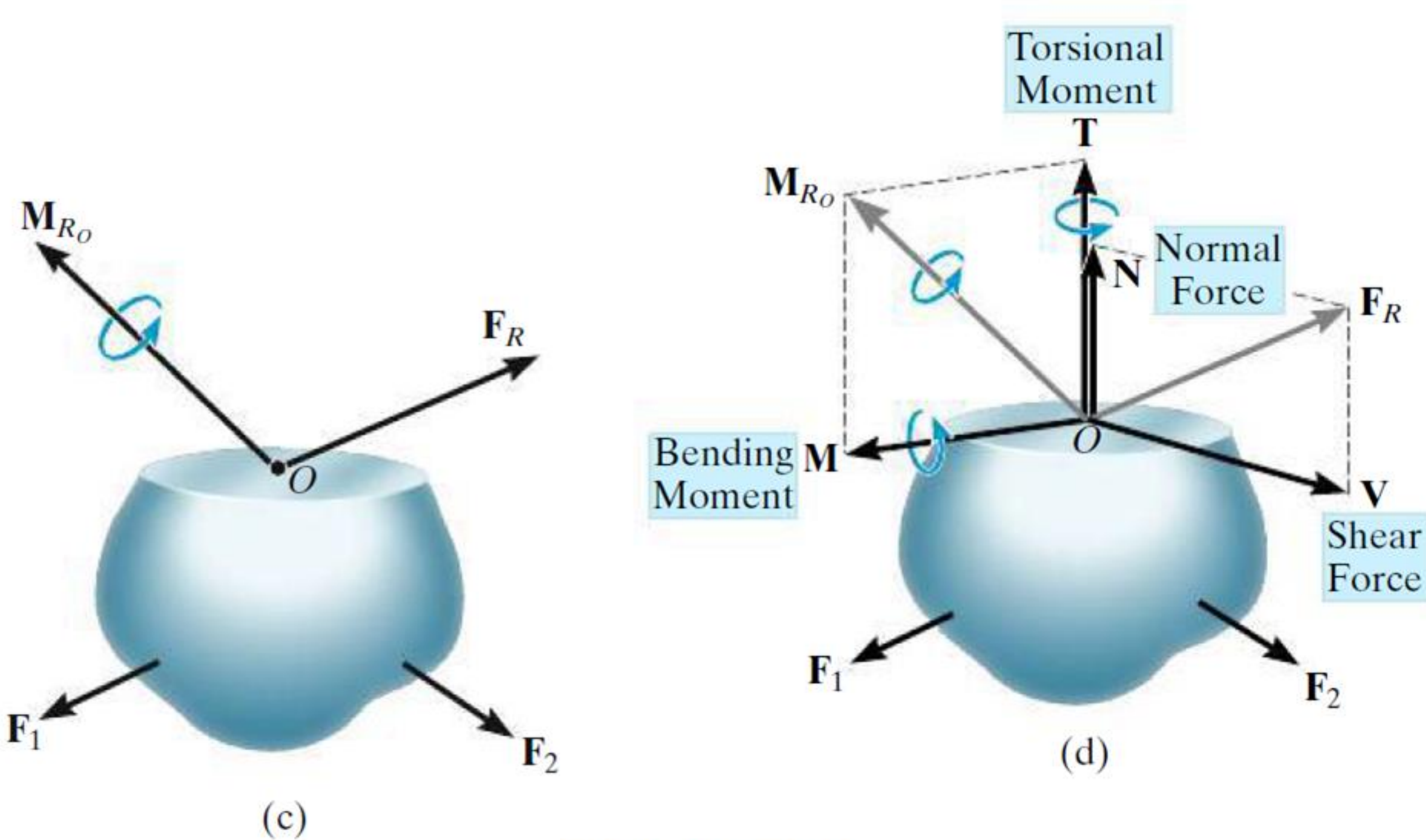
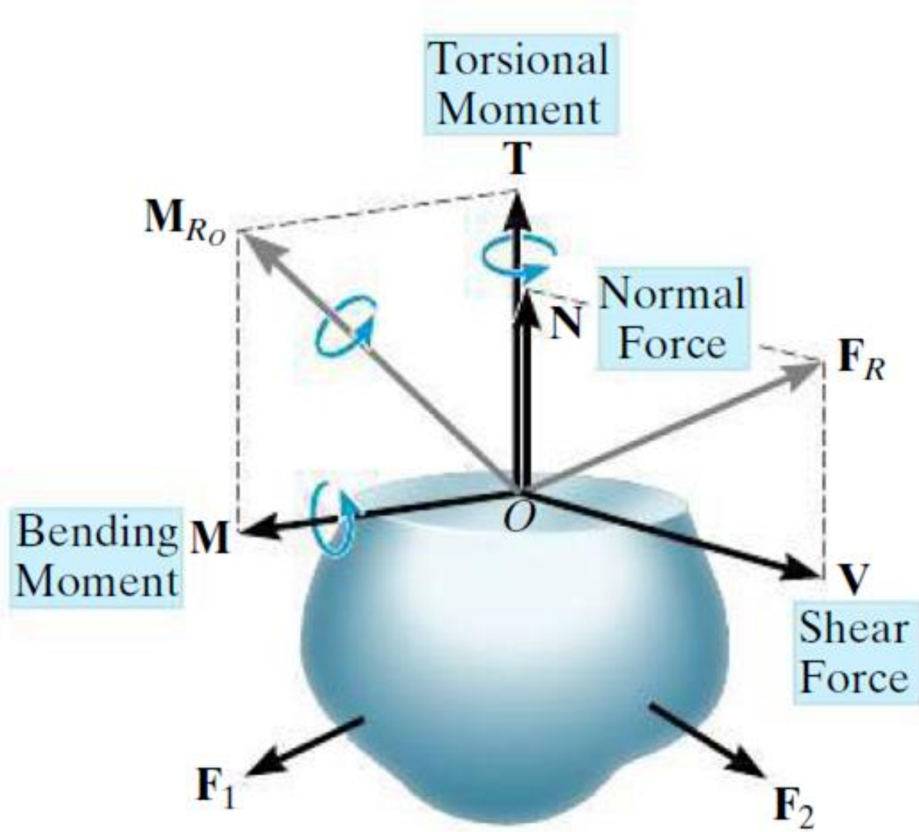


Fig. 1–2 (cont.)

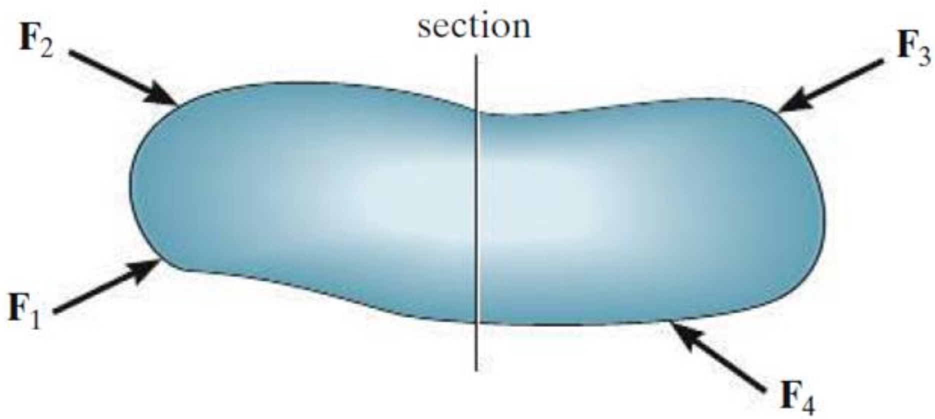
1.2 Equilibrium of a Deformable Body

- **Normal Force (*N*):** Acts perpendicular to the area
- **Shear Force (*V*):** Lies on the plane of the area
- **Torsional moment or torque (*T*):** When external load tend to twist one segment of the body with respect to other about an axis perpendicular to the area
- **Bending Moment (*M*):** It caused by the external loads that tend to bend the body about an axis lying within the plane after area

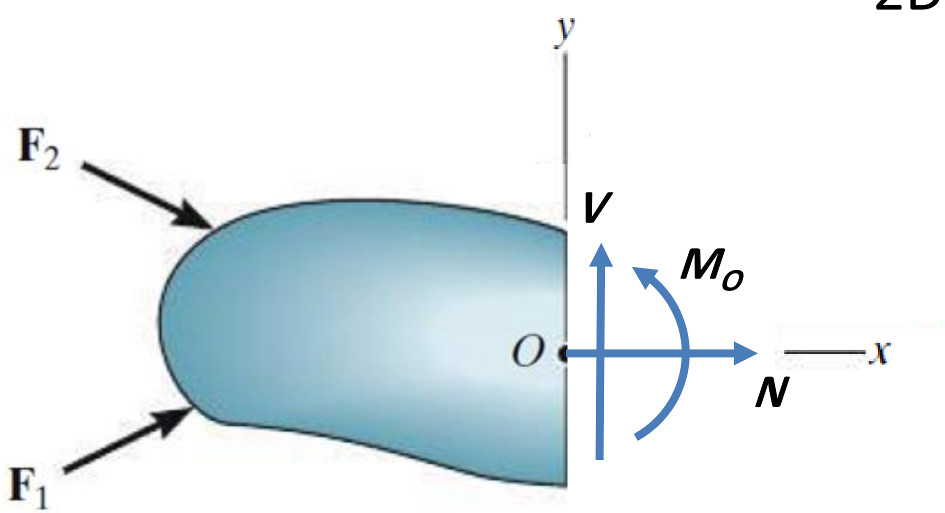


3D- Dimensional

1.2 Equilibrium of a Deformable Body



2D- Dimensional



Problem 1-6

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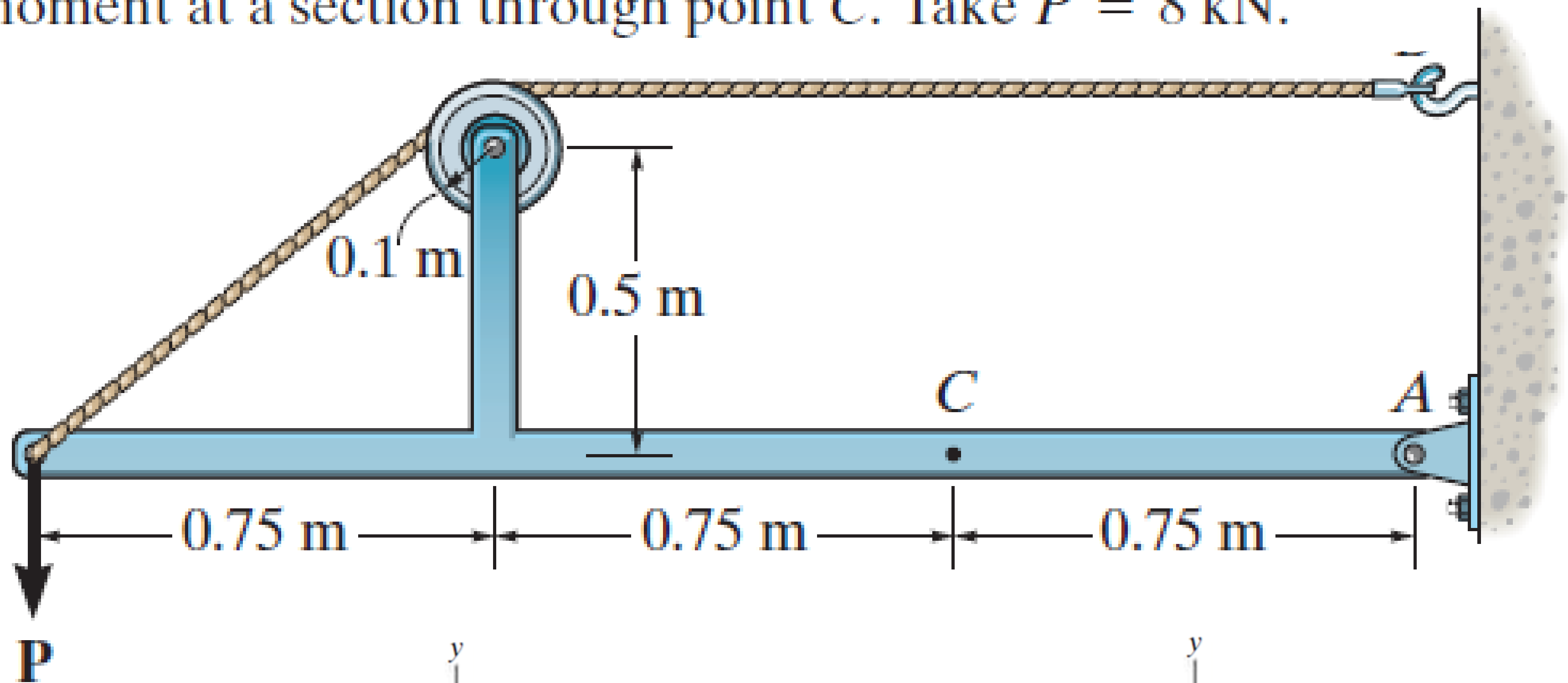
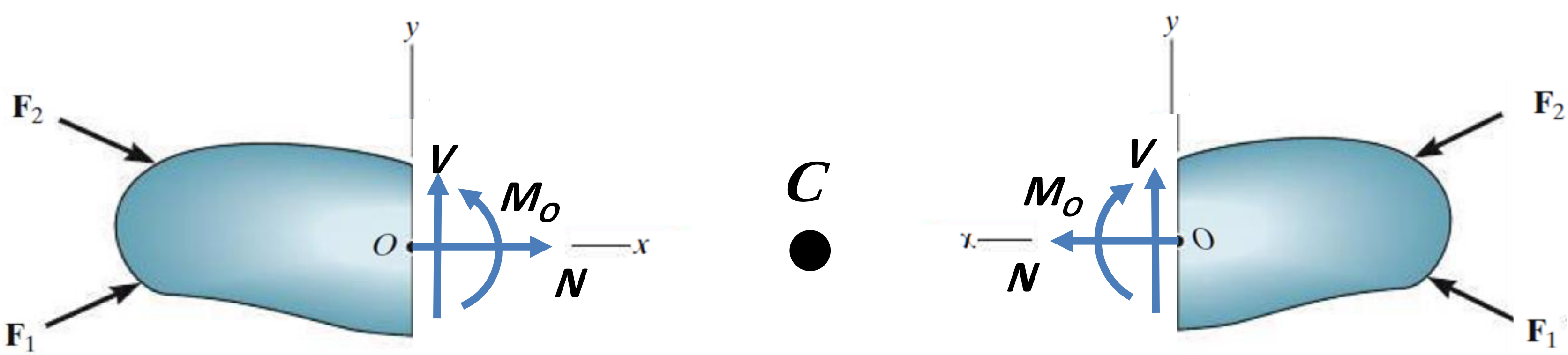
1–6. Determine the normal force, shear force, and moment at a section through point *C*. Take $P = 8$ kN.





Problem 1-6

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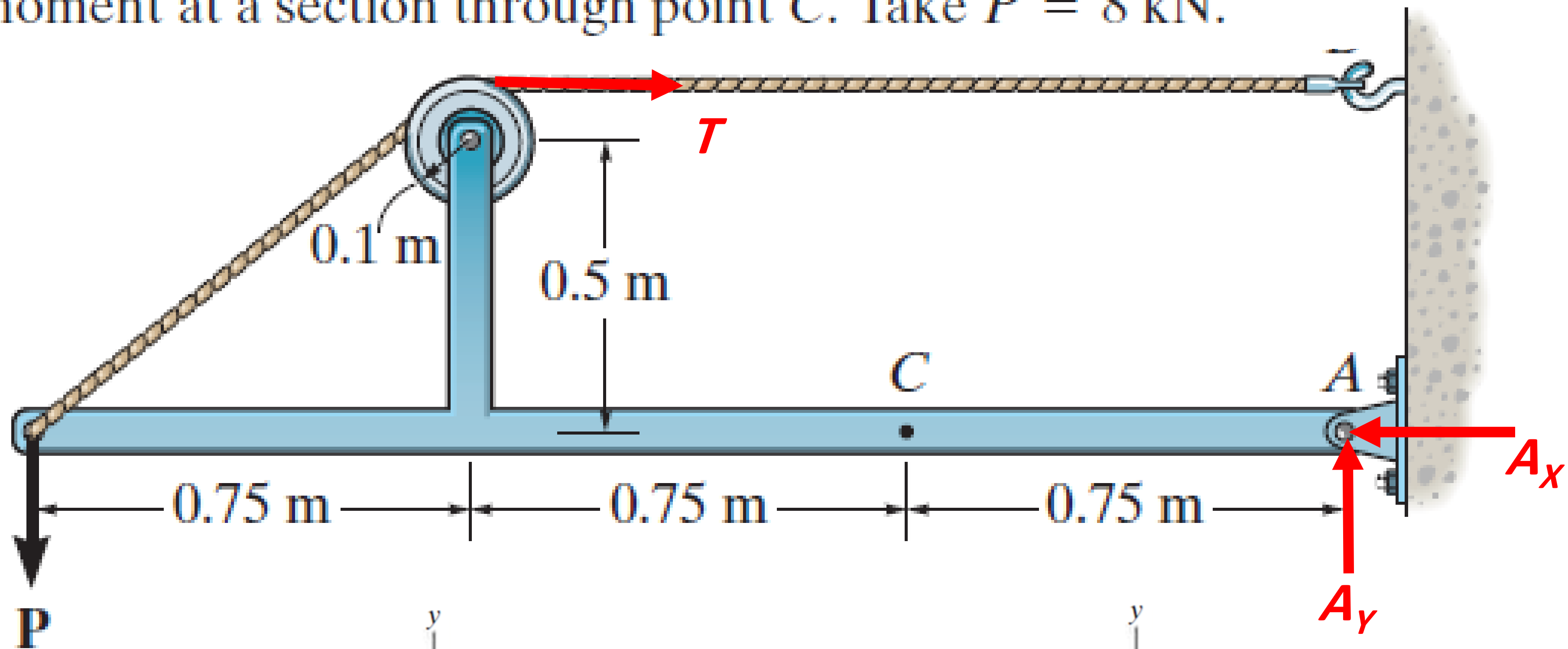
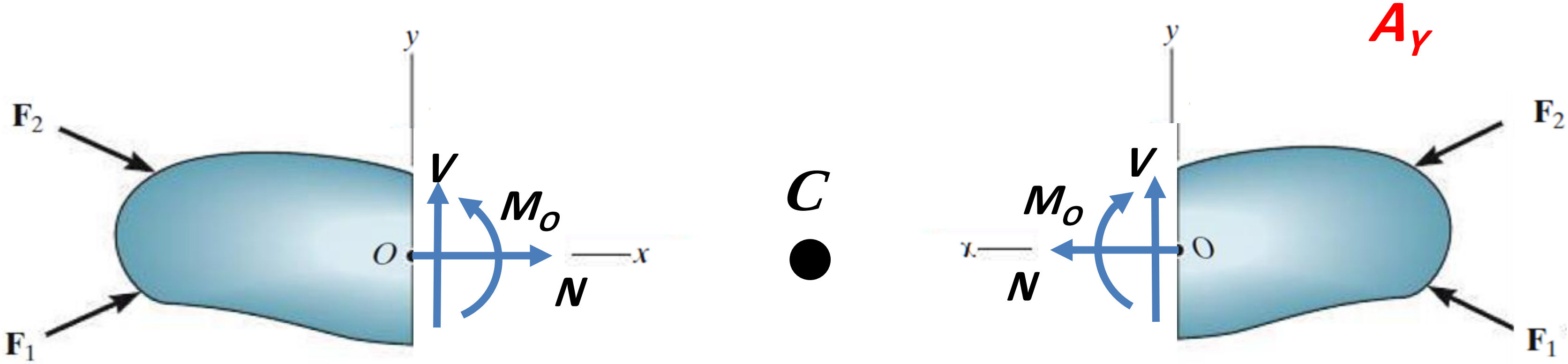
1–6. Determine the normal force, shear force, and moment at a section through point C. Take $P = 8$ kN.





Problem 1-22

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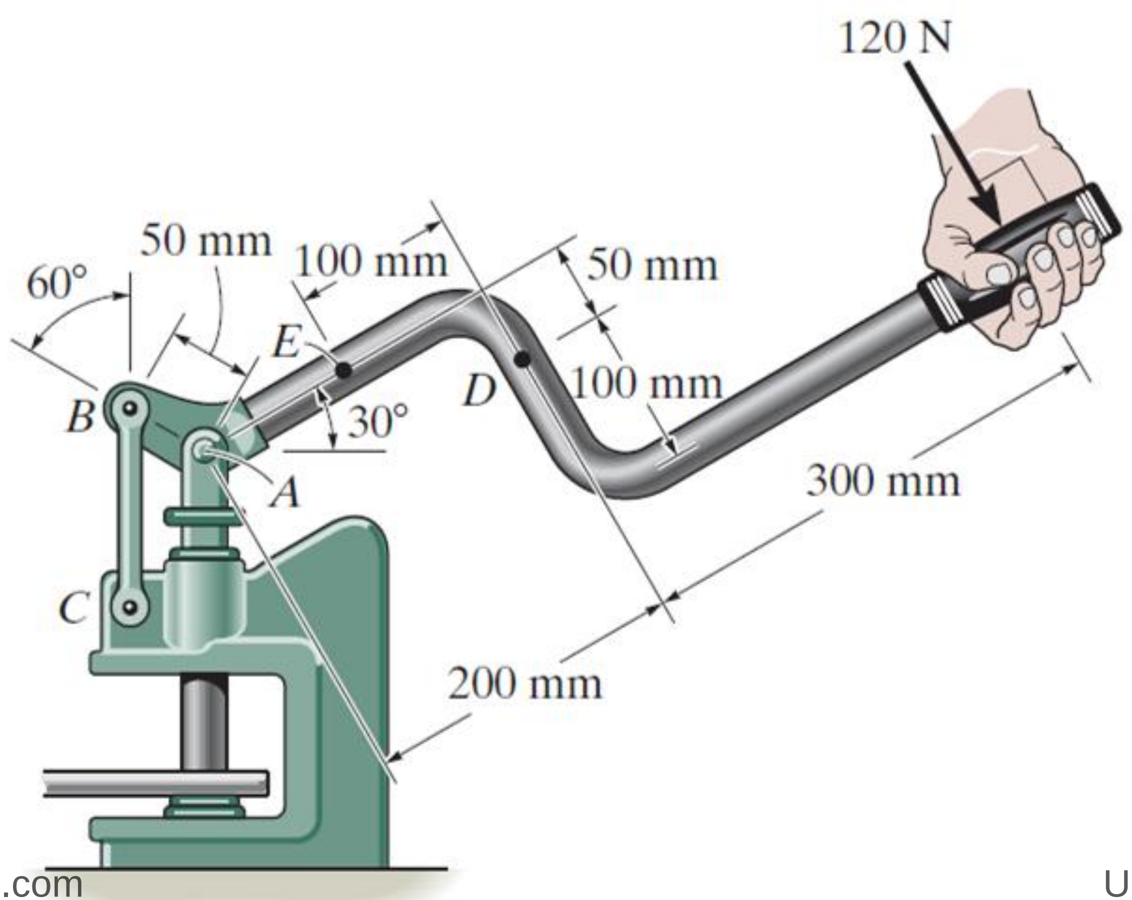
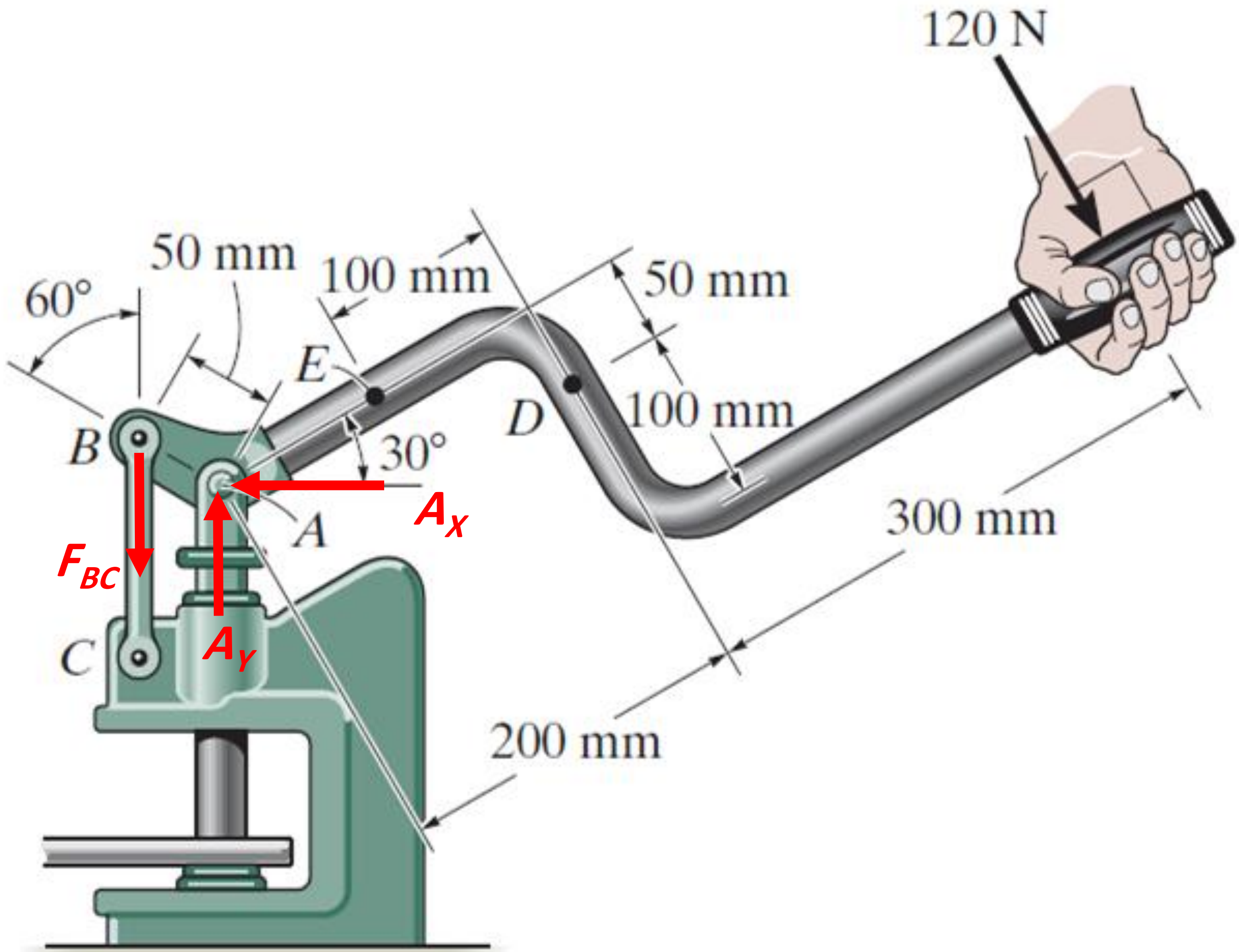
1–22. The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin *A* and in the short link *BC*. Also, determine the internal resultant loadings acting on the cross section passing through the handle arm at *D*.



Problem 1-22



1.3 Stress

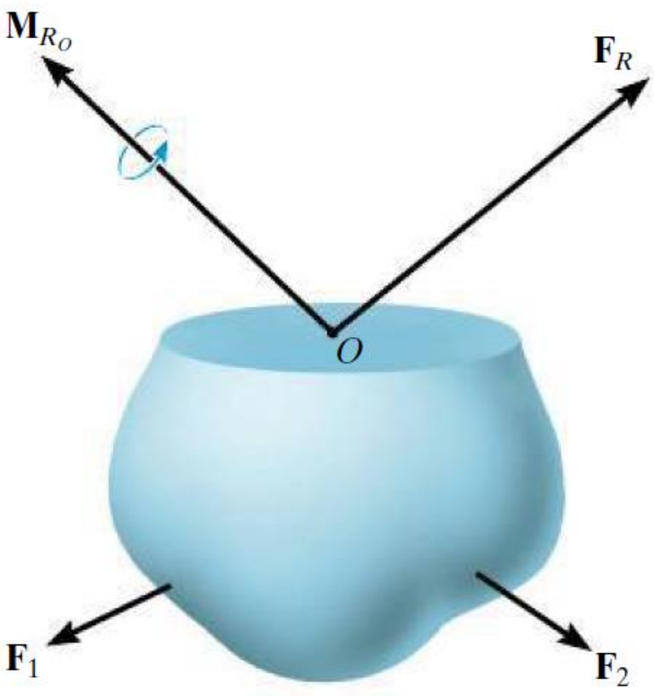
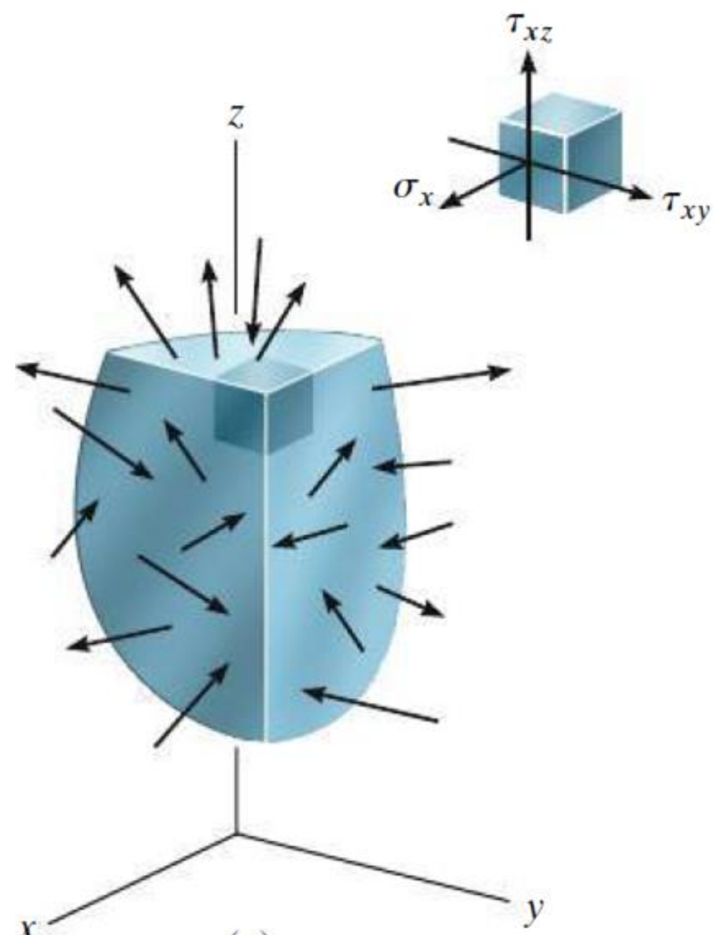
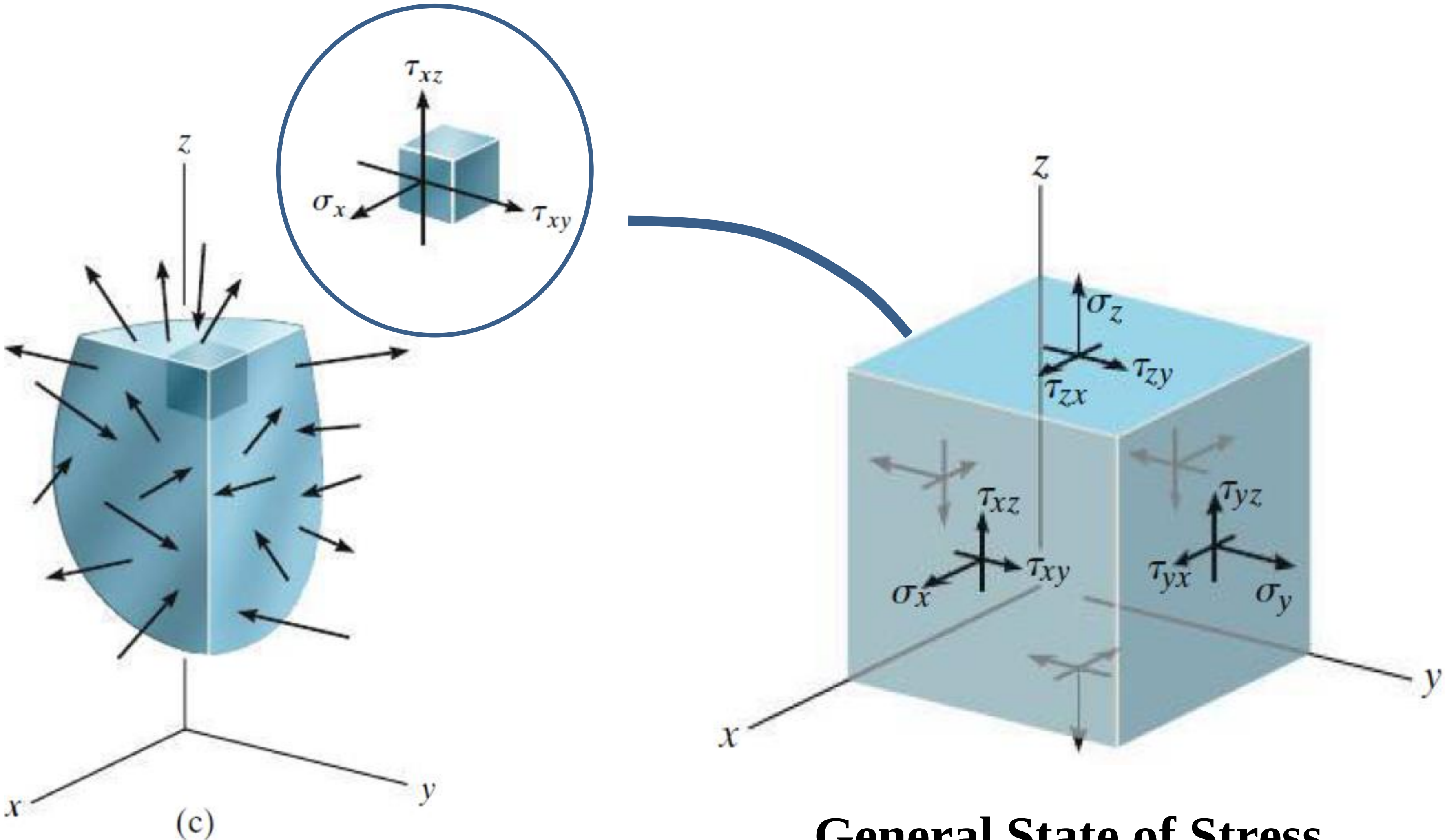


Fig. 1–8



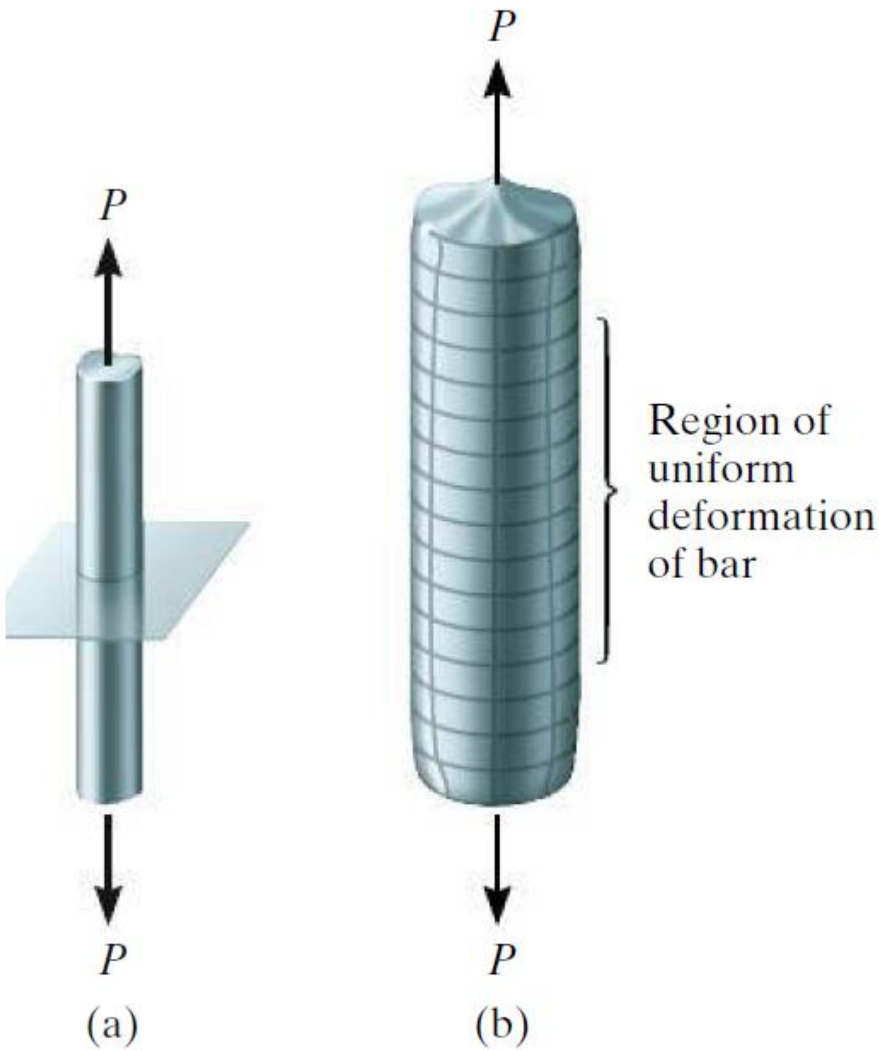
(c)

1.3 Stress



General State of Stress

1.3 Average Normal Stress in an Axially Loaded Bar



$$\sigma = \frac{P}{A}$$

1.5 Average Shear Stress

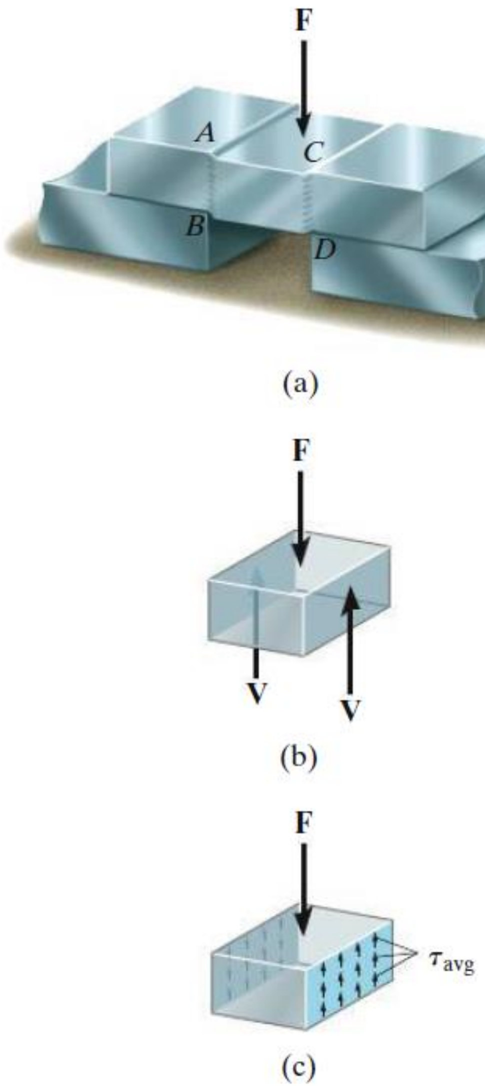


Fig. 1–19

$$\tau_{avg} = \frac{V}{A}$$

1.5 Average Shear Stress

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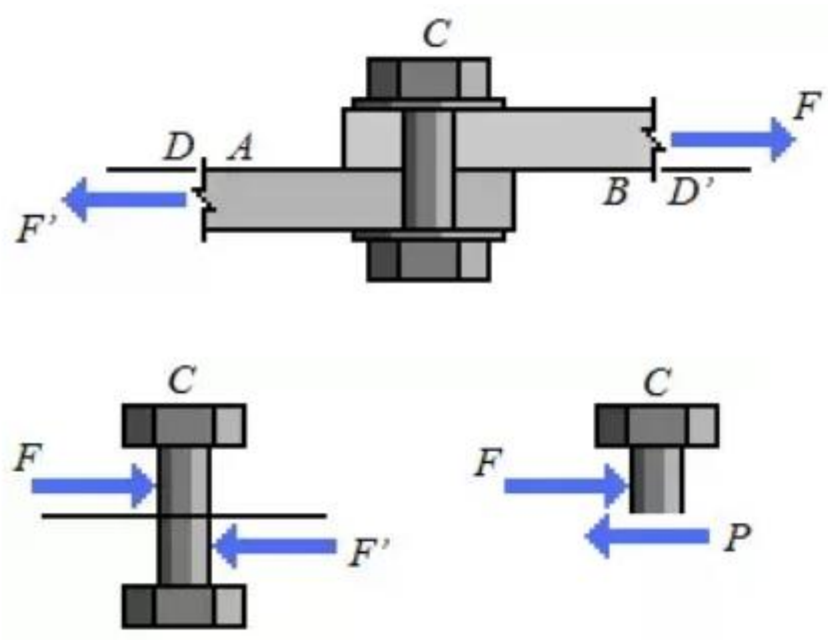
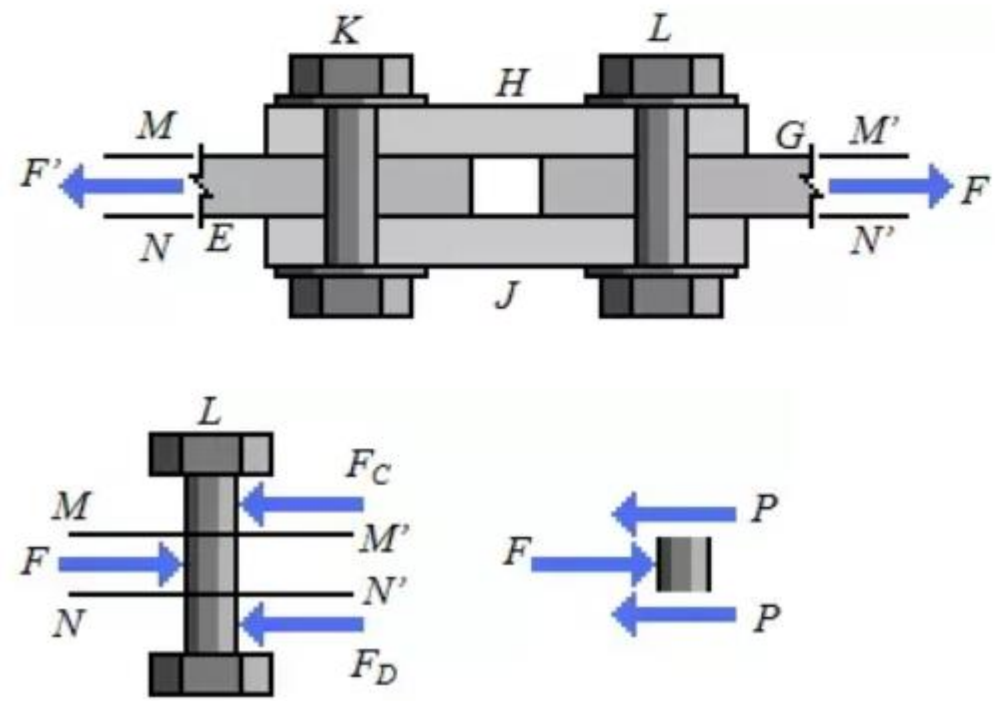
Single Shear



Bolt C: If plates *A* and *B* are connected by bolt *C*, shear will take place in bolt *C* in plane *DD'*.

