10.3: Inference about the difference between two population means, matched samples.

1. Independent sample design → 10.1 + 10.2.
2. Matched sample design → 10.3

Notes:

- μ_1 = Pop. 1 mean.
- μ_2 = Pop. 2 mean.
- $\mu_d = \mu_1 - \mu_2$
- $d_i = x_i^{I} - x_i^{II}$ (Difference in completion).
- $\bar{d} = \frac{\sum d_i}{n}$
- $s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n-1}}$
- n: sample size.
- n = no. of elements.

- $H_0: \mu_d = \mu_{d,0}$ — $\mu_{d,0}$: hypothesized value.
  $H_1: \mu_d \neq \mu_{d,0}$.

- test statistic: $\frac{\bar{d} - \mu_{d,0}}{\frac{s_d}{\sqrt{n}}}$, $df = n-1$.

- Reject H_0 if P-value $\leq \alpha$
  P-value = area in both tails

- Reject H_0 if $|t| \geq t_{\frac{\alpha}{2}}$, $df = n-1$

- $(1-\alpha)$ CI for $\mu_d = \bar{d} \mp \underbrace{t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}}_{\text{st. error}}$ → margin of error.

exp :

Worker	completion time Method I	completion time Method II
1	6	5.4
2	5	5.2
3	7	6.5
4	6.2	5.9
5	6	6
6	6.4	5.8

① perform $\mu_d = 0$

$\mu_d \neq 0$, $\alpha = 0.05$, $df = 5$.

$$t = \frac{\bar{d} - \mu_{d,0}}{\frac{s_d}{\sqrt{n}}} = \frac{0.3 - 0}{\frac{0.33}{\sqrt{6}}} = 2.227.$$

i	di (I - II)
1	0.6
2	-0.2
3	0.5
4	0.3
5	0
6	0.6

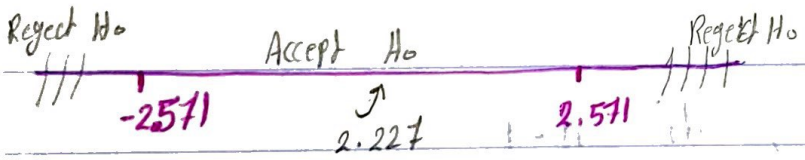
↳ $\bar{d} = 0.3$

$s_d = 0.33$

$n = 6$

critical values : $\mp t_{\frac{\alpha}{2}} = \pm t_{0.025}$

$= \mp 2.571.$



Don't Reject H_0 ($\alpha = 0.05$)

$\mu_d = 0$ ($\alpha = 0.05$).

conclusion in words : with significance 5%, there is No significant difference in completion times of method I and method II.

Cont ①:

p-value:

df	0.05	0.025
5	2.015	2.571

2.227

upper tail test (area) $\in$ (0.025, 0.05)

p-value $\in$ (0.05, 0.10). → 1.2 → fig

• two tailed test.

$\rightsquigarrow$ p-value $> \alpha$

so Don't Reject H_0 ($\alpha = 0.05$)

② 95% CI for $\mu_1 - \mu_2$.

$$95\%\ CI = \bar{d} \mp t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$$

$$= 0.3 \mp 2.571 \left(\frac{0.33}{\sqrt{6}}\right)$$

$$= 0.3 \mp 0.35$$

$$= [-0.05, 0.65].$$

Conclusion: we are 95% confident, that the defferent of the population means for the completion time of Method I and Method II is between -0.05 and 0.65.