Double Shear



Bolt C: If splice plates *H* and *J* are used to connect plates *E* and *G*, shear will take place in bolts *K* and *L* in each of the two planes *MM'* and *NN'*.

Problem 1-33

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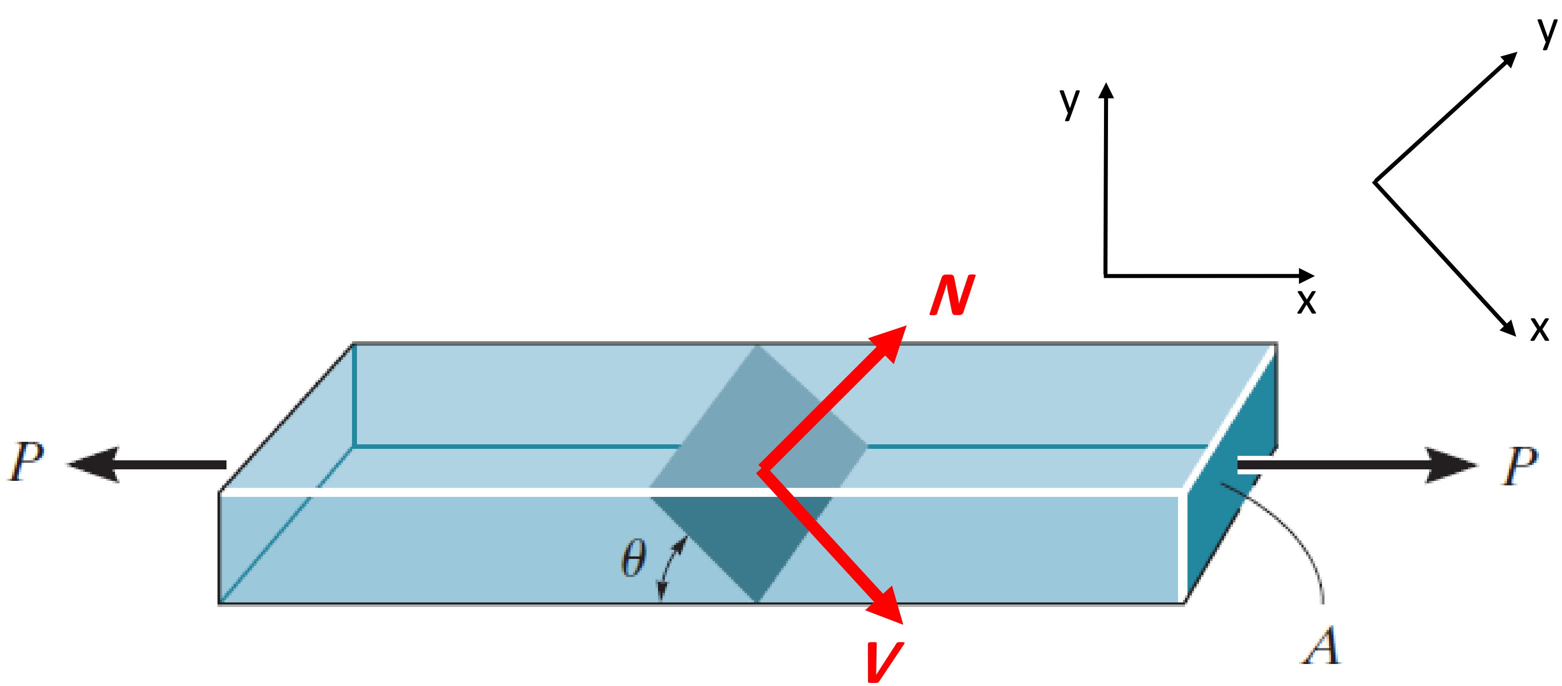
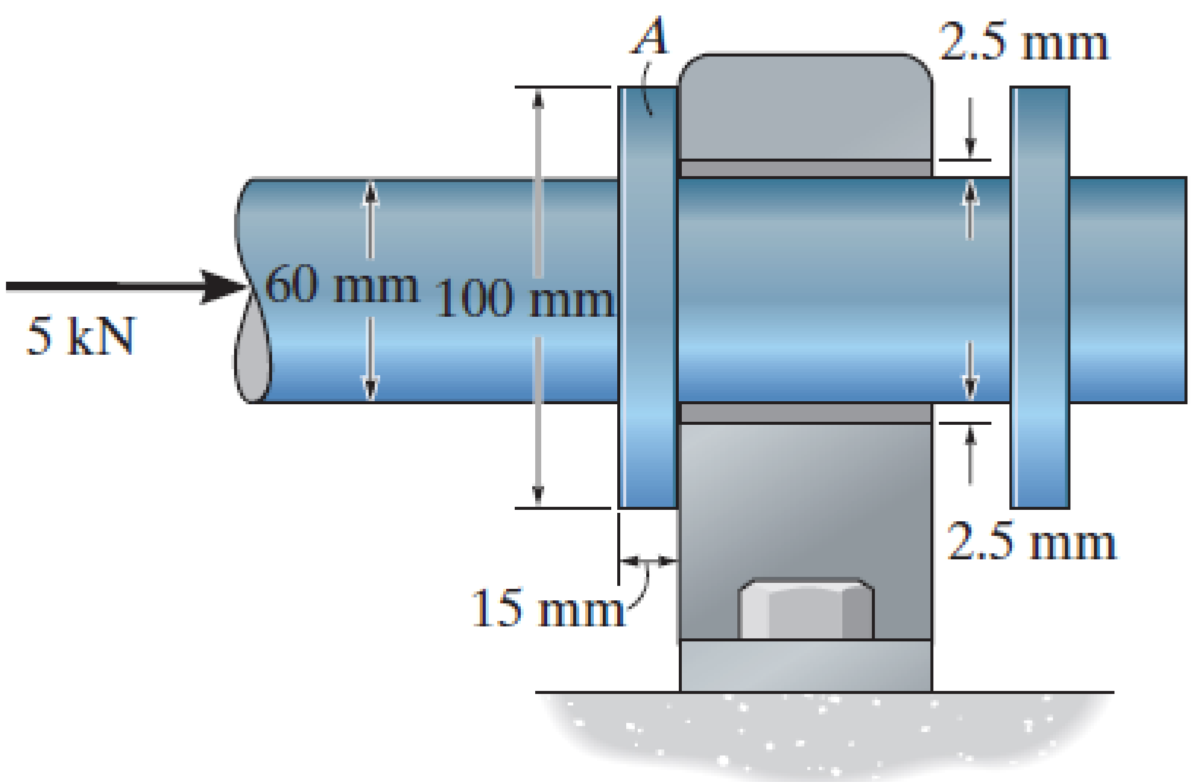
1–33. The bar has a cross-sectional area A and is subjected to the axial load P. Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ $(0 \leq \theta \leq 90°)$.



Problem 1-60

***1–60.** If the shaft is subjected to an axial force of 5 kN, determine the bearing stress acting on the collar *A*.



1.6 Allowable Stress Design

The ***factor of safety*** (F.S.) is a ratio of the failure load F_{fail} to the allowable load F_{allow}. Here F_{fail} is found from experimental testing of the material, and the factor of safety is selected based on experience so that all the above mentioned uncertainties are accounted for when the member is used under similar conditions of loading and geometry

$$\text{F.S.} = \frac{F_{\text{fail}}}{F_{\text{allow}}}$$

Problem 1-94

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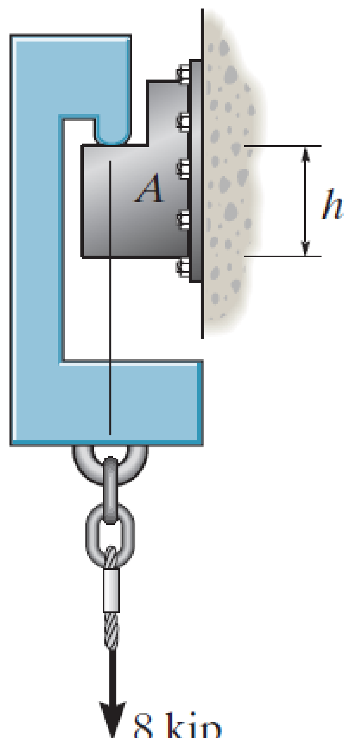
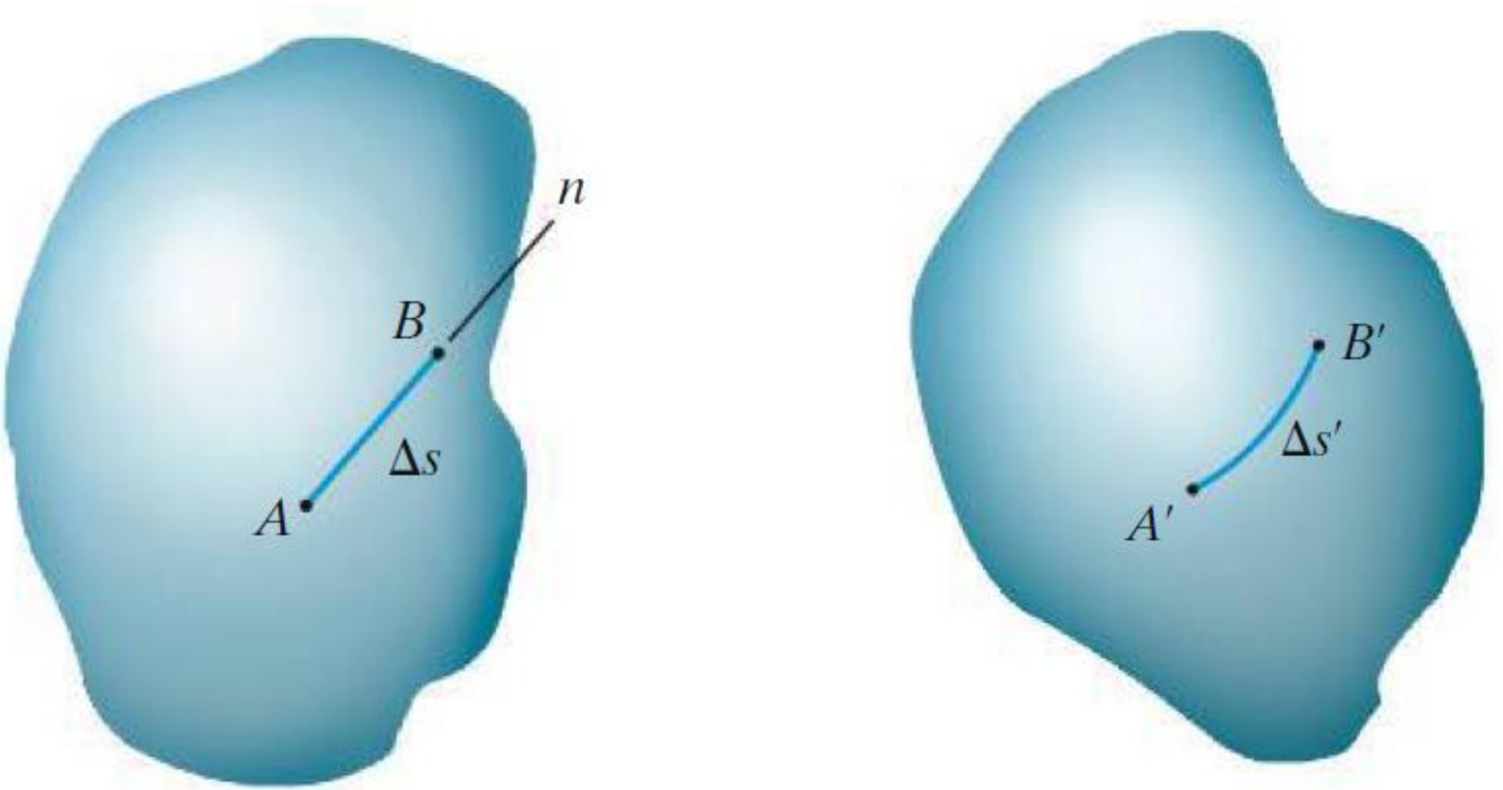
1–94. The aluminum bracket *A* is used to support the centrally applied load of 8 kip. If it has a constant thickness of 0.5 in., determine the smallest height *h* in order to prevent a shear failure. The failure shear stress is $\tau_{fail} = 23$ ksi. Use a factor of safety for shear of F.S. = 2.5.



2.1 Deformation

2.2 Strain – Normal Strain



Undeformed body

(a)

Deformed body

(b)

Fig. 2–1

2.2 Strain – Shear Strain

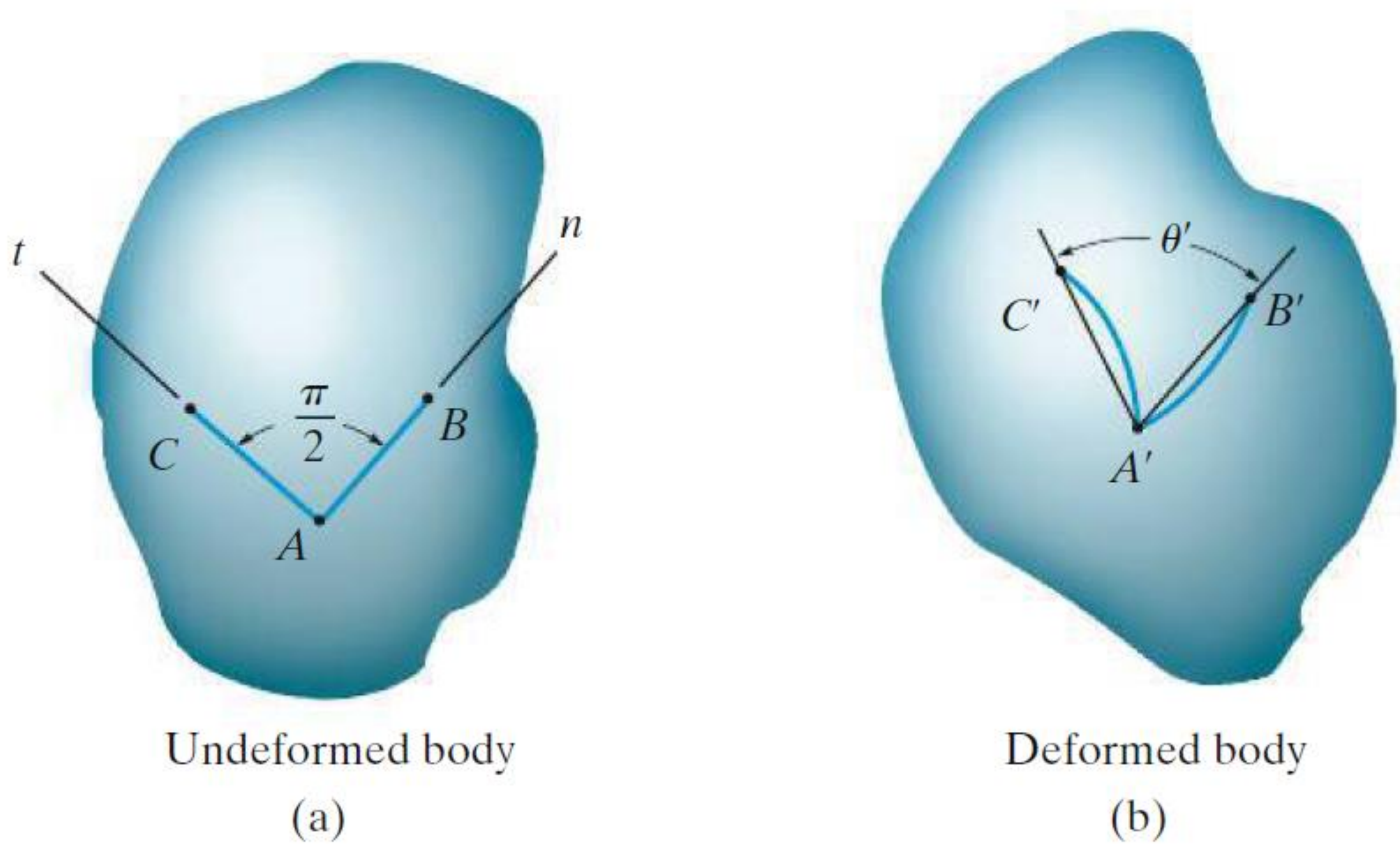
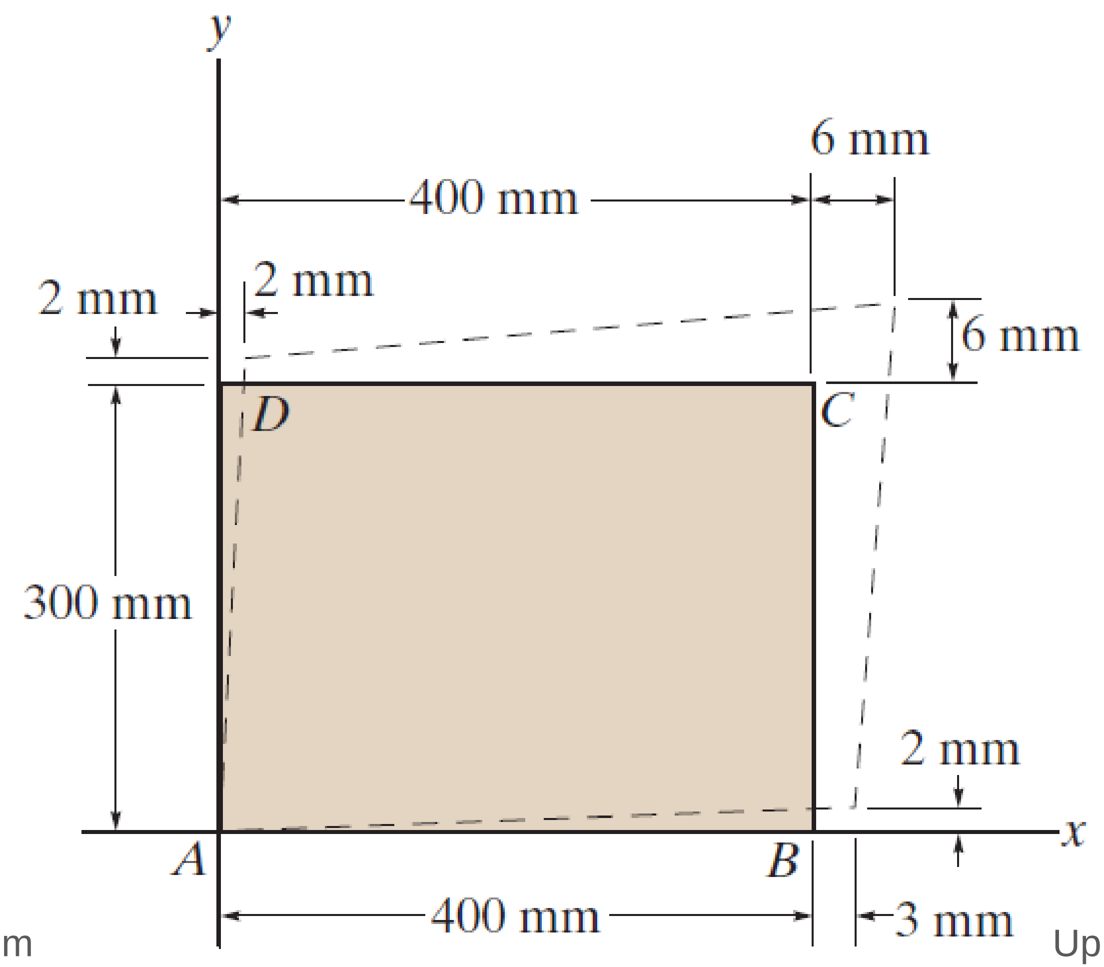


Fig. 2–2

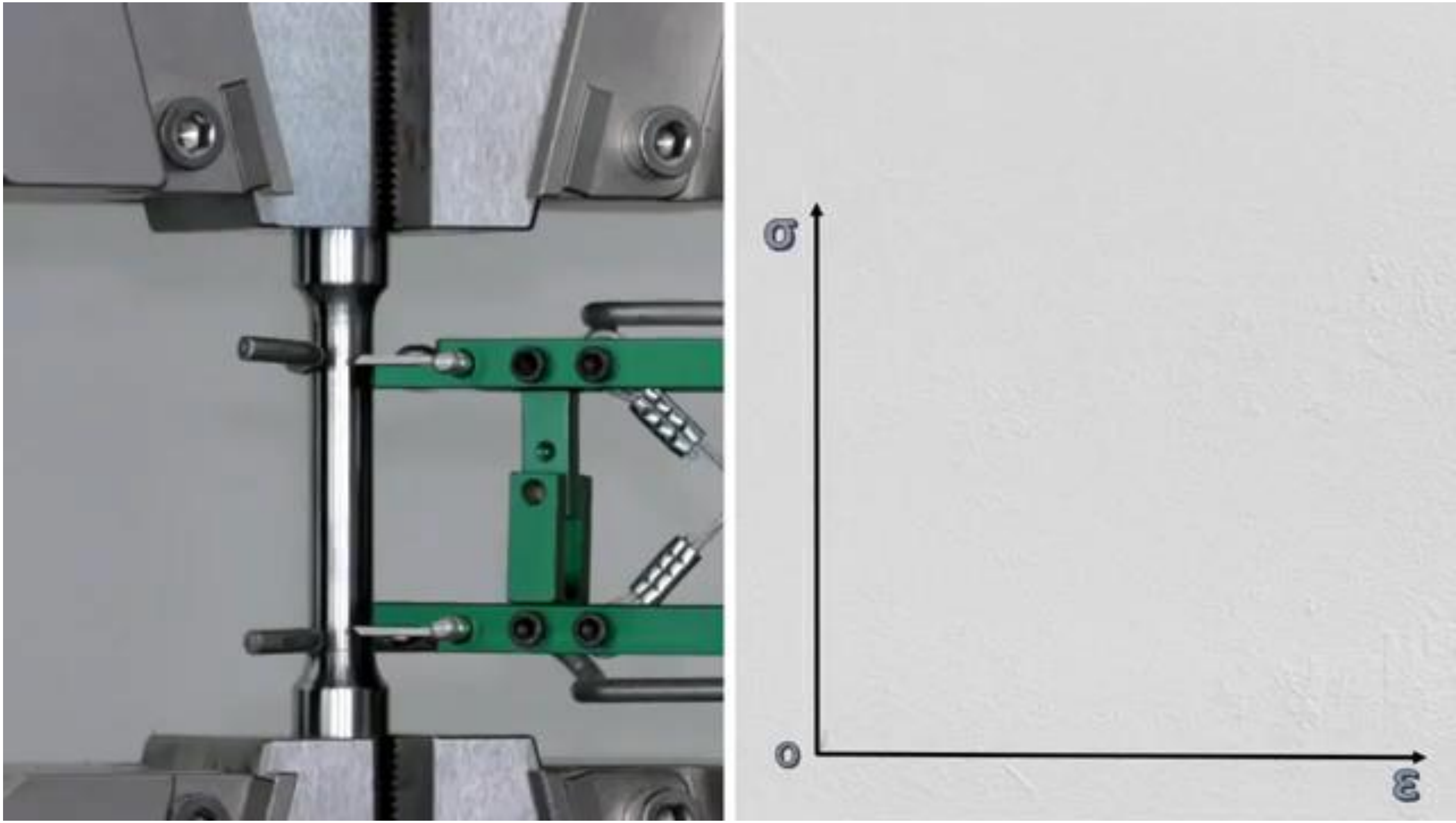
Problem 2-30

2–30. The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal BD, and the average shear strain at corner B.



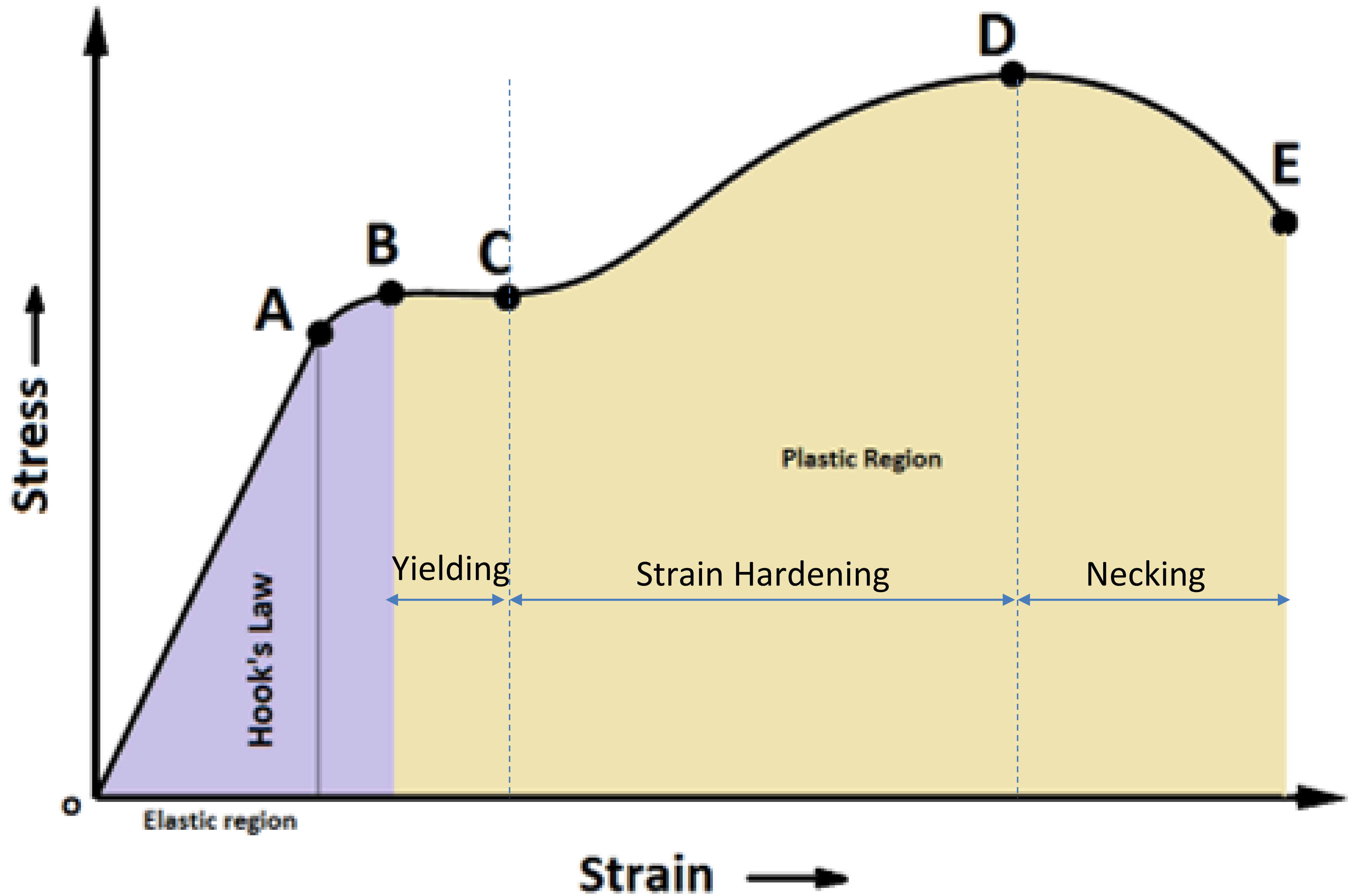
Stress-Strain Diagram

Stress-Strain Diagram



https://www.youtube.com/watch?v=RY9X_O8is-k

Stress-Strain Diagram



3.5 Strain Energy

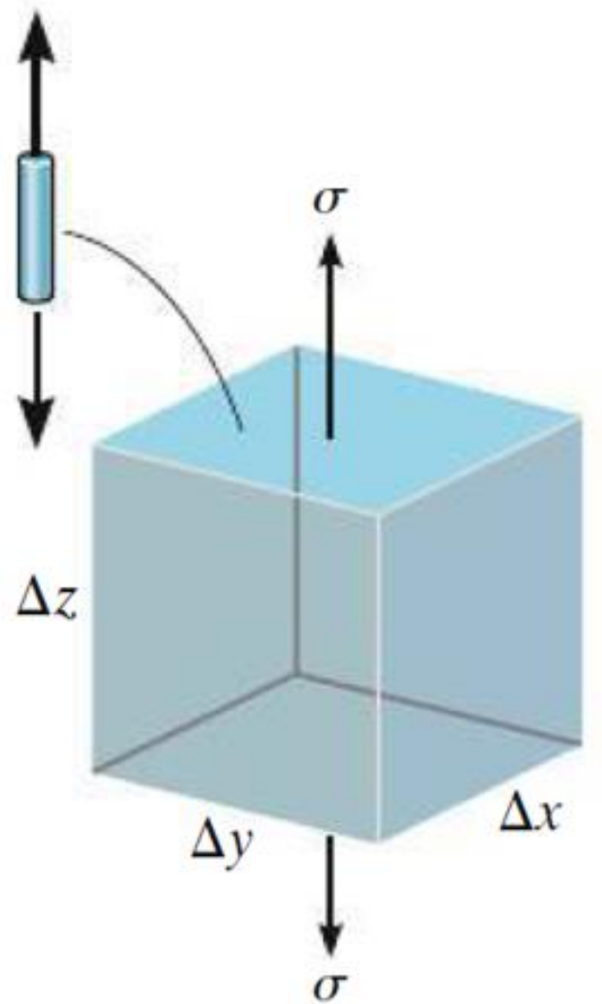
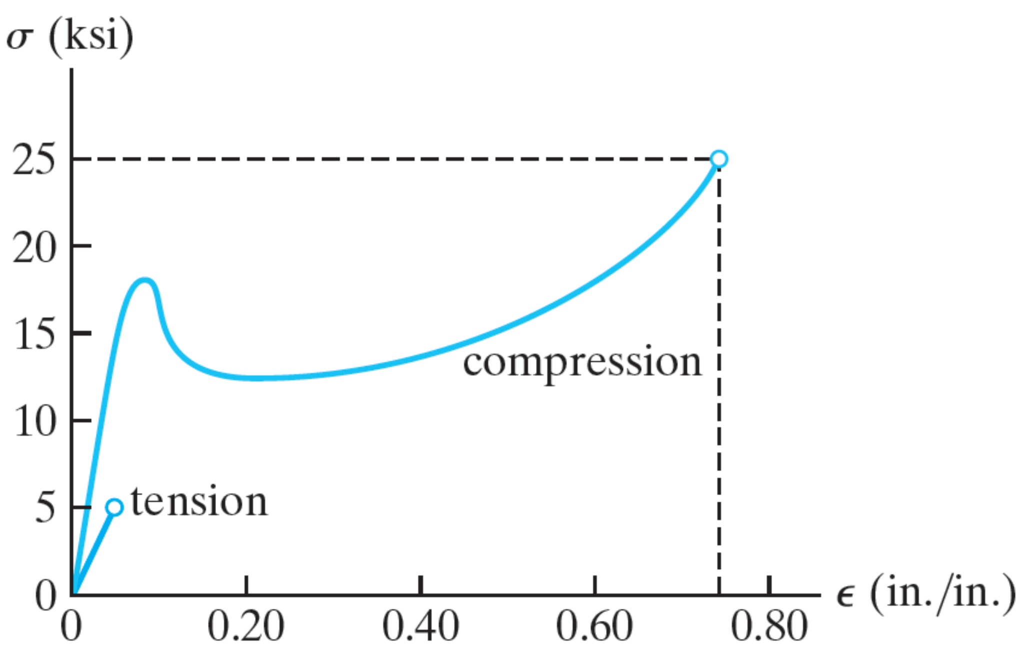
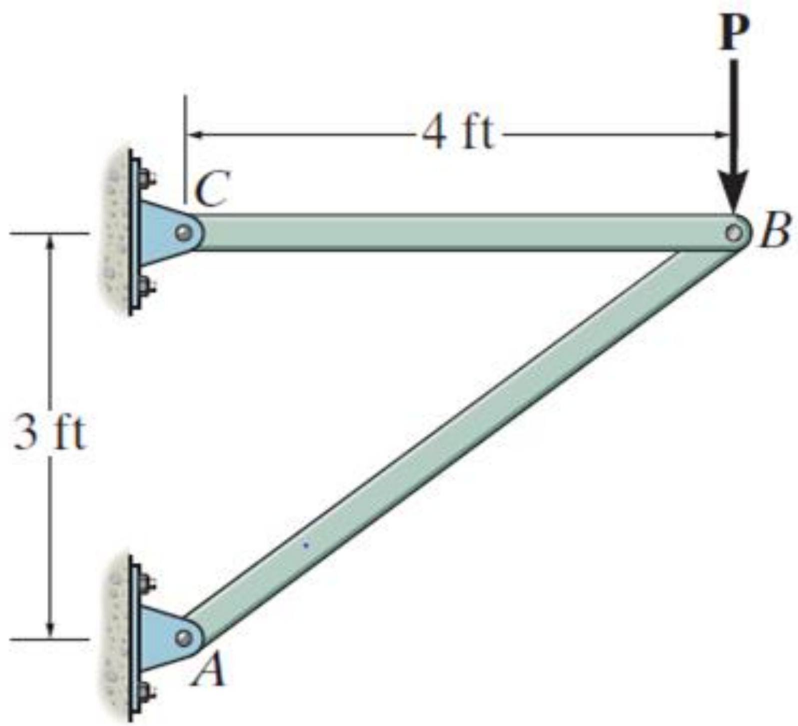


Fig. 3–15

Problem 3-22

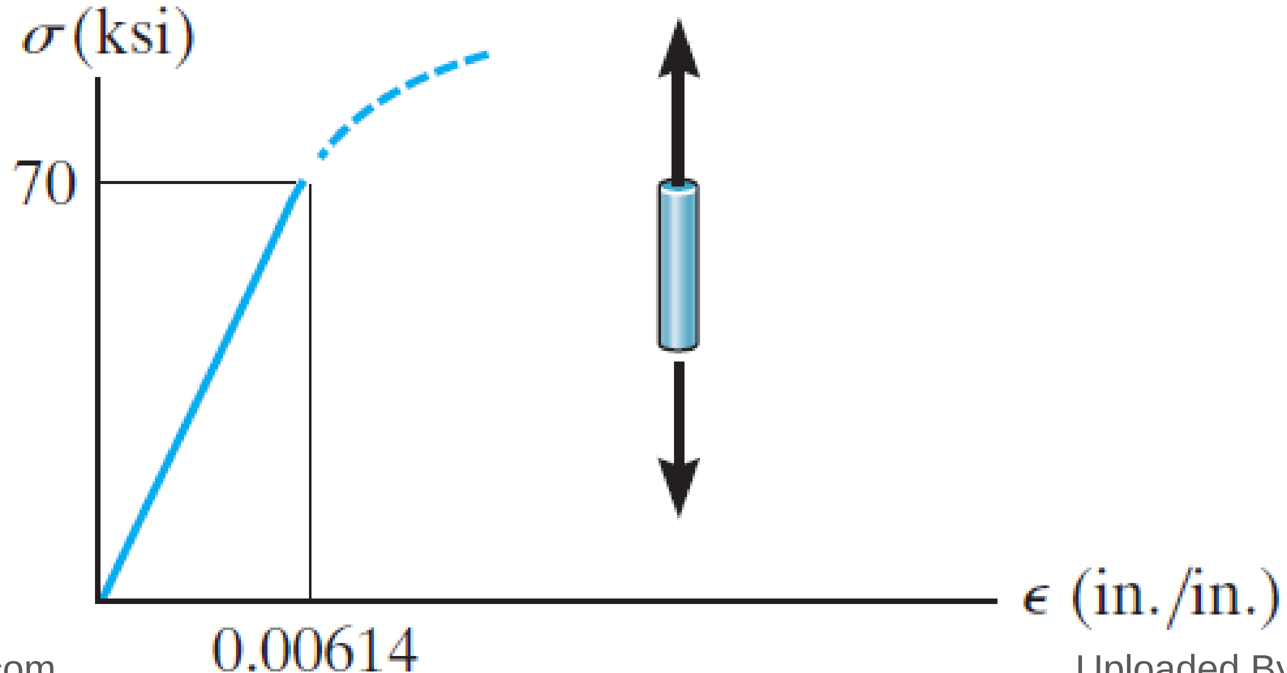
3–22. The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 3$ kip. Assume that buckling does not occur.





Problem 3-36

*3–36. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



Average Mechanical Properties of Typical Engineering Materials- Hibbeler

SI Units

Materials		Density ρ (Mg/m^3)	Moduls of Elasticity E (GPa)	Modulus of Rigidity G (GPa)	Yield Strength (MPa) σ_Y			Ultimate Strength (MPa) σ_u			% Elongation in 50 mm specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°C$
					Tens.	Comp.[b]	Shear	Tens.	Comp.[b]	Shear			
Metallic													
Aluminum Wrought Alloys	2014-T6	2.79	73.1	27	414	414	172	469	469	290	10	0.35	23
	6061-T6	2.71	68.9	26	255	255	131	290	290	186	12	0.35	24
Cast Iron Alloys	Gray ASTM 20	7.19	67.0	27	–	–	–	179	669	–	0.6	0.28	12
	Malleable ASTM A-197	7.28	172	68	–	–	–	276	572	–	5	0.28	12
Copper Alloys	Red Brass C83400	8.74	101	37	70.0	70.0	–	241	241	–	35	0.35	18
	Bronze C86100	8.83	103	38	345	345	–	655	655	–	20	0.34	17
Magnesium Alloy	[Am 1004-T61]	1.83	44.7	18	152	152	–	276	276	152	1	0.30	26
Steel Alloys	Structural A-36	7.85	200	75	250	250	–	400	400	–	30	0.32	12
	Structural A992	7.85	200	75	345	345	–	450	450	–	30	0.32	12
	Stainless 304	7.86	193	75	207	207	–	517	517	–	40	0.27	17
	Tool L2	8.16	200	75	703	703	–	800	800	–	22	0.32	12
Titanium Alloy	[Ti-6Al-4V]	4.43	120	44	924	924	–	1,000	1,000	–	16	0.36	9.4
Nonmetallic													
Concrete	Low Strength	2.38	22.1	–	–	–	12	–	–	–	–	0.15	11
	High Strength	2.37	29.0	–	–	–	38	–	–	–	–	0.15	11
Plastic Reinforced	Kevlar 49	1.45	131	–	–	–	–	717	483	20.3	2.8	0.34	–
	30% Glass	1.45	72.4	–	–	–	–	90	131	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.47	13.1	–	–	–	–	2.1[c]	26[d]	6.2[d]	–	0.29[e]	–
	White Spruce	3.60	9.65	–	–	–	–	2.5[c]	36[d]	6.7[d]	–	0.31[e]	–

US Customary Units

Materials		Specific Weight (lb/in³)	Moduls of Elasticity E (10³) ksi	Modulus of Rigidity G (10³) ksi	Yield Strength (ksi) σ_Y			Ultimate Strength (ksi) σ_u			% Elongation in 2 in. specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α (10⁻⁶)/°F
					Tens.	Comp.[b]	Shear	Tens.	Comp.[b]	Shear			
Metallic													
Aluminum Wrought Alloys	2014-T6	0.101	10.6	3.9	60	60	25	68	68	42	10	0.35	12.8
	6061-T6	0.098	10.0	3.7	37	37	19	42	42	27	12	0.35	13.1
Cast Iron Alloys	Gray ASTM 20	0.260	10.0	3.9	–	–	–	26	96	–	0.6	0.28	6.70
	Malleable ASTM A-197	0.263	25.0	9.8	–	–	–	40	83	–	5	0.28	6.60
Copper Alloys	Red Brass C83400	0.316	14.6	5.4	11.4	11.4	–	35	35	–	35	0.35	9.80
	Bronze C86100	0.319	15.0	5.6	50	50	–	35	35	–	20	0.34	9.60
Magnesium Alloy	[Am 1004-T61]	0.066	6.48	2.5	22	22	–	40	40	22	1	0.30	14.3
Steel Alloys	Structural A-36	0.284	29.0	11.0	36	36	–	58	58	–	30	0.32	6.60
	Structural A992	0.284	29.0	11.0	50	50	–	65	65	–	30	0.32	6.60
	Stainless 304	0.284	28.0	11.0	30	30	–	75	75	–	40	0.27	9.60
	Tool L2	0.295	29.0	11.0	102	102	–	116	116	–	22	0.32	6.50
Titanium Alloy	[Ti-6Al-4V]	0.160	17.4	6.4	134	134	–	145	145	–	16	0.36	5.20
Nonmetallic													
Concrete	Low Strength	0.086	3.20	–	–	–	1.8	–	–	–	–	0.15	6.0
	High Strength	0.086	4.20	–	–	–	5.5	–	–	–	–	0.15	6.0
Plastic Reinforced	Kevlar 49	0.0524	19.0	–	–	–	–	104	70	10.2	2.8	0.34	–
	30% Glass	0.0524	10.5	–	–	–	–	13	19	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.017	1.90	–	–	–	–	0.30[c]	3.78[d]	0.90[d]	–	0.29[e]	–
	White Spruce	0.130	1.40	–	–	–	–	0.36[c]	5.18[d]	0.97[d]	–	0.31[e]	–

4.2 Elastic Deformation of an Axially Loaded Member

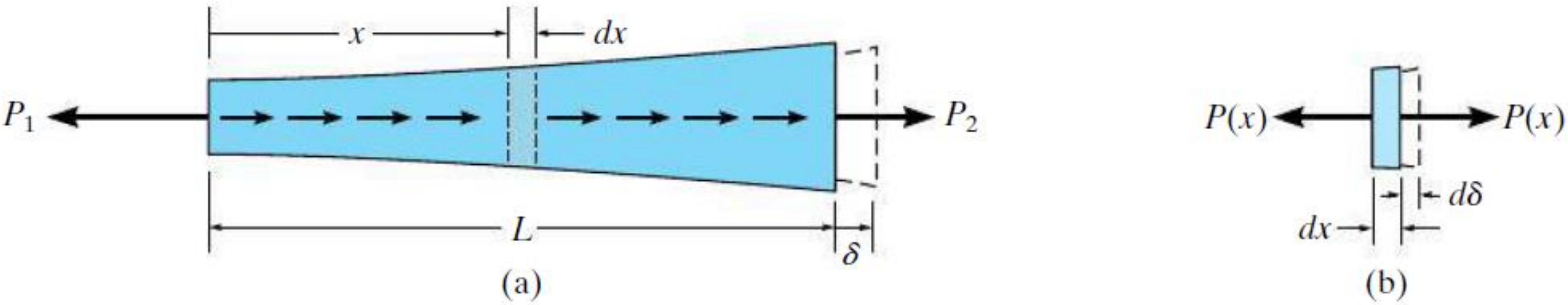


Fig. 4–2

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E(x)} \qquad (4\text{–}1)$$

where

δ = displacement of one point on the bar relative to the other point

L = original length of bar

$P(x)$ = internal axial force at the section, located a distance x from one end

$A(x)$ = cross-sectional area of the bar expressed as a function of x

$E(x)$ = modulus of elasticity for the material expressed as a function of x.

4.2 Elastic Deformation of an Axially Loaded Member

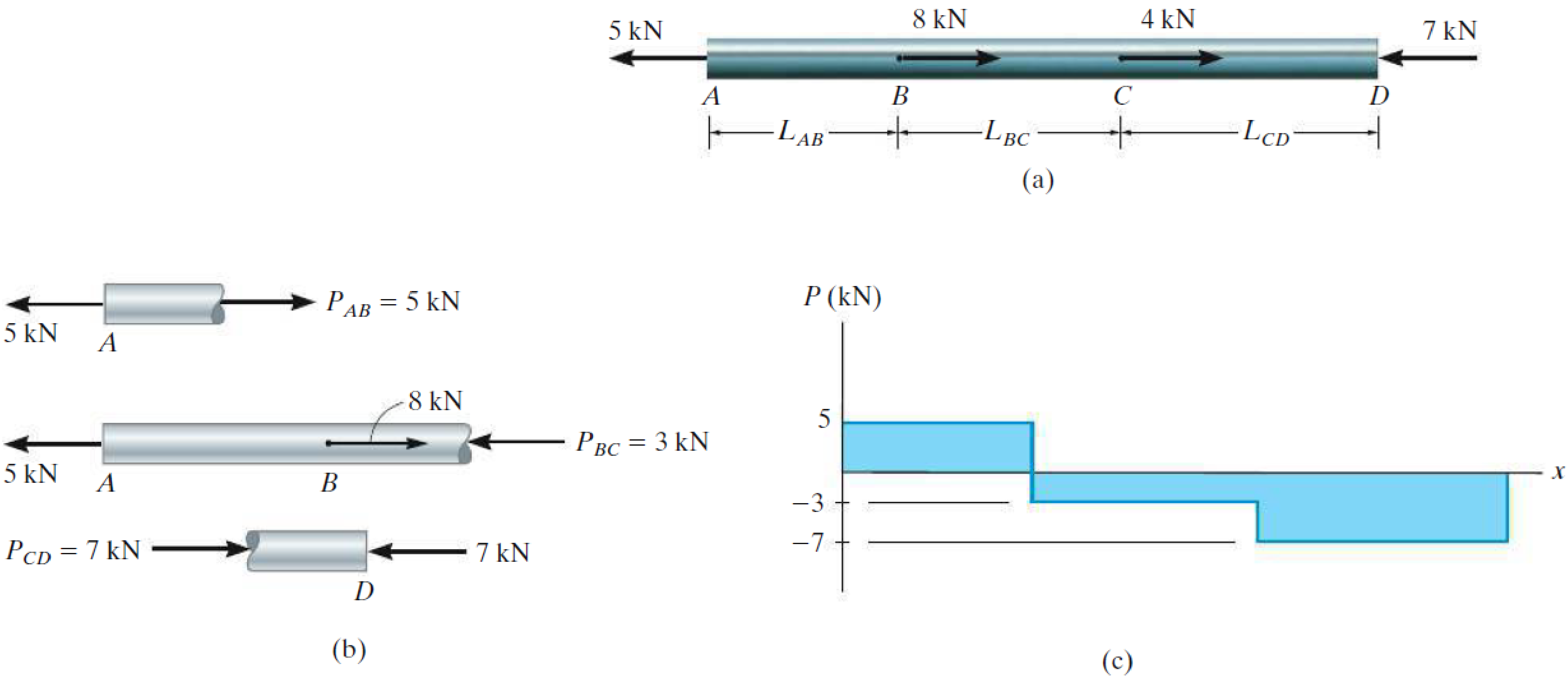


Fig. 4–5

$$\delta = \sum \frac{PL}{AE}$$

$$\delta_{A/D} = \sum \frac{PL}{AE} = \frac{(5\text{ kN})L_{AB}}{AE} + \frac{(-3\text{ kN})L_{BC}}{AE} + \frac{(-7\text{ kN})L_{CD}}{AE}$$

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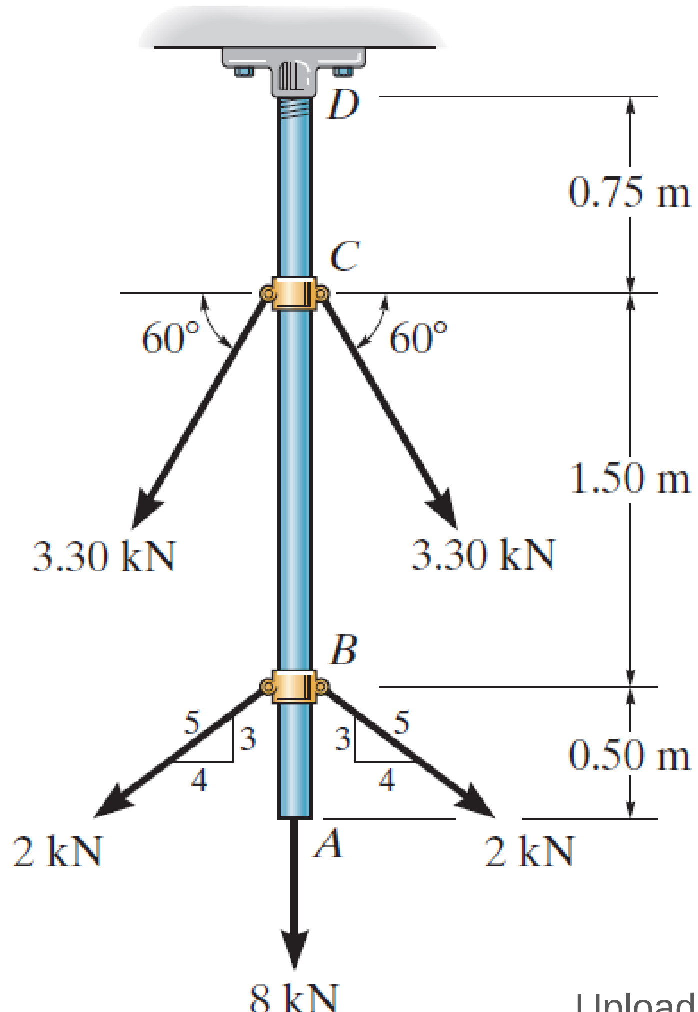
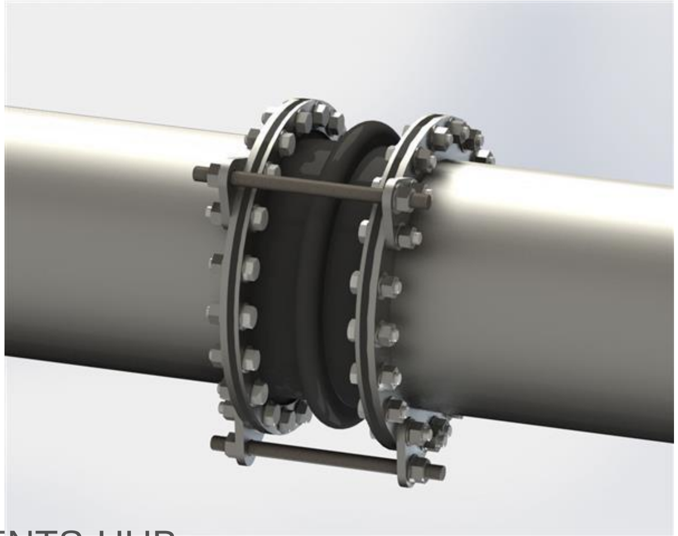
Problem 4-1

4–1. The A992 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm^2, determine the displacement of *B* and *A*, Neglect the size of the couplings at *B*, *C*, and *D*.



4.6 Thermal Stress

4.6 Thermal Stress



EXAMPLE METAL DUCT JOINT
ROUND UNIVERSAL FLANGED

END FITTING CHOICES
(Angle Flanges shown - Plate Flanges also available)

4.6 Thermal Stress

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$$\delta_T = \alpha \, \Delta T L \qquad (4\text{–}4)$$

where

α = a property of the material, referred to as the ***linear coefficient of thermal expansion***. The units measure strain per degree of temperature. They are 1/°F (Fahrenheit) in the FPS system, and 1/°C (Celsius) or 1/K (Kelvin) in the SI system. Typical values are given on the inside back cover

ΔT = the algebraic change in temperature of the member

L = the original length of the member

δ_T = the algebraic change in the length of the member

Problem 4-85

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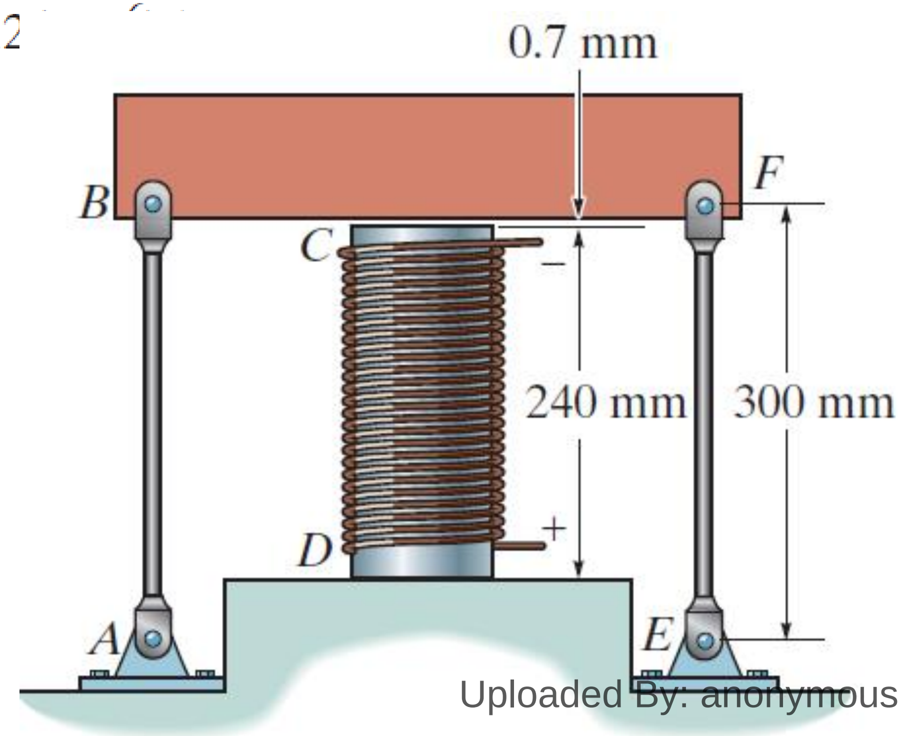
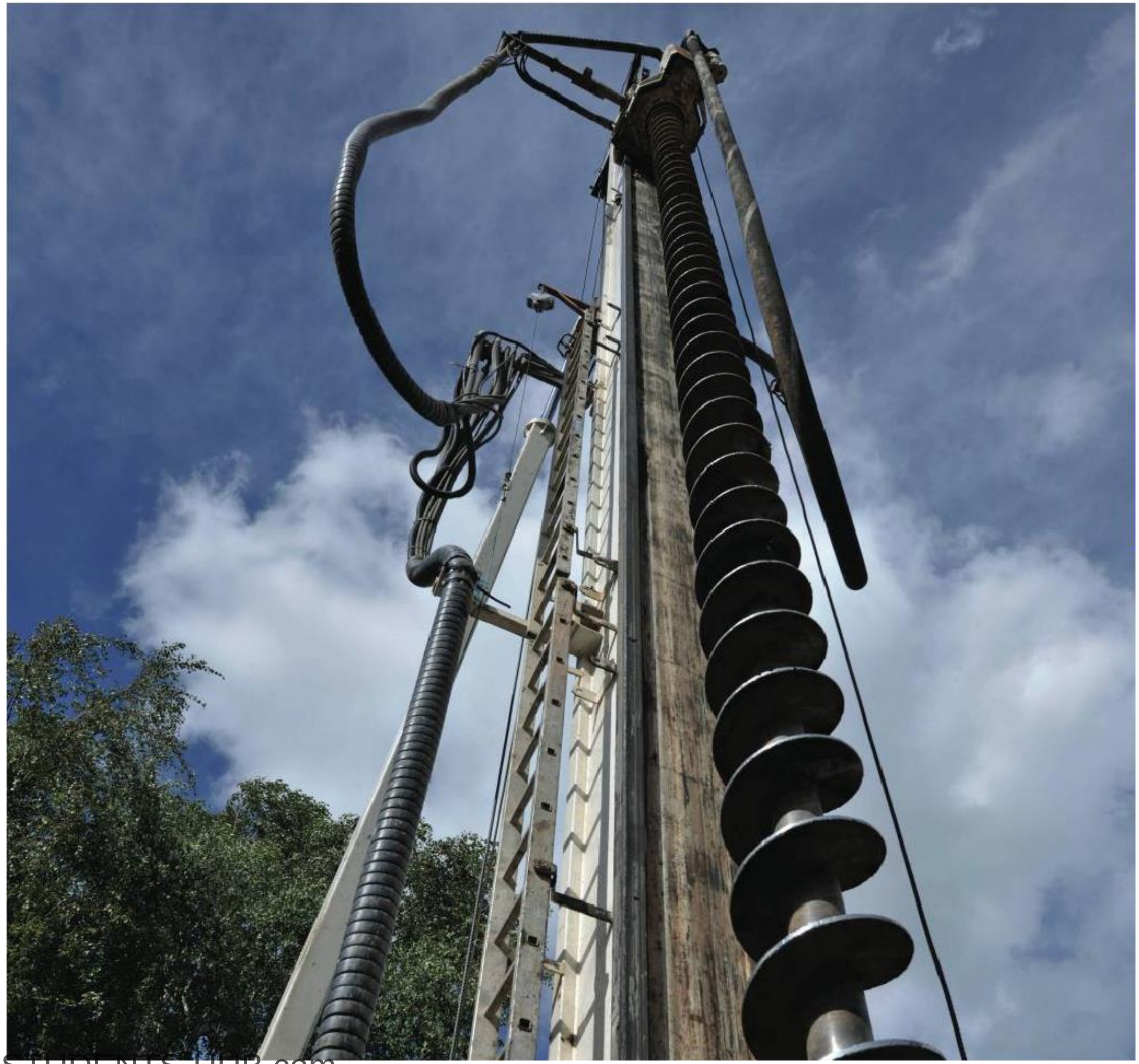
4–85. The center rod *CD* of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance heating. Also, the two end rods *AB* and *EF* are heated from $T_1 = 30°C$ to $T_2 = 50°C$. At the lower temperature T_1 the gap between *C* and the rigid bar is 0.7 mm. Determine the force in rods *AB* and *EF* caused by the increase in temperature. Rods *AB* and *EF* are made of steel, and each has a cross-sectional area of 125 mm². *CD* is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12$ $\alpha_{al} = 23(10^{-6})/°C$.



Torsion

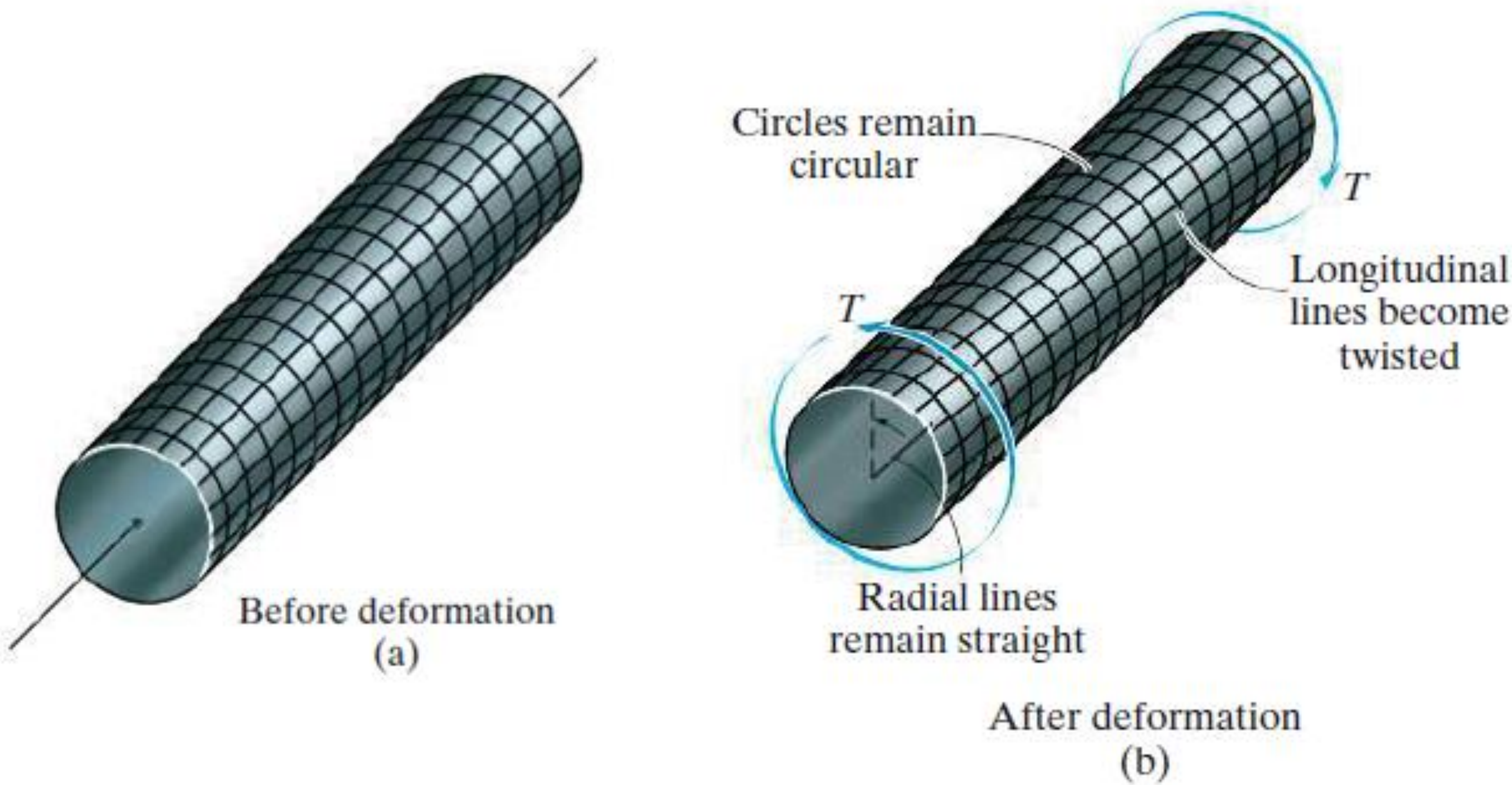


**Torsio
n**

Torsion

https://www.youtube.com/watch?v=85THvfChSNU

5.1 Torsional Deformation of a Circular Shaft



5.1 Torsional Deformation of a Circular Shaft

5.1 Torsional Deformation of a Circular Shaft

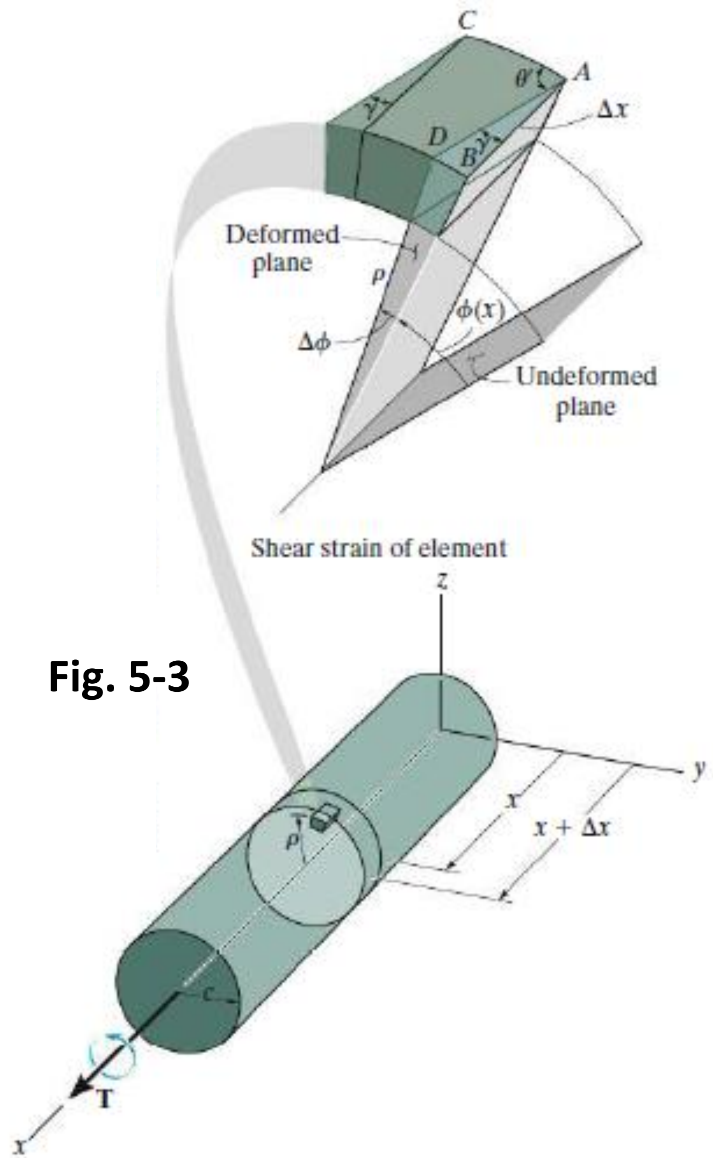


Fig. 5-3

5.2 The Torsional Formula

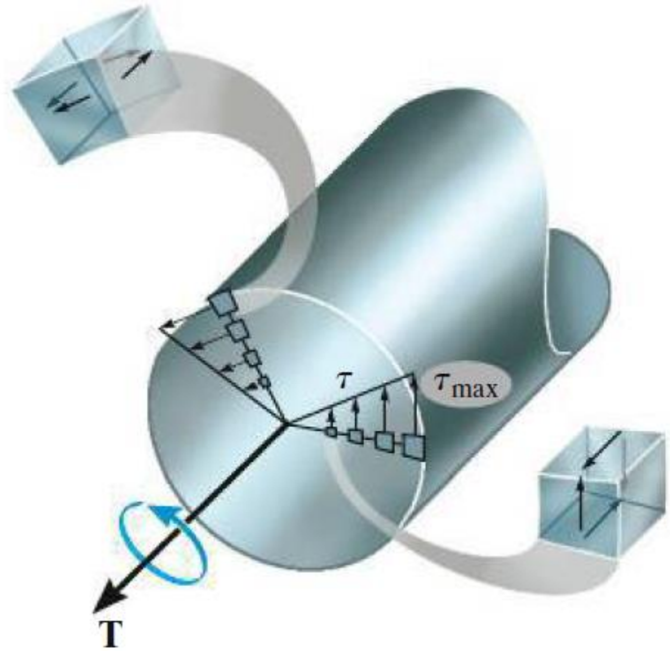


Fig. 5-7 (a)

$$\tau_{max} = \frac{Tc}{J}$$ **Eq. 5-6**

$$\tau = \frac{T\rho}{J}$$ **Eq. 5-7**

$$J = \frac{\pi}{2} c^4$$ **Eq. 5-8**

$$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$ **Eq. 5-9**

τ_{max} = the maximum shear stress in the shaft, which occurs at the outer surface

T = the resultant *internal torque* acting at the cross section. Its value is determined from the method of sections and the equation of moment equilibrium applied about the shaft's longitudinal axis

J = the polar moment of inertia of the cross-sectional area

c = the outer radius of the shaft

5.2 Angle of Twist

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$$\phi = \int_0^L \frac{T(x)\,dx}{J(x)G(x)} \qquad (5\text{–}14)$$

Here

ϕ = the angle of twist of one end of the shaft with respect to the other end, measured in radians

$T(x)$ = the *internal torque* at the arbitrary position x, found from the method of sections and the equation of moment equilibrium applied about the shaft's axis

$J(x)$ = the shaft's polar moment of inertia expressed as a function of x.

$G(x)$ = the shear modulus of elasticity for the material expressed as a function of x.

Constant torque and Cross-Sectional Area

$$\phi = \frac{TL}{JG}$$

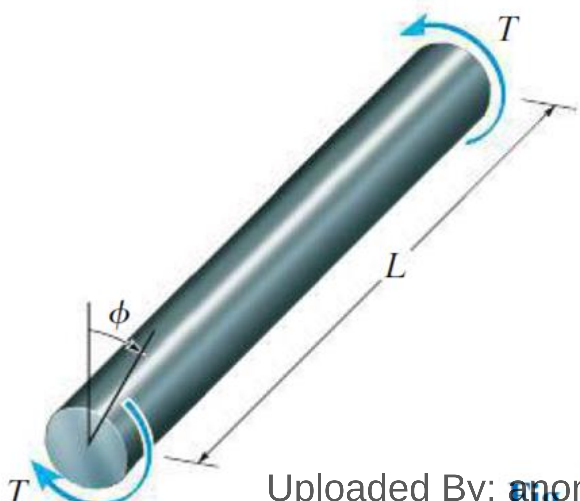
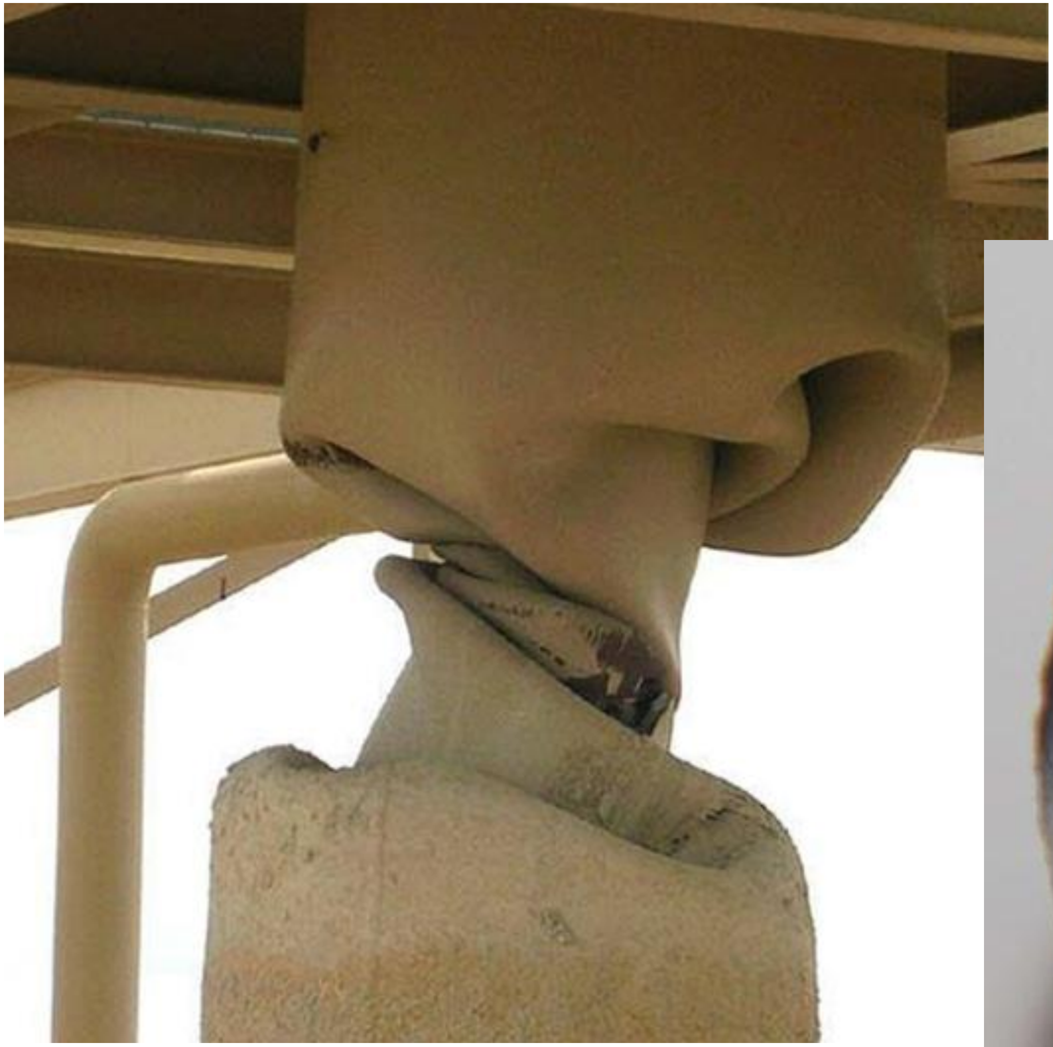


Fig. 5-15

Failure when high Torsion occurs



Problem 5-20

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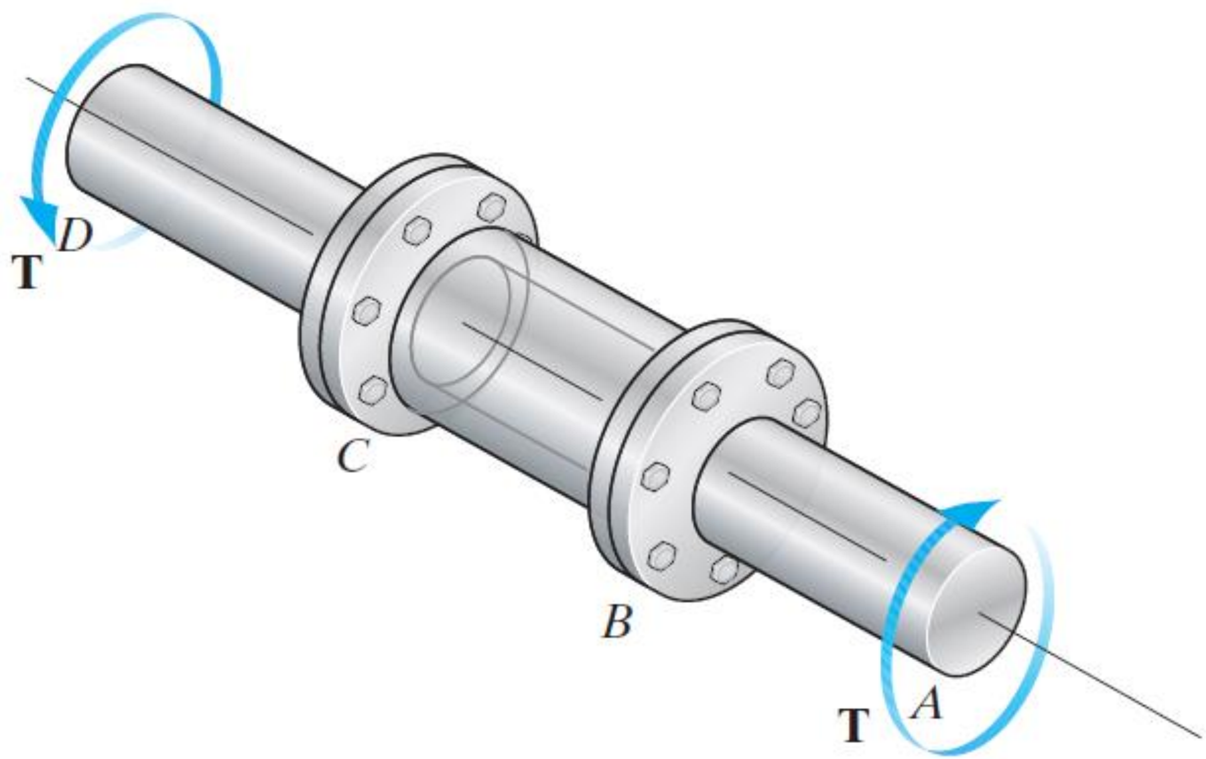
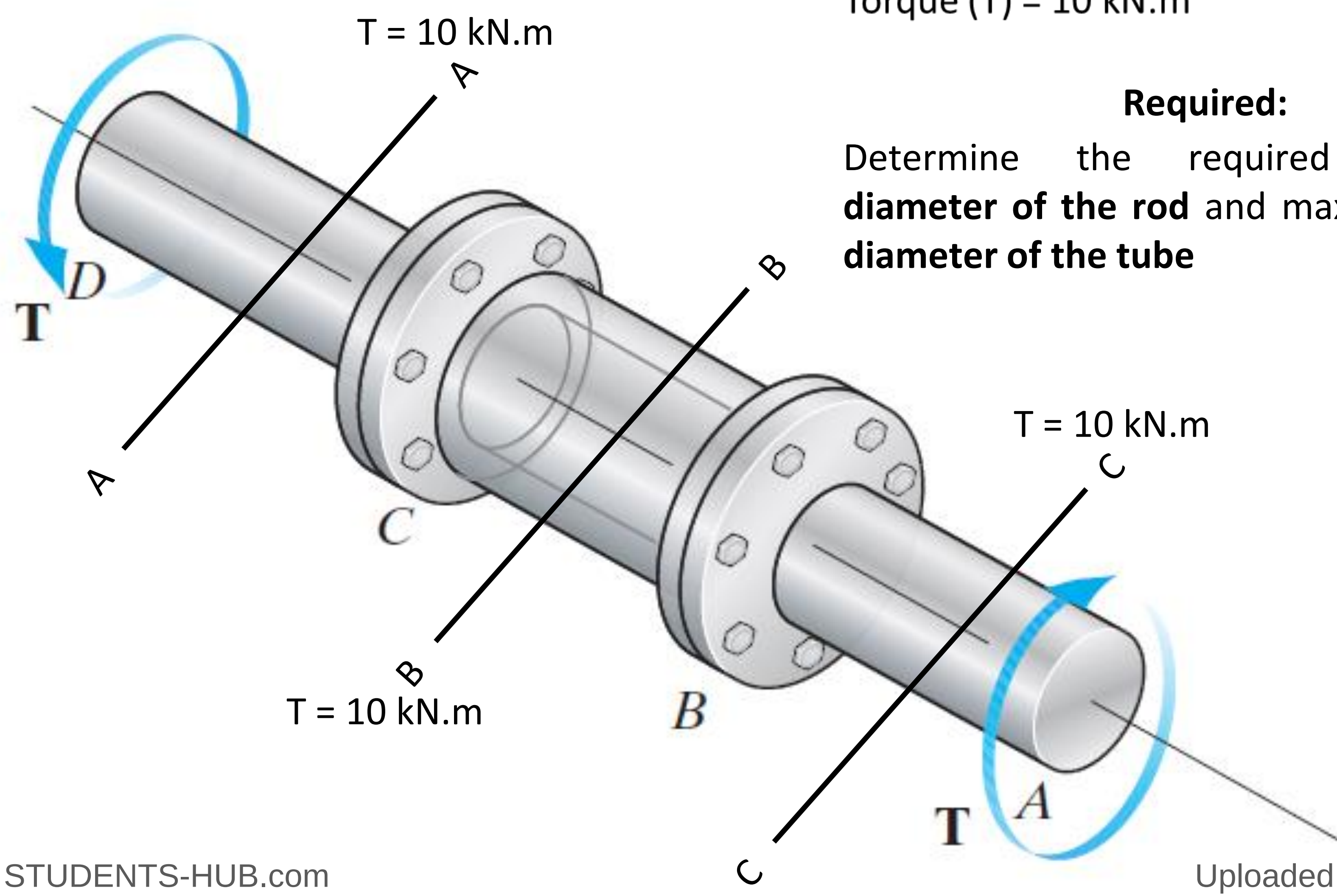
*5–20. The shaft consists of rod segments AB and CD, and the tubular segment BC. If the torque $T = 10\ \text{kN}\cdot\text{m}$ is applied to the shaft, determine the required minimum diameter of the rod and the maximum inner diameter of the tube. The outer diameter of the tube is 120 mm, and the material has an allowable shear stress of $\tau_{allow} = 75$ MPa.



Problem 5-20

BIRZEIT UNIVERSITY



Given:

Allowable shear stress $\tau_{allow} = 75\ MPa$

Outer Diameter of the Tube is 120 mm

Torque (T) = 10 kN.m

Required:

Determine the required **minimum diameter of the rod** and maximum **inner diameter of the tube**

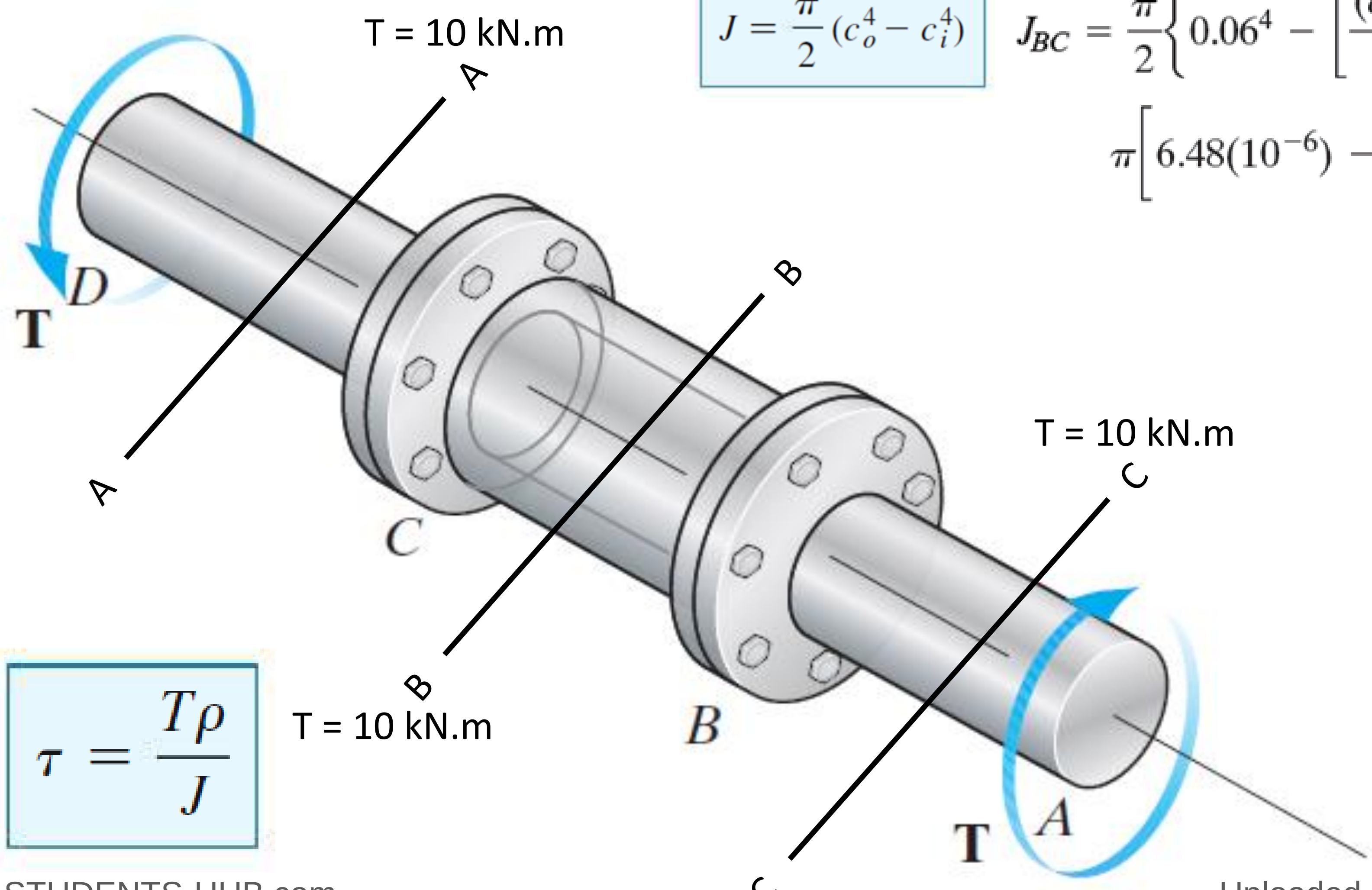
Problem 5-20

$$J = \frac{\pi}{2} c^4$$

$$J_{CD} = \frac{\pi}{2}\left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{32} d_{CD}{}^4$$

$$J = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$J_{BC} = \frac{\pi}{2}\left\{0.06^4 - \left[\frac{(d_{BC})_i}{2}\right]^4\right\} = \pi\left[6.48(10^{-6}) - \frac{(d_{BC})^4{}_i}{32}\right]$$



$$\tau = \frac{T\rho}{J}$$

T = 10 kN.m

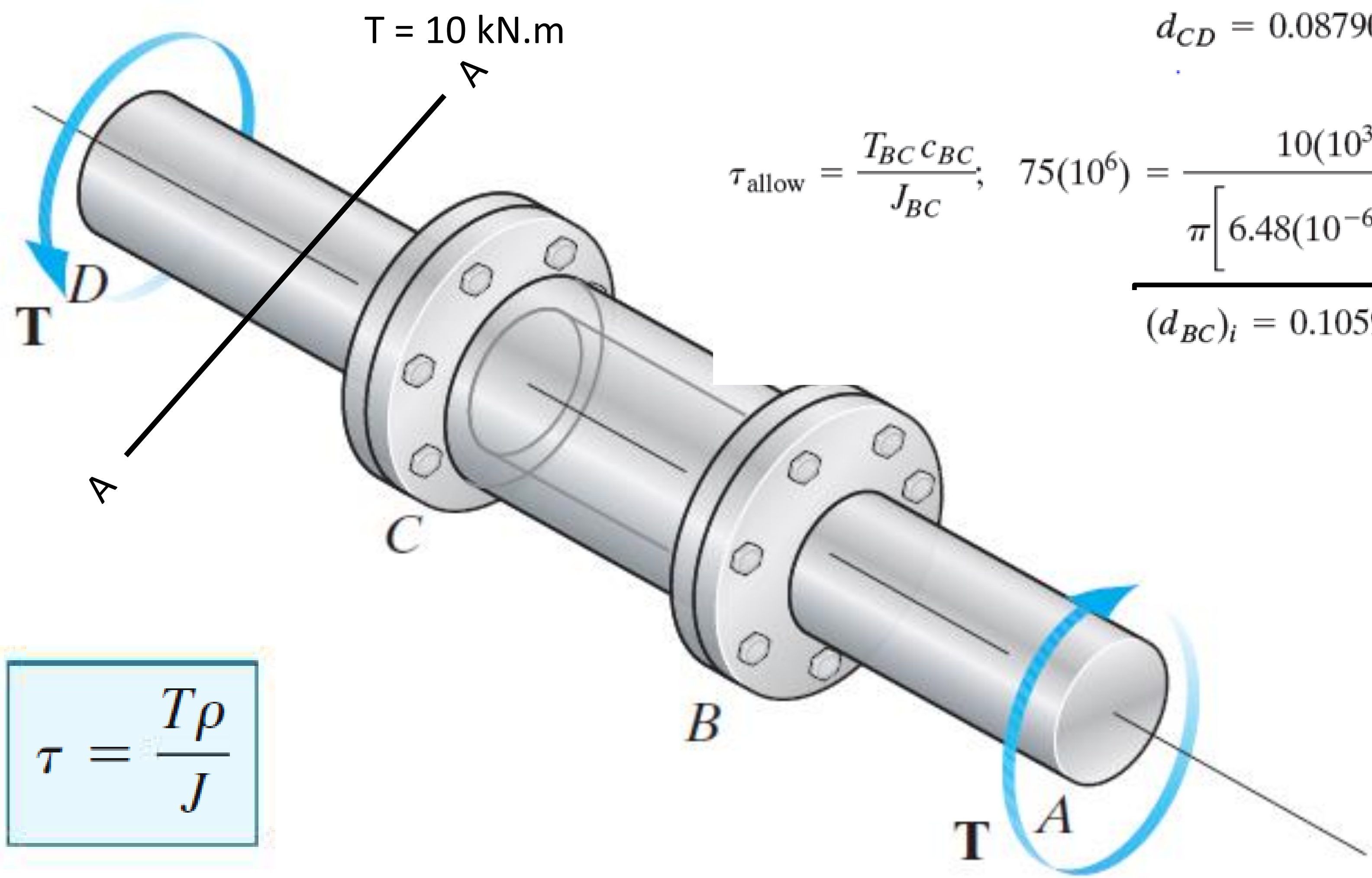
Problem 5-20

$$\tau_{allow} = \frac{T_{CD}c_{CD}}{J_{CD}}; \quad 75(10^6) = \frac{10(10^3)(d_{CD}/2)}{\frac{\pi}{32}d_{CD}{}^4}$$

$$d_{CD} = 0.08790 \text{ m} = 87.9 \text{ mm}$$

$$\tau_{allow} = \frac{T_{BC}c_{BC}}{J_{BC}}; \quad 75(10^6) = \frac{10(10^3)(0.06)}{\pi\left[6.48(10^{-6}) - \frac{(d_{BC})^4{}_i}{32}\right]}$$

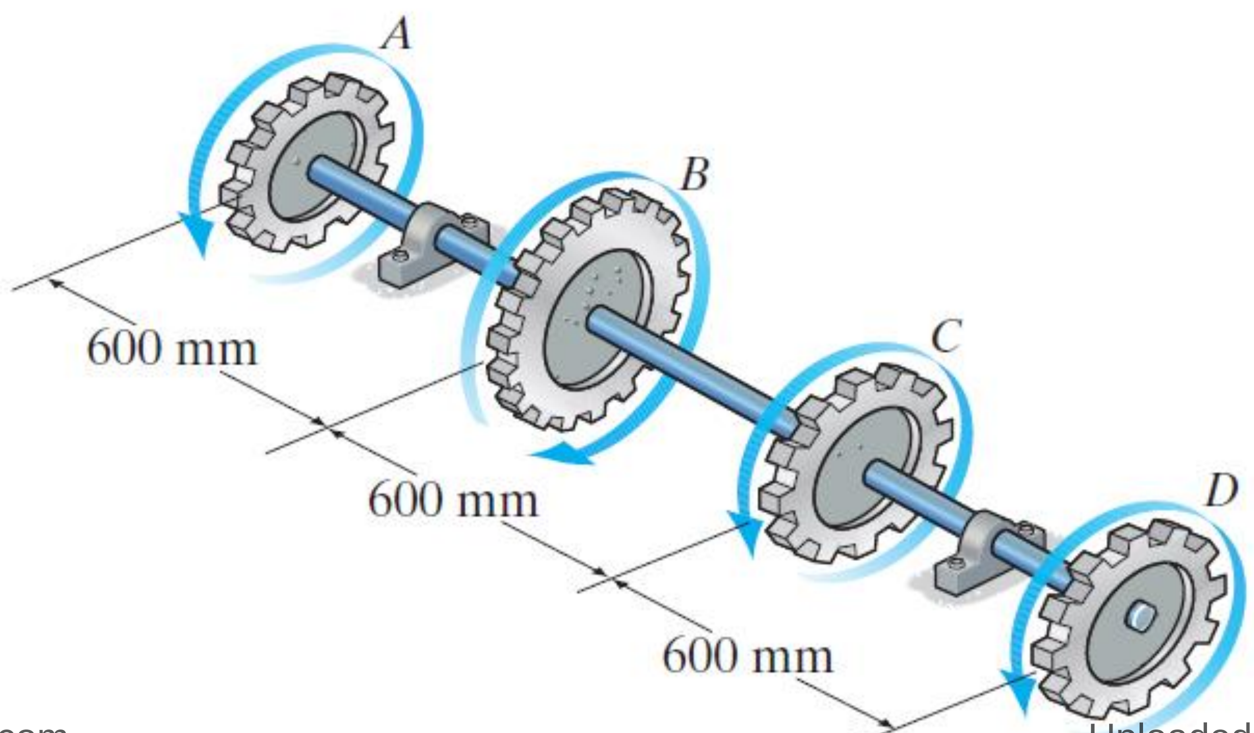
$$(d_{BC})_i = 0.1059 \text{ m} = 106 \text{ mm}$$



$$\tau = \frac{T\rho}{J}$$

Problem 5-54

5–54. The shaft is made of A992 steel with the allowable shear stress of $\tau_{allow} = 75$ MPa. If gear *B* supplies 15 kW of power, while gears *A*, *C*, and *D* withdraw 6 kW, 4 kW, and 5 kW, respectively, determine the required minimum diameter *d* of the shaft to the nearest millimeter. Also, find the corresponding angle of twist of gear *A* relative to gear *D*. The shaft is rotating at 600 rpm.



Bending

Bending

Failure when high Torsion occurs

Beam Sign Convention

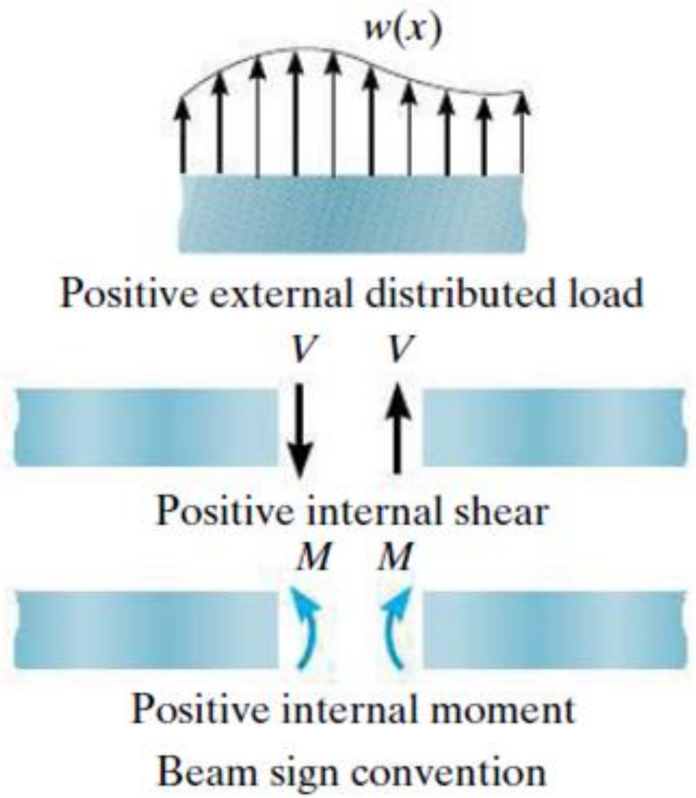
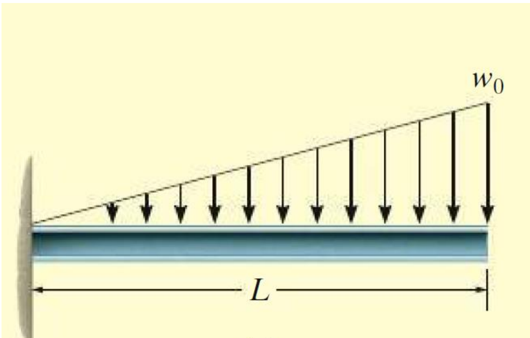


Fig. 6–3

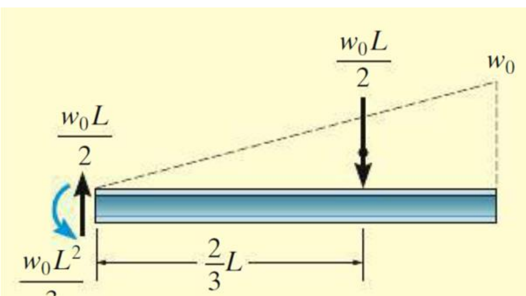
Beam Sign Convention. Before presenting a method for determining the shear and moment as functions of x and later plotting these functions (shear and moment diagrams), it is first necessary to establish a *sign convention* so as to define "positive" and "negative" values for V and M. Although the choice of a sign convention is arbitrary, here we will use the one often used in engineering practice and shown in Fig. 6–3. The *positive directions* are as follows: the *distributed load* acts *upward* on the beam; the internal *shear force* causes a *clockwise* rotation of the beam segment on which it acts; and the internal *moment* causes *compression* in the *top fibers* of the segment such that it bends the segment so that it "holds water". Loadings that are opposite to these are considered negative.

Shear and Moment Diagrams Example 6-8

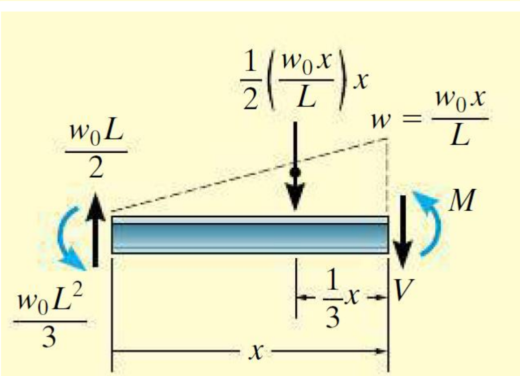
Example: Draw the shear and moment diagrams for the beam shown in figure below



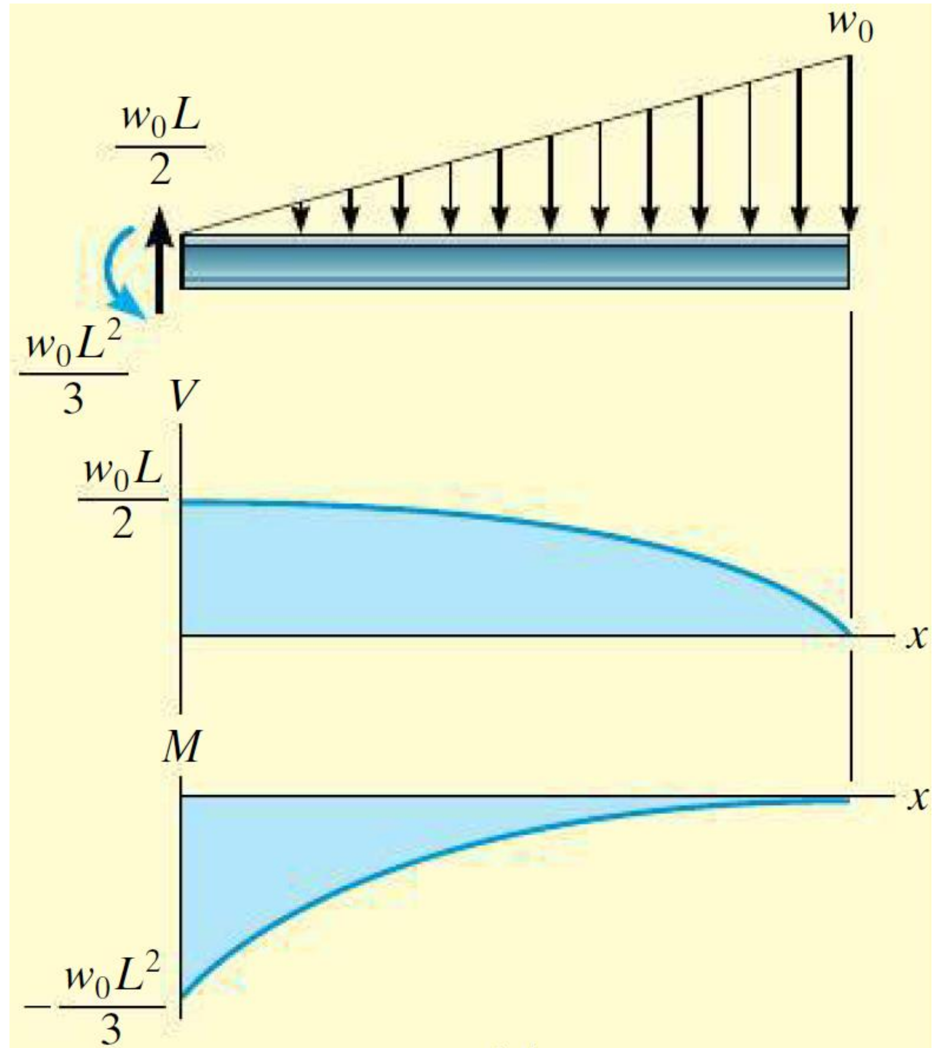
(a)



(b)

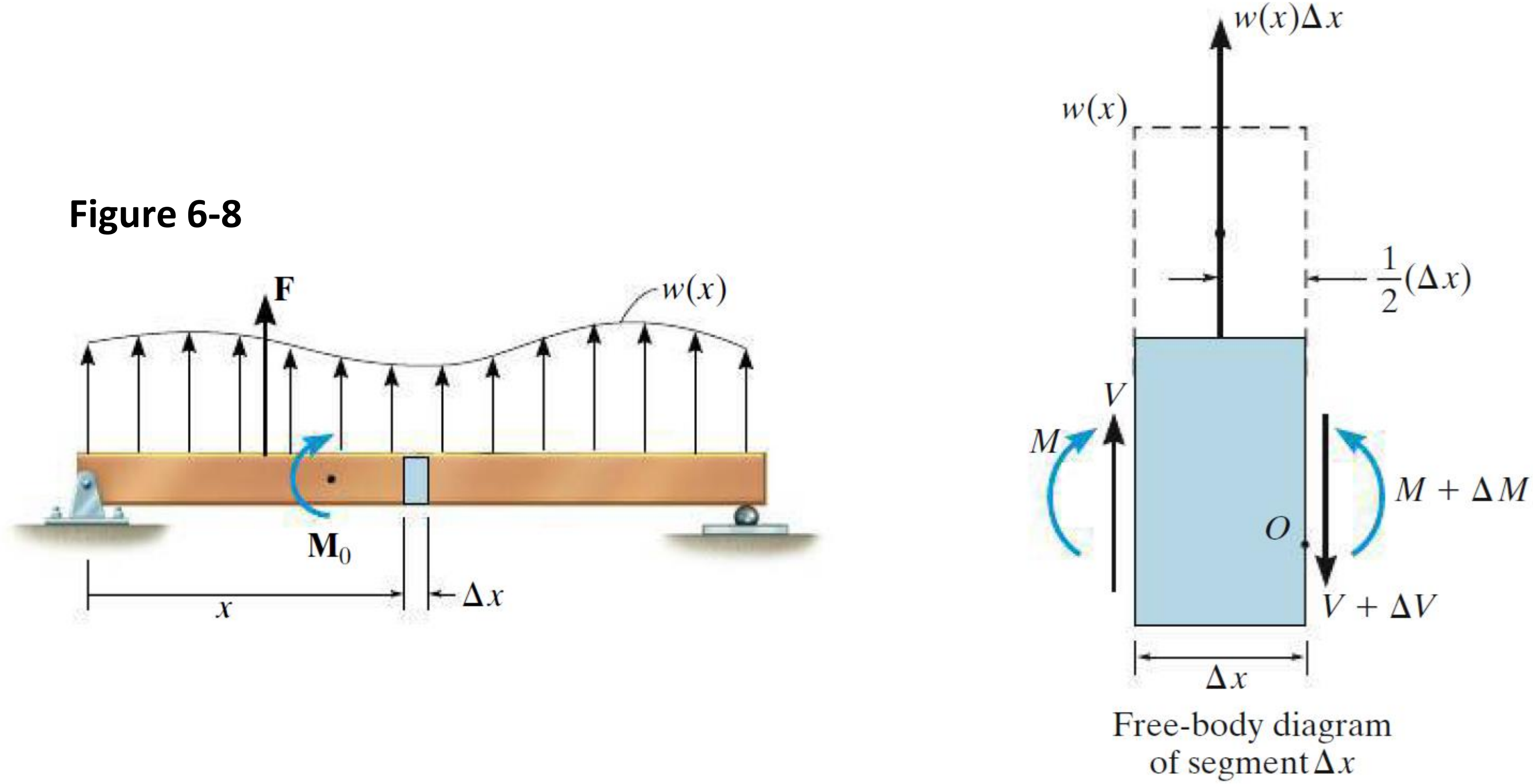


(c)



Graphical method for constructing shear and moment diagrams

Figure 6-8



Graphical method for constructing shear and moment diagrams

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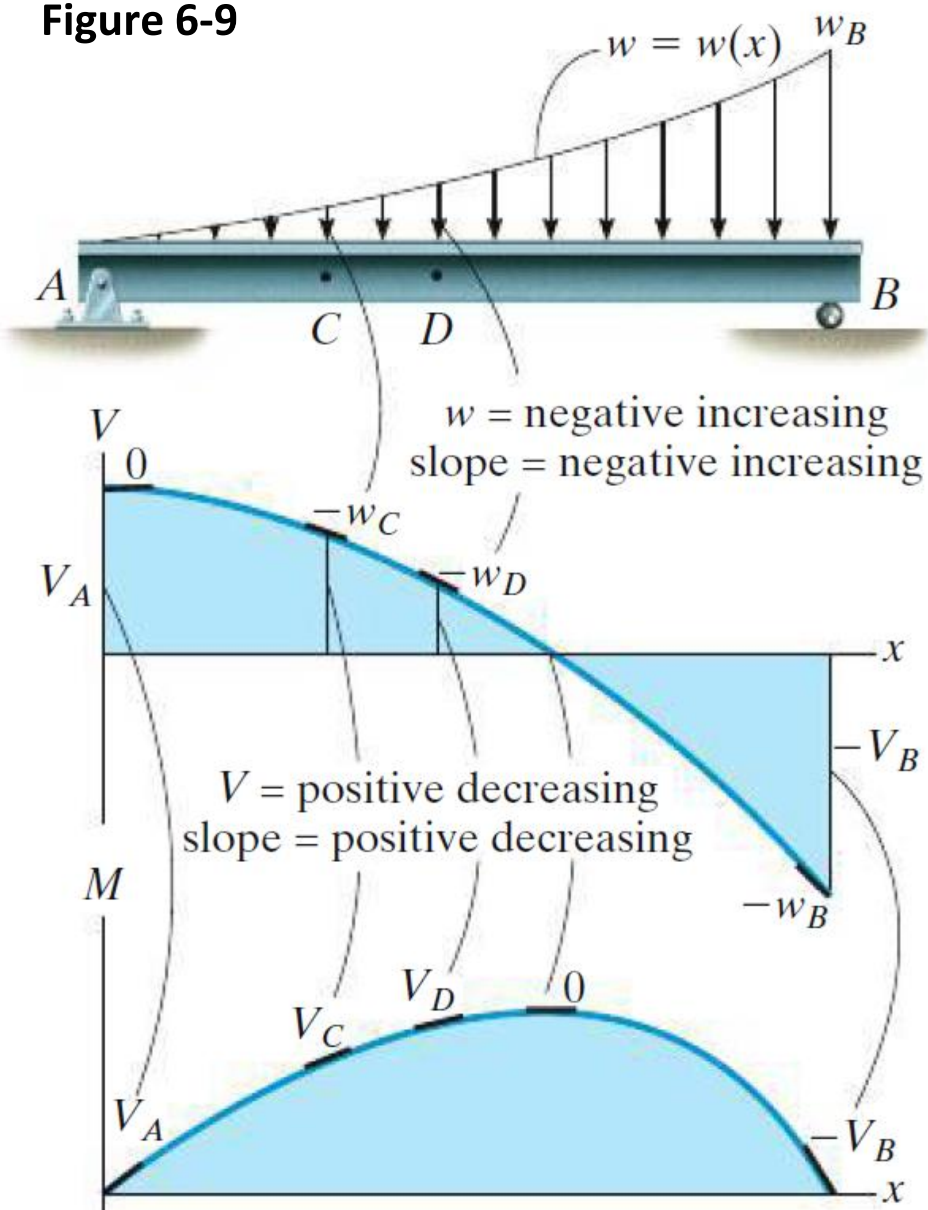
Figure 6-9



$$\frac{dV}{dx} = w(x) \quad (6\text{–}1)$$

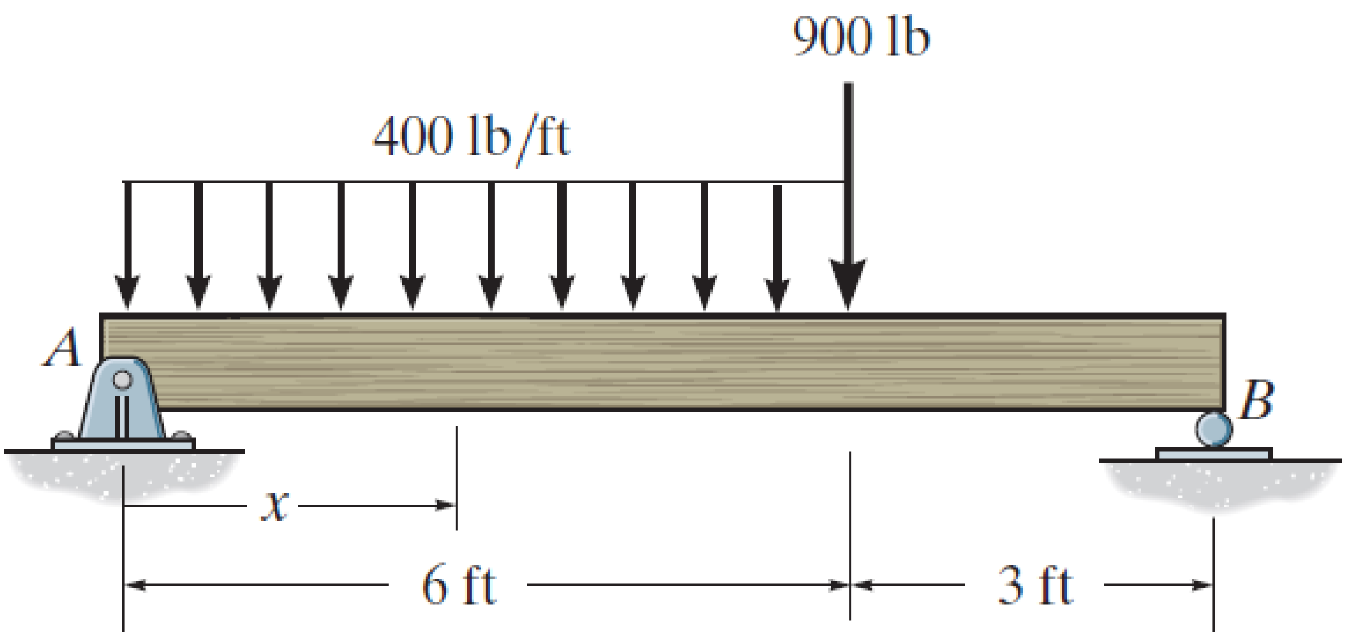
slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V(x) \quad (6\text{–}2)$$

slope of moment diagram at each point = shear at each point

Problem 6-8

***6–8.** Express the internal shear and moment in terms of x and then draw the shear and moment diagrams for the beam.



Problem 6-8

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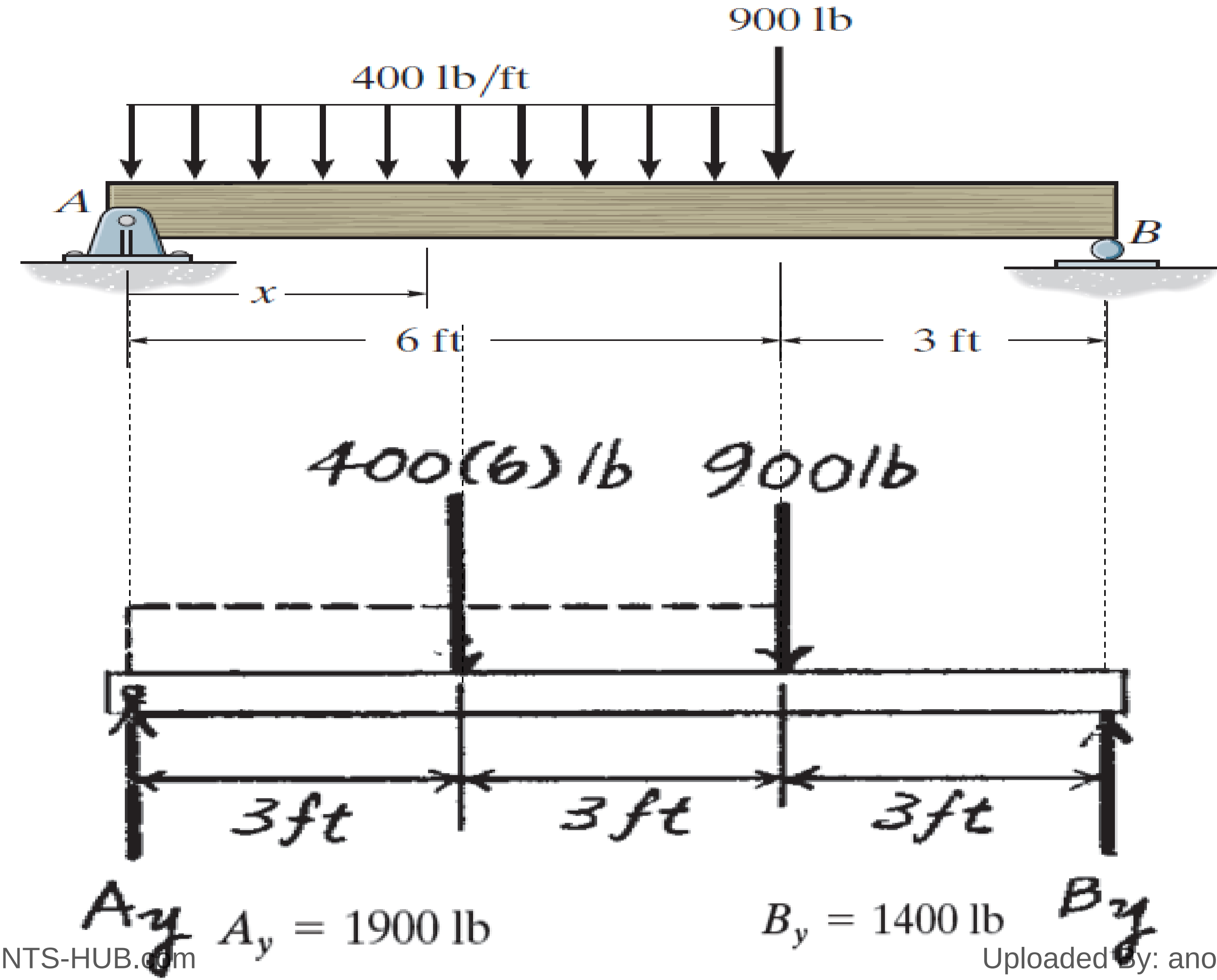
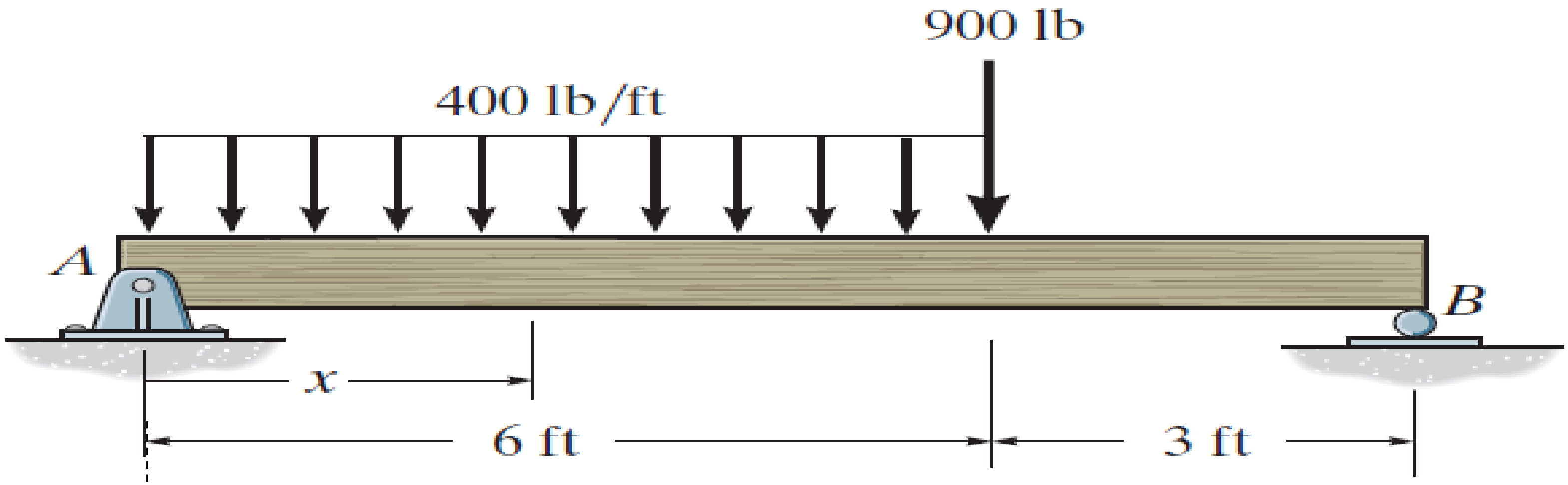
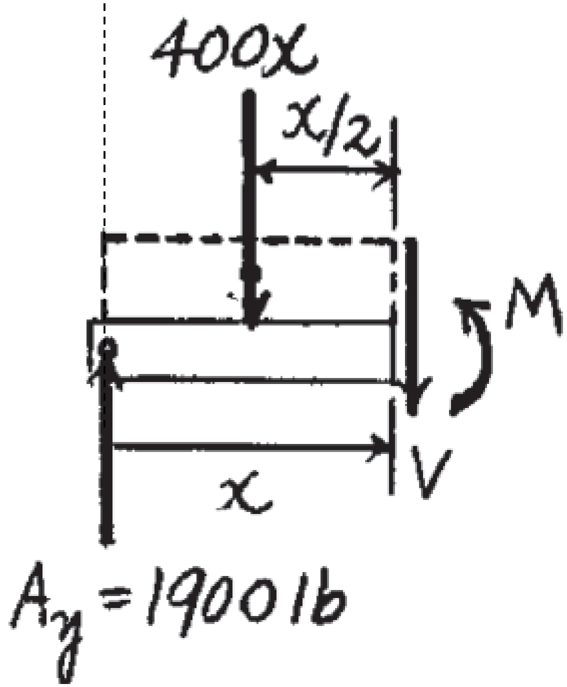


$A_y = 1900$ lb

$B_y = 1400$ lb

Problem 6-8





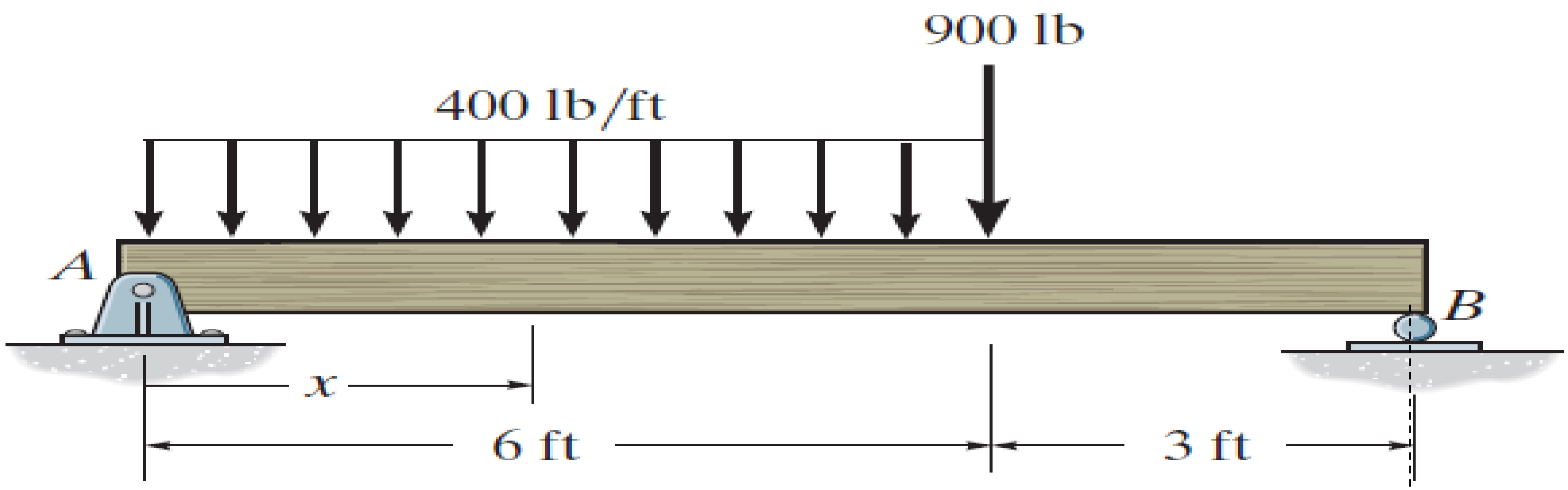
$+\uparrow \Sigma F_y = 0;\qquad 1900 - 400x - V = 0$

$$V = \{1900 - 400x\}\ \text{lb}$$

$\circlearrowleft + \Sigma M = 0;\qquad M + 400x\left(\frac{x}{2}\right) - 1900x = 0$

$$M = \{1900x - 200x^2\}\ \text{lb}\cdot\text{ft}$$

Problem 6-8

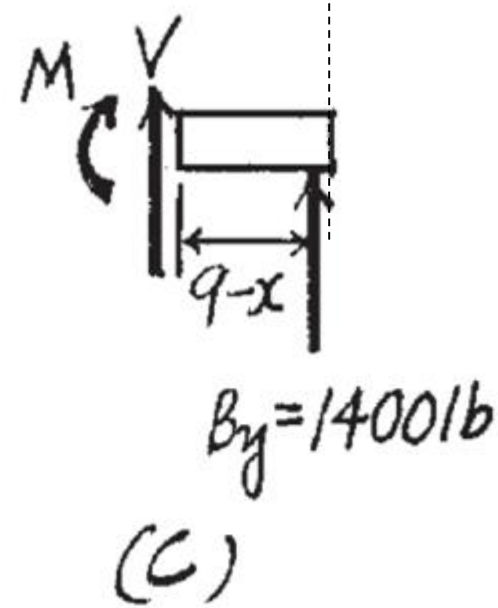


$$+\uparrow \Sigma F_y = 0; \qquad V + 1400 = 0$$

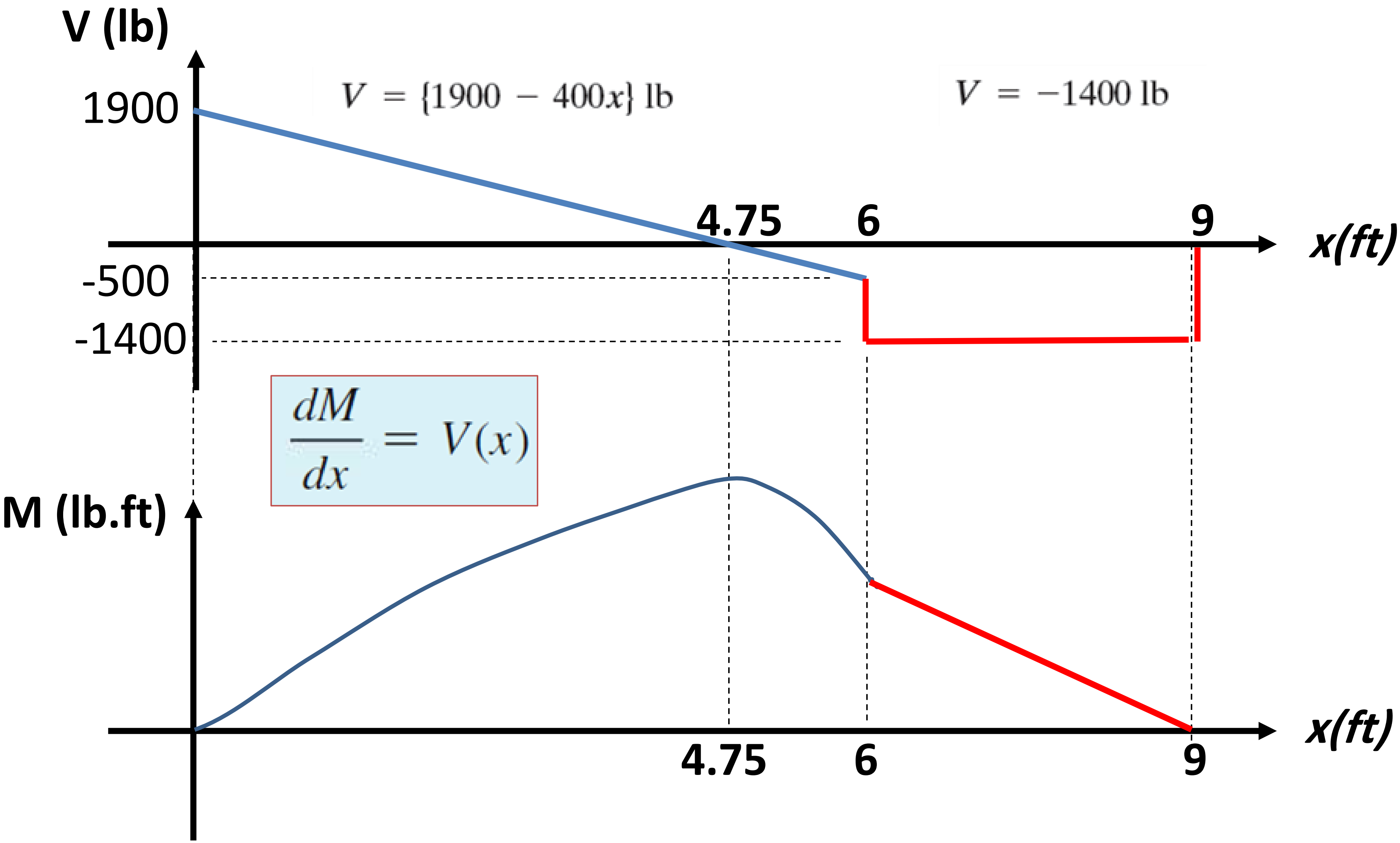
$$V = -1400 \text{ lb}$$

$$\circlearrowleft + \Sigma M = 0; \qquad 1400(9 - x) - M = 0$$

$$M = \{1400(9 - x)\} \text{ lb} \cdot \text{ft}$$



Problem 6-8



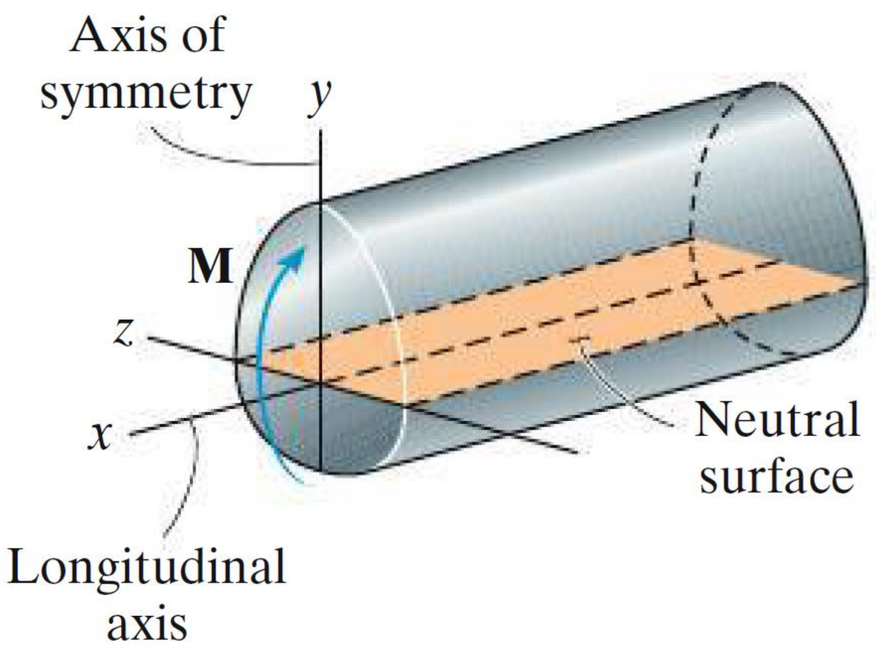
$$\frac{dM}{dx} = V(x)$$

Bending Deformation of a Straigh Member

Main Condition:

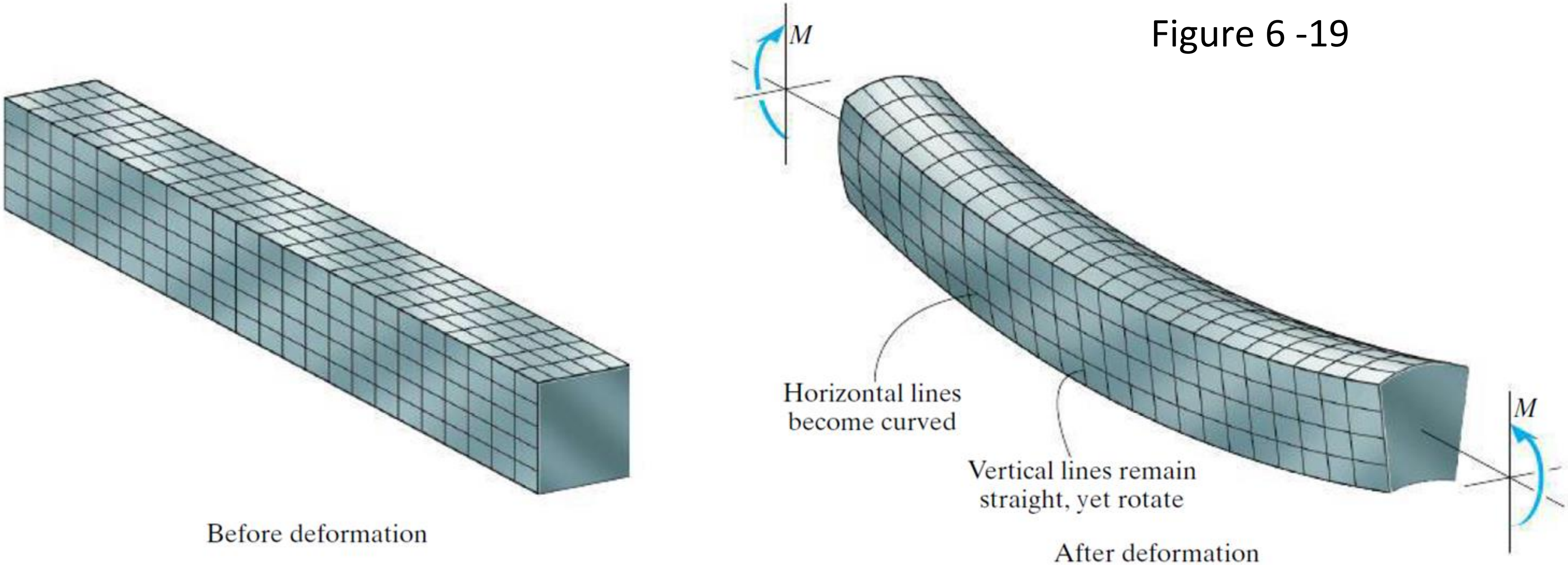
Figure 6-18

- Straight Prismatic beam, homogeneous
- Symmetrical cross-sectional area with respect to axis of symmetry
- Bending moment is applied about an axis perpendicular to the axis of symmetry



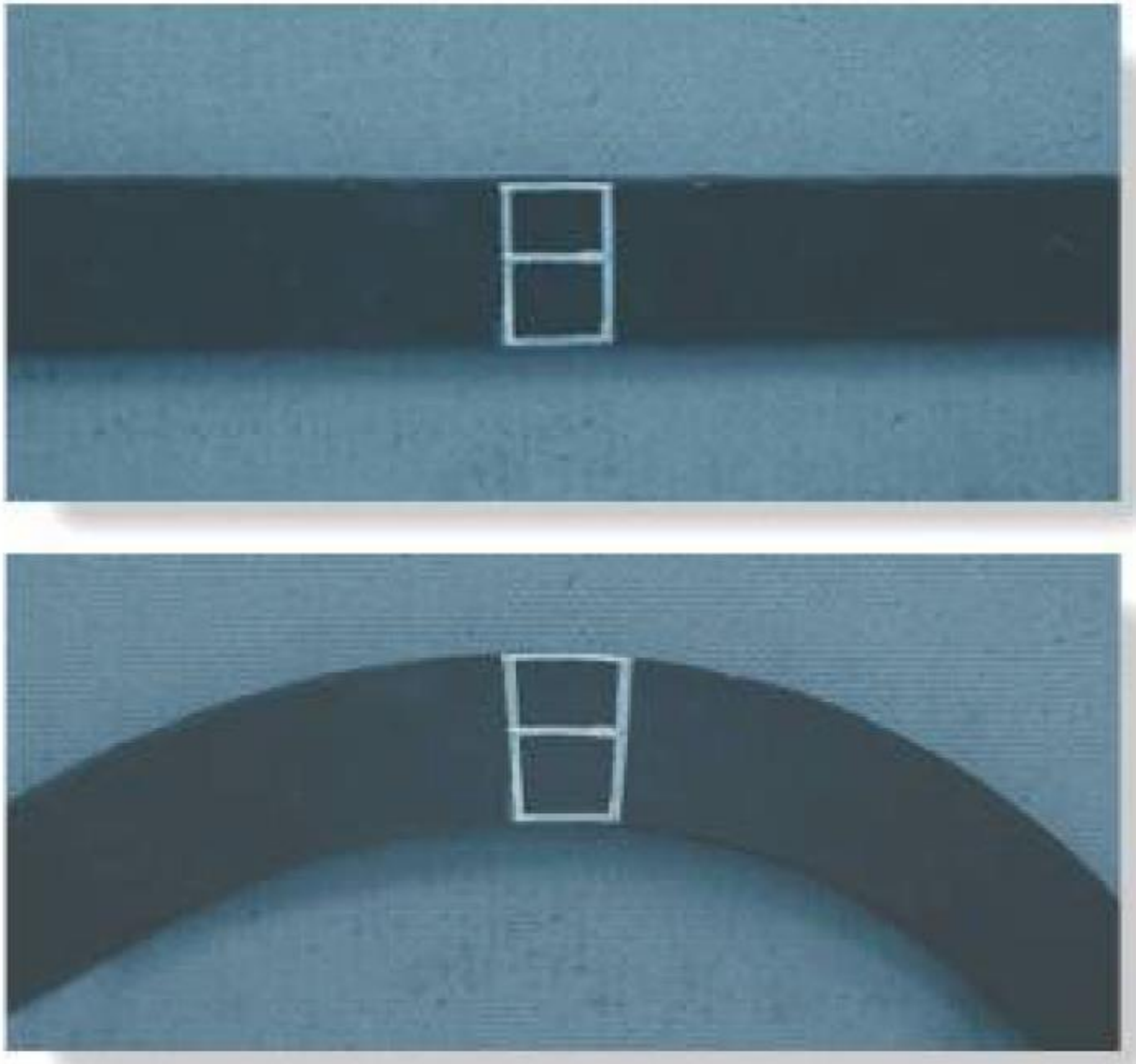
Bending Deformation of a Straigh Member

Figure 6 -19

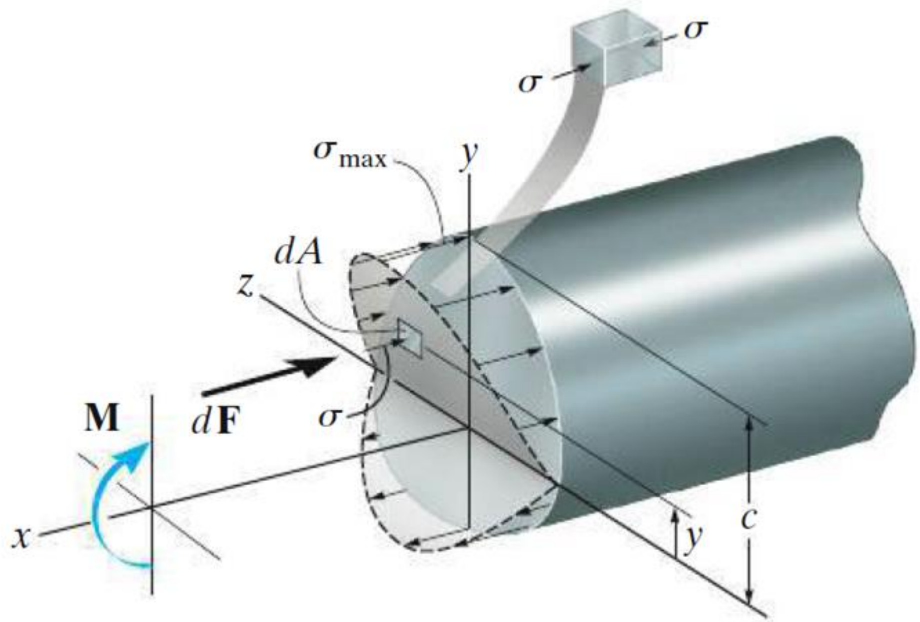


- The bending moment causes the material within the bottom portion of the bar to stretch and the material within the top portion to compress
- The longitudinal fibers in the neutral surface will not undergo a change in length

Bending Deformation of a Straigh Member

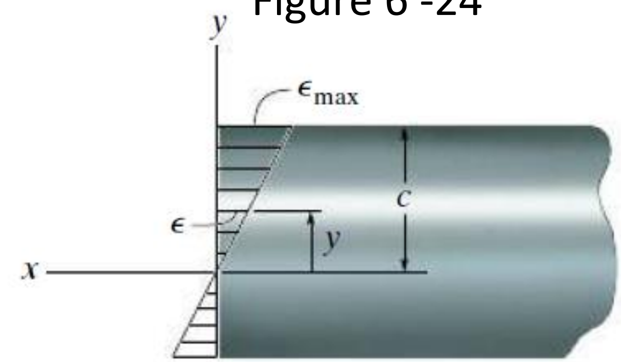


The Flexure Formula



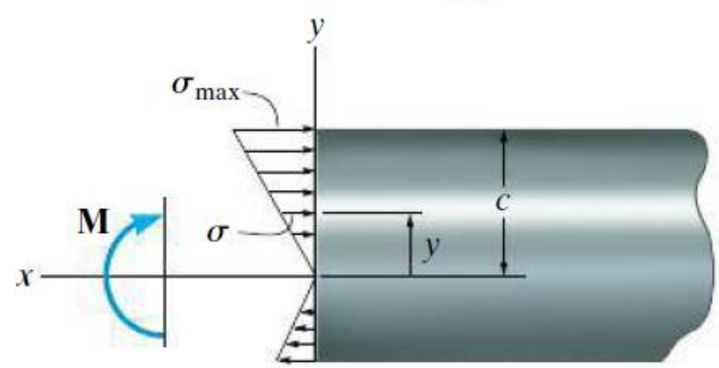
$$\sigma_{\max} = \frac{Mc}{I}$$

Figure 6 -24



Normal strain variation
(profile view)

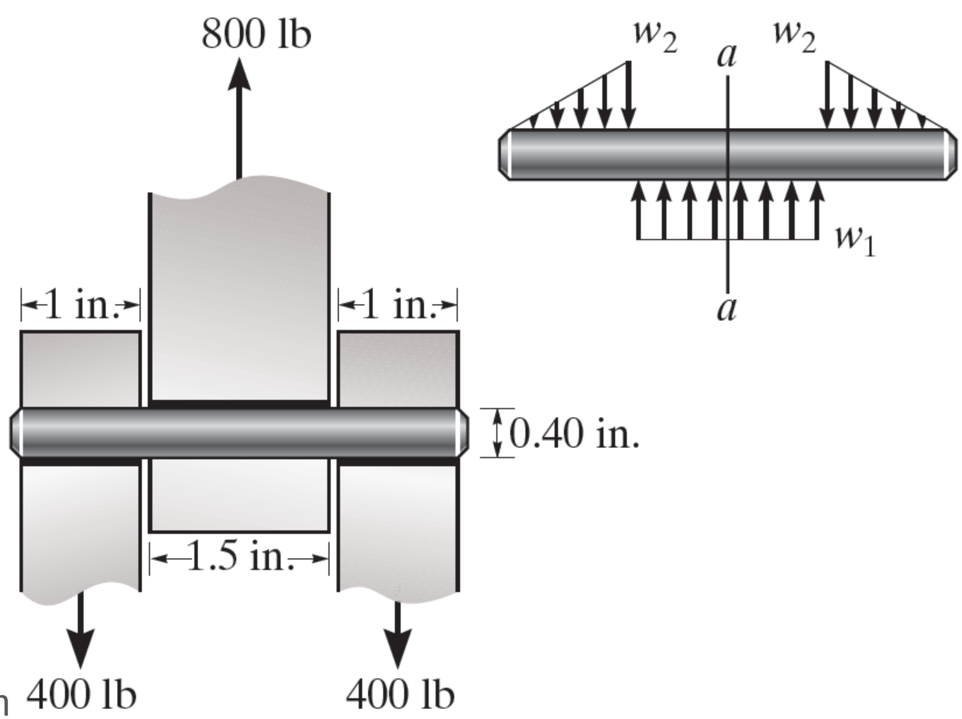
(a)



Bending stress variation
(profile view)

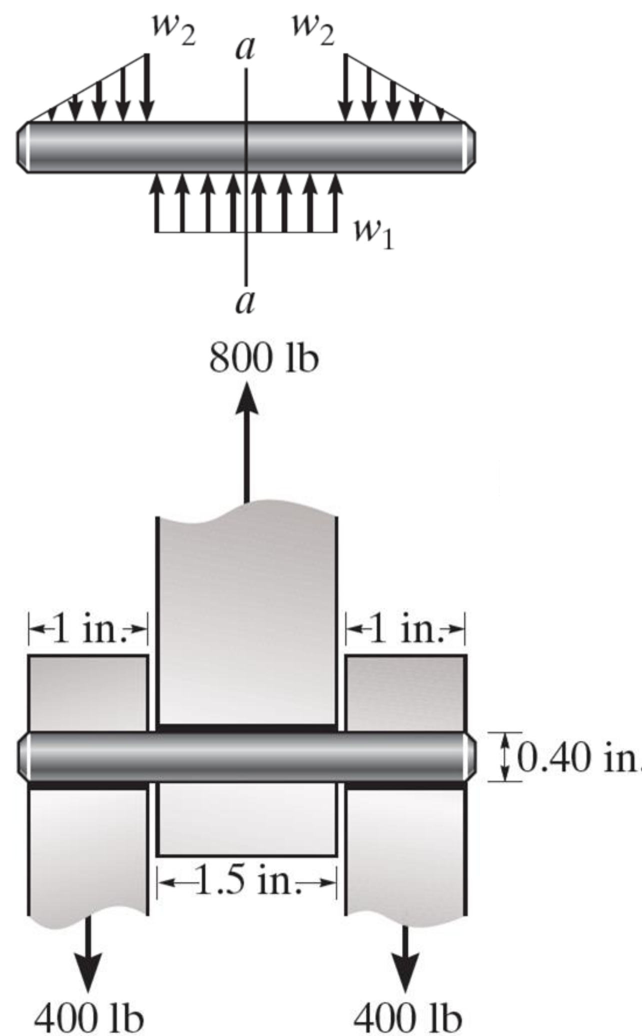
Problem 6-74

6–74. The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 0.40 in., determine the maximum bending stress on the cross-sectional area at the center section *a–a*. For the solution it is first necessary to determine the load intensities w_1 and w_2.



Problem 6-74

Step 1:



$$\frac{1}{2}w_2(1) = 400; \qquad w_2 = 800 \text{ lb/in.}$$

$$w_1(1.5) = 800; \qquad w_1 = 533 \text{ lb/in.}$$

Step 2:

$$\sigma_{\max} = \frac{Mc}{I}$$

$$I = \frac{1}{4}\pi(0.2^4) = 0.0012566 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{283.33\,(0.2)}{0.0012566}$$
$$= 45.1 \text{ ksi}$$

Transverse Shear

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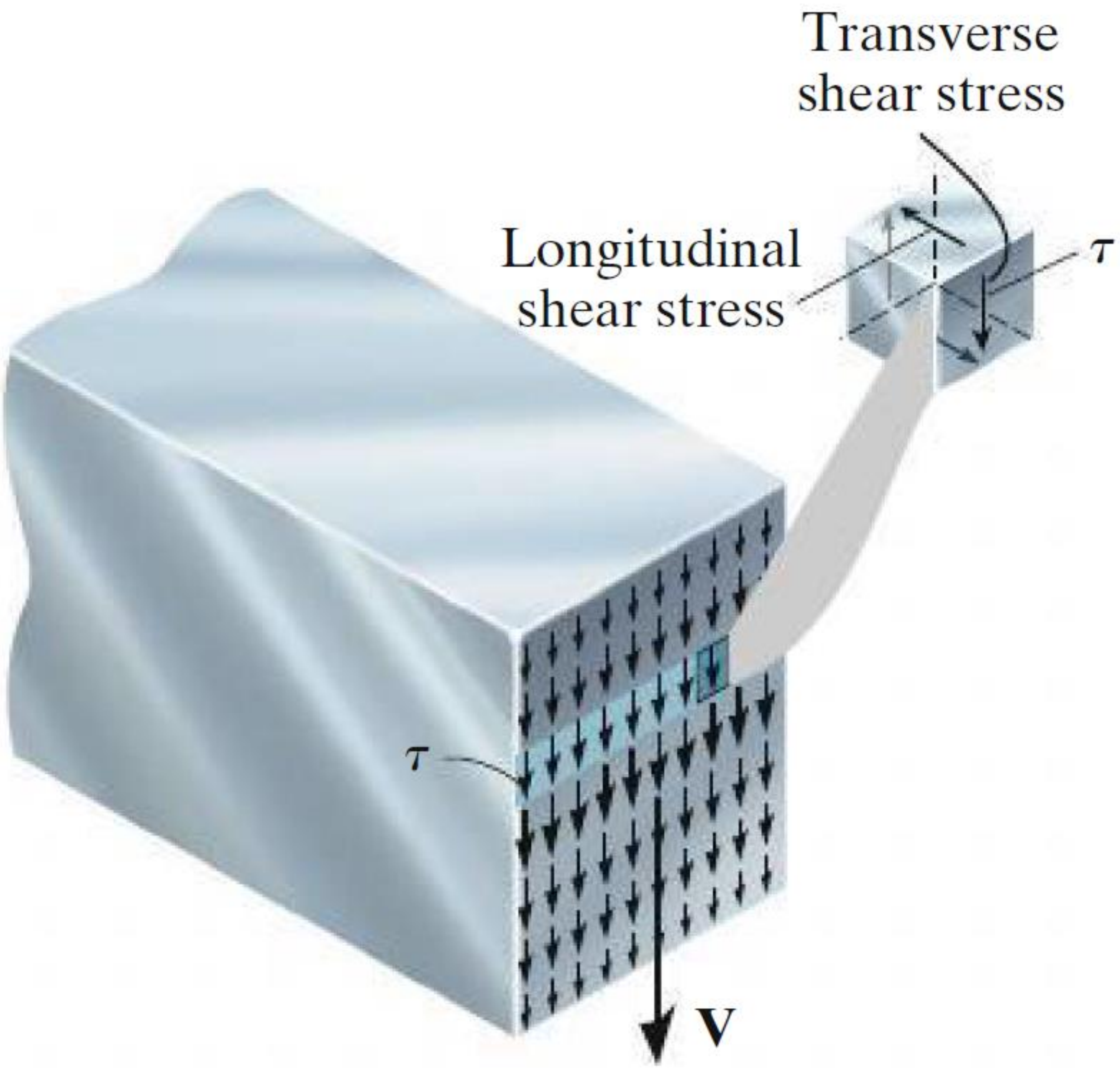
Transverse Shear

Shear in Straigh Members

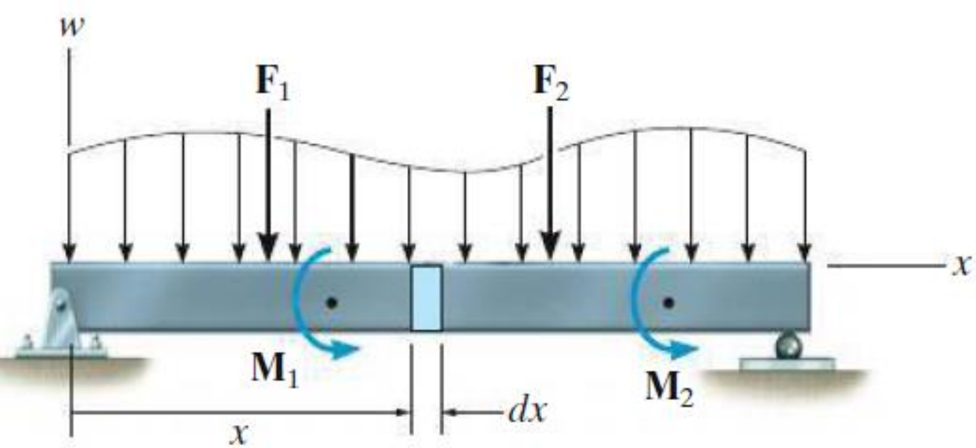


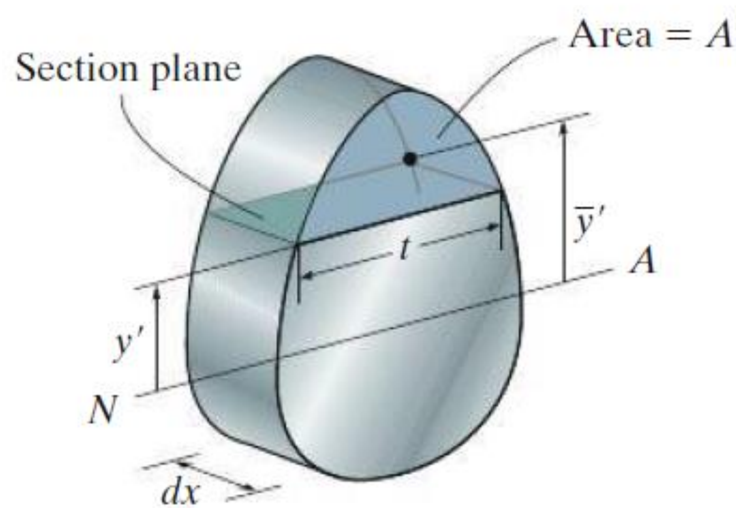
The Shear Formula

$$\tau = \frac{VQ}{It}$$

Main Condition:

- Straight Prismatic beam, homogeneous
- Internal resultant shear force must be directed along an axis of symmetry for the cross section area





The Shear Formula

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$$\tau = \frac{VQ}{It}$$

Main Condition:

- Straight Prismatic beam, homogeneous
- Internal resultant shear force must be directed along an axis of symmetry for the cross section area

τ = the shear stress in the member at the point located a distance y' from the neutral axis. This stress is assumed to be constant and therefore *averaged* across the width t of the member

V = the internal resultant shear force, determined from the method of sections and the equations of equilibrium

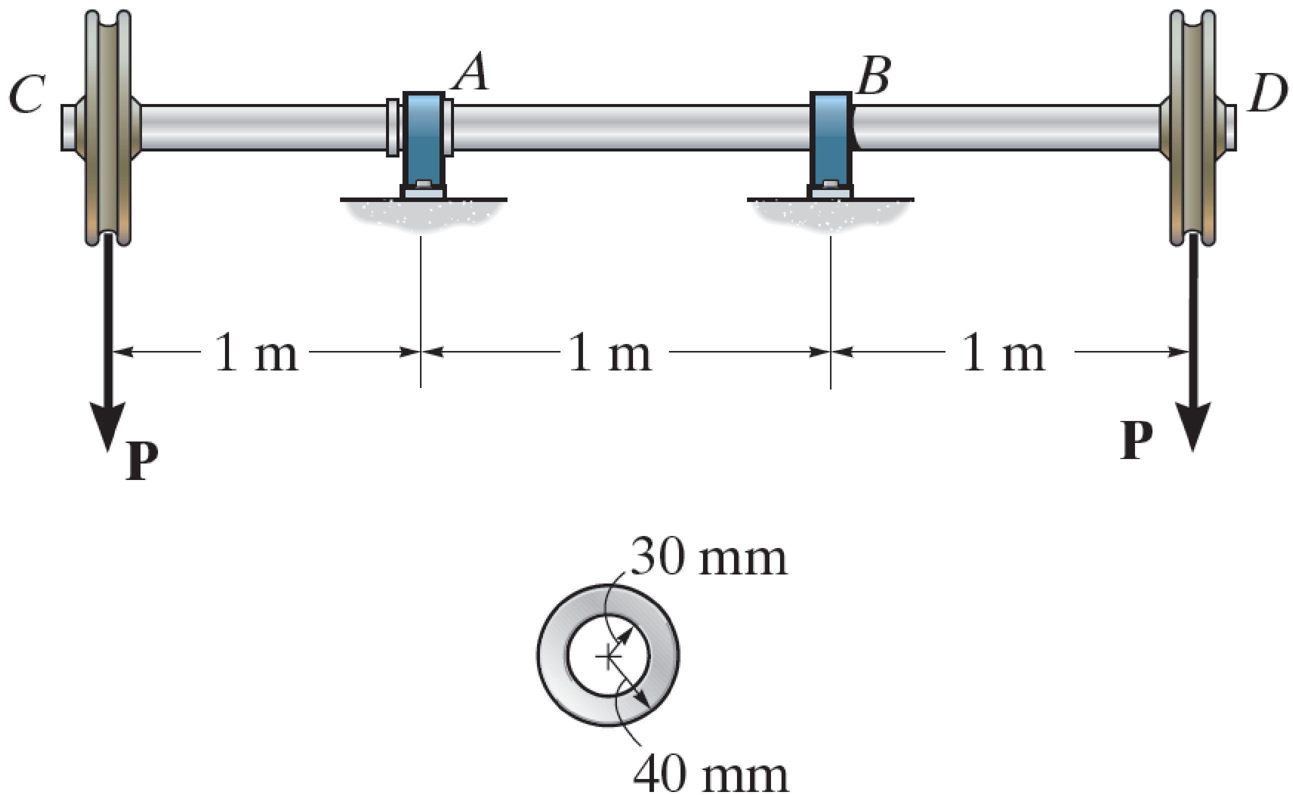
I = the moment of inertia of the *entire* cross-sectional area calculated about the neutral axis

t = the width of the member's cross-sectional area, measured at the point where τ is to be determined

$Q = \overline{y}'A'$, where A' is the area of the top (or bottom) portion of the member's cross-sectional area, above (or below) the section plane where t is measured, and $\overline{y}'$ is the distance from the neutral axis to the centroid of A'

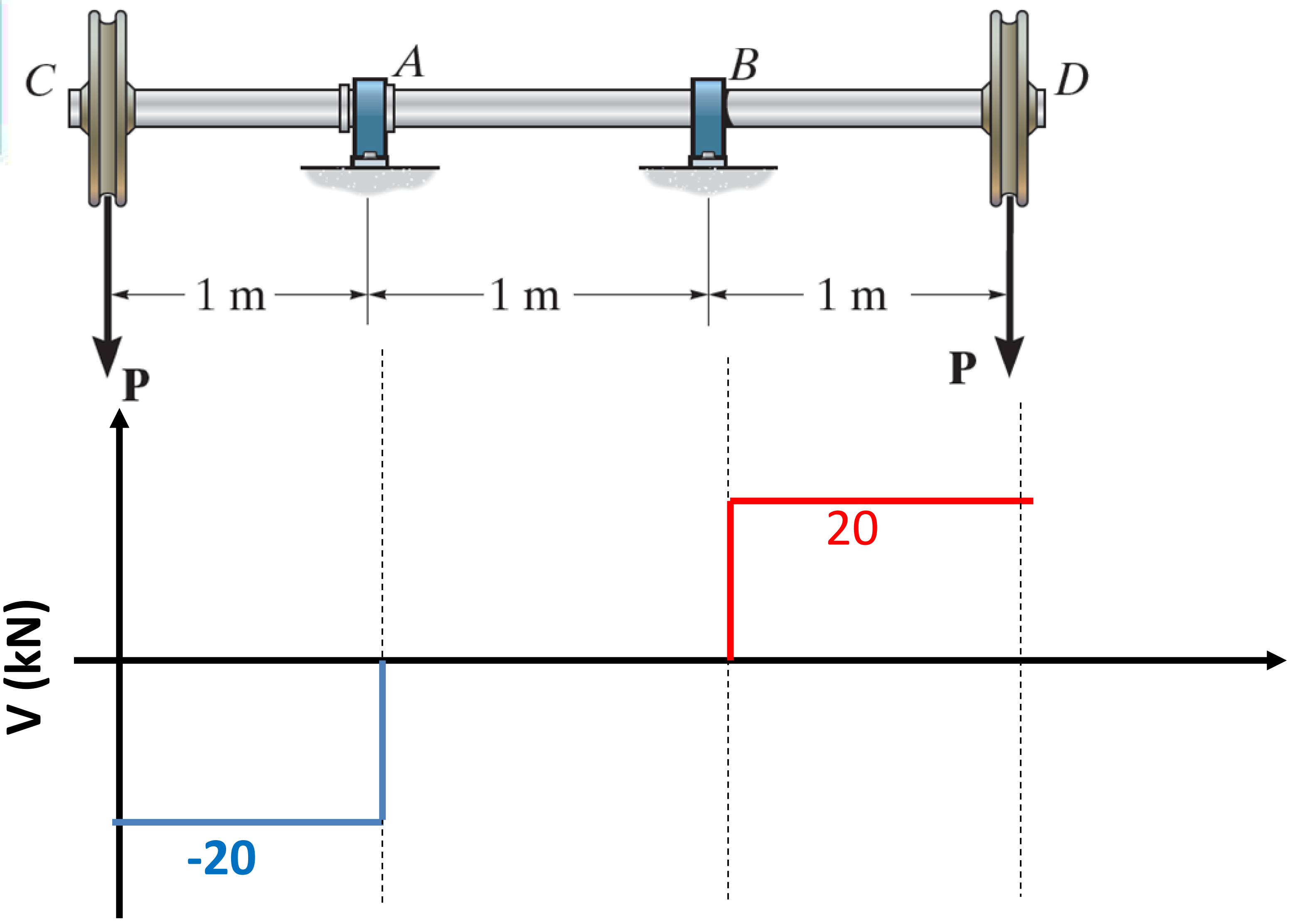
Problem 7-7

7–7. The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. If $P = 20$ kN, determine the absolute maximum shear stress in the shaft.



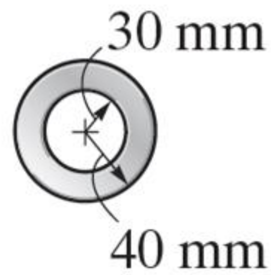
Problem 7-7

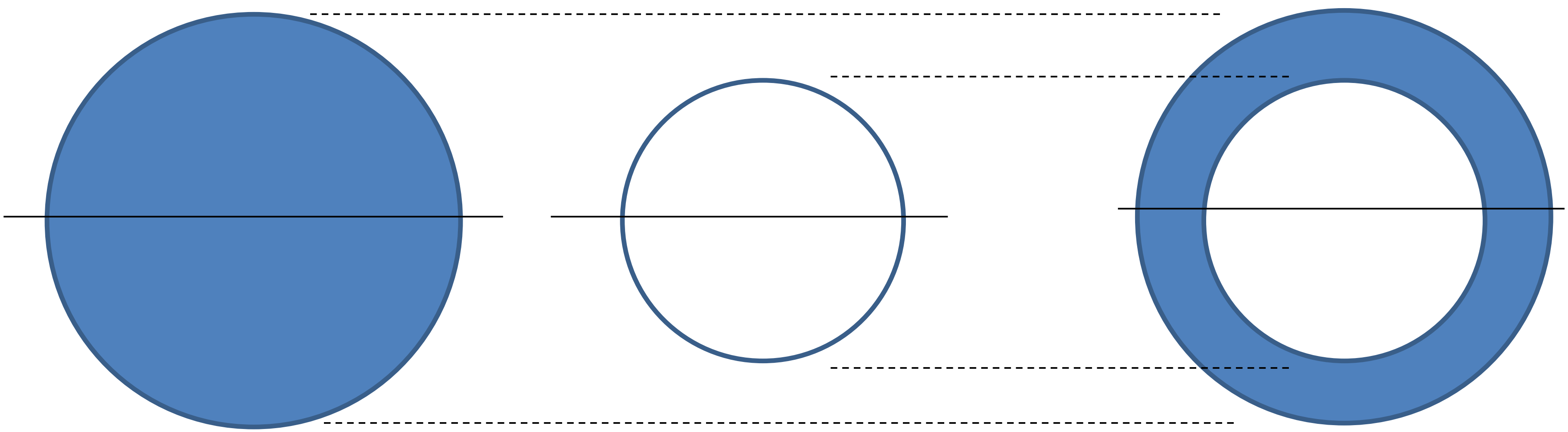
$$\tau = \frac{VQ}{It}$$



Problem 7-7

$$\tau = \frac{VQ}{It}$$

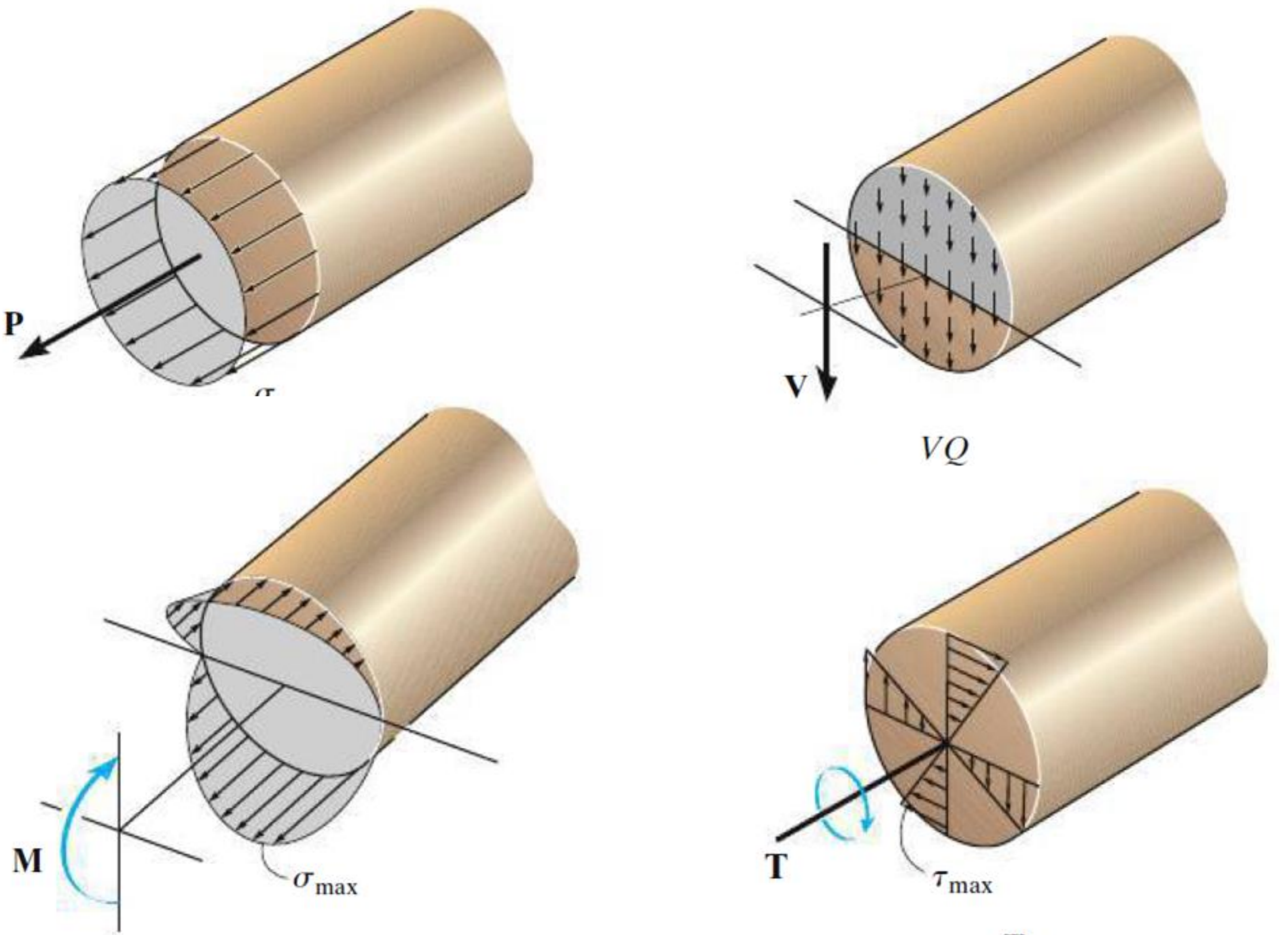




Combined Loading

Combined Loading

State of Stress Caused by Combined Loadings



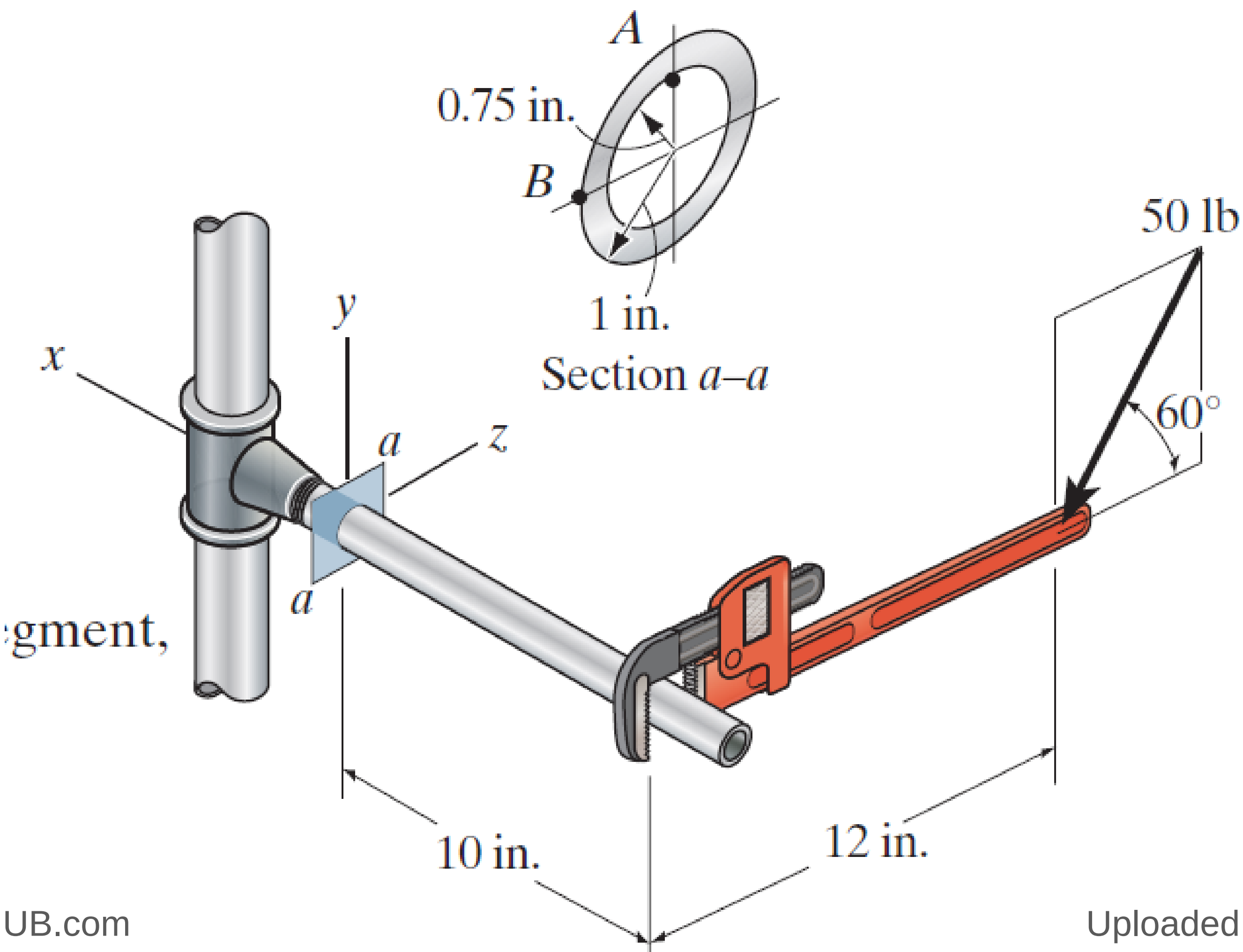
$\frac{VQ}{}$

$\sigma = -\frac{My}{I}$

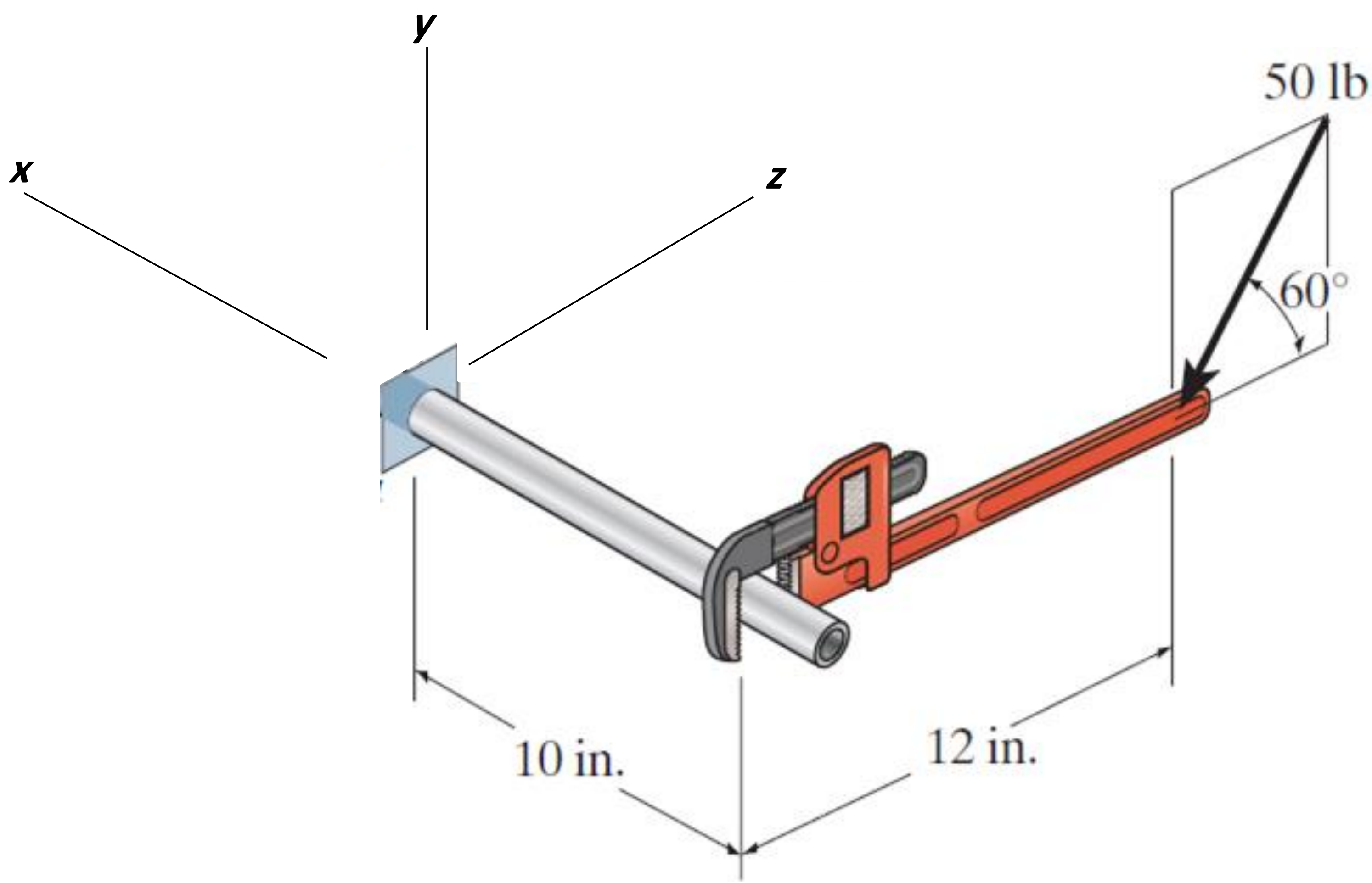
$\tau = \frac{T\rho}{J}$

Problem 8-65 and 66

8–66. Determine the state of stress at point *B* on the cross section of the pipe at section *a–a*.



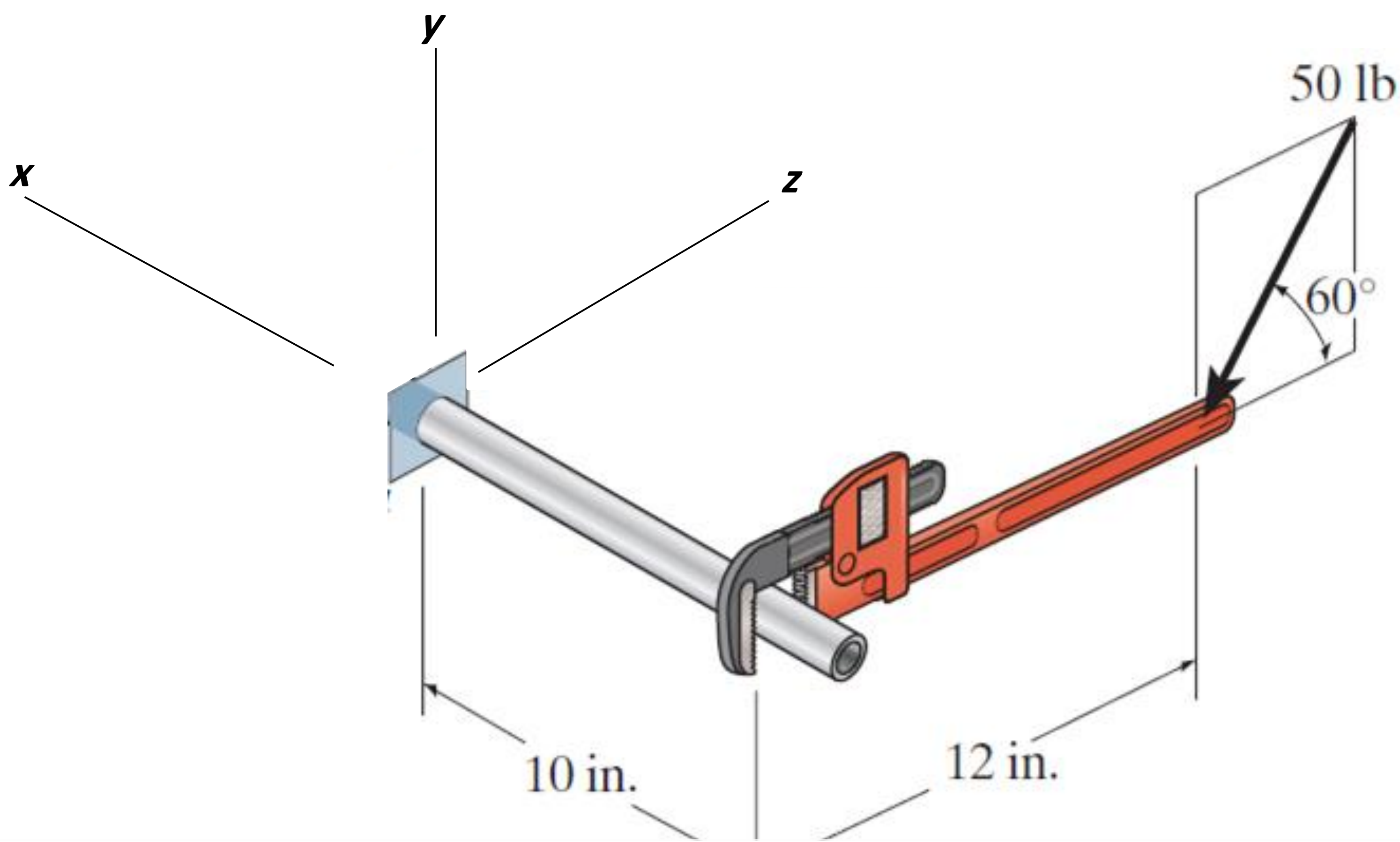
Problem 8-65 and 66



$\Sigma F_y = 0; \quad V_y - 50 \sin 60° = 0$ $\qquad V_y = 43.30 \text{ lb}$

$\Sigma F_z = 0; \quad V_z - 50 \cos 60° = 0$ $\qquad V_z = 25 \text{ lb}$

Problem 8-65 and 66

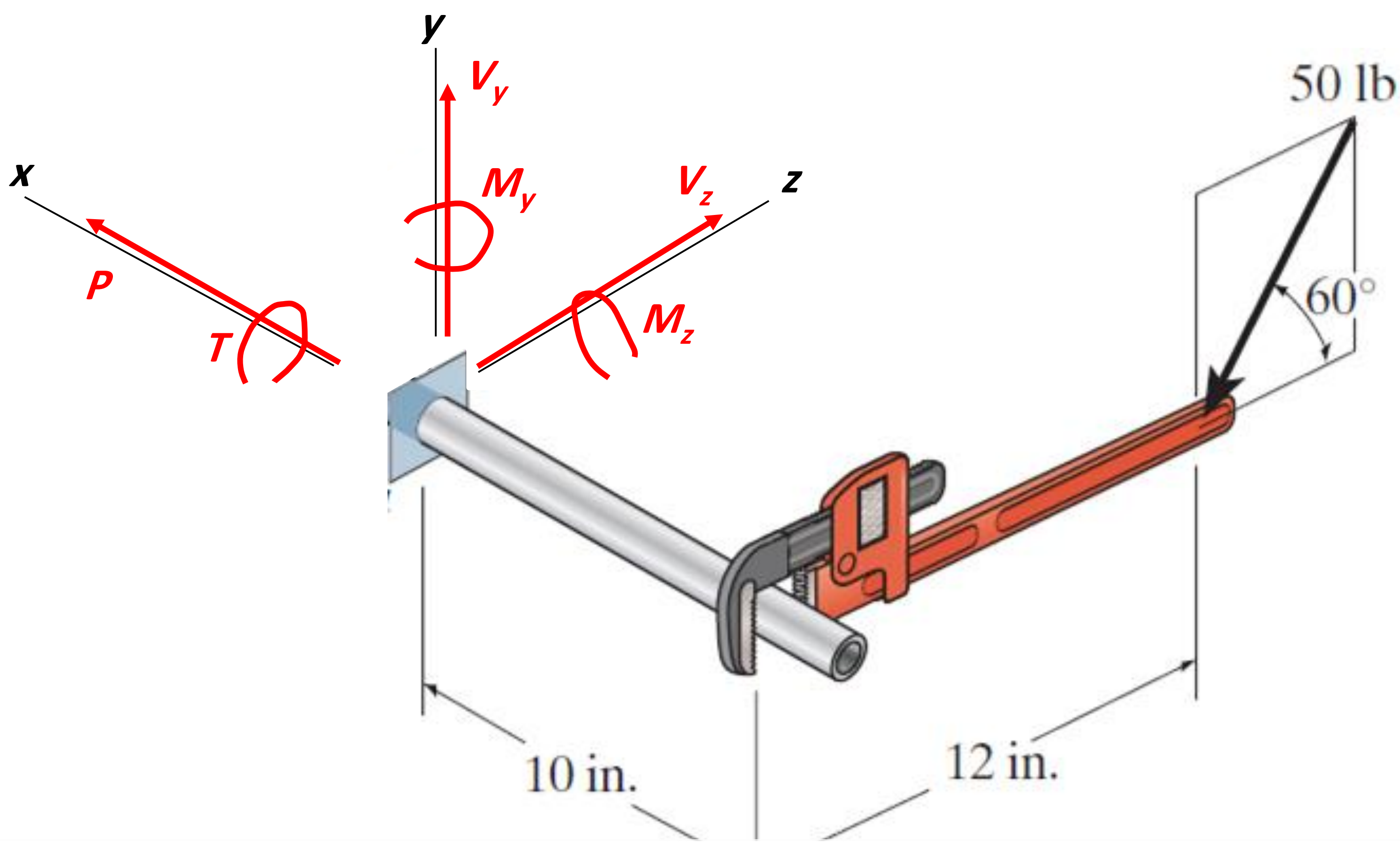


$$\Sigma M_x = 0; \quad T + 50\sin 60°(12) = 0 \qquad T = -519.62 \text{ lb} \cdot \text{in}$$

$$\Sigma M_y = 0; \quad M_y - 50\cos 60°(10) = 0 \qquad M_y = 250 \text{ lb} \cdot \text{in}$$

$$\Sigma M_z = 0; \quad M_z + 50\sin 60°(10) = 0 \qquad M_z = -433.01 \text{ lb} \cdot \text{in}$$

Problem 8-65 and 66



$\Sigma M_x = 0; \quad T + 50\sin 60°(12) = 0 \qquad T = -519.62 \text{ lb} \cdot \text{in}$

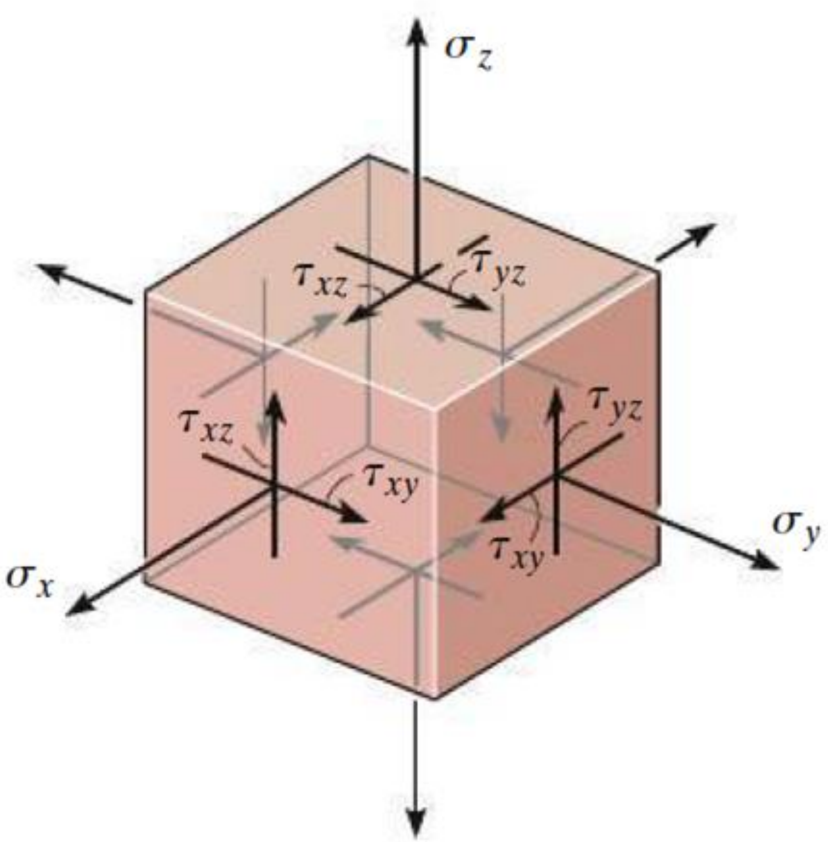
$\Sigma M_y = 0; \quad M_y - 50\cos 60°(10) = 0 \qquad M_y = 250 \text{ lb} \cdot \text{in}$

$\Sigma M_z = 0; \quad M_z + 50\sin 60°(10) = 0 \qquad M_z = -433.01 \text{ lb} \cdot \text{in}$

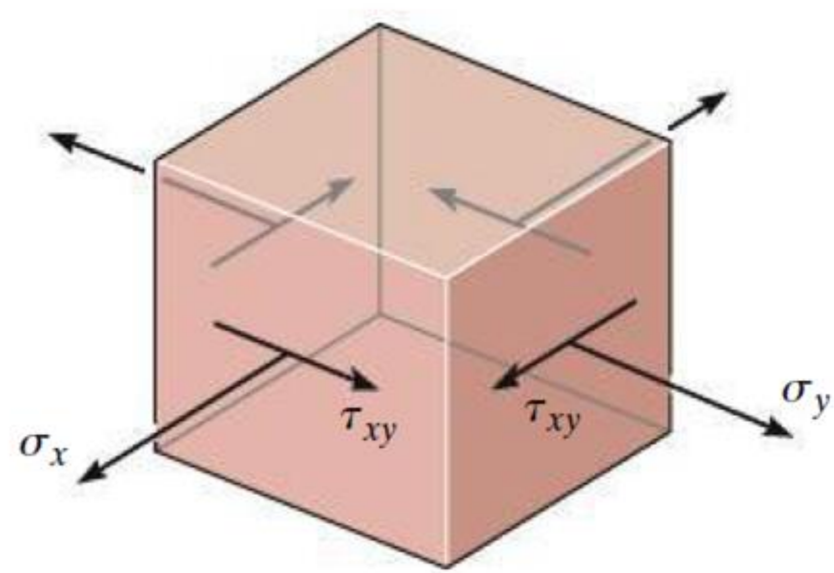
Stress Transformation

Stress Transformation

Introduction

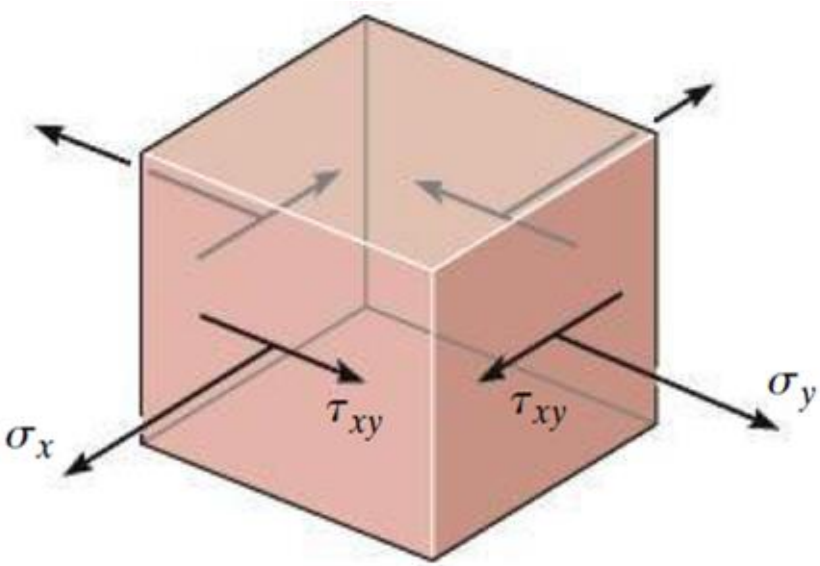


General state of stress

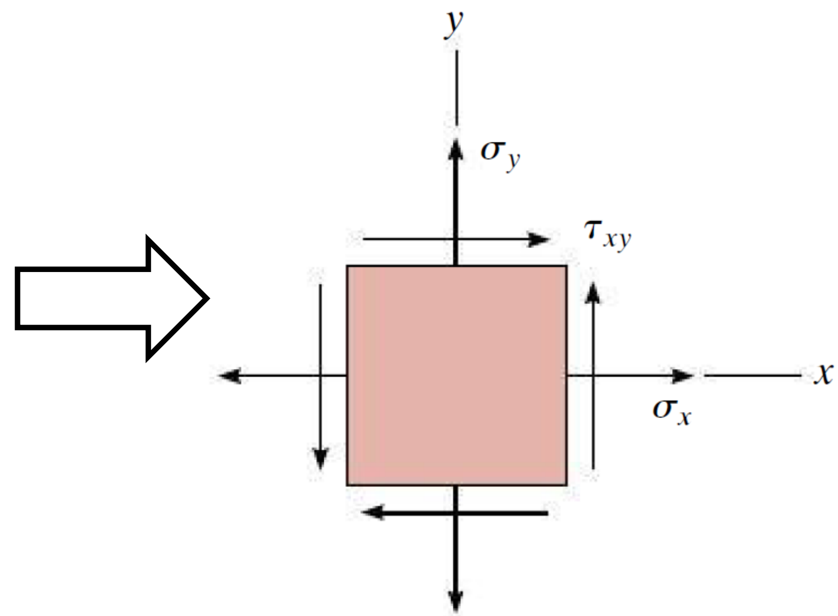


Plane stress

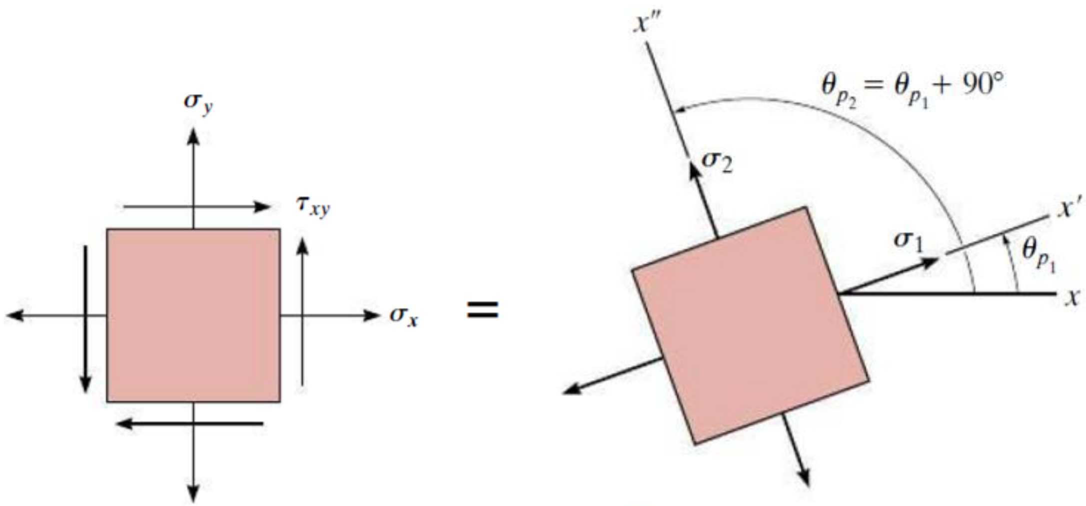
Introduction



Plane stress

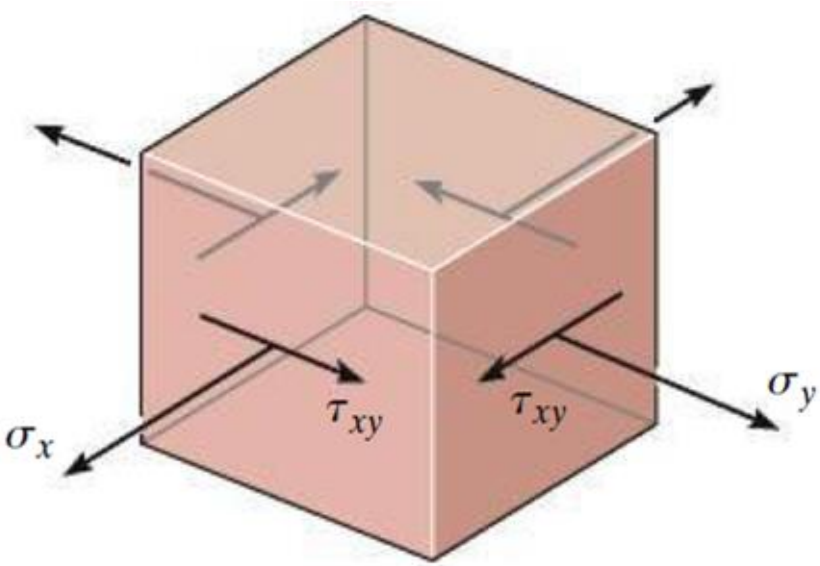


(a)

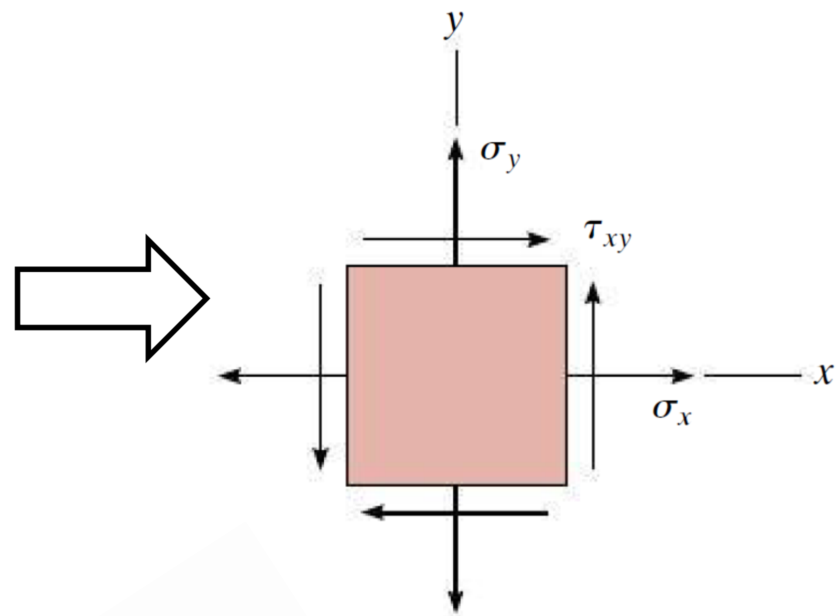


In-plane principal stresses

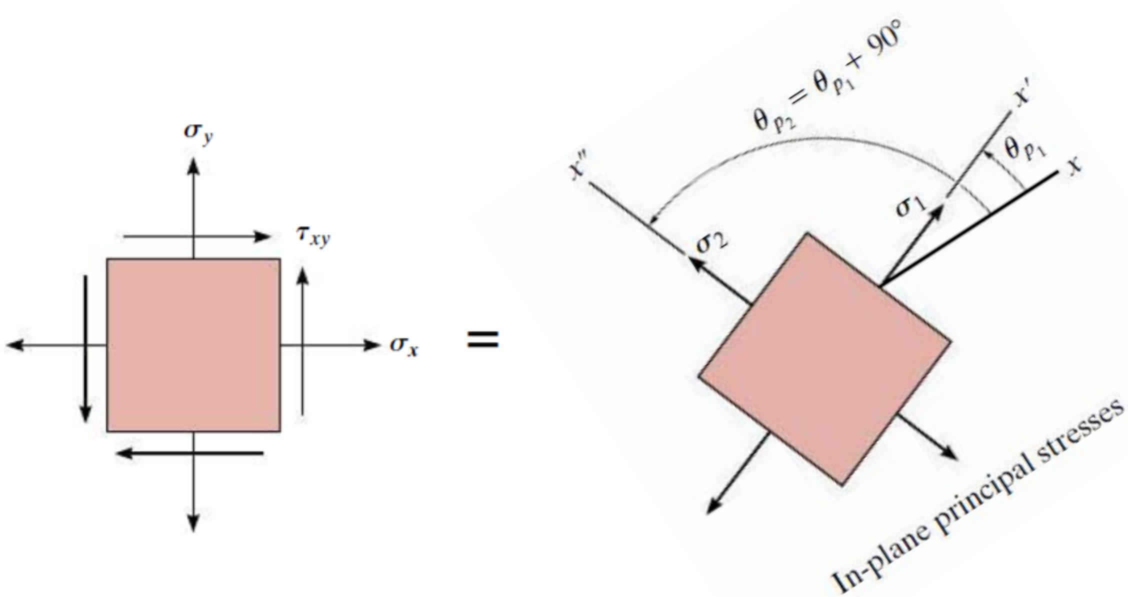
Introduction



Plane stress



(a)

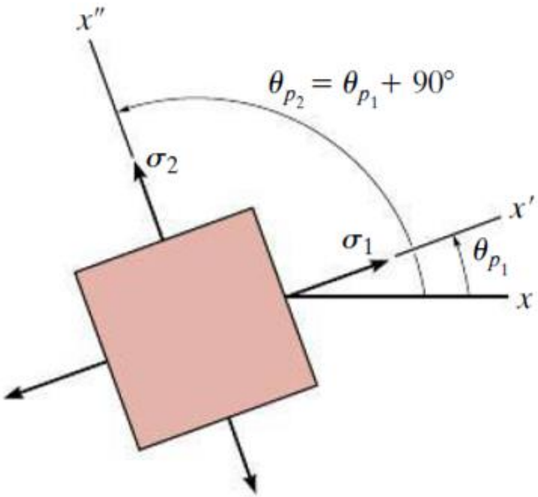


9.3 Principal Stresses and Max. In-Plane Shear Stress

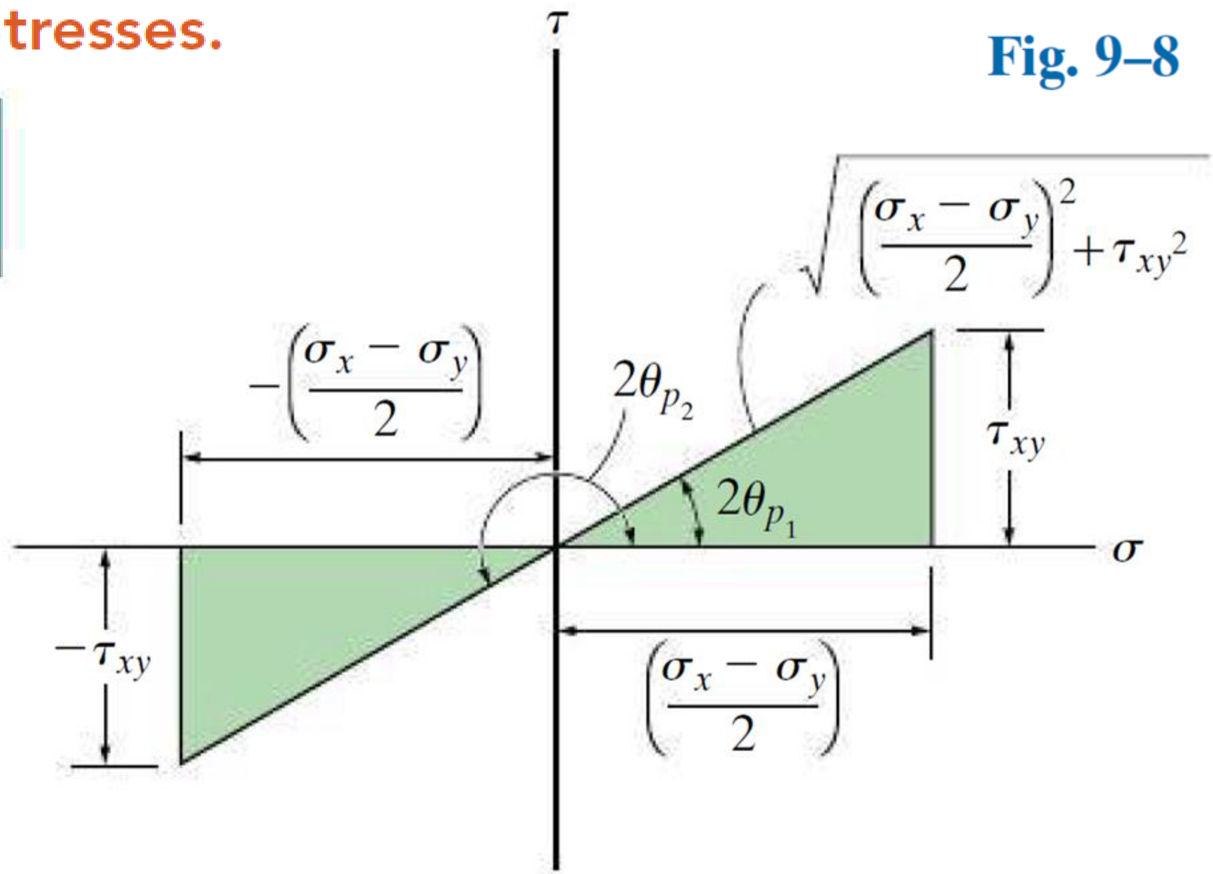
In-Plane Principal Stresses.

Fig. 9–8

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$



In-plane principal stresses



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

9.3 Principal Stresses and Max. In-Plane Shear Stress

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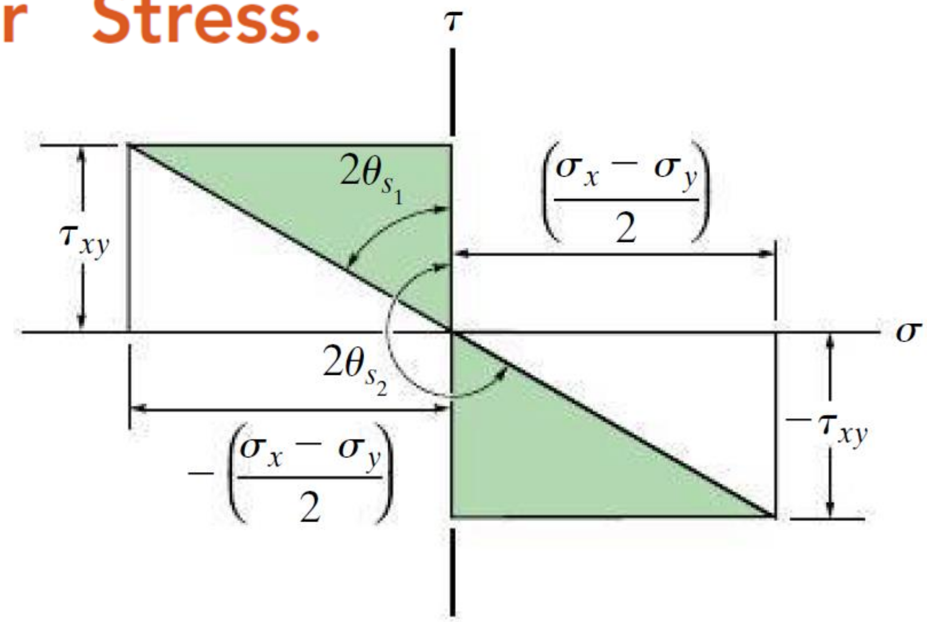
Maximum In-Plane Shear Stress.

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$



$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Problem 9-17

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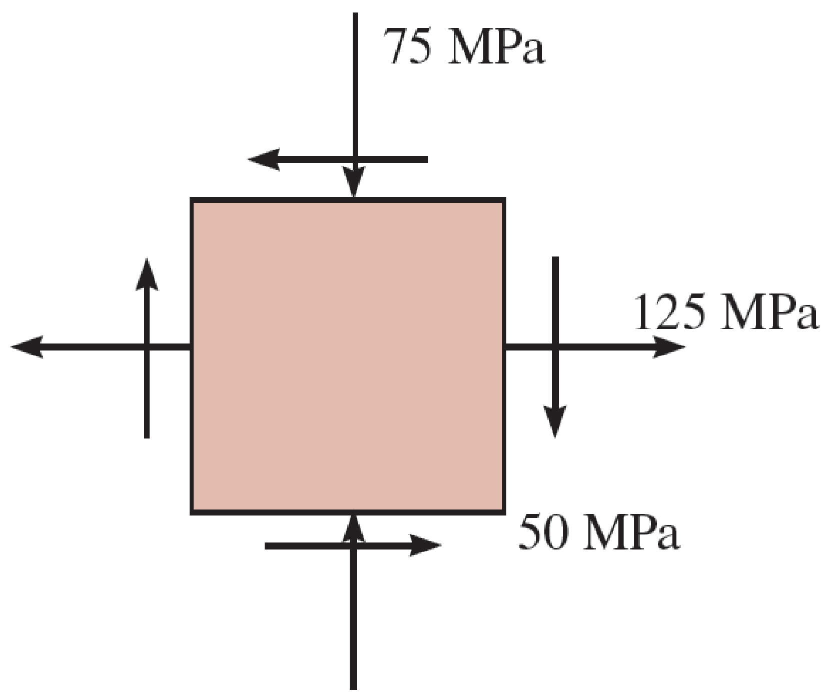
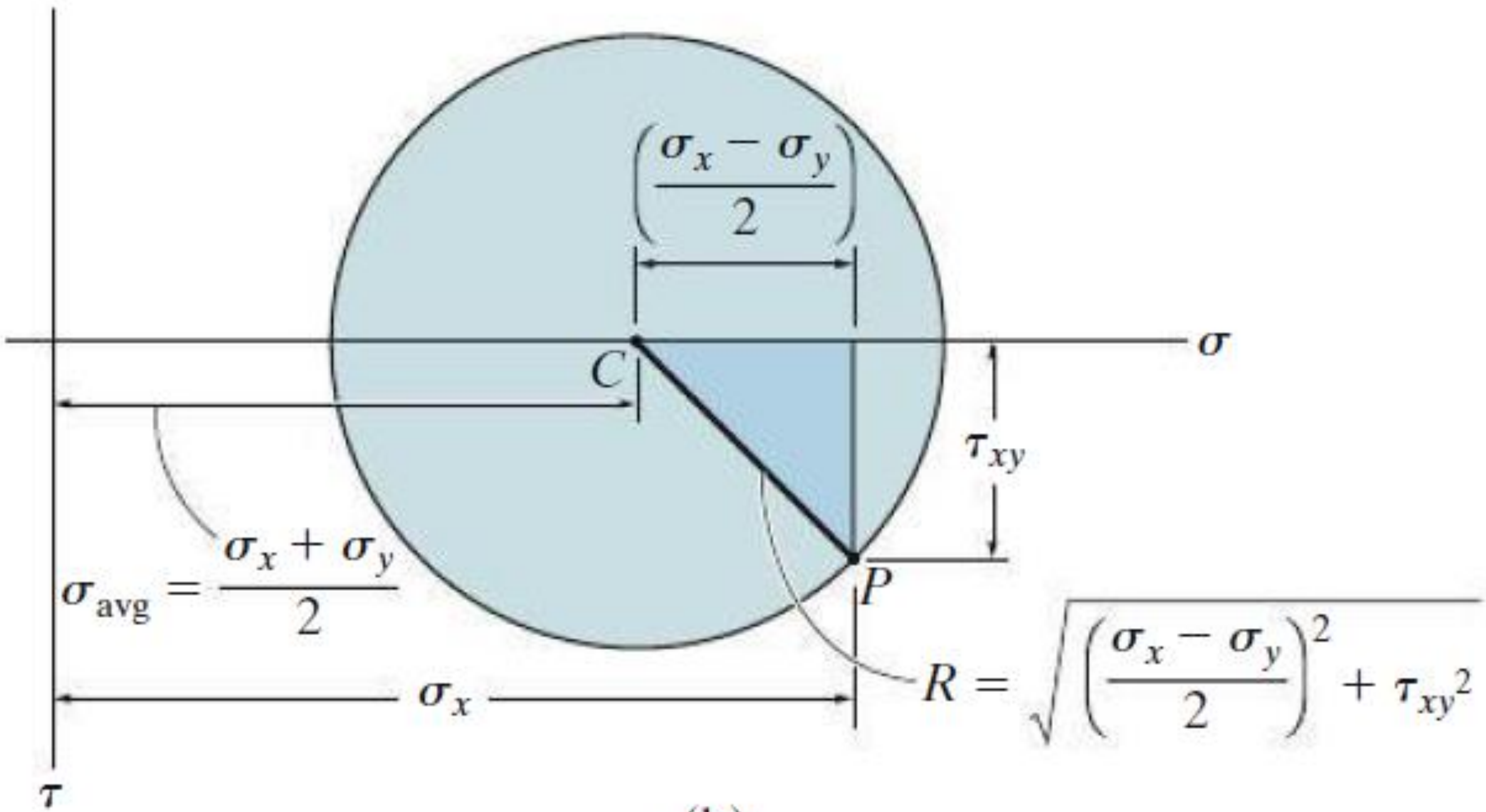
9–17. Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown. Sketch the results on each element.

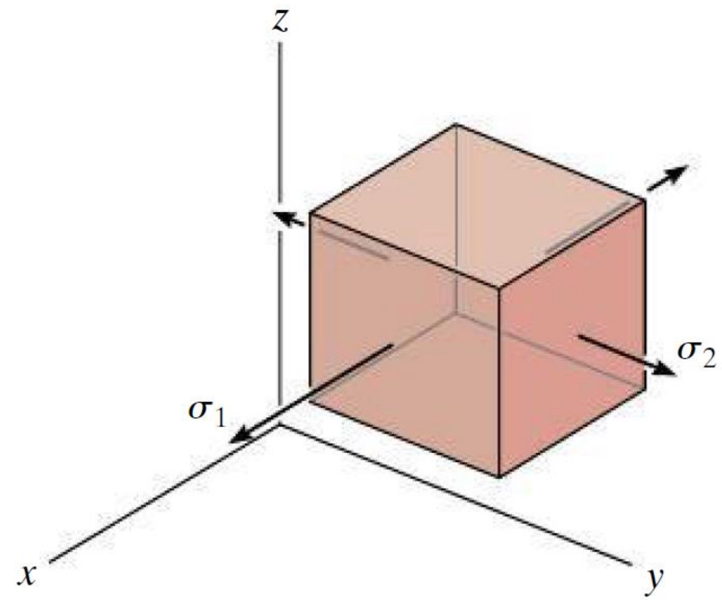


9.4 Mohr's Circle - Plane Stress

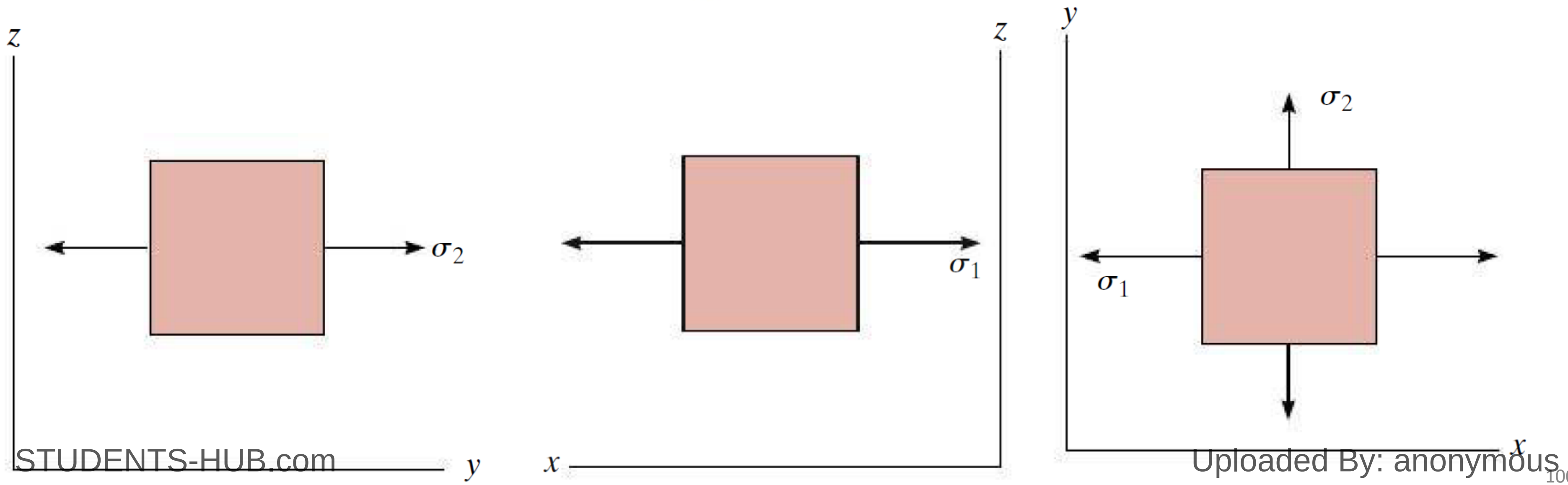


(b)

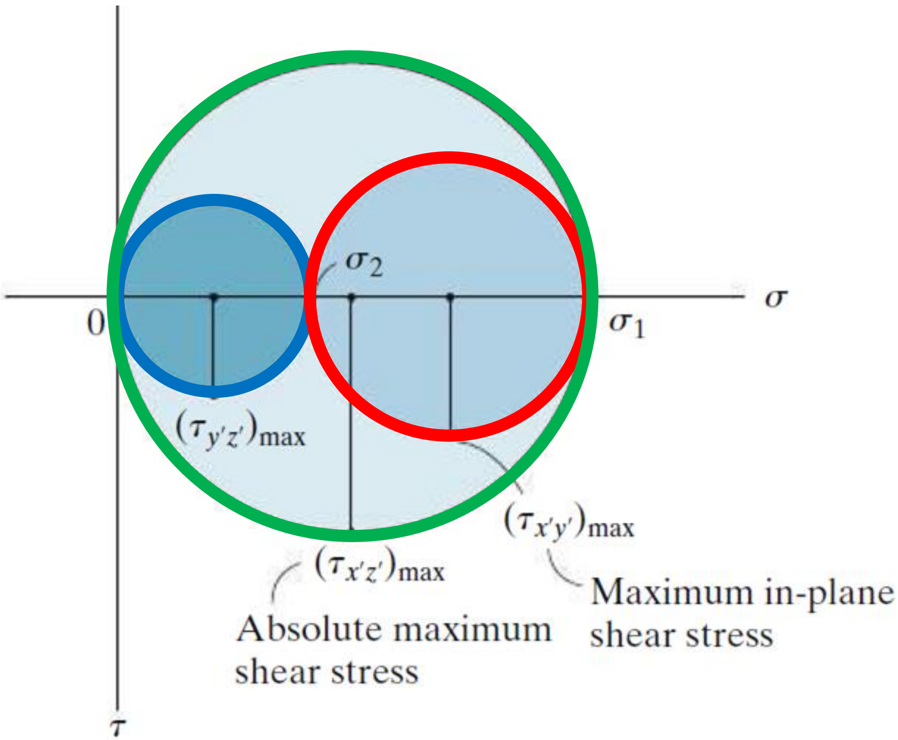
9.5 Absolute Maximum Shear Stress



x–y plane stress

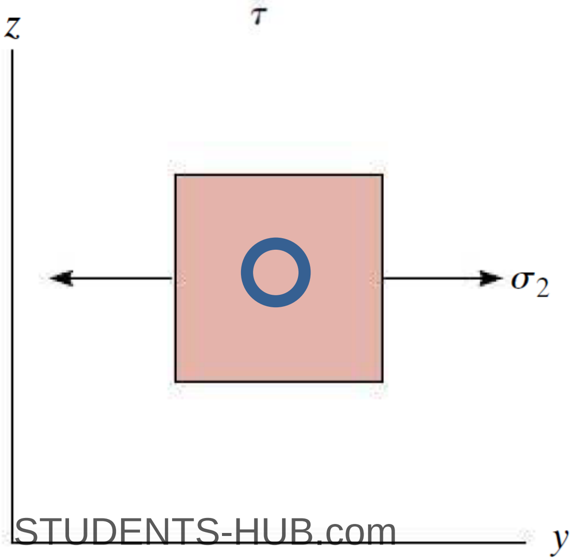


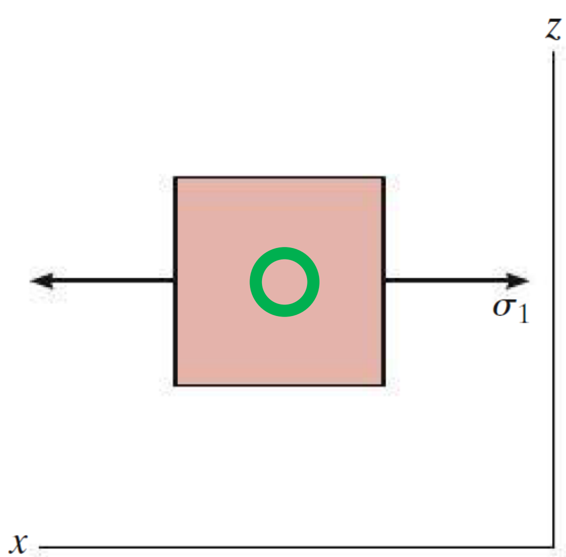
9.5 Absolute Maximum Shear Stress

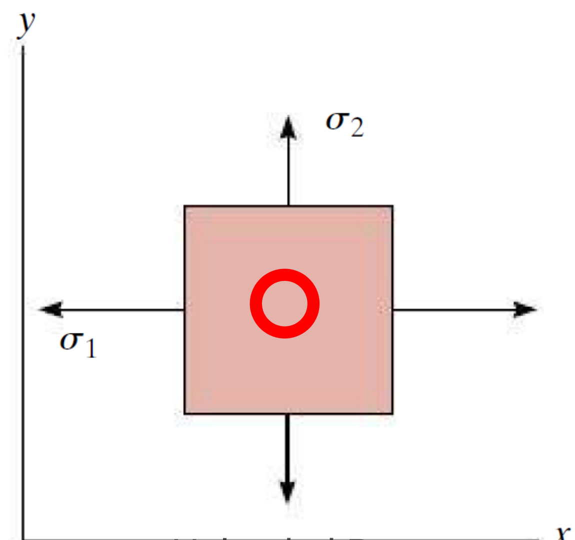


$$\tau_{\substack{abs\\max}} = \frac{\sigma_1}{2}$$

σ_1 and σ_2 have the same sign

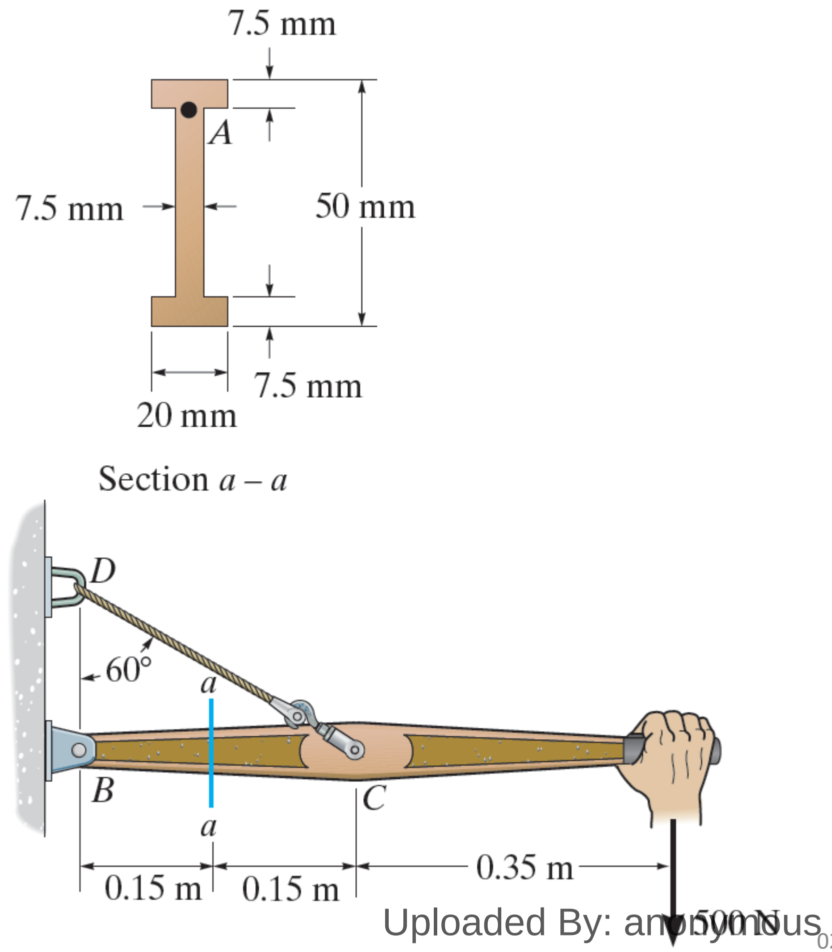




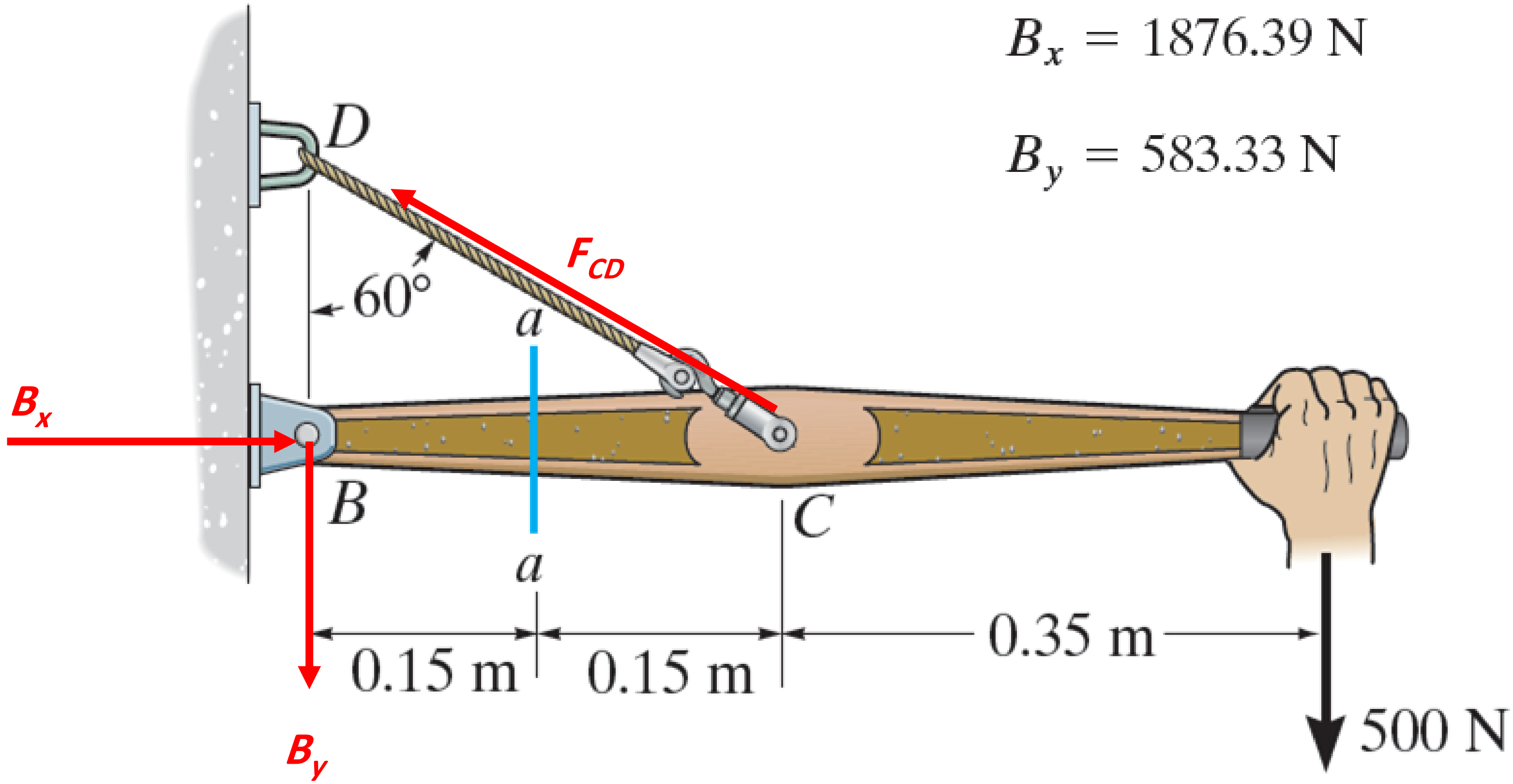


Problem 9-32

***9–32.** Determine the maximum in-plane shear stress developed at point *A* on the cross section of the arm at section *a–a*. Specify the orientation of this state of stress and indicate the results on an element at the point.



Problem 9-32

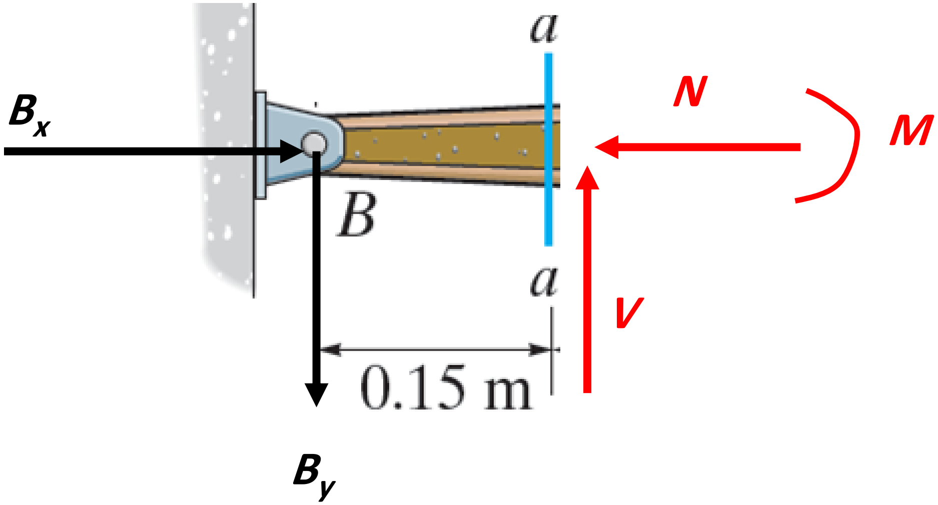


$$F_{CD} = 2166.67\text{ N}$$

$$B_x = 1876.39\text{ N}$$

$$B_y = 583.33\text{ N}$$

Problem 9-32



External Forces

$$F_{CD} = 2166.67\text{ N}$$

$$B_x = 1876.39\text{ N}$$

$$B_y = 583.33\text{ N}$$

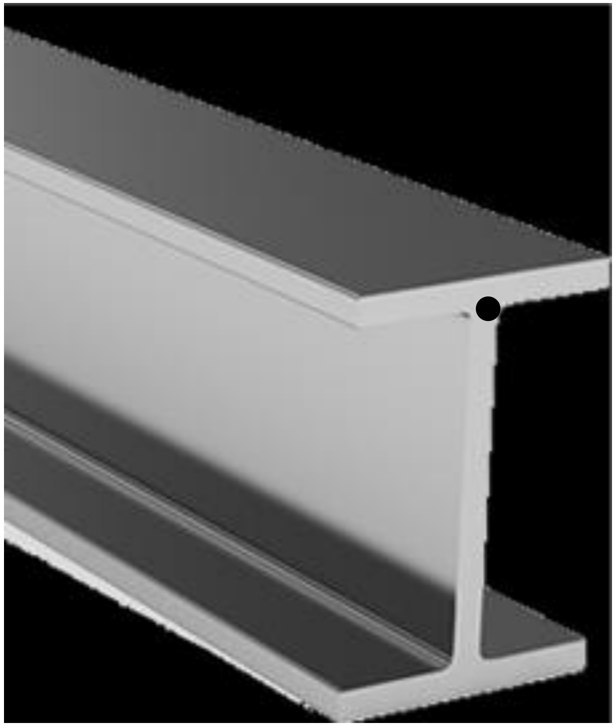
Internal Forces

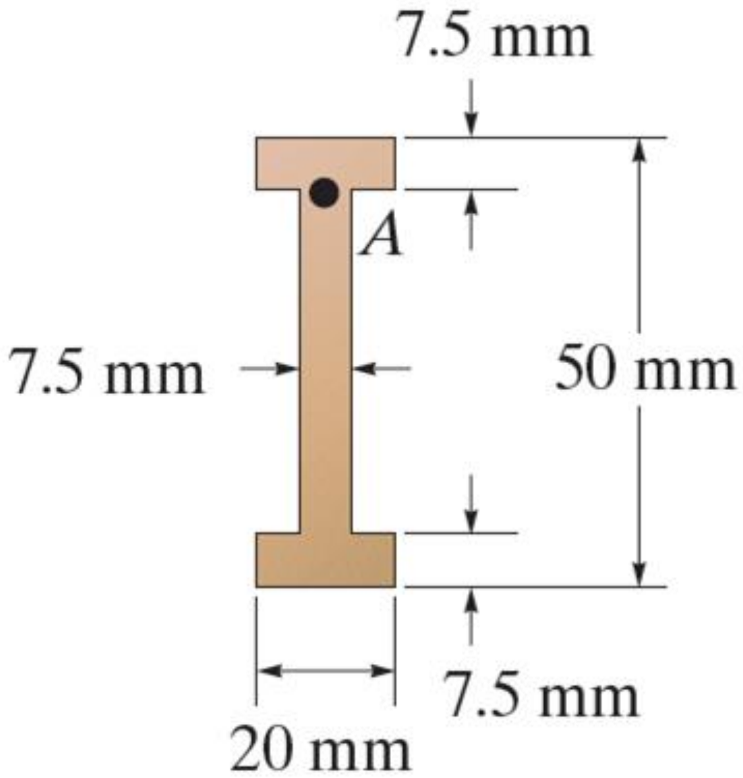
$$N = 1876.39\text{ N}$$

$$V = 583.33\text{ N}$$

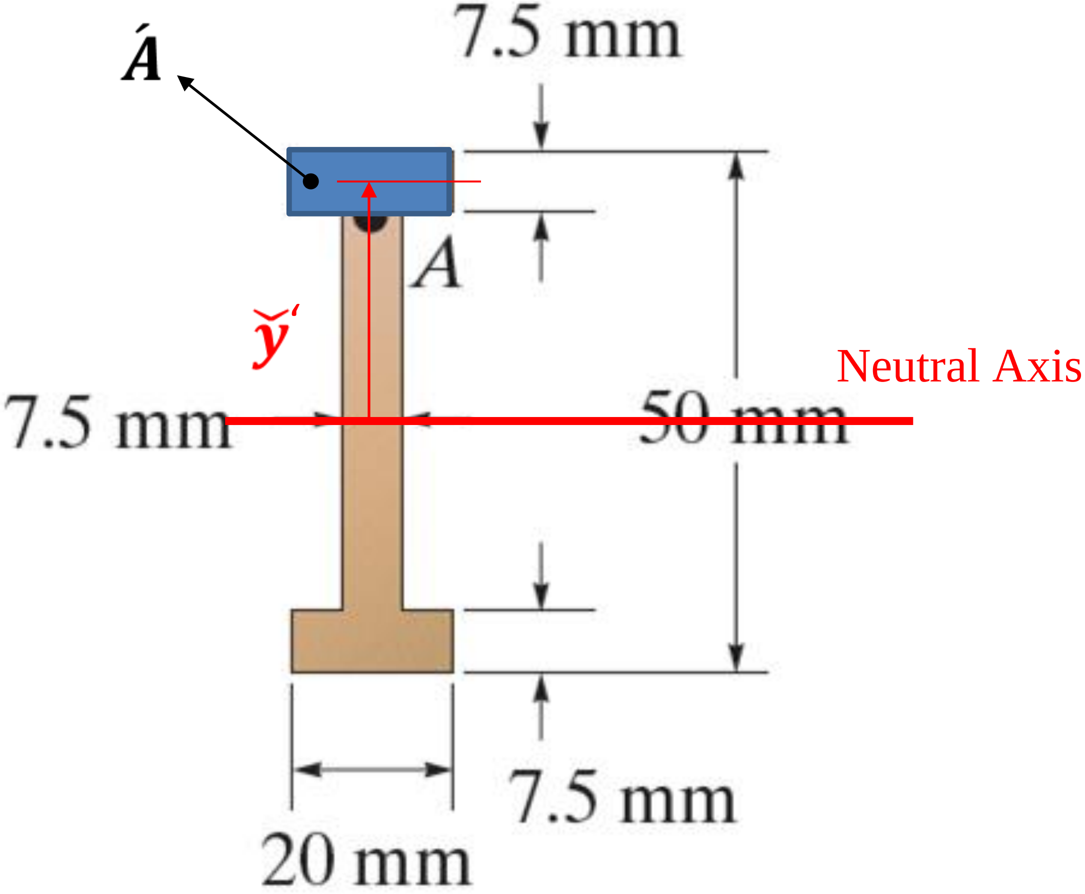
$$M = 87.5\text{ N}\cdot\text{m}$$

Problem 9-32



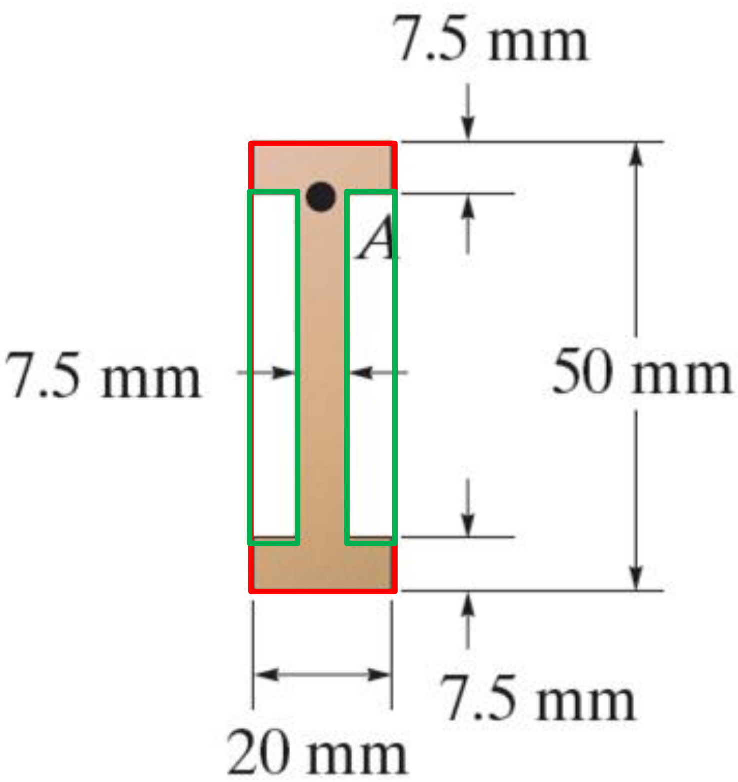


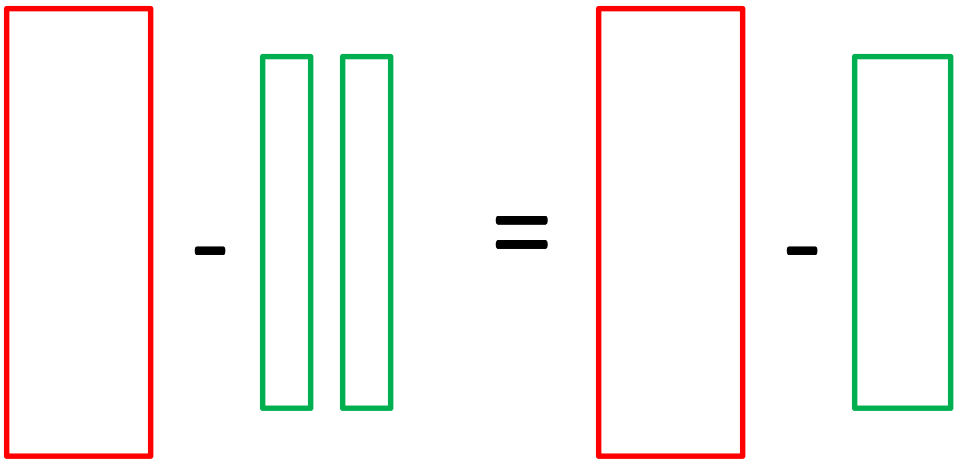
Problem 9-32



$$Q_A = \bar{y}' A'$$

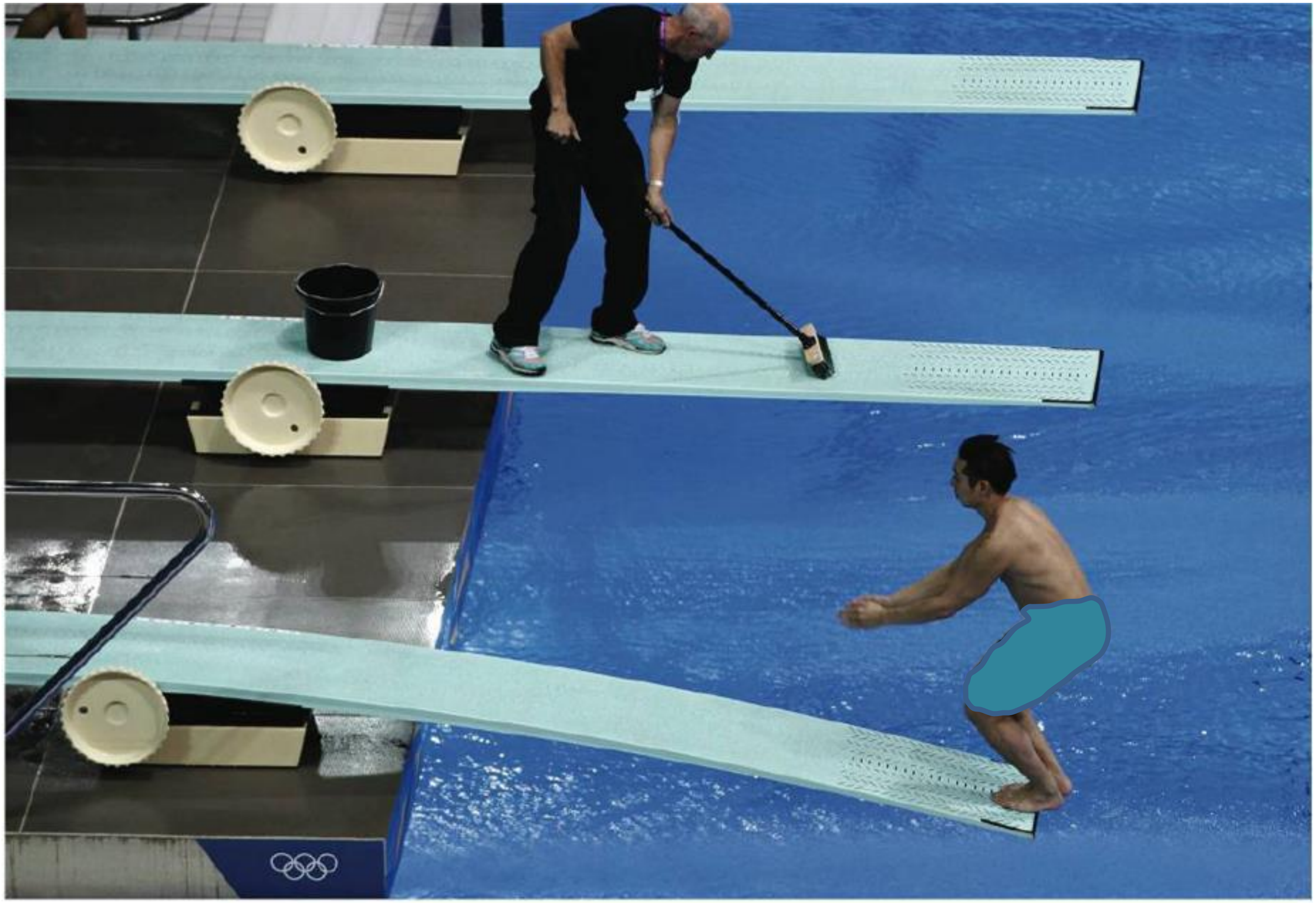
Problem 9-32



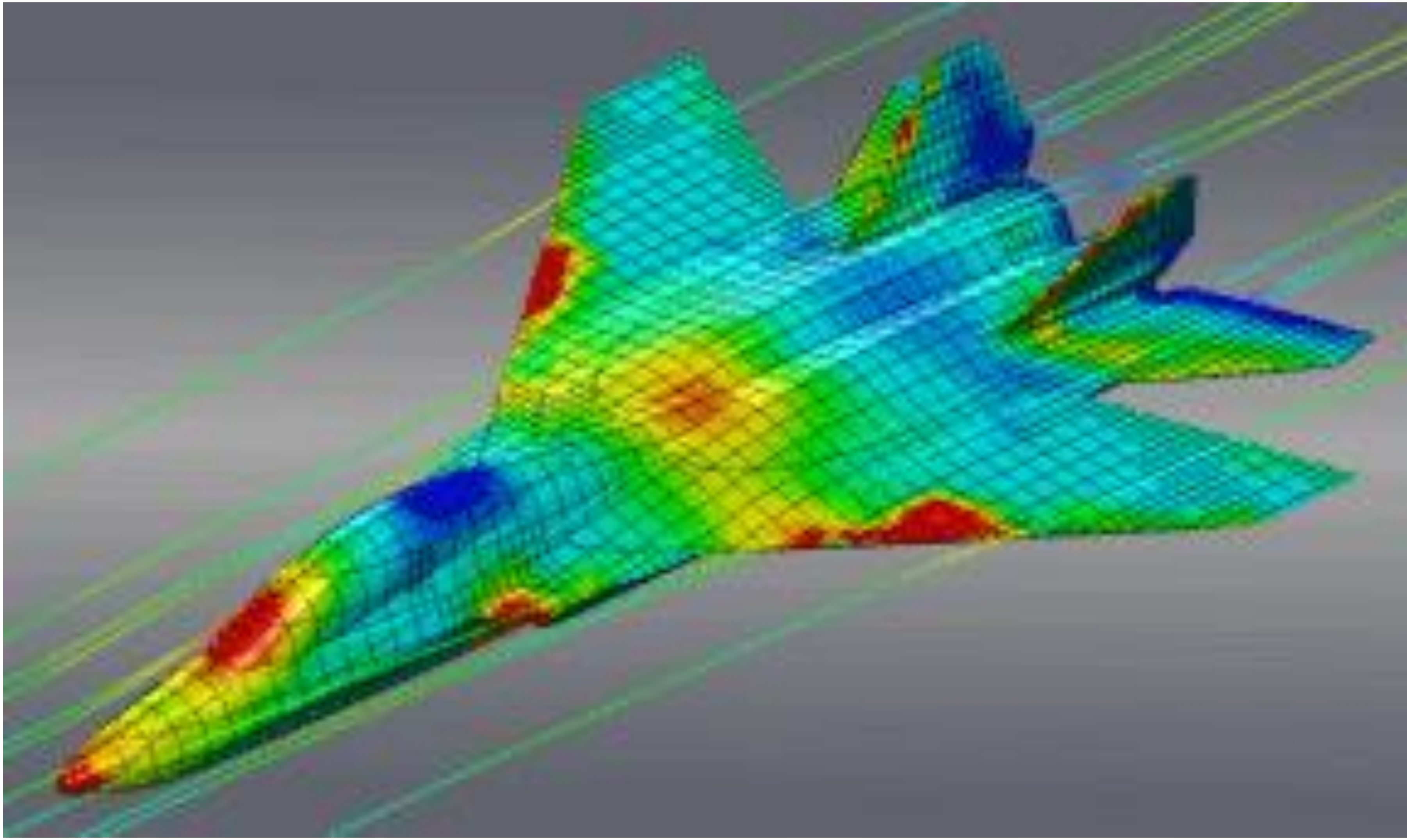


$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right)$$

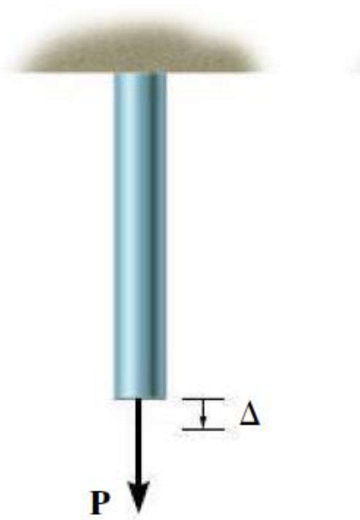
Energy Methods

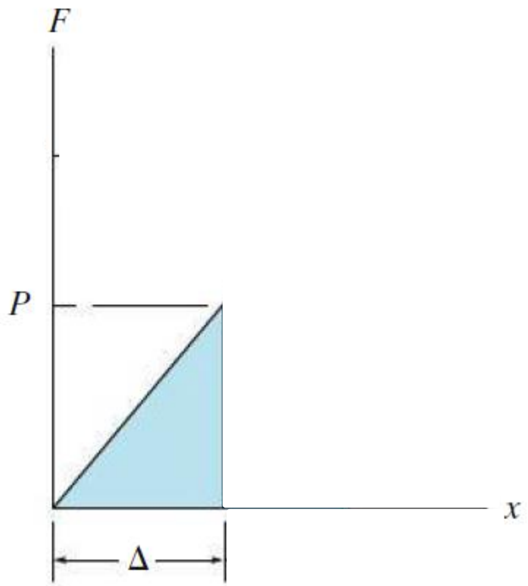


Energy Methods



14.1 External Work and Strain Energy



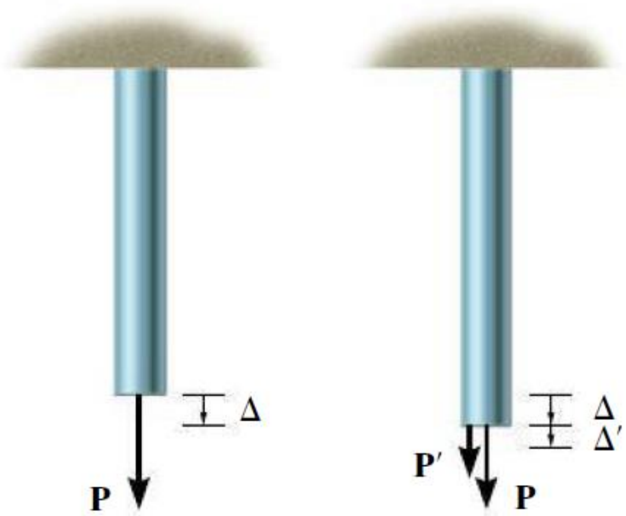


Work of a force

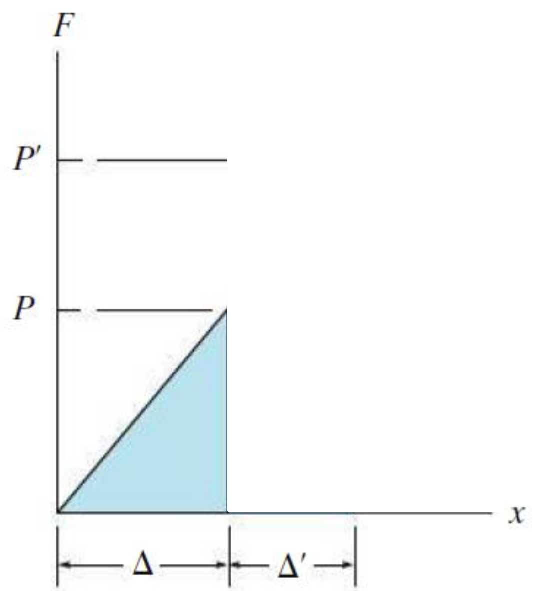
$$U_e = \int_0^{\Delta} F\,dx$$

$$U_e = \frac{1}{2} P \Delta$$

14.1 External Work and Strain Energy



(a) (b)



(c)

Work of a force

$$U_e = \int_0^{\Delta} F\,dx$$

$$U_e = \frac{1}{2}P\Delta$$

$$U_e' = P\Delta'$$

14.1 External Work and Strain Energy

Work of a Couple of Moment

$$U_e = \int_0^{\theta} M\, d\theta$$

$$U_e = \frac{1}{2} M\theta$$

$$U'_e = M\theta'$$

Work of a force

$$U_e = \int_0^{\Delta} F\, dx$$

$$U_e = \frac{1}{2} P\Delta$$

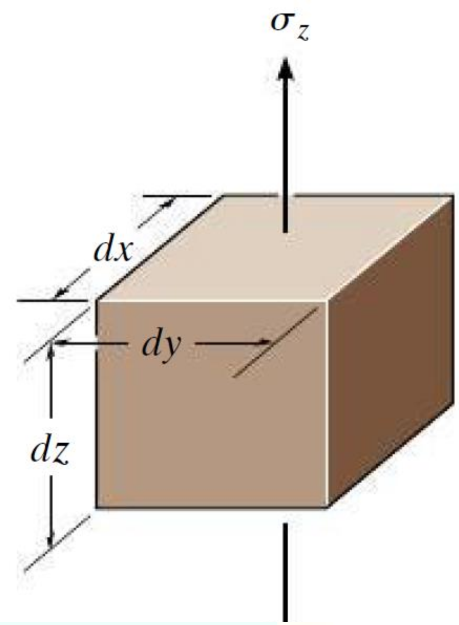
$$U'_e = P\Delta'$$

14.1 External Work and Strain Energy

Normal Stress & Strain Energy

$$d\Delta_z = \epsilon_z \, dz$$

$$dU_i = \frac{1}{2} dF_z \, d\Delta_z = \frac{1}{2}[\sigma_z \, dx \, dy]\epsilon_z \, dz$$



$$dV = dx \, dy \, dz$$

$$dU_i = \frac{1}{2}\sigma_z \epsilon_z \, dV$$

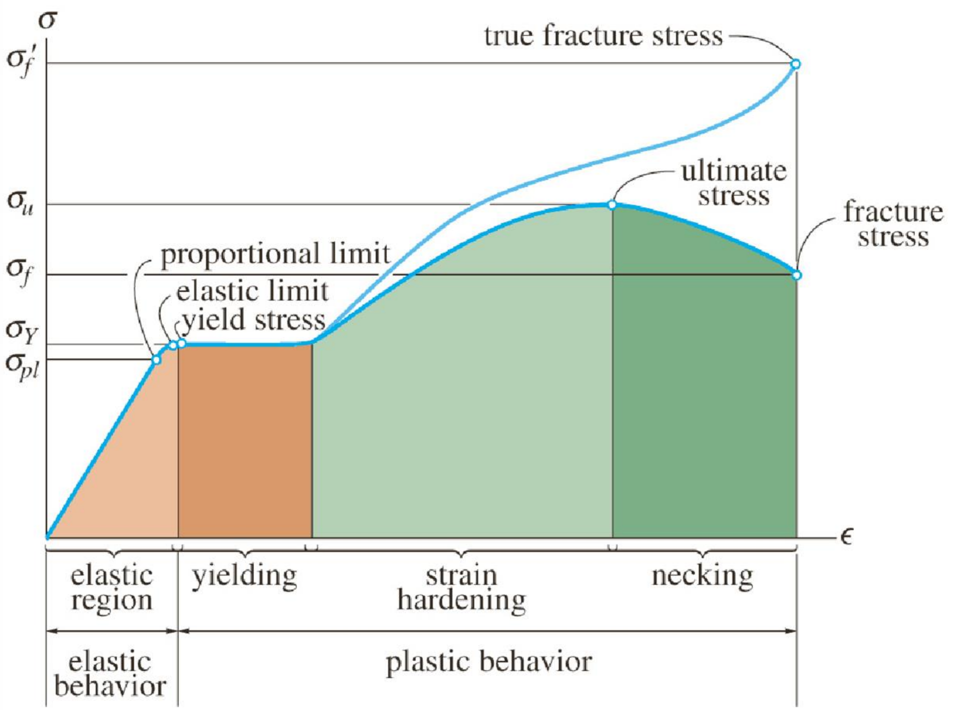
$$\sigma = E\epsilon$$

Question

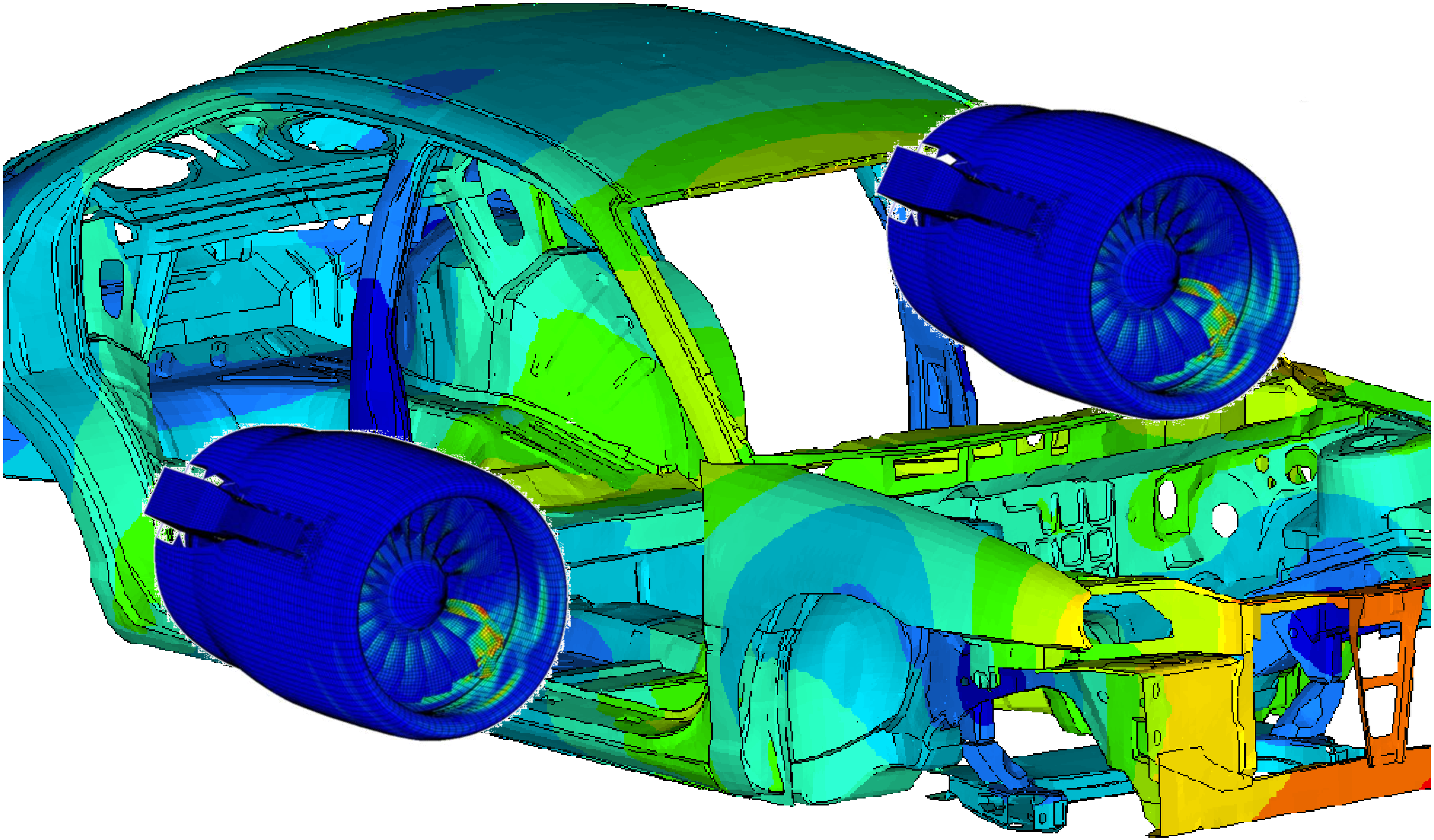
$$U_i = \int_V \frac{\sigma^2}{2E} dV$$

$$U_i = \int_V \frac{\tau^2}{2G} dV$$

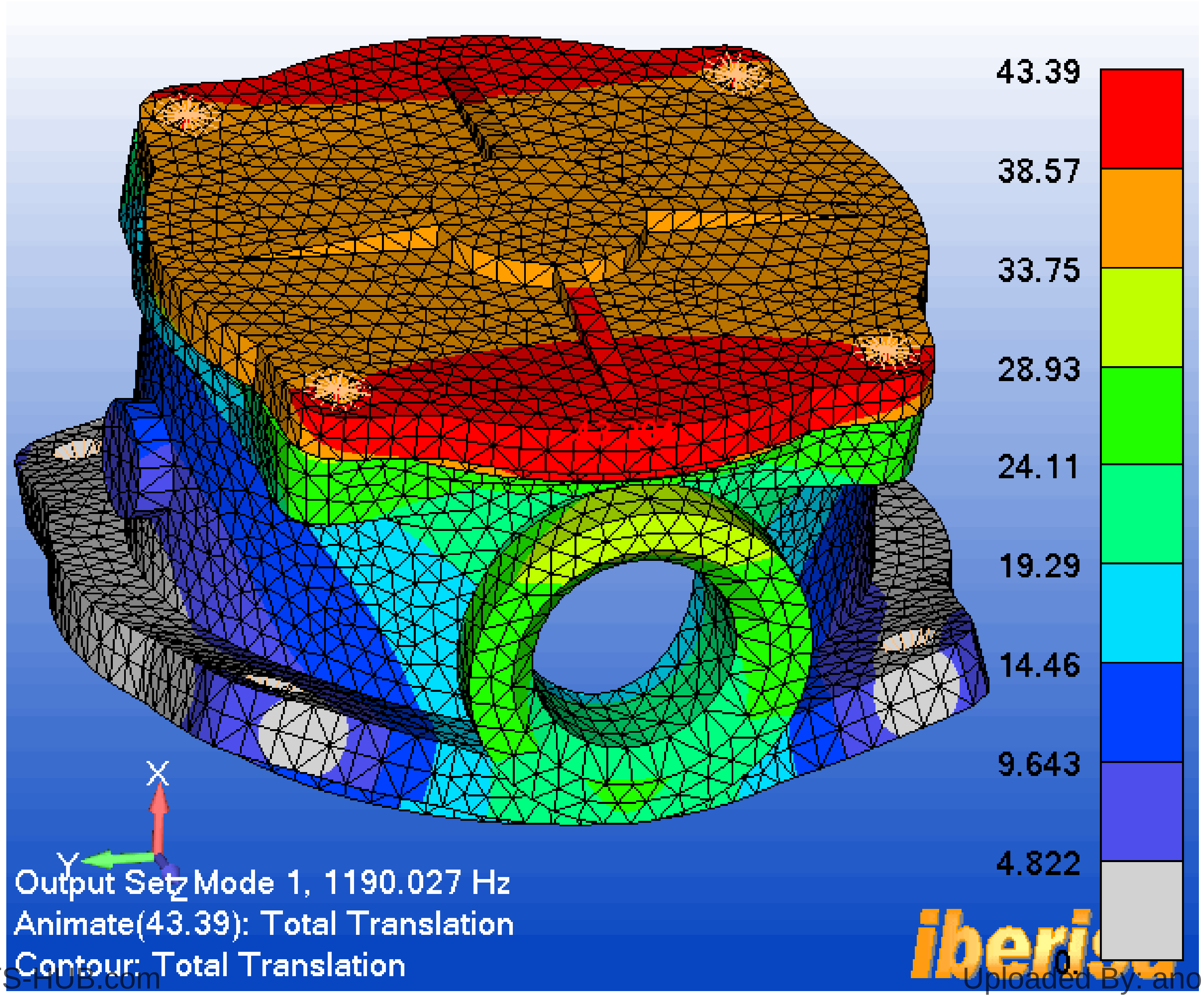
14.1 External Work and Strain Energy



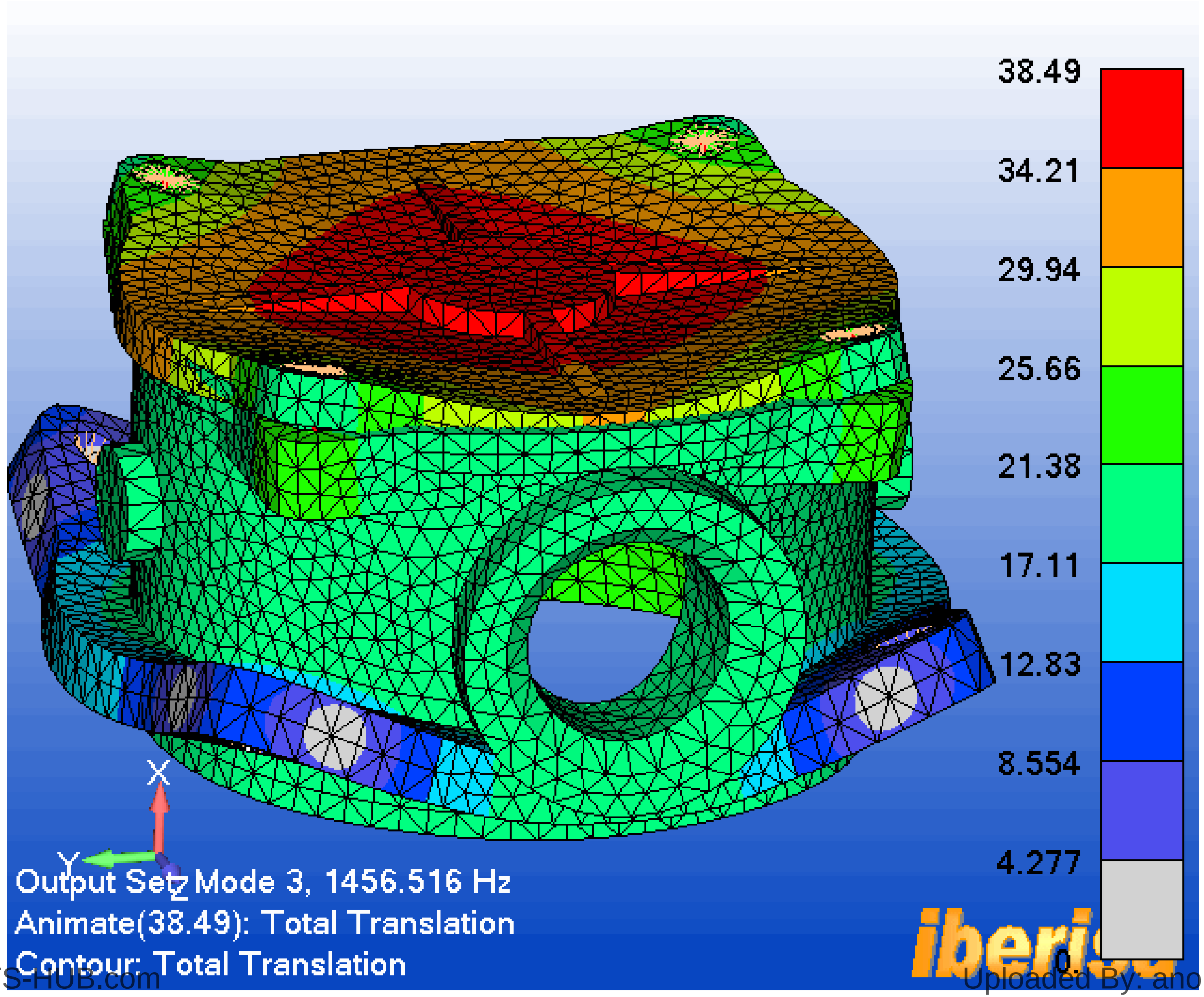
14.1 External Work and Strain Energy



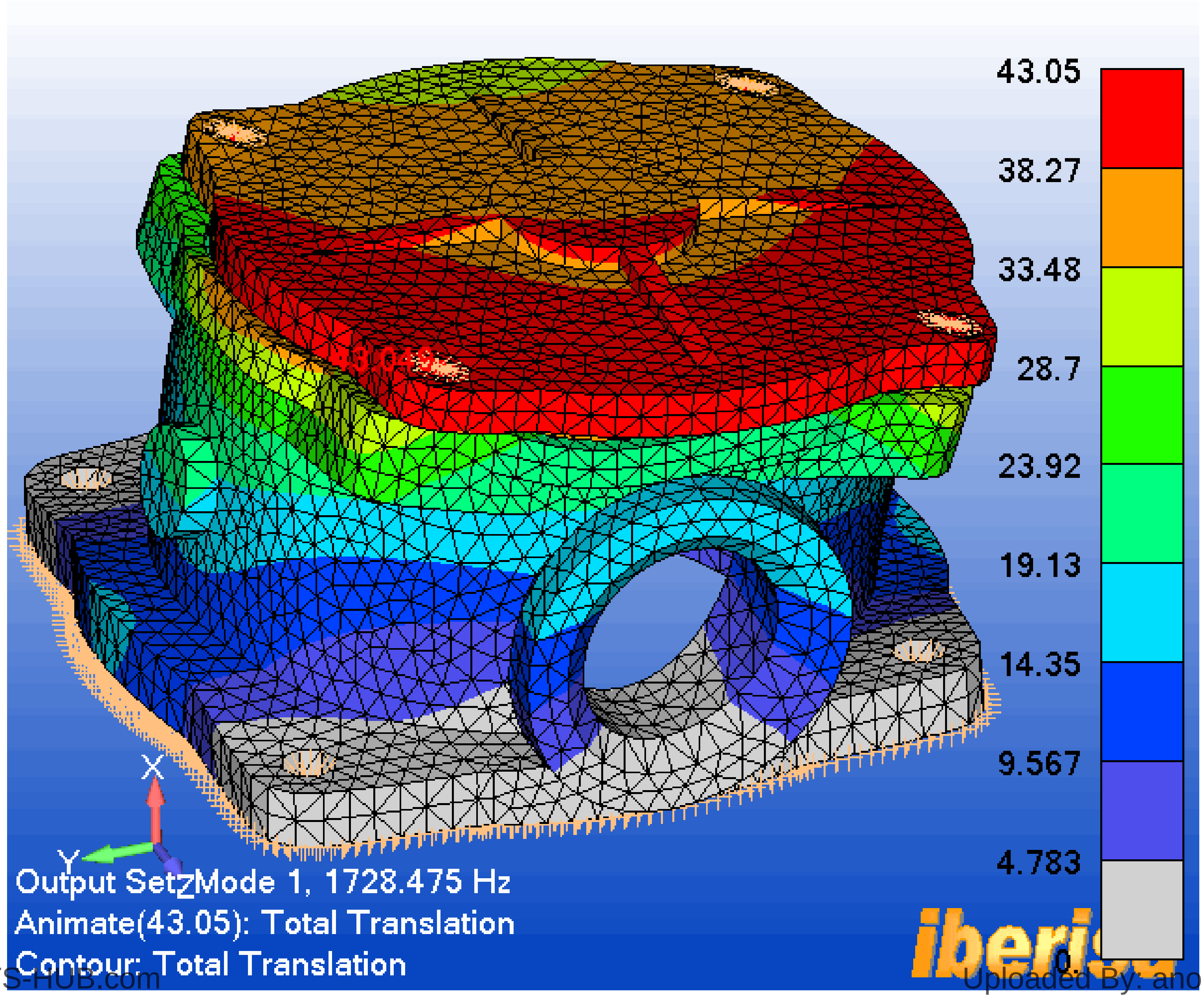
14.1 External Work and Strain Energy



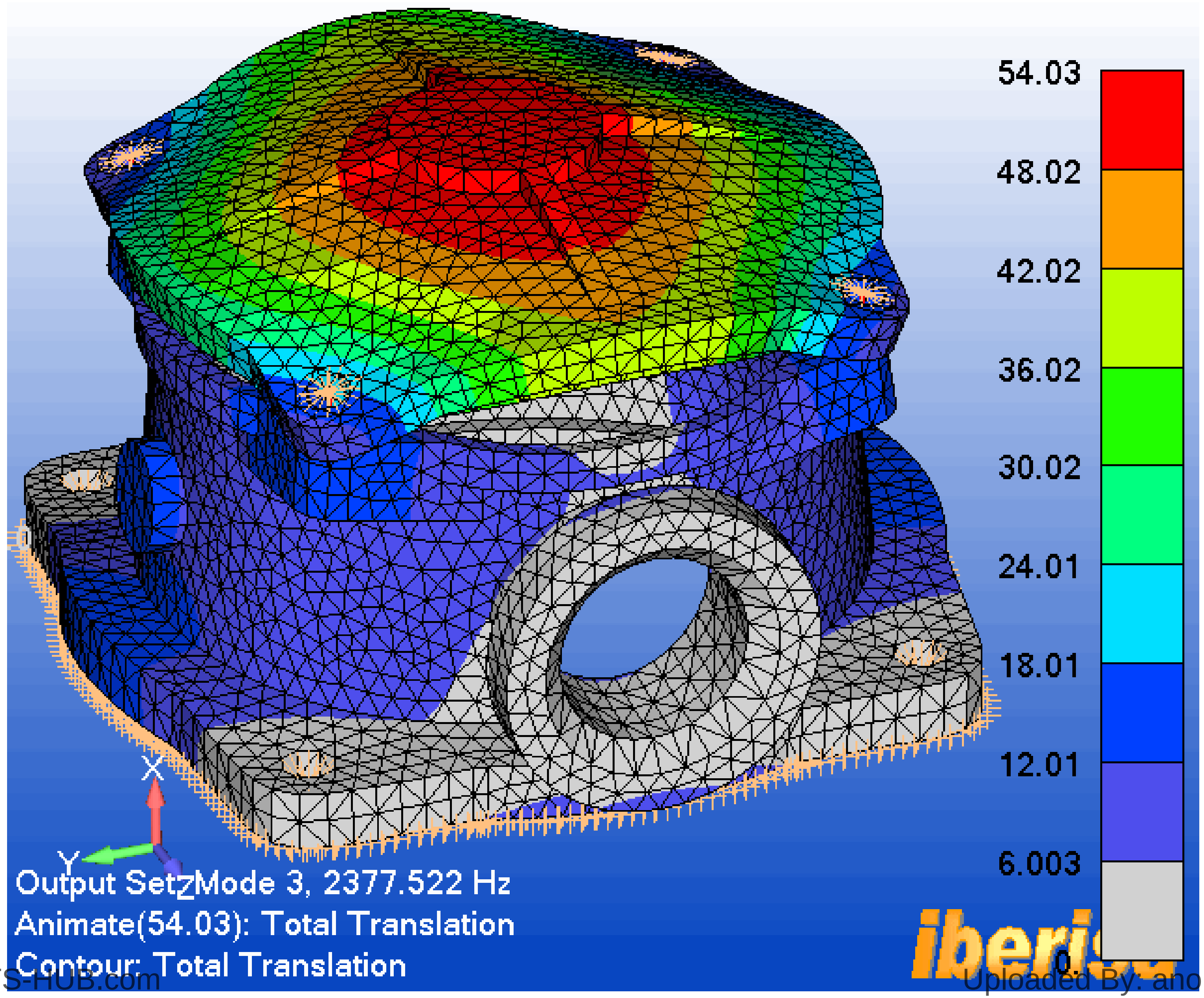
14.1 External Work and Strain Energy



14.1 External Work and Strain Energy



14.1 External Work and Strain Energy



14.2 Elastic Strain Energy for Various Loading

$$U_i = \int_V \frac{\sigma^2}{2E} dV$$

Axial Load

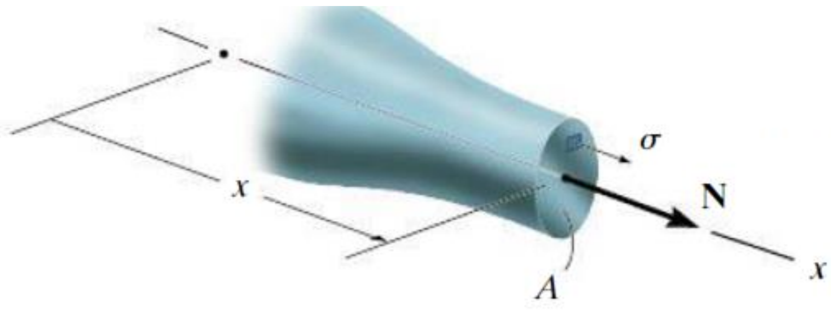


Fig. 14–6

$$U_i = \int_V \frac{{\sigma_x}^2}{2E} dV = \int_V \frac{N^2}{2EA^2} dV$$

$$U_i = \int_0^L \frac{N^2}{2AE} dx$$

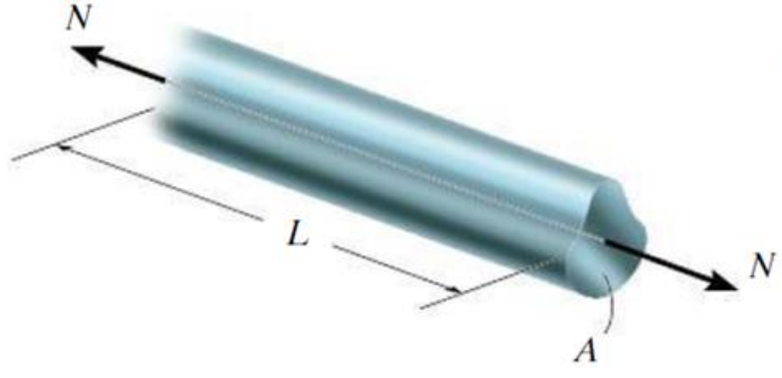


Fig. 14–7

$$U_i = \frac{N^2 L}{2AE}$$

14.2 Elastic Strain Energy for Various Loading

$$U_i = \int_V \frac{\sigma^2}{2E} dV$$

Bending Moment

$$U_i = \int_V \frac{\sigma^2}{2E} dV = \int_V \frac{1}{2E}\left(\frac{My}{I}\right)^2 dA\ dx$$

$$U_i = \int_0^L \frac{M^2}{2EI^2}\left(\int_A y^2\, dA\right) dx$$

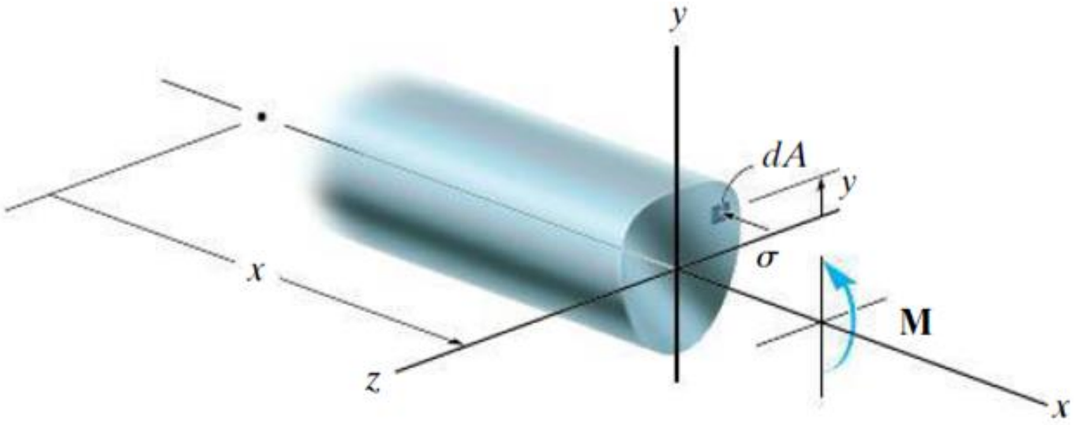


Fig. 14–9

$$U_i = \int_0^L \frac{M^2\, dx}{2EI}$$

14.2 Elastic Strain Energy for Various Loading

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Transverse Shear

$$U_i = \int_0^L \frac{f_s V^2 \, dx}{2GA}$$

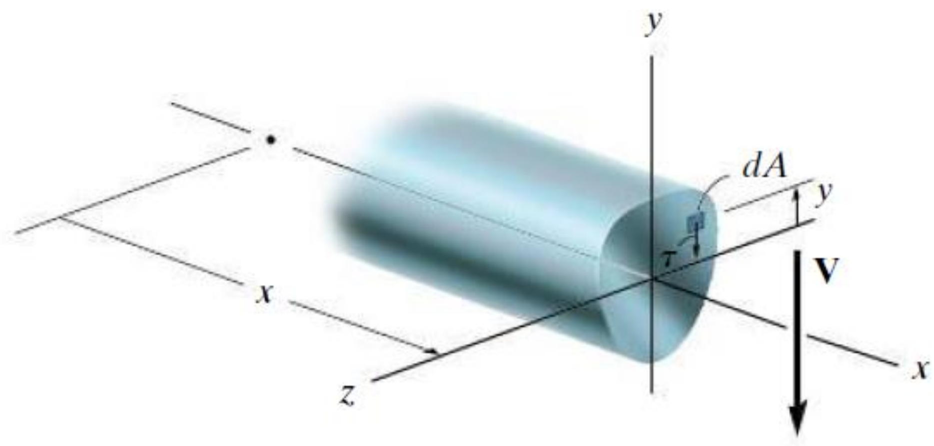

Fig. 14–12

Torsional Moment

$$U_i = \int_0^L \frac{T^2}{2GJ} dx$$

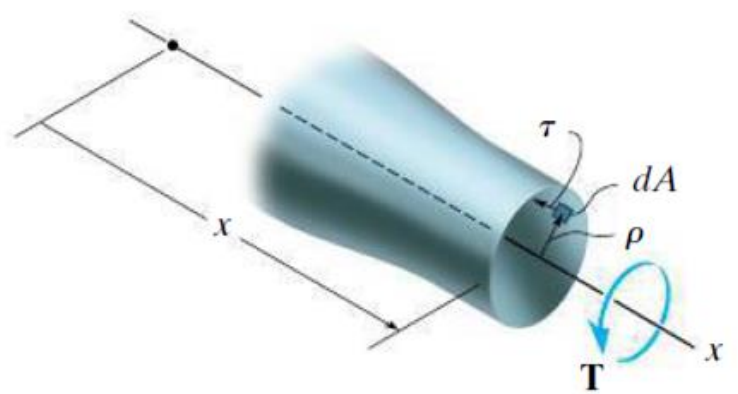

Fig. 14–15

4.8 Castigliano's Theorem

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Castigliano's theorem states that *when forces act on elastic systems subject to small displacements, the displacement corresponding to any force, in the direction of the force, is equal to the partial derivative of the total strain energy with respect to that force.*

$$\delta_i = \frac{\partial U}{\partial F_i}$$

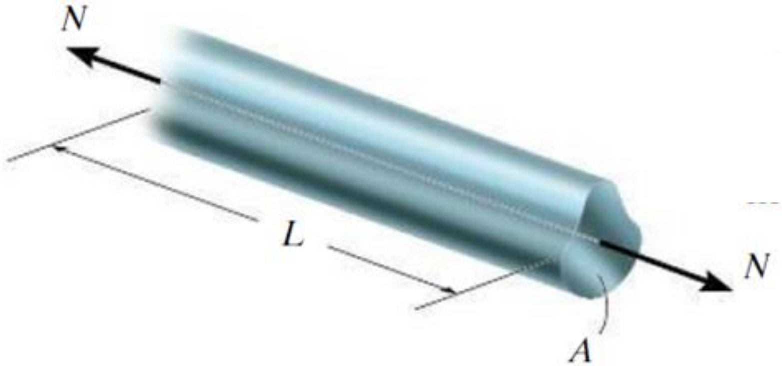


Fig. 14–7

$$U_i = \frac{N^2 L}{2AE}$$

$$\delta = \frac{\partial}{\partial F}\left(\frac{F^2 l}{2AE}\right) = \frac{Fl}{AE}$$

4.8 Castigliano's Theorem

Castigliano's theorem states that *when forces act on elastic systems subject to small displacements, the displacement corresponding to any force, in the direction of the force, is equal to the partial derivative of the total strain energy with respect to that force.*

$$\theta_i = \frac{\partial U}{\partial M_i}$$

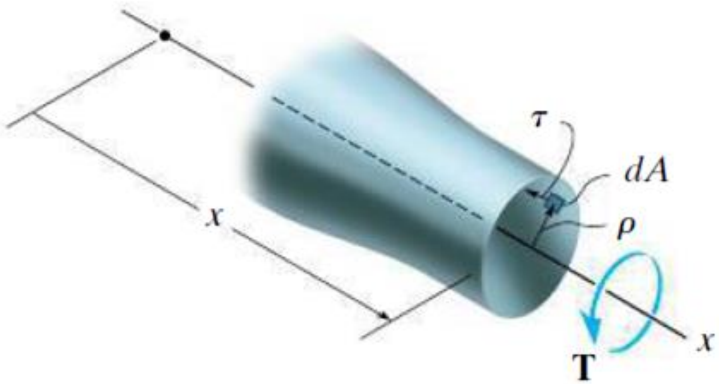


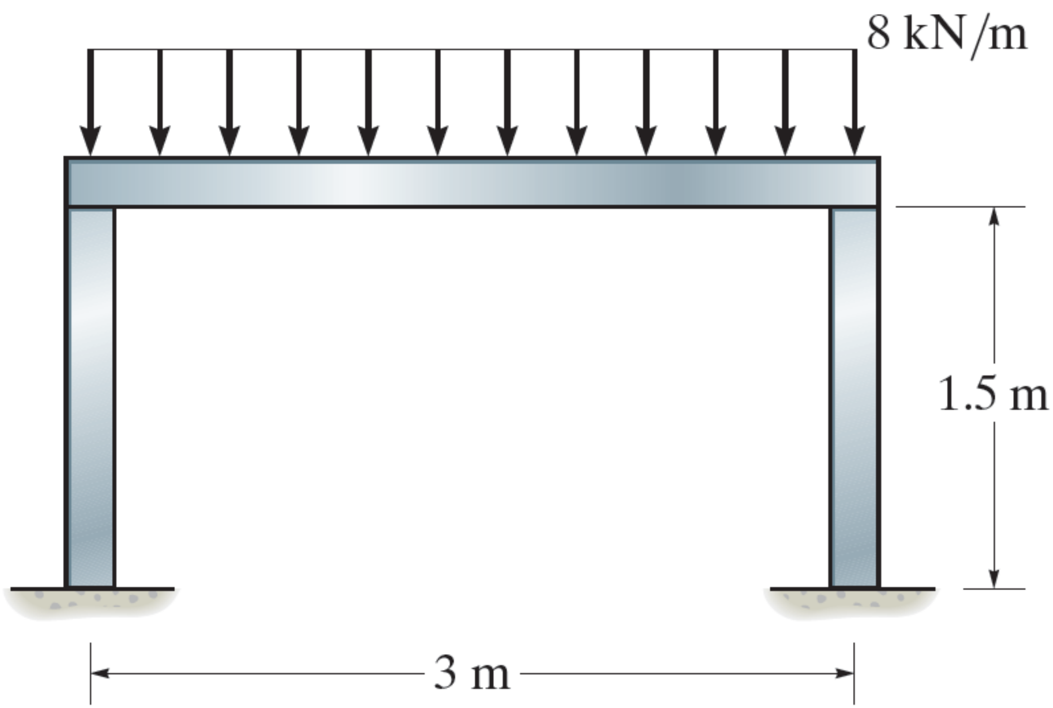
Fig. 14–15

$$U_i = \int_0^L \frac{T^2}{2GJ} dx$$

$$\theta = \frac{\partial}{\partial T}\left(\frac{T^2 l}{2GJ}\right) = \frac{Tl}{GJ}$$

Problem 14-22

14–22. Determine the bending strain energy in the beam and the axial strain energy in each of the two posts. All members are made of aluminum and have a square cross section 50 mm by 50 mm. Assume the posts only support an axial load. $E_{al} = 70$ GPa.



Problem 14-22

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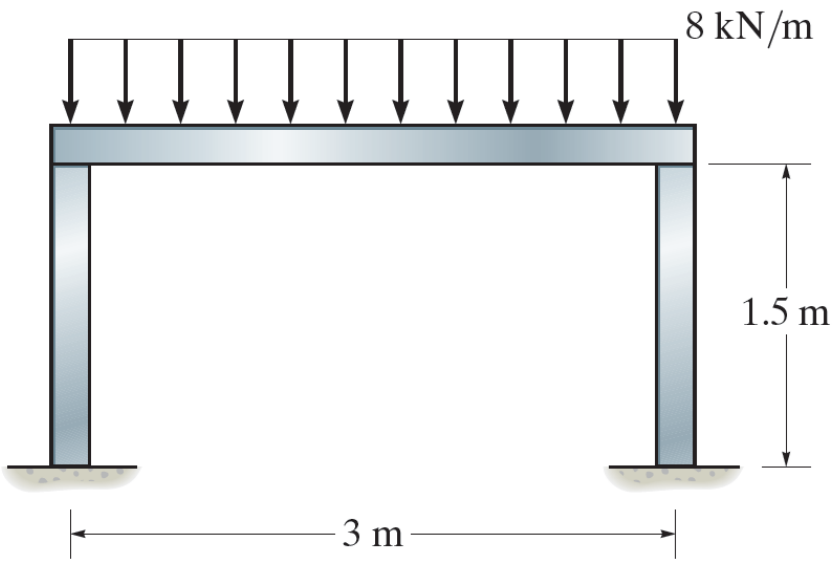
Bending Strain Energy in the beam

$$U_i = \int_0^L \frac{M^2\,dx}{2EI}$$

Axial Strain Energy of the two post

$$U_i = \frac{N^2 L}{2AE}$$



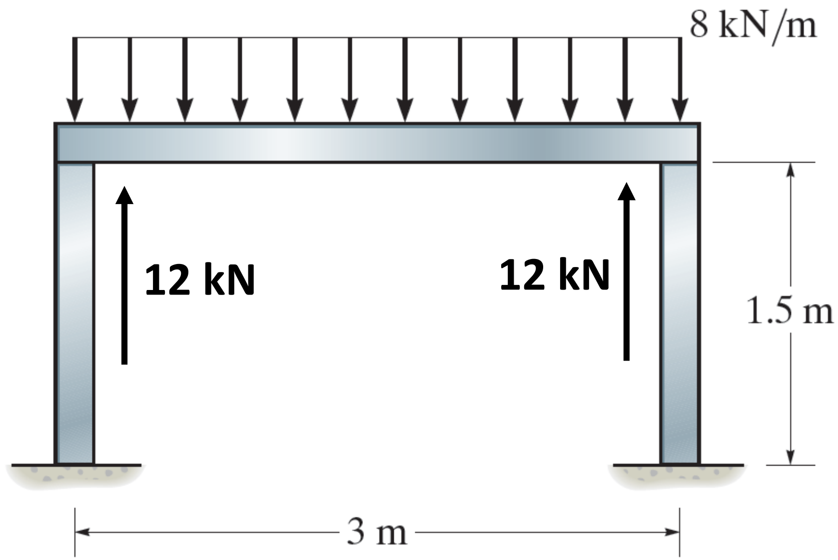
Section Properties:

$$A = (0.05)(0.05) = 2.5(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.05)(0.05)^3 = 0.52083(10^{-6})\ \text{m}^4$$

Problem 14-22

Axial Strain Energy of the two post



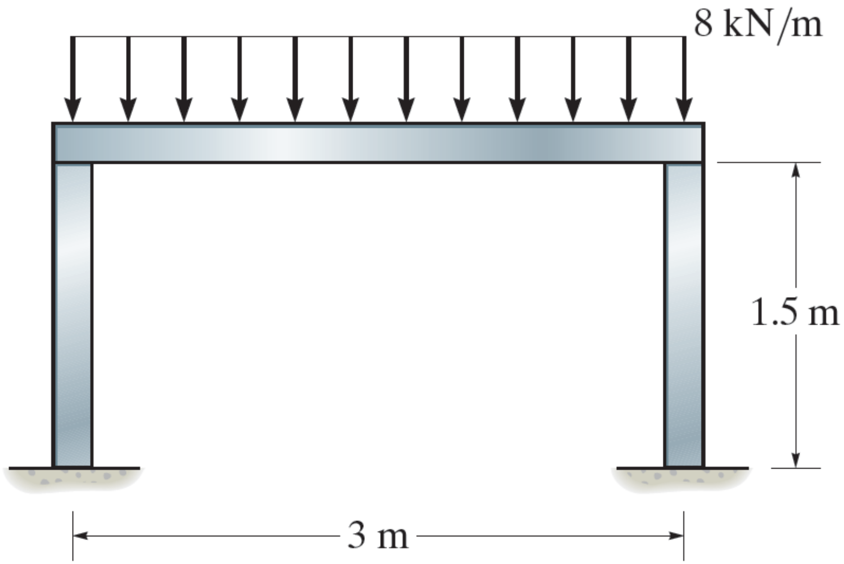
$$U_i = \frac{N^2 L}{2AE}$$

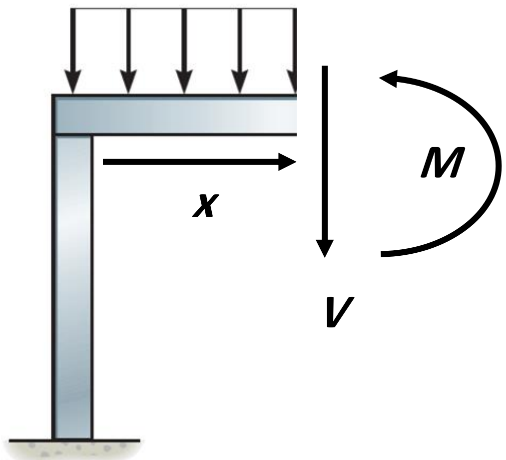
Problem 14-22

BIRZEIT UNIVERSITY

Bending Strain Energy in the beam

$$U_i = \int_0^L \frac{M^2\,dx}{2EI}$$





Problems

Chapter 1: 4, 5, 6, 21, 22, 25, 27, 28, 60, 80, 84

Chapter 2: 29

Chapter 3: 21, 36

Chapter 4: 1, 10, 85

Chapter 5: 11, 16, 20, 23, 39, 54, 65, 83

Chapter 6: 3, 8, 18, 24, 52, 74, 75

Chapter 7: 8, 11, 18

Chapter 8: 35, 36, 40, 57

Chapter 9: 17, 29, 67, 80

Chapter 14: 141, 190